W magnetic moment in electroweak mixing and composite models and radiation zeros in $q_i\bar{q}_j \to W\gamma$

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The same experimental and theoretical constraints that are used to set limits on the masses of the weak bosons in the composite-model versions of electroweak mixing models are used to bound the magnetic-moment parameter of the W. We find in general electroweak mixing schemes, ones with additional heavy isospin-1 bosons (W'), that off-diagonal weak-boson electromagnetic interactions ($WW'\gamma$ vertices) are allowed. The cross section for the process $q_i\bar{q}_j \to W\gamma$, which has been suggested as the best way to measure the W moments, is generalized to include the effects of these off-diagonal couplings as well as arbitrary and independent W magnetic dipole and electric quadrupole moments.

I. INTRODUCTION

The standard $SU(2)_L \times U(1)_Y$ electroweak gauge-theory model is not only consistent with all low-energy experimental data, but predicts the properties of the weak bosons, their masses, decay widths, branching ratios, and gauge-boson couplings. The simplest of the multiboson gauge couplings describes the electromagnetic interaction of the W, the $WW\gamma$ vertex. This interaction has been parametrized by Lee and Yang[1] in terms of a "magnetic-moment" parameter κ which gives

$$\mu_W = e(1+\kappa)/2M_W\ , \tag{1.1}$$

$$Q_W = -e\kappa/M_W{}^2\ , \tag{1.2}$$

$$R_W{}^2 = \kappa/M_W{}^2 \tag{1.3}$$

for the magnetic dipole moment, the electric quadrupole moment, and the charge radius of the W, respectively. The gauge-theory prediction is $\kappa=1$. (Bardeen, Gastmans, and Lautrup[2] have calculated the static magnetic dipole and electric quadrupole moments of the W in the standard model to second order and find that the anomalous moments are of order α/π and depend on $\sin^2\theta_W$ and the Higgs-boson mass.) Mikaelian, Samuel, and Sahdev[3] have shown that the differential cross section for the process

$$q_i\bar{q}_j \to W^{\pm}\gamma \tag{1.4}$$

(hereafter process I) exhibits a zero at tree level at $\cos\theta = -(1+2Q_i)$ (independent of s or $M_W{}^2$) due to the presence of the canonical gauge-theory $WW\gamma$ vertex. For non-gauge-theory values of κ ($\kappa \neq 1$), the zero is filled in so process I provides a sensitive test of the trilinear gauge coupling. Even when embedded in the experimentally relevant process, $p\bar{p} \to W\gamma X$ at $p\bar{p}$ colliders, a dramatic dip persists. Careful measurements of this process in a second-generation experiment may thereby provide the first direct test of a non-Abelian gauge-boson coupling in the weak interactions. The WWZ vertex will only be tested in $q_i\bar{q}_j \to WZ$ and $e^+e^- \to W^+W^-$ at higher energies.

Extended electroweak theories with extra U(1)'s,[4] SU(2)'s,[5] or left-right symmetry[6] can also be made to fit the low-energy data but allow the masses of the lightest gauge bosons to differ from (even lie above[7]) their standard-model values. The electromagnetic interactions of the W mass eigenstates are, however, given by the same form as in the standard model. Thus the presence of radiation zeros in process I tests the gauge-theory description of the weak interactions independent of the effective low-energy gauge group.

Electroweak mixing models,[8–10] not based on spontaneously broken gauge theories, but rather on W^0-γ mixing, can also reproduce the quantitative predictions of the standard model at low energy and allow for differing values at the weak-boson masses. In addition, such models also allow for the possibility of non-gauge-theory W electromagnetic interactions, i.e., $\kappa \neq 1$. Such mixing models have been extensively used in composite models,[11–13] where the weak interactions are thought of as being mediated by bound states of fundamental constituents bound by a new confining hypercolor force (just as the strong interactions of composite nucleons are mediated by ρ mesons, QCD $q\bar{q}$ bound states). In this context additional heavier W's and Z's are expected to naturally appear as excited states.

The parameters that characterize the magnetic-moment parameter κ in these schemes are also related to the masses of the weak bosons so that the same experimental and theoretical constraints that have been used to bound the masses can be used to constrain κ.

In this note we examine the bounds that are imposed on κ in the simplest W^0-γ mixing model,[9] its left-right-symmetric extension[14] [and extensions with arbitrarily many distinct SU(2) factors], and in general electroweak mixing schemes[10,15] which allow for additional heavy left-handed bosons (W') as might be expected in composite models. We use as inputs low-energy charged- and neutral-current physics and theoretical prejudices from composite models. The bounds on κ can then be used to limit the extent to which the zero in process I can be washed out. In the general electroweak mixing scheme we find additional off-diagonal weak-boson electromagnetic vertices ($WW'\gamma$) which also contribute to I and can be used to probe the extent to which the lowest-lying vector bosons saturate the effective weak-interaction strength.

