Quark model of baryon magnetic moments

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(Received 2 November 1982; revised manuscript received 16 May 1983)

The magnetic moments of the baryon octet and the $\Sigma^0 \to \Lambda^0$ transition moment are calculated for the most general form of wave function describing an essentially nonrelativistic constitutent quark model. For definiteness the quarks are taken to have Dirac moments. Flavor symmetry is assumed, that is, the Landé g factor is taken to be the same for all the baryons, an assumption that is justified by the small probabilities $(^2P)^2$, $(^4P)^2$, and $(^4D)^2$ suggested by our final results. All of the measured moments can be fitted by this model. Results of making the fit are the following: (1) the ratios of intrinsic quark moments are $\mu_u/\mu_d = -1.93 \pm 0.09$, $\mu_s/\mu_d = 0.79 \pm 0.12$, (2) the total admixture of decimet and singlet states with the octet amounts to $10 \pm 14\%$, (3) if $|\mu_d|$ is assumed to be 1 nuclear magneton (μ_N), the probabilities of admixed angular momentum states satisfy the condition $\frac{2}{3}(^2P)^2 - \frac{1}{3}(^4P)^2 + (^4D)^2 = 0.05 \pm 0.09$, and (4) within the context of the model there is an upper bound on the absolute value of the $\Sigma^0 \to \Lambda^0$ transition moment equal to $(1.97 \pm 0.14)\mu_N$ and depending only on the measured values of the seven magnetic moments. A more precise measurement of the transition moment is needed to confirm that it does not exceed this bound. Numerical values are also specified for two matrix elements that may be useful for determining more detailed properties of the wave function in specific models but they will be very model dependent. At present, all of the qualitative features of these results would appear to be consistent with a dynamical model based on simple potentials between constituent quarks augmented by spin-orbit couplings and SU(3)-breaking interactions of the order of magnitude of the "hyperfine" interaction of De Rújula, Georgi, and Glashow.

I. INTRODUCTION

The recently measured magnetic moments of members of the baryon octet appear to be in surprisingly good agreement with the sum rule

$$\mu(\Sigma^+) + \mu(\Xi^0) - \mu(\Sigma^-) - \mu(\Xi^-) = 2.640\mu_N \quad (1a)$$

($\mu_N =$ nuclear magneton), which was predicted[1] on the basis of the constituent quark model by making use of the assumption of flavor symmetry.[2] The experimental values of the moments given in Table I yield

$$\mu_{\text{exp}}(\Sigma^+) + \mu_{\text{exp}}(\Xi^0) - \mu_{\text{exp}}(\Sigma^-) - \mu_{\text{exp}}(\Xi^-) = (2.78 \pm 0.18)\mu_N \ . \quad (1b)$$

This good agreement suggests that an attempt be made to extend the concept of flavor symmetry to include the other two baryons of the octet, the Λ^0 and Σ^0, and to determine whether this extended model is capable of accounting for each of the observed moments.

The purpose of this paper is to show that not only is the model capable of accounting for the seven measured magnetic moments and the $\Sigma^0 \to \Lambda^0$ transition moment but also there are interesting restrictions placed on the wave functions of the model by fitting it to the data. The restrictions are as follows.

(I) The amount by which symmetry breaking mixes the decimet and singlet representations of SU(3) with the octet totals $(10 \pm 14)\%$.

(II) If the intrinsic moment of the d quark is assumed to have the magnitude of 1 nuclear magneton (μ_N), the probabilities of the mixing of ^2P, ^4P, and ^4D states with the basic ^2S state are restricted by the condition

$$\tfrac{2}{3}(^2P)^2 - \tfrac{1}{3}(^4P)^2 + (^4D)^2 = 0.05 \pm 0.09 \ , \quad (2)$$

where the term symbols represent the amplitudes of the state, which are real.[3]

(III) There is a large contribution to the moment from a cross term between the octet and decimet, providing independent evidence for mixing of the ^2P, ^4P, or ^4D states *if* the ^2S state is assumed to be dominated by the usual 56 representation of SU(6).

The only assumptions made in order to obtain these results are the following.

(A) The state of each baryon is described by an essentially nonrelativistic constituent quark model.

(B) The g factor for the spin of a quark is twice the orbital g factor and there are no exchange moments.

(C) The wave functions are eigenfunctions of isotopic spin.

(D) The Landé g factors, which depend only on the distribution of total spin and total orbital angular momentum, are approximately the same for the octet, decimet, and singlet SU(3) states. This condition defines "flavor symmetry."

Because the assumptions are so general, the essential qualitative features of the constraints I—III would appear to be incontrovertible within the meaning of three-particle constituent quark models which treat the ^2S, SU(6) state as

TABLE I. Measured magnetic moments and transition moment for the octet of baryons.

