Ratio of lifetimes for the D^+ and D^0 mesons

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Using the SLAC lattice QCD theory, we derive an effective interaction density for the process $c+\bar{u}\to s+\bar{d}+G$, where G is a gluon. In this way, we calculate the ratio of lifetimes for the D^+ and D^0 mesons. Our result for this ratio is close to unity.

The question of the ratio of lifetimes for the charmed particles D^+ and D^0 has been given renewed interest with the recent result of the SLAC bubble-chamber experiment,[1] which finds $\tau(D^+)/\tau(D^0) \simeq 1.2^{+0.9}_{-0.5}$, where $\tau(A) \equiv$ lifetime of A, $A=D^+,D^0$. This should be compared with previous experimentation,[2] which had suggested that $\tau(D^+)$ may be more than five times $\tau(D^0)$. Thus, it is of considerable importance to understand theoretically just what this ratio should be.

In this latter connection, we should mention that, prior to the result in Ref. 1, several theoretical analyses[3] showed that, in the context of the standard Glashow, Salam, and Weinberg model,[4] augmented with (perturbative) QCD,[5] one could indeed accomodate a large value of $\tau(D^+)/\tau(D^0)$ due to the annihilation process

$$c+\bar{u}\to s+\bar{d}+G \quad , \tag{1}$$

where G is a gluon, provided certain parameters, primarily the D-meson decay constant f_D or constants related to it, were of the appropriate size. Hence, with the advent of the result in Ref. 1, it is of some interest to analyze (1) in models wherein it can be computed with reasonable accuracy. We will present in this Brief Report the result of such an analysis in the context of the SLAC lattice QCD theory.[6]

More specifically, in Ref. 6 it has been shown that, to leading order in $1/g^2$, the SLAC lattice theory results in the effective interaction Hamiltonian

$$H_{\rm eff}^{(2)}(g,a)=a^5\sum_{\vec{j},n,\hat{\mu}}\frac{\delta'(n)\delta'(-n)}{\frac{1}{2}g^2|n|C_F}\psi^{\dagger\alpha f}_{\vec{j}}\alpha_\mu\psi^{\beta f}_{\vec{j}+n\hat{\mu}}\psi^{\dagger\beta f'}_{\vec{j}+n\hat{\mu}}\alpha_\mu\psi^{\alpha f'}_{\vec{j}} \quad , \tag{2}$$

where a is the lattice constant, g is the QCD gauge coupling constant, $\psi^{\alpha f'}_{\vec{j}}(t)$ is the quark field of color α and flavor f' at lattice site $\vec{j}a$ at time t, and $\alpha_\mu=\gamma^0\gamma^\mu$ is the Dirac matrix for the direction $\hat{\mu}$ in the notation of Bjorken and Drell.[7] The function $\delta'(n)$ is the defining property of the SLAC theory[6] and is given by

$$\delta'(n)=(-1)^{n+1}/n \tag{3}$$

in the infinite-volume limit in which we shall work. Finally, we note that C_F is the value of the quadratic Casimir operator for the fundamental representation of $SU(N_c)$ of color and is given by $C_F=(N_c{}^2-1)/2N_c$. To analyze (1), we use (2) in conjunction with the QCD-augmented $SU(2)\times U(1)$ effective interaction[4,5]

$$\mathcal{L}_{W,\,\rm eff}=\frac{-G_F}{\sqrt{2}}\left[\tfrac{1}{2}(f_++f_-)\bar{\psi}^{\alpha u}\gamma_\mu(1-\gamma_5)\psi^{\alpha d}\bar{\psi}^{\beta s}\gamma^\mu(1-\gamma_5)\psi^{\beta c}+\tfrac{1}{2}(f_+-f_-)\bar{\psi}^{\alpha s}\gamma_\mu(1-\gamma_5)\psi^{\alpha d}\bar{\psi}^{\beta u}\gamma^\mu(1-\gamma_5)\psi^{\beta c}\right] \quad , \tag{4}$$

where,[5] assuming five quark flavors with[8] $m_c=1.84$ GeV, we have

$$f_+\cong 0.74,\quad f_-\cong 1.84 \quad , \tag{5}$$

for a one-loop value of 0.34 GeV for the QCD scale[5,9,10] $\Lambda_{\rm QCD}$.