Finally, we generalize the expression for the cross section for process I to include the effects of arbitrary and *independent* W magnetic dipole ($M1$) and electric quadrupole ($E2$) moments and make some comments.

II. BOUNDS ON κ IN ELECTROWEAK MIXING MODELS

Hung and Sakurai[9] have described the electroweak interactions by an $SU(2)_L$-symmetric Lagrangian where a single triplet of W_L bosons is assumed to mediate the weak interactions and the $W_L^{\pm,0}$ are degenerate in mass before mixing. The global $SU(2)_L$ symmetry is broken by mixing between the photon and W_L^0 and the resulting low-energy effective Lagrangian can successfully describe weak charged- and neutral-current data while allowing for less restrictive boson-mass relations than in the standard model. Barbieri and Mohapatra[14] have extended this W^0-γ mixing analysis to a left-right-symmetric electroweak theory describing a certain class of composite models.[13] The weak interactions in this scheme are obtained by adding a single right-handed triplet of W_R bosons.

We can easily extend these mixing analyses further to more general electroweak mixing Lagrangians with a global symmetry consisting of arbitrarily many distinct SU(2) factors, i.e., $G=\prod_{i=1}^{N}SU(2)_i$ [with $SU(2)_1 \equiv SU(2)_L$]. Such theories with $N>2$ might represent, for example, the composite-model—electroweak-mixing versions of the "petite unification" electroweak models of Hung, Buras, and Bjorken[16] based on $[SU(2)]^4$. We begin by assuming a Lagrangian

$$\mathscr{L}=-\tfrac{1}{4}(\tilde{F}_{\mu\nu})^2-\tfrac{1}{4}\vec{W}^{(i)}_{\mu\nu}\cdot\vec{W}^{(i)}_{\mu\nu}-\tfrac{1}{2}m_i^2\vec{W}^{(i)}_{\mu}\cdot\vec{W}^{(i)}_{\mu} +g_i\vec{J}^{(i)}_{\mu}\cdot\vec{W}^{(i)}_{\mu}+eJ^{EM}_{\mu}A_\mu+\bar{\psi}_r(i\gamma_\mu\partial_\mu-m_r)\psi_r \tag{2.1}$$

and mixing terms

$$\mathscr{L}_{\text{mix}}=-\tfrac{1}{2}\lambda_i\tilde{F}_{\mu\nu}W^{0(i)}_{\mu\nu}\ , \tag{2.2}$$

where $\tilde{F}_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ and the N currents $\vec{J}^{(i)}_{\mu}$, $i=1,\ldots,N$, are generated by N distinct SU(2) $\vec{T}^{(i)}$'s (with $\vec{T}^{(1)}\equiv\vec{T}_L$). Some of the additional $\vec{T}^{(i)}$ may well be inert with respect to ordinary fermions and act nontrivially only on heavier multiplets (e.g., mirror fermions[16]). We then have

$$Q=\sum_i T^{3(i)}+T^D\ , \tag{2.3}$$

where T^D accounts for any additional non-SU(2) diagonal generators.

The charged-current interactions at $q^2\simeq 0$ will then be given essentially as always if we make the identification

$$\frac{g_1^2}{8m_1^2}=\frac{G_F}{\sqrt{2}} \tag{2.4}$$

[since $SU(2)_1\equiv SU(2)_L$] and if all of the additional charged bosons which interact with ordinary fermions are 2—3 times heavier than m_1. Diagonalizing the neutral-current interactions using the propagator matrix formalism[9] gives an effective Lagrangian at low energies,

$$\mathscr{L}^{\text{eff}}=\frac{1}{2}\left[\frac{e^2}{q^2}(J^{\text{EM}})^2+\sum_{i=1}^{N}\frac{g_i^2}{m_i^2}\left[J^{3(i)}-\frac{e\lambda_i}{g_i}J^{\text{EM}}\right]^2\right]. \tag{2.5}$$

Assuming that the only current that couples to ν_L is J^{3L}, we can reproduce neutrino neutral-current data if we set

$$e\lambda_1/g_1=\sin^2\theta_W\ . \tag{2.6}$$

We then obtain

$$\mathscr{L}^{\text{eff}}_{\text{NC}}=\frac{4G_F}{\sqrt{2}}\left[(J^{3L}-\sin^2\theta_W J^{\text{EM}})^2+\sum_{i=2}^{N}\epsilon_i\left[J^{3(i)}-\frac{e\lambda_i}{g_i}J^{\text{EM}}\right]^2\right], \tag{2.7}$$

where $\epsilon_i=(g_i m_1/g_1 m_i)^2$ and the ϵ_i can be bounded by other neutral-current experimental data.