Baryon	Measured moment	Source
p	$2.793 \cdots \pm 0.000$	Particle Data Group[a]
n	$-1.913 \cdots \pm 0.000$	Particle Data Group[a]
Λ	-0.613 ± 0.005	Schachinger,[b] Cox[c]
Σ^+	2.33 ± 0.13	Particle Data Group[a]
Σ^-	-1.01 ± 0.12	World average[d]
Ξ^0	-1.250 ± 0.014	Bunce,[e] Cox[c]
Ξ^-	-0.69 ± 0.04	Handler[f]
$\lvert \Sigma^0 \to \Lambda^0 \rvert$	$1.82^{+0.25}_{-0.18}$	Dydak[g]

[a]Particle Data Group, Phys. Lett. 111B, 1 (1982).
[b]L. Schachinger *et al.*, Phys. Rev. Lett. 41, 1348 (1978).
[c]P. T. Cox *et al.*, Phys. Rev. Lett. 46, 877 (1981).
[d]L. Deck *et al.*, Phys. Rev. D 28, 1 (1983).
[e]G. Bunce *et al.*, Phys. Lett. 86B, 386 (1979).
[f]R. Handler *et al.*, in *High Energy Spin Physics—1982,* proceeding of the 5th International Symposium, Brookhaven National Laboratory, edited by G. Bunce (AIP, New York, 1983).
[g]F. Dydak *et al.*, Nucl. Phys. B118, 1 (1977).

dominant. Although the use of anomalous quark moments would introduce additional parameters, the qualitative features of the constraints would not be changed, nor would they be changed by relaxing the condition $|\mu_d| = \mu_N$ within the range of values permitted by ^2S dominance.

Despite the small probability of mixing, individual moments may deviate sharply from the values predicated on simple SU(6) symmetry because the cross terms between the dominant and mixed states are proportional to the *amplitudes* of mixing, which, of course, may be appreciable even when the probabilities are small. The cross terms are also sensitive to details of the wave functions but, unfortunately, these details cannot be pinpointed by use of the measured moments because there are so many ways in which the imposed constraints can be realized.

In general, then, it is a qualitative picture that emerges. However, the mirror properties[1] will be shown to be completely independent of the cross terms and the consequent sum rule Eq. (1a) depends only on the smallness of the mixing probabilities. Thus, the values of the ratios of quark moments, which are obtained by applying these relationships to the data, have a reasonably high degree of reliability within the broadest context of the model. On the basis of the results presented in Table I, these ratios are

$$\mu_u/\mu_d = -1.93 \pm 0.09 \,, \tag{3a}$$

$$\mu_s/\mu_d = 0.79 \pm 0.12 \,. \tag{3b}$$

II. DESCRIPTION OF THE STATES

The wave functions of the three-body system may be classified in general by the associated representations of SU(3). In terms of functions of the flavor variable the irreducible representations are 8, 8, 10, and 1. Because the Pauli principle requires that the space-spin-flavor function be totally symmetric (the color function is antisymmetric) under permutations, the space-spin functions ψ_8, $\bar{\psi}_8$, ψ_{10}, and ψ_1 associated with these flavor functions have the same permutation properties as those of the flavor functions.

Since flavor symmetry is based on the permutation properties of the wave functions, we specify the choice of the bases for the two 8 representations by their properties under permutation of the particle labels 1,2,3 (Ref. 4)

$$P_{12}\psi_8 = -\psi_8 \,, \tag{4a}$$

$$P_{12}\bar{\psi}_8 = \bar{\psi}_8 \,, \tag{4b}$$

$$P_{13}\psi_8 = \tfrac{1}{2}(\psi_8 + \sqrt{3}\bar{\psi}_8) \,, \tag{4c}$$

$$P_{13}\bar{\psi}_8 = \tfrac{1}{2}(\sqrt{3}\psi_8 - \bar{\psi}_8) \,, \tag{4d}$$

$$P_{23}\psi_8 = \tfrac{1}{2}(\psi_8 - \sqrt{3}\bar{\psi}_8) \,, \tag{4e}$$

$$P_{23}\bar{\psi}_8 = -\tfrac{1}{2}(\sqrt{3}\psi_8 + \bar{\psi}_8) \,. \tag{4f}$$