Our method of analysis is closely related to that in Ref. 11, wherein one looks at the appropriate amplitude in terms of the fundamental fields $\psi^{\alpha f}$ in order to discover the effective interaction density for the process in question. Following Refs. 3, we see that, in our case, the respective amplitude is as shown in Fig. 1. Using (2) and (4) we find that the amplitude is, using the method of Ref. 10 and the notation of Ref. 7 (the kinematics is summarized in Fig. 1),

$$\mathcal{A} = (2\pi)^4\delta^4(P_c + P_{\bar{u}} - P_s - P_{\bar{d}} - P_q - P_{\bar{q}})\frac{i\sqrt{2}a^2G_F}{g^2C_F}$$
$$\times \sum_{n,\hat{\mu}_1} \frac{\cos(na\hat{\mu}_1\cdot\vec{\mathrm{P}}_{\bar{q}})}{|n|^3}\Bigg[2(f_+ + f_-)\bar{u}_s^{\nu}\gamma^{\mu_2}(1-\gamma_5)u_c^{\sigma'}\delta^{\sigma'\nu}\bar{v}_{\bar{u}}^{\sigma}\lambda^{a'}_{\sigma\nu'}\gamma_0\alpha_{\mu_1}\frac{1}{\not{P}_q + \not{P}_{\bar{q}} - \not{P}_{\bar{u}} - m_u}\gamma_{\mu_2}(1-\gamma_5)v_{\bar{d}}^{\nu'}$$
$$- 2(f_+ - f_-)\bar{u}_s^{\nu}\gamma^{\mu_2}(1-\gamma_5)v_{\bar{d}}^{\nu'}\delta^{\nu\nu'}\bar{v}_{\bar{u}}^{\sigma}\lambda^{a'}_{\sigma\sigma'}\gamma_0\alpha_{\mu_1}$$
$$\times \frac{1}{\not{P}_q + \not{P}_{\bar{q}} - \not{P}_{\bar{u}} - m_u}\gamma_{\mu_2}(1-\gamma_5)u_c^{\sigma'}\Bigg]\bar{u}_q^{\xi}\lambda^{a'}_{\xi\xi'}\gamma_0\alpha_{\mu_1}v_{\bar{q}}^{\xi'} \quad , \tag{6}$$

where we have omitted all Wick contractions in Fig. 1 which are included in the standard spectator model[3,5] of D^0 decay; further, we have introduced the $\mathrm{U}(N_c)$ matrices $\lambda^{a'}$, $a'=0,\ldots,N_c^2-1$, such that the $\mathrm{SU}(N_c)$ quark color matrices are $\lambda^{a'}$, $a'=1,\ldots,N_c^2-1$. $\delta^{\nu\nu'}$ is the Kronecker delta function.

To relate (6) to (1), we must infer the effective interaction for (1) implied by (6). We view the quarks and gluons in (1) as constituents so that, for application to (1), we are interested in the regime[12,13] (the large-distance regime)

$$\vec{\mathrm{P}}_q, \vec{\mathrm{P}}_{\bar{q}} \to \vec{0} \ ,$$
$$P_{\bar{u}} = (m_u - \epsilon_1, \vec{0}), \quad P_c = (m_c - \epsilon_2, \vec{0}) \ , \tag{7}$$

where $\epsilon_1/\epsilon_2 = m_c/m_u$ and

$$1.863\ \mathrm{GeV} = m_{D^0} = m_u + m_c - \epsilon_1 - \epsilon_2 \ . \tag{8}$$