From Eqs. (2.4) and (2.6) we have

$$m_L^2\equiv m_1^2=M_W^2\lambda_1^2/\sin^2\theta_W\ , \tag{2.8}$$

where $M_W^2=(\alpha\pi/\sqrt{2}G_F\sin^2\theta_W)^2\simeq(79\ \text{GeV})^2$ (for $\sin^2\theta_W\simeq 0.23$) is the standard model-$W$ mass. The experimental lower limit[17] is $m_L\gtrsim 19$ GeV. Demanding that all $Z^{(i)}$ masses be real after mixing requires that

$$\left[1-\sum_i\lambda_i^2\right]>0 \tag{2.9}$$

and if all mixing parameters λ_i are equal, we have the bound (using $\sin^2\theta_W=0.23$)

$$m_L^2<M_W^2/N\sin^2\theta_W\simeq(165/\sqrt{N})^2\ \text{GeV}^2\ . \tag{2.10}$$

The fermion interaction terms in (2.1) can be obtained by a "minimal" substitution

$$\partial_\mu\psi_r\to(\partial_\mu-ieQA_\mu-ig_i\vec{T}^{(i)}\cdot\vec{W}^{(i)}_{\mu})\psi_r\ . \tag{2.11}$$

The prescription applied to the $W^{(i)}$ bosons gives for the electromagnetic coupling of the $W^{(i)}$

$$\mathscr{L}^{W^{(i)}W^{(i)}\gamma}=eA_\mu\left[(\vec{W}^{(i)}_{\mu\nu}\times\vec{W}^{(i)}_{\nu})_3+\frac{g_i\lambda_i}{e}M^{(i)}_{\mu}\right], \tag{2.12}$$

where

$$M^{(i)}_{\mu}\equiv i\partial_\nu(W^{(i)-}_{\nu}W^{(i)+}_{\mu}-W^{(i)+}_{\nu}W^{(i)-}_{\mu})$$

and

$$W^{3(i)}_{\mu}\equiv W^{0(i)}_{\mu}\ .$$

The first term of Eq. (2.12) comes from the $W^{(i)}$ kinetic term after minimal substitution while the second piece, which contributes only to the W $M1$ and $E2$ moments, arises from the mixing terms (2.2). The interaction (2.12) is then identical to the Lee-Yang[1] parametrization if we identify $\kappa_i=g_i\lambda_i/e$. In particular, there are no off-diagonal electromagnetic weak-boson interactions ($W^{(k)}W^{(j)}\gamma$, $k\neq j$) because the group generators are distinct. This will not be the case in the general electroweak mixing scheme discussed below where additional heavy W's all couple to the same current.

Using (2.6) and (2.8), we then find that

$$\kappa_L \equiv \kappa_1 = m_L{}^2/M_W{}^2 , \qquad (2.13)$$

and using the theoretical upper limit (2.10) for N equal λ_i's and the experimental lower limit, we have

$$0.06 \lesssim \kappa_L \lesssim 4.4/N . \qquad (2.14)$$

If the so-called unification conditions $\lambda_i g_i = e$, $i=1,\ldots,N$, are satisfied, the electroweak interactions have an asymptotic

$$G\times \mathrm{U}(1)_D = \prod_{i=1}^{N} \mathrm{SU}(2)_i \times \mathrm{U}(1)_D$$

symmetry, i.e.,

$$\mathscr{L}(q^2\to\infty) \simeq \frac{1}{2q^2}\left[\frac{e^2(J^D)^2}{(1-\sum_i \lambda_i{}^2)} + \sum_i g_i{}^2(\vec{\mathrm{J}}^{\,(i)})^2\right] , \qquad (2.15)$$

where $J^D = J^{\mathrm{EM}} - \sum_i J^{3(i)}$. In this limit each $W^{(i)}$ interacts electromagnetically with the gauge-theoretic value $\kappa_i = \lambda_i g_i/e = 1$.

In the original Hung-Sakurai scheme we have $N=1$ and $G=\mathrm{SU}(2)_L$, the neutral-current interactions are the same as in the standard model, and we have the bounds

$$19 \lesssim m_L \lesssim 165 \text{ GeV} , \qquad (2.16)$$

$$0.06 \lesssim \kappa_L \lesssim 4.4 . \qquad (2.17)$$

In the limit where the unification condition holds we have asymptotic $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ symmetry, i.e.,

$$\mathscr{L}(q^2\to\infty) \simeq \frac{1}{2q^2}[(g')^2(J^Y)^2 + g^2(\vec{\mathrm{J}}^{\,L})^2] , \qquad (2.18)$$

where $J^Y = J^{\mathrm{EM}} - J^{3L}$ is the usual hypercharge current and $(g')^2 \equiv e^2/\cos^2\theta_W$. The standard-model mass relations are also recovered in this limit,

$$m_L = M_W, \quad m_{ZL} = M_{Z^0} . \qquad (2.19)$$

For the model of Barbieri and Mohapatra we have $N=2$ and $G=\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. The couplings g_L, g_R and the mixing parameters λ_L, λ_R are taken to be equal but the masses m_L, m_R are allowed to differ. The weak neutral current is given by