Functions belonging to the 10 representations are totally symmetric:

$$P_{ij}\psi_{10} = \psi_{10} \,, \tag{5}$$

and those belonging to 1 are antisymmetric,

$$P_{ij}\psi_1 = -\psi_1 \,. \tag{6}$$

In order to determine which functions are to be associated with which hadrons, it is convenient to specify the flavors associated with the particle indices. When there are two like quarks, they are taken to be quarks 1 and 2, when all the quarks are different, $1 \equiv u$, $2 \equiv d$. Then the sextet of hadrons having two like quarks (flavor α) and one unlike (flavor β) may be represented as a linear combination of functions symmetric in 1 and 2:

$$\Psi(\alpha^2,\beta) = \bar{a}_8\bar{\psi}_8 + a_{10}\psi_{10} \,, \tag{7a}$$

with[3]

$$\bar{a}_8{}^2 + a_{10}{}^2 = 1 \,. \tag{7b}$$

It is to be noted that the only nonstrange component of ψ_{10} belongs to $I = \frac{3}{2}$; therefore $a_{10} = 0$ for the nucleon. The coefficient a_{10} measures the deviation from flavor SU(3) and it is assumed that that is due to the difference between the masses of the s quark and the other two. The isotopic-spin assignments of $I=1$ for the $\Sigma^{\pm}$ and $I=\frac{1}{2}$ for the Ξ's are met by both $\bar{\psi}_8$ and ψ_{10}.

The wave function of the Σ^0 is given by the $I_3=0$ component associated with $\Psi(\Sigma^{\pm})$:

$$\Psi(\Sigma^0)=\bar{a}_8\bar{\psi}_8+a_{10}\psi_{10} , \tag{8}$$

with quark indices $1\equiv u$, $2\equiv d$, as prescribed.

On the other hand, the $I=0$ Λ^0 state is antisymmetric in u and d, that is in 1 and 2, and can include only those functions that are antisymmetric under P_{12}:

$$\Psi(\Lambda^0)=a_8\psi_8+a_1\psi_1 , \tag{9a}$$

where

$$a_8{}^2+a_1{}^2=1 . \tag{9b}$$

The coefficient a_1 is also a measure of deviations from flavor SU(3) but there is no simple connection between a_1 and a_{10}. We treat them as independent parameters to be determined by experiments.

III. MAGNETIC MOMENTS

Under our assumption B the magnetic-moment operator for the baryons is

$$\vec{\mathbf{M}}=\mu_1(\vec{\sigma}_1+\vec{l}_1)+\mu_2(\vec{\sigma}_2+\vec{l}_2)+\mu_3(\vec{\sigma}_3+\vec{l}_3) , \tag{10}$$

where $\vec{\sigma}_1$ and $\vec{l}_1$ are the Pauli spin operator and orbital-angular-momentum operator for the ith quark and μ_i is its intrinsic moment. Equation (10) can be rewritten in the form

$$\vec{\mathbf{M}}=\tfrac{1}{2}(\mu_1+\mu_2)(2\vec{\mathbf{S}}+\vec{\mathbf{L}})$$
$$+\tfrac{1}{2}(\mu_1-\mu_2)[\vec{\sigma}_1+\vec{l}_1-(\vec{\sigma}_2+\vec{l}_2)]$$
$$+\tfrac{1}{2}(2\mu_3-\mu_1-\mu_2)(\vec{\sigma}_3+\vec{l}_3) . \tag{11}$$

The magnetic moment of a baryon in state Ψ (with magnetic quantum number $\frac{1}{2}$) is given by the expectation value of M_z in that state and the $\Sigma^0\to\Lambda^0$ transition moment is the Σ^0,Λ^0 matrix element of M_z. Therefore, the moments are linear combinations of the matrix elements

$$\langle N\,|\,2S_z+L_z\,|\,N'\rangle=A(N)D(N,N') , \tag{12a}$$

$$\langle N\,|\,\sigma_{3z}+l_{3z}\,|\,N'\rangle=B(N,N') , \tag{12b}$$

$$\langle N\,|\,\sigma_{1z}+l_{1z}-(\sigma_{2z}+l_{2z})\,|\,N'\rangle=C(N,N') . \tag{12c}$$

Here, N and N' take on the values 8, $\bar{8}$, 10, and 1 corresponding to ψ_8, $\bar{\psi}_8$, ψ_{10}, and ψ_1. By making use of their time-reversal properties it can easily be shown that these Hermitian matrices are symmetric if the phases are chosen in accordance with the Wigner convention.