Here, we will use $m_u = 0.33$ GeV so that $\epsilon_1 \simeq 0.26$ GeV. From (7) and (6) we obtain

$$\mathcal{A} = (2\pi)^4\delta^4(P_c + P_{\bar{u}} - P_s - P_{\bar{d}} - P_q - P_{\bar{q}})\frac{i\sqrt{2}a^2G_F[2\zeta(3)]}{g^2C_F\epsilon_1}$$
$$\times [2(f_+ + f_-)\bar{u}_s^{\nu}\gamma^{\mu_2}(1-\gamma_5)u_c^{\sigma'}\delta^{\sigma'\nu}\bar{v}_{\bar{u}}^{\sigma}\lambda^{a'}_{\sigma\nu'}\gamma^k\gamma_{\mu_2}(1-\gamma_5)v_{\bar{d}}^{\nu'}$$
$$-2(f_+ - f_-)\bar{u}_s^{\nu}\gamma^{\mu_2}(1-\gamma_5)v_{\bar{d}}^{\nu'}\delta^{\nu\nu'}\bar{v}_{\bar{u}}^{\sigma}\lambda^{a'}_{\sigma\sigma'}\gamma^k\gamma_{\mu_2}(1-\gamma_5)u_c^{\sigma'}]\bar{u}_q^{\xi}\lambda^{a'}_{\xi\xi'}\gamma^k v_{\bar{q}}^{\xi'} \quad , \tag{9}$$

where $\zeta(3) \simeq 1.202$ is the Riemann ζ function of argument 3.

The corresponding effective annihilation (ann) interaction implied by (9) is

$$\mathcal{L}_{\mathrm{ann,\,eff}} = \frac{2\sqrt{2}[2\zeta(3)]a^2G_F}{g^2C_F\epsilon_1}[(f_+ + f_-)\bar{\psi}^u\lambda^{a'}\gamma^k\gamma_{\mu_2}(1-\gamma_5)\psi^d\bar{\psi}^s\gamma^{\mu_2}(1-\gamma_5)\psi^c$$
$$+ (f_+ - f_-)\bar{\psi}^u\lambda^{a'}\gamma^k\gamma_{\mu_2}(1-\gamma_5)\psi^c\bar{\psi}^s\gamma^{\mu_2}(1-\gamma_5)\psi^d]\bar{\psi}^q\lambda^{a'}\gamma^k\psi^q \quad , \tag{10}$$

where we now suppress most sums over color. Presuming (10) to have been abstracted[10,14] to the continuum, we recall the identification

$$\frac{1}{g}(\partial_\nu\delta^{a'b} + g\epsilon_{a'bc}A^c_\nu)F^{b\nu\mu} = \bar{\psi}\gamma^\mu\lambda^{a'}\psi \quad , \tag{11}$$

where

$$F^{b\nu\mu} \equiv \partial^\nu A^{b\mu} - \partial^\mu A^{b\nu} - g\epsilon_{bcd}A^{c\nu}A^{d\mu} \tag{12}$$

is the familiar Yang-Mills field strength tensor for QCD, so that ϵ_{abc} are the $\mathrm{SU}(N_c)$ structure constants. A^b_μ is the QCD gluon field. We have in mind here that the effect of the large-distance interactions is to give the quanta of A^b_μ a constituent (as opposed to current) mass[15] m_G. Thus, this mass m_G does not appear explicitly in (11),[15,16] for the current mass of A^b_μ is zero. For comparison, we note that, from Ref. 15, we have

$$m_G \cong 0.7\ \mathrm{GeV} \ . \tag{13}$$

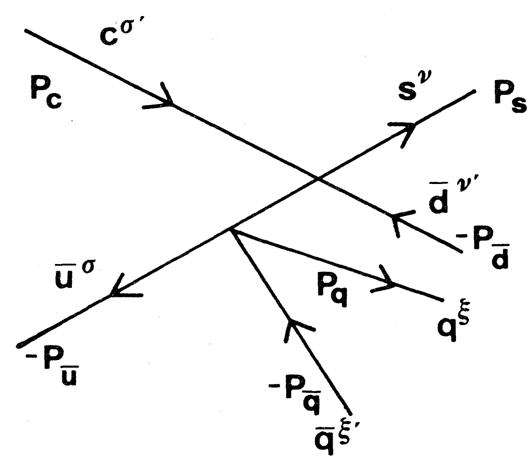

FIG. 1. SLAC lattice QCD model for the annihilation mechanism to lowest order in $1/g^2$. The superscripts are color indices.