$$\mathscr{L}^{\mathrm{eff}}_{\mathrm{NC}} = \frac{4G_F}{\sqrt{2}}[(J^{3L} - \sin^2\theta_W J^{\mathrm{EM}})^2 + \epsilon(J^{3R} - \sin^2\theta_W J^{\mathrm{EM}})^2] , \qquad (2.20)$$

where $\epsilon = (m_L/m_R)^2$. Almost all low-energy data, including the results of the cesium atomic parity-violation experiment,[18] are consistent with $m_R/m_L \gtrsim 3$; an older experiment on atomic bismuth[19] implies that $m_R/m_L \gtrsim 7.5$. The limits on the W_L mass and κ_L are then

$$19 \lesssim m_L \lesssim 117 \text{ GeV} , \qquad (2.21)$$

$$0.06 \lesssim \kappa_L \lesssim 2.2 , \qquad (2.22)$$

more restrictive than (2.16) and (2.17). When $\lambda g = e$, the electroweak interactions are asymptotically $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ symmetric, i.e.,

$$\mathscr{L}(q^2\to\infty) \simeq \frac{1}{2q^2}[(g_{B-L}J^{B-L})^2 + g^2(\vec{\mathrm{J}}^{\,L})^2 + g^2(\vec{\mathrm{J}}^{\,R})^2] , \qquad (2.23)$$

where $(g_{B-L})^2 = e^2/\cos(2\theta_W)$. In this limit the left-handed boson masses are[14]

$$m_L = M_W, \quad m_{ZL} \simeq M_{Z^0}(1 - \epsilon\tan^4\theta_W) \qquad (2.24)$$

and the lightest Z can differ in mass from the standard-model prediction by less than 1%.

We note that for $N=4$ (as in Ref. 16) and when all λ's are equal the W_L mass and κ_L upper bounds are very close to the standard-model predictions,

$$m_L \lesssim 82.5 \text{ GeV}, \quad \kappa_L \lesssim 1.1 . \qquad (2.25)$$

These generalizations of the simplest mixing model assume that the interactions corresponding to each added symmetry factor are dominated by the exchange of the lowest-lying vector mesons compatible with that symmetry. In composite models where the weak bosons are considered as bound states of more fundamental constituents one expects a spectrum of excited states. Such more general electroweak mixing schemes, where there are several bosons all coupled to the usual isospin current, have been considered by de Groot, Schildknecht, and Kuroda[10,15] and found to have neutral-current interactions resembling those in extended gauge theories.[4,5] Kögerler, Schildknecht, and Kuroda[20,21] have also considered such schemes more in the context of composite models and we will have occasion to use several of their assumptions in deriving bounds on the form of the electromagnetic interactions of the W's in these models.

For this case of a spectrum of composite left-handed bosons we assume the form of the Lagrangians (2.1) and (2.2) except that the N currents $\vec{\mathrm{J}}^{\,(i)}_{\mu}$ are now all taken to be generated by the usual $\mathrm{SU}(2)_L$ $\vec{\mathrm{T}}_L$. For simplicity we also parametrize the spectrum of excited states by a single additional W so we only have two weak bosons, W^1 and W^2, both coupled to the isospin current and a total of six parameters, g_i, λ_i, and m_i, $i=1,2$.

The charged currents at $q^2 \simeq 0$ will have the correct normalization if

$$\frac{g_1{}^2}{m_1{}^2} + \frac{g_2{}^2}{m_2{}^2} = \frac{8G_F}{\sqrt{2}} . \qquad (2.26)$$

Diagonalizing the neutral currents at $q^2 \simeq 0$ gives

$$\mathscr{L}^{\mathrm{eff}} = \frac{1}{2}\left[\frac{e^2}{q^2}(J^{\mathrm{EM}})^2 + \frac{8G_F}{\sqrt{2}}[(J^3 - \sin^2\theta_W J^{\mathrm{EM}})^2 + C(J^{\mathrm{EM}})^2]\right] \qquad (2.27)$$

if we require that

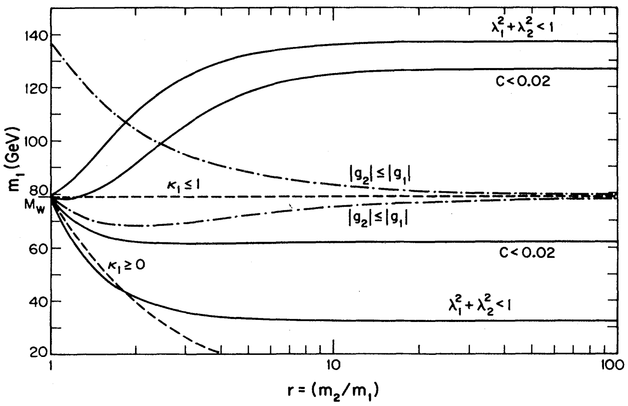

FIG. 1. Bounds on m_1 as a function of $r=m_2/m_1$ for the various experimental and theoretical constraints discussed in Sec. II.