Because the quark indices label dummy variables, the matrix elements are invariant under permutations of the indices and the following useful relationships among them may be derived by use of the permutation properties of the wave functions, Eqs. (4)—(6):

$$D(N,N')=\delta_{NN'} , \tag{13a}$$

$$A(8)=A(\bar{8}) , \tag{13b}$$

$$B(\bar{8},\bar{8})+B(8,8)=\tfrac{2}{3}A(8) , \tag{13c}$$

$$C(\bar{8},8)=\frac{\sqrt{3}}{2}[B(8,8)-B(\bar{8},\bar{8})]$$
$$=\sqrt{3}[\tfrac{1}{3}A(8)-B(\bar{8},\bar{8})] , \tag{13d}$$

$$B(1,1)=\tfrac{1}{3}A(1) , \tag{13e}$$

$$C(1,\bar{8})=-\sqrt{3}B(1,8) , \tag{13f}$$

$$C(8,10)=\sqrt{3}B(\bar{8},10) , \tag{13g}$$

$$C(N,N)=C(1,8)=C(1,10)=0 . \tag{13h}$$

When the flavors are assigned as specified earlier, the magnetic moments of the baryons and the transition moment are found from Eqs. (7)—(9) and (11)—(13) to be

$$\mu(p)=\mu_uA(\bar{8})+(\mu_d-\mu_u)B(\bar{8},\bar{8}) , \tag{14a}$$

$$\mu(n)=\mu_dA(\bar{8})+(\mu_u-\mu_d)B(\bar{8},\bar{8}) , \tag{14b}$$

$$\mu(\Sigma^+)=\mu_uA+(\mu_s-\mu_u)B , \tag{14c}$$

$$\mu(\Sigma^0)=\tfrac{1}{2}(\mu_u+\mu_d)A+\tfrac{1}{2}(2\mu_s-\mu_u-\mu_d)B , \tag{14d}$$

$$\mu(\Sigma^-)=\mu_dA+(\mu_s-\mu_d)B , \tag{14e}$$

$$\mu(\Xi^0)=\mu_sA+(\mu_u-\mu_s)B , \tag{14f}$$

$$\mu(\Xi^-)=\mu_sA+(\mu_d-\mu_s)B , \tag{14g}$$

$$\mu(\Lambda^0)=\tfrac{1}{2}(\mu_u+\mu_d)A'+\tfrac{1}{2}(2\mu_s-\mu_u-\mu_d)B' , \tag{14h}$$

and

$$\mu(\Sigma^0\to\Lambda^0)=\frac{\sqrt{3}}{2}(\mu_u-\mu_d)B'' , \tag{14i}$$

where

$$A=\bar{a}_8{}^2A(\bar{8})+a_{10}{}^2A(10) , \tag{15a}$$

$$A'=a_8{}^2A(\bar{8})+a_1{}^2A(1) , \tag{15b}$$

$$B=\bar{a}_8{}^2B(\bar{8},\bar{8})+2\bar{a}_8a_{10}B(\bar{8},10)+\tfrac{1}{3}a_{10}{}^2A(10) , \tag{15c}$$

$$B'=a_8{}^2[\tfrac{2}{3}A(\bar{8})-B(\bar{8},\bar{8})]+2a_8a_1B(1,8)+\tfrac{1}{3}a_1{}^2A(1) , \tag{15d}$$

and

$$B''=\bar{a}_8a_8[\tfrac{1}{3}A(\bar{8})-B(\bar{8},\bar{8})]$$
$$-\bar{a}_8a_1B(1,8)+a_8a_{10}B(\bar{8},10) . \tag{15e}$$

The appearance of the common coefficient B in Eqs. (14c) and (14e)—(14g) implies that there is a consistency condition among the equations relating $\mu(\Sigma^+)-\mu(\Sigma^-)$ with $\mu(\Xi^0)-\mu(\Xi^-)$. This condition takes the form of a sum rule,

$$\mu(\Sigma^+)-\mu(\Sigma^-)+\mu(\Xi^0)-\mu(\Xi^-)=(\mu_u-\mu_d)A . \tag{16}$$

The matrix elements $A(N)$ given by Eq. (12a) are diagonal in total spin and total orbital angular momentum and are therefore a linear combination of the probabilities of the allowed terms in the state ψ_N, the coefficients being the well-known Landé g factors:

$$A(N)=[1-\tfrac{2}{3}(^2P)^2+\tfrac{1}{3}(^4P)^2-(^4D)^2]_N \ . \tag{17a}$$

The amplitudes of the four allowed terms are subject to the normalization condition:

$$(^2S)^2+(^2P)^2+(^4P)^2+(^4D)^2=1 \ , \tag{17b}$$

which has been used to eliminate the amplitude of the (dominant) ^2S state from the expression for $A(N)$.

IV. EXTENDED FLAVOR SYMMETRY

The notion of flavor symmetry[1] is based on two assumptions. The first is an extension of charge symmetry, the "mirror" property of baryons made up of two like quarks and one unlike quark:

$$\mu(\alpha^2,\beta)+\mu(\beta^2,\alpha)=(\mu_\alpha+\mu_\beta)A(\alpha,\beta) \ , \tag{18}$$

which follows immediately, and in general, for the sextet of baryons from Eqs. (14a)–(14c) and (14e)–(14g) with $A(\alpha,\beta)=A(\overline{8})$ for the nucleons and $A(\alpha,\beta)=A$ for the other baryons. For the Σ's and Ξ's it is another consequence of the appearance of the common coefficient B in Eqs. (14).