On introducing (11) into (10), we would thus have the effective Lagrangian for the process (1) for G at rest; to obtain the analogous quantity for G not at rest, one simply has to replace the sum on k in (10) with its Lorentz-invariant form, after the introduction of (11). In this way, we find

$$\mathcal{L}_{\mathrm{ann,eff}}=\frac{-2\sqrt{2}[2\zeta(3)]a^2G_F}{C_F g^2\epsilon_1}[(f_++f_-)\bar{\psi}^u\lambda^{a'}\gamma_{\mu_1}\gamma_{\mu_2}(1-\gamma_5)\psi^d\bar{\psi}^s\gamma^{\mu_2}(1-\gamma_5)\psi^c$$
$$+(f_+-f_-)\bar{\psi}^u\lambda^{a'}\gamma_{\mu_1}\gamma_{\mu_2}(1-\gamma_5)\psi^c\bar{\psi}^s\gamma^{\mu_2}(1-\gamma_5)\psi^d]\frac{1}{g}(\partial_\nu\delta^{a'b}+g\epsilon_{a'bc}A_\nu^c)F^{b\nu\mu_1}\ . \tag{14}$$

Here, we note that the λ^0 term in (10) has been dropped because it can be shown to be negligible by the standard Fermi phase-space considerations for D^0 decay. We may now evaluate the process (1) in the context of D^0 decay.

More precisely, from (14) we find the amplitude[12]

$$\mathcal{A}(D^0\to s+\bar{d}+G)=(2\pi)^4\delta^4(P_c+P_{\bar{u}}-P_s-P_{\bar{d}}-P_G)\frac{-i2\sqrt{2}[2\zeta(3)]a^2G_F}{C_F g^2\epsilon_1}\left(\frac{-m_G^2}{g}\right)\frac{1}{\sqrt{2N_c}}$$
$$\times(f_++f_-)[\bar{u}_s^\nu\gamma^{\mu_2}(1-\gamma_5)u_c^\nu(\underline{n})\bar{v}_{\bar{u}}^\nu(-\underline{n})\not{e}\lambda_{\nu\nu'}^{a'}\gamma_{\mu_2}(1-\gamma_5)v_{\bar{d}}^{\nu'}$$
$$-\bar{u}_s^\nu\gamma^{\mu_2}(1-\gamma_5)u_c^\nu(-\underline{n})\bar{v}_{\bar{u}}^\nu(\underline{n})\not{e}\lambda_{\nu\nu'}^{a'}\gamma_{\mu_2}(1-\gamma_5)v_{\bar{d}}^{\nu'}]\psi_{D^0}(0)\ , \tag{15}$$

where P_G is the gluon four-momentum, e^μ is the gluon polarization four-vector, $\underline{n}$ is an appropriate spin four-vector in the D^0 rest frame, and the labels ν, ν', and a' are all color indices: $\nu,\nu'=$ red, white, blue, etc.; $a'=1,\ldots,N_c^2-1$. The quantity $\psi_{D^0}(0)$ is the value of the D^0 wave function at the origin.[12] In the SLAC model[6] for the D^0, the c and $\bar{u}$ quarks reside at a single site $\vec{j}a$. Thus, in this model, the transcription of the definition in Ref. 12 for $\psi_{D^0}(0)$ leads to the identification