$$\frac{\lambda_1 g_1}{m_1{}^2}+\frac{\lambda_2 g_2}{m_2{}^2}=\frac{8G_F\sin^2\theta_W}{e\sqrt{2}} . \tag{2.28}$$

The constant C is given by

$$C=\frac{\alpha\pi}{\sqrt{2}G_F}\left[\frac{\lambda_1{}^2}{m_1{}^2}+\frac{\lambda_2{}^2}{m_2{}^2}\right]-\sin^4\theta_W \tag{2.29}$$

and is constrained only by parity-nonviolating effects in $e^+e^-\to l^+l^-$ as in extended gauge theories. If we further assume the generalized unification condition

$$\lambda_1 g_1+\lambda_2 g_2=e , \tag{2.30}$$

the electroweak interactions formally have asymptotic $SU(2)_L\times U(1)_Y$ symmetry. (The authors of Refs. 20 and 21 find this condition as a natural constraint in composite models by postulating the saturation, at low energies, of the composite fermion form factors by one[20] or several[21] W bosons.) To further constrain the parameters, we assume a duality relationship among the mixing parameters implied by compositeness,[21,22]

$$\lambda_1 m_1=\lambda_2 m_2 . \tag{2.31}$$

Bounds on the remaining free parameters m_1,m_2 (or equivalently $R\equiv m_1/M_W$ and $r\equiv m_2/m_1$) can then be obtained from various experimental and theoretical constraints. (We follow Kuroda and Schildknecht[21] in obtaining the mass bounds that follow with minor differences.)

Both Z masses can be shown to be real only if

$$\lambda_1{}^2+\lambda_2{}^2<1 , \tag{2.32}$$

which gives the upper and lower bounds on m_1 as a function of r plotted in Fig. 1; this condition allows $32\lesssim m_1\lesssim 137$ GeV. The C parameter is bounded by PETRA measurements of the $e^+e^-\to l^+l^-$ cross sections to be[23]

$$C\lesssim 0.02 \quad (95\% \text{ C.L.}) \tag{2.33}$$

and the corresponding limits on m_1 versus r are also plotted in Fig. 1. The most general limits on m_1 from this constraint are $62\lesssim m_1\lesssim 127$ GeV. If we assume that the W couplings satisfy

$$|g_2|\le|g_1| \tag{2.34}$$

as might be expected in bound-state models, we have the even more restrictive bounds shown in Fig. 1. Combining the constraints from (2.32), (2.33), and (2.34), we find that

$$68\lesssim m_1\lesssim 99 \text{ GeV} \tag{2.35}$$

and even with an asymptotically $SU(2)_L\times U(1)_Y$ symmetric theory we can have significant deviations from the standard-model mass relations in composite models.

To see how these mass bounds are related to the bounds on the magnetic-moment parameters, we first apply the minimal-coupling prescription

$$\partial_\mu\to\partial_\mu-ieQA_\mu-i\vec{T}_L\cdot(g_1\vec{W}^1_\mu+g_2\vec{W}^2_\mu) \tag{2.36}$$

(which gives the correct fermion electromagnetic-interaction terms) to the W bosons. The same $W^{(i)}W^{(i)}\gamma$ interactions as in Eq. (2.12) are found so that $\kappa_1=\lambda_1 g_1/e$ and $\kappa_2=\lambda_2 g_2/e$. The unification condition (2.30) now ensures that both W bosons cannot have the canonical gauge-theory value of $\kappa=1$. Solving for κ_1 in terms of the parameters $R=m_1/M_W$ and $x\equiv m_1/m_2=1/r$, we find

$$\kappa_1=(R^2-x^2)/(1-x^2) , \tag{2.37}$$

so that $\kappa_2=(1-R^2)/(1-x^2)$ by Eq. (2.30). The limits on m_1 as a function of $r=1/x$ set by the constraints (2.32), (2.33), and (2.34) and plotted in Fig. 1 can now be translated into limits on κ_1 using (2.37) and are plotted in Fig. 2. If only (2.33) is used, we have the bound

$$0.25\lesssim\kappa_1\lesssim 2.6 , \tag{2.38}$$

while if we add the more restrictive condition (2.34) we have

$$0.5\lesssim\kappa_1\lesssim 1.7 . \tag{2.39}$$

We might here make the additional theoretical assumption that the quadrupole moments and (charge radii)^2 of the W bosons (assumed to be similar bound states of the same constituents) are, if not equal, at least of the same sign,