The fact that this result is independent of SU(3) symmetry breaking may appear to be surprising for a mirror pair such as the Σ^+ and Ξ^0 which have different numbers of s quarks. The reason for the generality of the result is that the symmetry breaking can mix only the state ψ_{10} with $\overline{\psi}_8$ because of the constraints imposed on the colored quarks by the Pauli principle. This guarantees that there is only the one admixed state, ψ_{10}, in the sextet of baryons with two like quarks.

The second assumption underlying the original notion of flavor symmetry is that the Landé g factors, Eq. (17a), which depend only on the probability distribution of total spin and total orbital angular momentum, are approximately the same for all members of the sextet; the distribution is independent of the symmetry breaking. The justification for this assumption is that symmetry breaking due to the difference in mass between the s quark and the u and d quarks, while causing energy (mass) splitting, would not be expected to change the angular momentum characteristics of a wave function in first order. In fact, the mixing of states of different flavor representations with the octet would be expected to be of the same order as the mixing of the different angular momentum terms with the ^2S, if the spin-orbit couplings and SU(3)-breaking interactions are of the same order of magnitude as suggested by the relativistic "hyperfine" interaction of De Rújula, Georgi, and Glashow.[5] We shall find that they are indeed of the same order.

Since the $A(N)$ are given by Eq. (17a), which may be written as $A(N)=1-\Delta(N)$, dominance of the ^2S state means that $\Delta(N)$ is small, of order 10% as it turns out. Then Eqs. (15a) and (7b) show that A differs from $A(\overline{8})$ by a term $a_{10}{}^2[\Delta(10)-\Delta(\overline{8})]$. The dominance of the octet implies that $a_{10}{}^2$ is also small, of the same order as Δ and the difference between A and $A(\overline{8})$ is of the order of 1%, thereby justifying the second assumption, which may be written as

$$A(\overline{8})=A \ . \tag{19}$$

Therefore, $A(\alpha,\beta)=A$ in the three equations represented by the mirror relation Eq. (18) and A can be eliminated by taking the two ratios of the three mirror equations.[1] Then the ratios of the intrinsic quark moments can be determined from the six observed moments with the results given in Eqs. (3).

Equation (3a) is consistent with the assumption that μ_u and μ_d are in the ratio of the electric charges,

$$\mu_u/\mu_d=-2 \tag{20}$$

and Eq. (3b) may be interpreted as the ratio of the constituent quark masses m_d/m_s.

Again by making use of Eqs. (18) and (19) it is possible to express A directly in terms of the very well-known value of $\mu(p)+\mu(n)$ and the ratio μ_u/μ_d. Insertion of the values of A obtained when $\mu_u/\mu_d=-2$ into the consistency condition Eq. (16) leads immediately to the sum rule Eq. (1a). The fact that this sum rule is satisfied by the measured moments indicates not only that the consistency condition is satisfied but that it is satisfied under the assumption of flavor symmetry.

The straightforward extension of the notion of flavor symmetry is to assume that the Landé g factor of the Λ^0 is about the same as that of the other baryons. Since Eq. (13b) guarantees this equality for the octet component of the state, the only new assumption is

$$A(1)\approx A(10)\approx A \ . \tag{21}$$

Although these equalities are broken to first order in $\Delta(N)$, the coefficients $A(1)$ and $A(10)$ always appear with the small factors $a_1{}^2$ and $a_{10}{}^2$, making the corrections to the moments negligible.

The extended flavor symmetry reduces the number of independent parameters that can be fixed by comparing Eqs. (14) with the measured moments. There are exactly eight independent parameters and we choose them to be

$$\begin{gathered}\mu_u/\mu_d,\ \mu_s/\mu_d,\ \mu_d A,\ \mu_d B(\overline{8},\overline{8}) \ , \\ a_{10}\mu_d B(\overline{8},10),\ a_1\mu_d B(1,8),\ a_8,\ \overline{a}_8 \ .\end{gathered} \tag{22}$$

There are also just eight measured quantities because $\mu(\Sigma^0)$ is not directly measurable with existing techniques. If $\mu(\Sigma^0)$ were measurable, the relationship

$$\mu(\Sigma^0)=\tfrac{1}{2}[\mu(\Sigma^+)+\mu(\Sigma^-)] \tag{23}$$

that follows from Eqs. (14) would be a test of conservation of isotopic spin which, if confirmed, would provide no new information about the eight parameters, Eq. (22).