$$\psi_{D^0}(0)=a^{-3/2}\ . \tag{16}$$

Hence, using the standard methods, we find the annihilation rate

$$\Gamma_{\mathrm{ann}}(D^0\to s+\bar{d}+G)=\frac{8aG_F^2m_G^4[2\zeta(3)]^2|f_++f_-|^2i_0}{\pi^3g^6(m_G^2)C_F\epsilon_1^2}\ , \tag{17}$$

where i_0 is the phase-space integral ($E_s\equiv P_s^0$, $E_G\equiv P_G^0$)

$$i_0=\int_{\substack{\text{phase}\\ \text{space}}}dE_s dE_G(m_{D^0}-E_s-E_G)\{E_s+E_G[2m_{D^0}(E_s+E_G)-m_{D^0}^2+m_d^2-m_s^2-m_G^2]/m_G^2\}\ . \tag{18}$$

For $m_s=0.5$ GeV we find, numerically,

$$i_0\cong 0.037\ \mathrm{GeV}^4\ . \tag{19}$$

Further, for $\Lambda_{\mathrm{QCD}}=0.34$ GeV in a five-flavor one-loop[5] formula for $g^2(m_G^2)$, we have

$$g^2(m_G^2)=\frac{48\pi^2}{23\ln[(m_G/0.34\ \mathrm{GeV})^2]}\cong 14.3\ . \tag{20}$$

Finally, in the model of Refs. 8, 99.7% of the support of the D^0 wave function in momentum space lies in the region

$$|P_{\bar{u}}^i|\le 0.72\ \mathrm{GeV},\quad i=x,y,z\ . \tag{21}$$

This gives

$$\frac{\pi}{a}=0.72\ \mathrm{GeV}\ \text{ or }\ a\cong 4.4\ \mathrm{GeV}^{-1} \tag{22}$$

if we require that the internal momenta of the D^0 be essentially disjoint from the momenta on the lattice. Hence, using (19), (20), and (22), we arrive, for $N_c=3$, at

$$\Gamma_{\mathrm{ann}}(D^0\to s+\bar{d}+G)\cong 2.0\times 10^{-13}\mathrm{GeV}\ , \tag{23}$$

to be compared with the resonably-agreed-upon experimental result

$$\Gamma_{\mathrm{exp}}(D^+\to \mathrm{all})\cong 8.0\times 10^{-13}\ \mathrm{GeV}\ . \tag{24}$$

Presuming (24) to be *the* spectator rate for D^0 decay, we find

$$\tau(D^+)/\tau(D^0)\cong 1.24 \tag{25}$$

in this (SLAC) lattice QCD model.

The result (25) agrees with the result of Ref. 1.

Hence, we feel our calculation emphasizes the need for more statistics in all of the D-lifetime measurements so that a precise experimental value of the ratio in (25) can be obtained. Only in this way can we assess the true accuracy of the methods used in this paper.

Note added. For the parameters in the text, the theoretical spectator rate for D decay would give 1.06 for the ratio in (25). The implied uncertainty in the short-distance value of m_c is not a large-distance problem.

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[1]K. Abe *et al.*, Phys. Rev. Lett. 48, 1526 (1982).

[2]N. Ushida *et al.*, Phys. Rev. Lett. 45, 1049 (1980); 45, 1053 (1980); D. Allasia *et al.*, Nucl. Phys. B176, 13 (1980); A Fiorino *et al.*, Lett. Nuovo Cimento 30, 166 (1981); M. I. Adamovich *et al.*, Phys. Lett. 89B, 427 (1981); 99B, 271 (1981); W. Bacino *et al.*, Phys. Rev. Lett. 45, 329 (1980); R. Schindler *et al.*, Phys. Rev. D 24, 78 (1981); G. Trilling, Phys. Rep. 75, 57 (1981); G. Diambrini-Palazzi *et al.*, Report No. CERN-EP/82-46, 1982 (unpublished), and references therein.