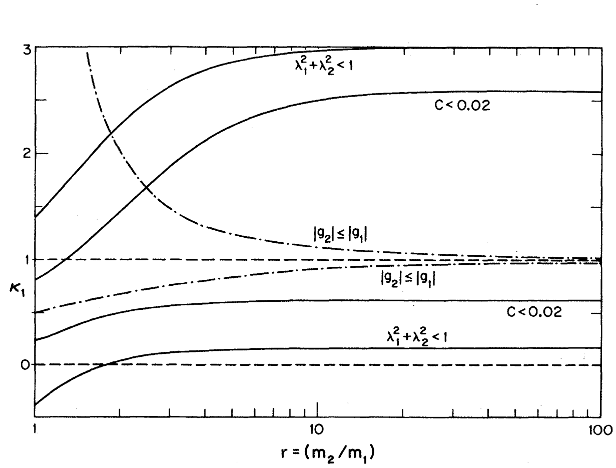

FIG. 2. Bounds on the magnetic-moment parameter κ_1 as a function of $r=m_2/m_1$.

i.e., that κ_1 and κ_2 have the same sign. Since the unification condition forces $\kappa_1+\kappa_2=1$, this implies that

$$0 \le \kappa_1 \le 1 \ , \tag{2.40}$$

which further restricts κ_1. Equation (2.40) then places bounds on m_1 via Eq. (2.37),

$$x \le R \le 1 \ , \tag{2.41}$$

which has been added in Fig. 1. The upper limit implies that $M_W \ge m_1$.

Because both W's are coupled to the same current, we also obtain off-diagonal weak-boson electromagnetic interactions from the mixing terms after minimal substitution. We find

$$\mathscr{L}^{W^1W^2\gamma} = e\tilde{\kappa}A_\mu[M_\mu^{(a)}+M_\mu^{(b)}] \ , \tag{2.42}$$

where

$$M_\mu^{(a)} = i\partial_\nu(W_\nu^{2-}W_\mu^{1+} - W_\mu^{2-}W_\nu^{1+})$$

($M_\mu^{(b)}$ is obtained by interchanging 1 and 2) and $\tilde{\kappa}=(g_1\lambda_2+g_2\lambda_1)/2e$. Such off-diagonal interactions are not usually present when a minimal-coupling prescription is applied to other more familiar charged composite systems, for instance, ground-state and excited ions. The existence of a higher-order electromagnetic interaction (dipole or higher) is needed to connect two such orthogonal states. The mixing terms in (2.2), which for the diagonal W electromagnetic couplings contribute only to their magnetic dipole and electric quadrupole interactions, play that role here.

This additional interaction, connecting the lightest W^1 with excited states, is important because it can contribute to process I via s-channel graphs where the W^2 is the intermediate state. Using the $W_\alpha^2(p+k)$-$\gamma_\mu(k)$-$W_\beta^1(p)$ vertex implied by (2.42),

$$V_{\alpha\beta\mu} = -ie\tilde{\kappa}(g_{\beta\mu}k_\alpha - g_{\alpha\mu}k_\beta) \ , \tag{2.43}$$

we can generalize the differential cross section for $q_i\bar{q}_j \to W^-\gamma$ in Ref. 3 to include such effects. We find

$$\frac{d\sigma}{dt}(q_i\bar{q}_j\to W^{1-}\gamma) = \frac{\alpha}{s^2}\frac{{g_1}^2}{8}g_{ij}^2\left[\left[Q_i+\frac{1}{1+t/u}\right]^2\frac{t^2+u^2+2s{m_1}^2}{ut} + \left[Q_i+\frac{1}{1+t/u}\right](u-t)\left[\frac{\kappa_1-1}{s-{m_1}^2}+\frac{g_2}{g_1}\frac{\tilde{\kappa}}{s-{m_2}^2}\right]\right.$$
$$\left. +\frac{1}{2}\left[ut+(t^2+u^2)\frac{s}{4{m_1}^2}\right]\left[\frac{\kappa_1-1}{s-{m_1}^2}+\frac{g_2}{g_1}\frac{\tilde{\kappa}}{s-{m_2}^2}\right]^2\right] \ , \tag{2.44}$$

where the g_{ij} are the Kobayashi-Maskawa mixings. Thus the presence of radiation zeros in process I can be used to test the off-diagonal electromagnetic couplings of a composite W to higher excitations.

Solving for the "transition magnetic moment" parameter $\tilde{\kappa}$ in terms of R and x, we find

$$\tilde{\kappa} = \frac{1}{2(1-x^2)}\left[x(R^2-x^2)+\frac{(1-R^2)}{x}\right] \tag{2.45}$$

and the bounds imposed on $\tilde{\kappa}$ implied by Eqs. (2.32), (2.33), (2.34), and (2.40) are plotted in Fig. 3; taken together they imply that $0.0 \le \tilde{\kappa} \le 0.5$.