The eight equations obtained by inserting experimental moments on the left side of Eqs. (14) are *not* a complete set of equations for the eight parameters because two of the undetermined parameters, $\overline{a}_8$ and $a_{10}\mu_\alpha B(\overline{8},10)$, appear in the coefficient B which is common to four of the equations. The consistency of these relationships expressed by Eq. (16) does not involve the undetermined parameters and it therefore eliminates one of the eight equations. Thus, the data yield only seven independent equations for the eight parameters.

V. INTERPRETATION OF MEASURED MOMENTS

The values of the measured moments given in Table I have already been used to determine two of the eight parameters, namely the ratios of intrinsic quark moments, Eqs. (3). Of the remaining five independent equations, Eq. (18) for the sum of proton and neutron moments yields

$$\mu_d A = -(0.95\pm0.09)\mu_N \,, \tag{24a}$$

where μ_N is the nuclear magneton. The difference of the proton and neutron moments given by Eqs. (14a) and (14b) yields

$$\mu_d B(\bar{8},\bar{8}) = (0.33\pm0.02)\mu_N \tag{24b}$$

and the difference of the Ξ moments or the Σ moments gives

$$\mu_d B = (0.191\pm0.014)\mu_N \,. \tag{24c}$$

The four remaining parameters are related by three equations, two of them following from Eqs. (15c) and (15d):

$$a_{10}\mu_d B(\bar{8},10) = (-0.323\pm0.005)\bar{a}_8 + (0.254\pm0.012)\bar{a}_8^{\,-1} \tag{25a}$$

and

$$a_1\mu_d B(1,8) = (0.323\pm0.005)a_8 - (0.262\pm0.042)a_8^{\,-1} \,. \tag{25b}$$

It is convenient to simplify the third equation by making use of the assumption that deviations from the octet representation are small, that is,

$$a_8 = 1-\epsilon, \quad \bar{a}_8 = 1-\bar{\epsilon} \,, \tag{26a}$$

with

$$0<\epsilon\ll 1, \quad 0<\bar{\epsilon}\ll 1 \,. \tag{26b}$$

Then Eqs. (15e) and (25) combine to give the final equation in the approximate form[6]

$$\epsilon+\bar{\epsilon}\simeq 0.05\pm0.07 \,. \tag{26c}$$

Although ϵ and $\bar{\epsilon}$ cannot be determined independently, it is clear from Eqs. (25a) and (25b) that neither of them can vanish:

$$\epsilon\neq 0, \quad \bar{\epsilon}\neq 0 \,. \tag{27}$$

From the normalization conditions, Eqs. (7b) and (9b), it follows that

$$a_{10}^{\,2} = 2\bar{\epsilon}, \quad a_1^{\,2} = 2\epsilon \,. \tag{28}$$

Therefore, the total probability for an admixture of states other than the octet is

$$a_{10}^{\,2} + a_1^{\,2} = 0.10\pm0.14 \,. \tag{29}$$

We conclude that the observed moments show that SU(3) is broken by inclusion of both ψ_{10} and ψ_1 in the octet but that the amount of either may be quite small (however, probably $>1\%$) and the total is of order 10%.

Another important result may be obtained by combining Eqs. (25a) and (25b) with the independent Eq. (15e) to show that $|\mu(\Sigma^0\to\Lambda^0)|$ is a monotone decreasing function of ϵ and $\bar{\epsilon}$ such that

$$|\mu(\Sigma^0\to\Lambda^0)| \leq 1.97\pm0.14 \,, \tag{30}$$

where the numerical value of the bound is based on the measured values of the seven magnetic moments. Although the value of $\mu(\Sigma^0\to\Lambda^0)$ given in Table I falls well within this limit, the estimated errors are large enough to permit a violation. Therefore, a more precise measurement of $\mu(\Sigma^0\to\Lambda^0)$ is of particular interest as a test of the model.

Additional information to be obtained concerning the distribution of angular momentum in the baryons depends on the magnitude of μ_d. If the value $|\mu_d| = \mu_N$ is used to provide an estimate of absolute values of the amplitudes and matrix elements, Eqs. (24a) and (17a) yield a relationship among the probabilities of states admixed with the ^2S:

$$\tfrac{2}{3}(^2P)^2 - \tfrac{1}{3}(^4P)^2 + (^4D)^2 = 0.05\pm0.09 \,. \tag{31}$$

Thus, the magnitude of the mixing of states of higher orbital angular momentum with the ^2S state is of the same order as that of the symmetry breaking, in agreement with our surmise based on the De Rújula—Georgi—Glashow[5] spin-orbit coupling.