[3]W. Bernreuther, O. Nachtmann, and B. Stech, Z. Phys. C 4, 257 (1980); H. Fritzsch and P. Minkowski, Phys. Lett. 90B, 455 (1980); S. P. Rosen, Phys. Rev. Lett. 44, 4 (1980); M. Bander, B. Silverman, and A. Soni, Phys. Rev. Lett. 44, 7 (1980); I. I. Y. Bigi, Nucl. Phys. B177, 395 (1981).

[4]Some of the early works on the gauge-theoretic view of weak and electromagnetic interactions are those by J. Schwinger, Ann. Phys. (N.Y.) 2, 407 (1957); A. Salam and J. Ward, Nuovo Cimento 11, 568 (1959); S. L. Glashow, Nucl. Phys. 22, 579 (1961); A. Salam and J. Ward, Phys. Lett. 13, 168 (1964); A. Salam, in *Elementary Particle Theory,* edited by R. Svartholm (Wiley, New York, 1968); S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).

[5]D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); H. D. Politzer, *ibid.* 30, 1346 (1973); G. 't Hooft (unpublished); M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. 52B, 351 (1974); J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B100, 313 (1975); N. Cabibbo and L. Maiani, Phys. Lett. 73B, 418 (1978).

[6]S. D. Drell, M. Weinstein, and S. Yankielowicz, Phys. Rev. D 14, 487 (1976); 14, 1627 (1976); M. Weinstein, S. D. Drell, H. R. Quinn, and B. Svetitsky, *ibid.* 22, 1190 (1980).

[7]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[8]E. Eichten, Phys. Rev. D 22, 1819 (1980); E. Eichten *et al., ibid.* 21, 203 (1980), and references therein.

[9]This value of Λ_{QCD} is an average of the results of several experiments; see Ref. 10 and references therein.

[10]B. F. L. Ward, Phys. Rev. D 25, 1330 (1982); *ibid.* 28, 1138 (1983).

[11]B. W. Lee, J. R. Primack, and S. B. Treiman, Phys. Rev. D 7, 510 (1973); M. K. Gaillard and B. W. Lee, *ibid.* 10, 897 (1974); M. K. Gaillard, B. W. Lee, and R. E. Shrock, *ibid.* 13, 2674 (1976); B. F. L. Ward, Nuovo Cimento 38A, 299 (1978).

[12]R. Van Royen and V. F. Weisskopf, Nuovo Cimento 50A, 617 (1967).

[13]S. J. Brodsky and J. R. Primack, Phys. Rev. 174, 2071 (1968).

[14]See, for example, H. Fritzsch and M. Gell-Mann, in *Broken Scale Invariance and the Light Cone,* proceedings of the 1971 Coral Gables Conference on Fundamental Interactions at High Energy, edited by M. Dal Cin, G. J. Iverson, and A. Perlmutter (Gordon and Breach, New York, 1971), Vol. 2, pp. 1–42, and references therein.

[15]See, for example, J. M. Cornwall and A. Soni, Report No. UCLA/82/TEP 13, 1982 (unpublished); P. G. O. Freund and Y. Nambu, Phys. Rev. Lett. 34, 1645 (1975); B. Berg, Phys. Lett. 97B, 401 (1980); J. F. Donoghue, K. Johnson, and B. A. Li, Phys. Lett. 99B, 416 (1981); K. Ishikawa, M. Teper, and G. Schierholz, *ibid.* 110B, 399 (1982); C. Bhanot and C. Rebbi, Nucl. Phys. B180 (FS2), 469 (1981); H. Hamber and G. Parisi, Phys. Rev. Lett. 47, 1792 (1981); C. Edwards *et al., ibid.* 48, 458 (1982); 49, 259 (1982); C. Bernard, Phys. Lett. 108B, 431 (1982); C. Peterson, T. H. Hansson, and K. Johnson, Phys. Rev. D 26, 415 (1982), and references therein.

[16]A. A. Slavnov, Teor. Mat. Fiz. 10, 305 (1972) [Theor. Math. Phys. 10, 201 (1972)]; J. M. Cornwall, Phys. Rev. D 10, 500 (1974); Nucl. Phys. B157, 392 (1979).