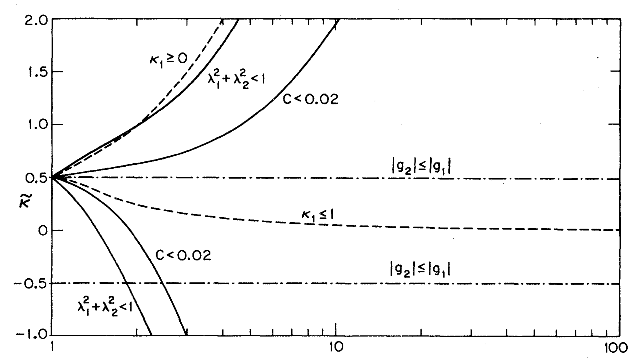

FIG. 3. Bounds on the "transition-magnetic-moment" parameter $\tilde{\kappa}$ as a function of $r=m_2/m_1$.

III. THE W MOMENTS AND $q_i\bar{q}_j \to W\gamma$

The W electromagnetic interactions discussed in these schemes, all obtained by a minimal-coupling prescription and parametrized by a single parameter $\kappa=g\lambda/e$, can be applied to the results of Mikaelian *et al.*,[3] who also use the Lee-Yang parametrization, for process I. Even for the off-diagonal interaction terms found in the general mixing model are essentially of this form, characterized by a single "transition moment" $\tilde{\kappa}$ and contributing to $q_i\bar{q}_j \to W^-\gamma$ as in Eq. (2.44). Assuming the C invariance of the electromagnetic interactions, however, the W has, *a priori*, three independent multipole moments ($E0$,$M1$,$E2$), where only the charge has been measured in low-energy experiments. Aronson[24] and Kim and Tsai[25] have shown that by adding a higher-derivative term to the Lagrangian,

$$\mathscr{L} = \frac{ie\rho}{{M_W}^2}F_{\mu\nu}W_{\mu\rho}^-W_{\nu\rho}^+ \ , \tag{3.1}$$

one obtains a $WW\gamma$ vertex with arbitrary and *independent* moments

$$\mu_W = e(1+\kappa+\rho)/2M_W \ , \tag{3.2}$$

$$Q_W = -e(\kappa-\rho)/{M_W}^2 \ , \tag{3.3}$$

and charge radius

$$R_W{}^2=(\kappa+\rho)/M_W{}^2 \;. \tag{3.4}$$

The gauge-theory prediction is $\kappa=1$ and $\rho=0$.

Assuming now only a single W of mass M_W and using this more general parametrization for the process $q_i\bar{q}_j \to W^-\gamma$,[26] we find that

$$\frac{d\sigma}{dt}(q_i\bar{q}_j\to W^-\gamma)=\frac{\alpha}{s^2}\frac{M_W{}^2G_F}{\sqrt{2}}g_{ij}{}^2\left[\left[Q_i+\frac{1}{1+t/u}\right]^2\frac{t^2+u^2+2sM_W{}^2}{tu}+(\kappa-1)\left[Q_i+\frac{1}{1+t/u}\right]\frac{t-u}{t+u}+\frac{(\kappa-1)^2}{2(t+u)^2}\left[tu+(t^2+u^2)\frac{s}{4M_W{}^2}\right]+\frac{(\kappa-1)\rho}{4M_W{}^2}\left[\frac{2uts+s(u+t)^2}{(u+t)^2}\right]+\frac{\rho^2 s}{8M_W{}^4}\left[\frac{4uts+M_W{}^2(u^2+t^2)}{(u+t)^2}\right]\right]. \tag{3.5}$$

As an example, we plot the cross section for $d\bar{u}\to W^-\gamma$ (with $\sqrt{s}=200$ GeV and $M_W=79$ GeV) for various κ (with $\rho=0$) in Fig. 4(a) and for several ρ's (with $\kappa=1$) in Fig. 4(b). Because of the higher-derivative nature of the additional coupling in (3.1), the high-energy behavior of the cross section is now more singular and the presence of radiation zeros depends more sensitively on deviations of ρ from its gauge-theory value than on κ. We estimate that in $p\bar{p}$ collisions at CERN energies ($\sqrt{s}=540$ GeV) the differential cross section *at the position of the zero* is 6–9 times more sensitive to ρ than to $\kappa-1$. At the higher energies expected at the Fermilab Tevatron ($\sqrt{s}\simeq 2000$ GeV), this ratio of sensitivities increases to a factor of roughly a hundred.

Laursen, Samuel, and Sen[27] have recently included the effects of quark anomalous magnetic moments in process I and find that large quark anomalies can also wash out the dip. Thus any future collider experiment measuring $p\bar{p}\to W\gamma X$ will set limits on some combination of quark (μ_u, $\mu_{\bar{d}}$, and $\mu_{\bar{s}}$) and W moments (μ_W, Q_W, or κ and ρ).