The introduction of this same value of μ_d into Eqs. (25) leads to values of the coefficients ranging from

$$B(\bar{8},10) = \pm0.45, \quad B(1,8) = \pm0.13 \tag{32a}$$

for the one extreme ($\bar{\epsilon}=0.01$, $\epsilon=0.04$) to

$$B(\bar{8},10) = \pm0.16, \quad B(1,8) = \pm0.49 \tag{32b}$$

at the other extreme ($\bar{\epsilon}=0.04$, $\epsilon=0.01$).

In order to extract further information from these values of $B(\bar{8},10)$ and $B(1,8)$, it is necessary to be more specific about the wave functions. The general structure of the possible functions can be formulated by the methods used for the nuclear three-body problem.[4] The calculation of the matrix elements of $(\sigma_{3z}+l_{3z})$ in terms of integrals over the (unknown) radial functions is straightforward.[7]

Since the ^2S state is dominant, we remark that there are three allowed forms that may be combined linearly to construct a ^2S state in the octet, one of them being the usual state belonging to the 56 representation associated with SU(6) symmetry.[8] The other two belong to the 70 and 20 representations of SU(6). There is only one form of ^2S for the decimet and one for the singlet, both of these belonging to the 70 representation of SU(6).

The contributions of these dominant ^2S states to $B(\bar{8},10)$ and $B(1,8)$ arise only from the overlap of functions belonging to the 70 representations. Therefore, if the ^2S state alone is to account for the rather large value of $B(\bar{8},10)$, a large breaking of SU(6) symmetry would be implied. However, that situation is not nearly as serious as it might seem because the contributions of the ^2P, ^4P, and ^4D states to these matrix elements are proportional to their *amplitudes* which may be quite large despite the limitation, Eq. (31), on their probabilities. In particular, these contributions depend on the overlap between pairs of radial functions and on derivatives of the radial functions[7], and they are therefore sensitive to the structure of these functions. To determine the radial functions would require a detailed dynamical model but there is no obvious reason for excluding radial functions that are capable of accounting for the data.[9] However, one can argue that the large value of $B(\overline{8},10)$ provides additional and independent evidence that the contribution of the ^2P, ^4P, or ^4D terms to the wave function must be at least of the order of 10%, *if* it is assumed that the ^2S state is dominated by SU(6) symmetry.

VI. CONCLUSION

That the eight measured moments can be fitted by a model involving the eight undetermined parameters Eq. (22) is not surprising but the significance of this fit should be emphasized. We have shown by Eqs. (14) that in fact ten parameters are implicit in *any* use of the constituent quark model. In all previous attempts to calculate the moments in this model[10] one or another of these parameters has been set equal to zero and that is the reason those attempts have not been generally successful. While the probabilities for selected states may be close to zero, in agreement with intuition, the amplitudes may not be assumed to vanish because they enter into the calculation of moments with factors $B(N,N')$ that are very sensitive to completely unknown details of the wave functions, such as overlap integrals of nonorthogonal functions, and can be quite large.

We have reduced the number of parameters from ten to eight, not by setting any of them equal to zero but by imposing the reasonable physical constraint of flavor symmetry, a step that clearly does not introduce a disagreement with experiment.

As we have already remarked, a distinction should be made between the quantitative results Eqs. (3) obtained here for the ratios of the quark moments and the more qualitative conclusions concerning the constraints on the wave functions imposed by the measured moments. The first of these may be adjusted as the result of new measurements, but the fact that only a modest amount of SU(3) symmetry breaking and only a modest deviation from the ^2S state belonging to the 56 representation of SU(6) are required to account for the moments is not likely to change.

Most remarkable in our view is the fact that these qualitative conditions can clearly be easily satisfied. It should be kept in mind that the opposite conclusion was not *a priori* unlikely. The model is subject to severe constraints, such as the mirror property Eq. (18), the consistency condition Eq. (16), and flavor symmetry. Had the data violated these constraints, or had the derived numerical values of the parameters stretched credulity based on physical intuition, the results could have been interpreted as evidence for exchange currents between quarks,[11] for contributions from the quark sea,[1] or for the total failure of the model. Instead the results suggest that an essentially nonrelativistic dynamical model corrected by interactions of the order of the "hyperfine" coupling of De Rújula, Georgi, and Glashow[5] is fully capable of accounting for all of the measured moments. However, these happy conclusions might be invalidated by more precise measurements of the moments, especially if the transition moment should violate the condition Eq. (30).

ACKNOWLEDGMENTS

The authors had the benefit of information from Professor T. Devlin on the preliminary results of the measurements of the Σ^- moment reported in Table I to encourage the undertaking of this work, which was supported by the Division of High Energy Physics of the U.S. Department of Energy under Research Contract No. DE-AC02-80ER10587.

[1]R. G. Sachs, Phys. Rev. D 23, 1148 (1981).

[2]A similar assumption of symmetry under exchange of flavors was introduced by J. Franklin [Phys. Rev. 182, 1607 (1969)] to obtain some sum rules. However, aside from a possible specific choice of the parameters $B(\overline{8},\overline{8})$ and $B(\overline{8},10)$ which does not agree with the measurements, his assumptions, and therefore his sum rules, include the requirement that there is no breaking of flavor SU(3) (in which case our flavor symmetry is trivial) while our assumption is specifically about SU(3) breaking. See Eqs. (19) and (21). Franklin's sum rules do not include Eq. (1a) but are of course consistent with it. As they should be, our results, Eqs. (14), are consistent with Franklin's sum rules when $a_{10}=a_1=0$.

[3]It is assumed throughout this paper that all wave functions satisfy Wigner's phase condition. Therefore, all amplitudes introduced here are real as in the case of the nuclear three-body problem. See R. G. Sachs, *Nuclear Theory* (Addison-Wesley, Reading, Mass., 1953), p. 187.

[4]The construction of the space and spin functions having these symmetry properties is the same as for the nuclear three-body problem [Sachs, *Nuclear Theory* (Ref. 3), pp. 182—186]. However, note that the functions given there are *not* normalized whereas the functions ψ_8, $\overline{\psi}_8$, ψ_{10}, and ψ_1 are normalized. Also, it has been called to our attention by J. Franklin that there is an important erroneous statement on p. 183 of *Nuclear Theory* to the effect that "There is no antisymmetric S function $\cdots$." As Franklin notes (private communication) there is, indeed, a totally antisymmetric S function of the form $(\vec{r}\cdot\vec{\rho})[(|\vec{\rho}|^2-3|\vec{r}|^2)^2-4(\vec{r}\cdot\vec{\rho})^2]$ in the notation of the reference. The significance of this point is that it makes possible the construction out of the totally antisymmetric ^4P function a totally symmetric function belonging to the 10-representation of SU(3).

[5]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[6]The estimated errors throughout these calculations are obtained by treating the quoted error for each of the measured moments entering into a given calculations as an independent standard deviation.

[7]Except for obvious modifications of the dependence on space coordinates, these matrix elements are given by R. G. Sachs, Phys. Rev. 72, 312 (1947). The matrix elements are also given in a different notation by J. Franklin, Phys. Rev. 172, 1807 (1968). However, since Franklin calculates them for eigenstates of Dalitz orbital-angular-momentum variables $\vec{l},\vec{L}$, an infinite series of his elements will usually be required to describe the matrix elements for a *bound* state. The reason is that the bound-state wave function decays exponentially in the distance between pairs of quarks at large distances and such a function is generally an infinite series in $\vec{l},\vec{L}$. The

only exception occurs if the radial function happens to depend only on the single variable $r_{12}^2+r_{13}^2+r_{23}^2$.

[8]F. Gursey and L. A. Radicati, Phys. Rev. Lett. 13, 173 (1964); B. Sakita, Phys. Rev. 136, B1756 (1964).

[9]In the special case that the radial functions depend only on $r_{12}^2+r_{13}^2+r_{23}^2$, cited in Ref. 7, the derivatives vanish and, for that unlikely situation, it might be necessary to require large SU(6) breaking to account for the large value of $B(\overline{8},10)$.

[10]See, for example, N. Isgur and G. Karl, Phys. Rev. D 21, 3175 (1980) where the SU(3)-breaking parameters are introduced but not the mixing of ^2P, ^4P, or ^4D states. For summaries see J. L. Rosner, in *High Energy Physics—1980,* proceedings of the XXth International Conference, Madison, Wisconsin, edited by L. Durand and L. G. Pondrom (AIP, New York, 1981), p. 540; also J. Franklin, in *High Energy Spin Physics—1982,* proceedings of the 5th International Symposium Brookhaven National Laboratory, edited by G. Bunce (AIP, New York, 1983).

[11]An example from past experience may be enlightening. The first convincing evidence for exchange moments in nuclei arose from the fact that the fit of the measured magnetic moments of ^{3}H and ^{3}He to the mirror condition strained credulity. See R. Avery and R. G. Sachs, Phys. Rev. 74, 1320 (1948). In the same connection, Isgur and Karl (Ref. 10) quote the dynamical calculation of F. Villars [Phys. Rev. 72, 257 (1947)] as providing the evidence for exchange currents. Although such a dynamical model can yield results that fit the data, it cannot rule out other ways of fitting the data while a general mirror condition can, and did.