The minimal-substitution prescription employed in all the versions of electroweak mixing discussed in Sec. II,

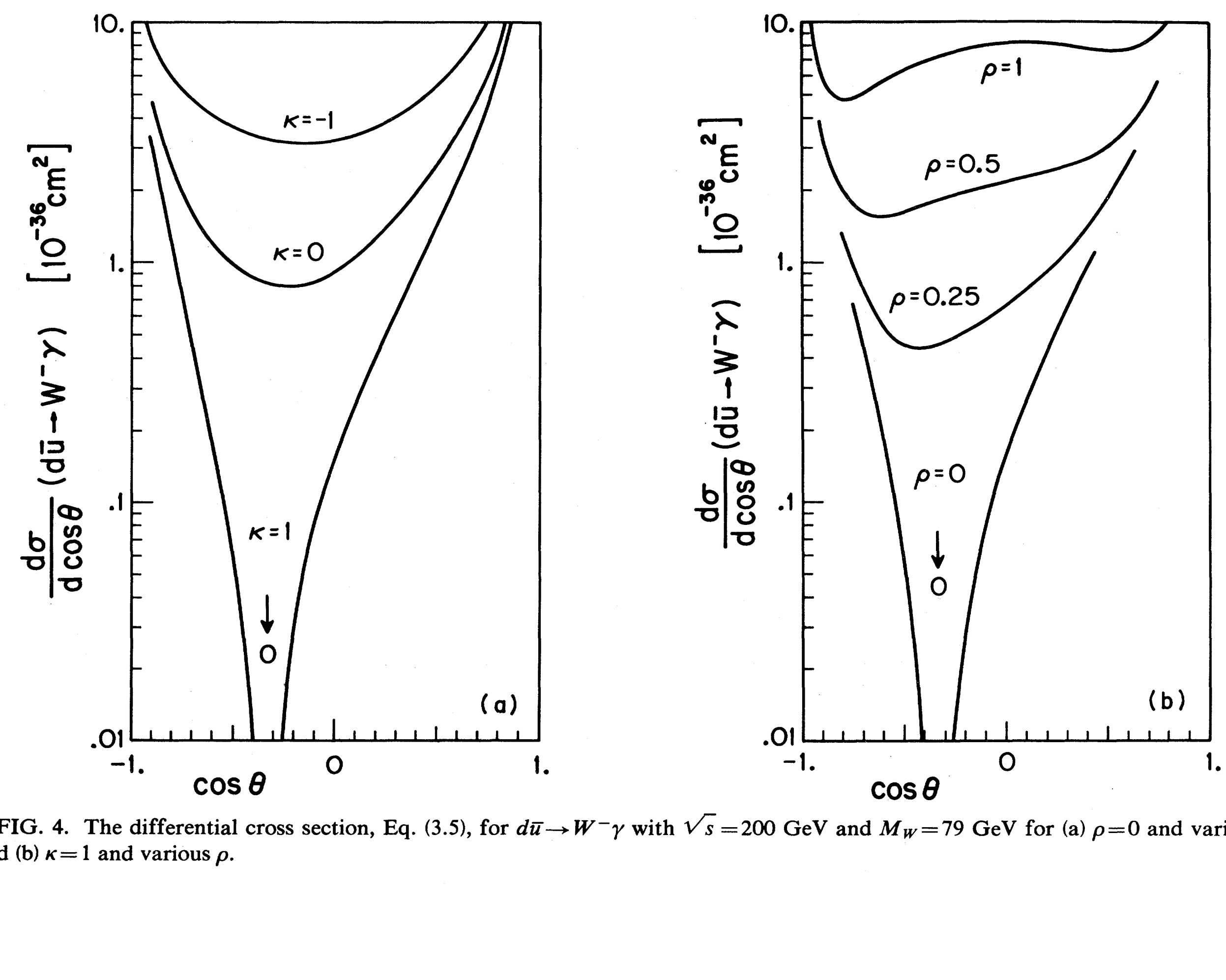

FIG. 4. The differential cross section, Eq. (3.5), for $d\bar{u}\to W^-\gamma$ with $\sqrt{s}=200$ GeV and $M_W=79$ GeV for (a) $\rho=0$ and various κ and (b) $\kappa=1$ and various ρ.

while allowing for nongauge W electromagnetic couplings, did not generate any anomalous $Z\gamma\gamma_V$ or $ZZ\gamma$ interactions, but nonstandard models could, in principle, allow such vertices. Renard[28] has discussed the form of such interactions allowed by charge-conjugation invariance and their possible effects on $e^+e^-\rightarrow Z\gamma$. While there are no dramatic amplitude zeros in this process that would make it a test of the gauge structure, we note that several authors[29] have suggested the reaction $e^+e^-\rightarrow \nu\bar{\nu}\gamma$ with Z^0 tagging as possibly the best way to precisely count the number of low-mass neutrino species. Thus, even if the weak-boson masses are found to be very close to their standard-model values, the production of weak bosons accompanied by photons ($W\gamma$,$Z\gamma$) in $p\bar{p}$ and e^+e^- colliders will provide important tests of the gauge-theory description and possible compositeness of the weak interactions.

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