B-meson decays and t-quark mass

G. L. Fogli
Istituto di Fisica, Università di Bari, Bari, Italy
and Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Bari, Italy
(Received 24 January 1983; revised manuscript received 27 May 1983)

On the basis of a recent left-right-symmetric model of electroweak interactions, in which natural flavor conservation is ensured and CP is spontaneously violated, an approach to generalized Cabibbo mixing in a six-quark picture is worked out. The specific form of the mixing-matrix elements in terms of the quark masses is derived and a consistent comparison with the present phenomenological knowledge of quark mixing is performed. In particular, B-meson decays are reviewed, by including nonspectator effects coming from annihilation and gluon-emission processes, with a careful treatment of the phase-space factors: the emerging picture on the one hand allows severely constraining the t-quark mass, and on the other hand leads to specific predictions about future measurements of the decay parameters.

I. INTRODUCTION

It is not difficult to predict that during the next few years a large amount of experimental physics will be concerned with the analysis of heavy-quark decays. Several reasons are at the basis of this interest.

(a) A better understanding of the bound-state structure of heavy hadrons: in particular, masses and leptonic widths of the bound states of the heaviest known quark, the b quark, are expected to provide crucial information on the dynamics of quark binding.[1,2]

(b) Even though the $SU(2)\times U(1)$ standard model[3] is generally expected to give the essential features of all weak-decay effects, the emerging experimental picture of charmed-meson decays (enhancement of D^0, and possibly F^+, nonleptonic decays, etc.) is not, or not fully, understood at present,[4] and seems to require a careful introduction of strong-interaction effects. There are valid reasons to think that these effects manifest themselves in a cleaner way when heavier quarks are involved, as in B-meson (or T-meson) decays.

(c) The t quark, expected in the standard model, is presently unseen at the highest energy available in e^+e^- annihilation. This implies $m_t > 19$ GeV. If $20 \leq m_t \leq 60$ GeV, then the CERN $p\bar{p}$ collider will provide the possibility of discovering the top flavor in the near future.[5] The nonexistence of the t quark, being related to the possibility that the b quark is an isosinglet, has well defined implications of the level of B decays[6] (essentially, the Glashow-Iliopoulos-Maiani mechanism no longer can be called to ensure the absence of flavor-changing neutral-current decays). Experimental data seem to indicate that these topless models are to be excluded.[7] As will be seen later, under reasonable assumptions B decays can give further indications about the t quark, more specifically about its mass.

(d) The possibility of accounting for the generalized Cabibbo mixing, at least at a numerical level, is one of the most interesting results connected to B-meson decays. This mixing has certainly an important physical meaning, and presumably is strongly related to the origin of the quark masses. Our knowledge of the generalized Cabibbo mixing, deduced from the available experimental data under reasonable theoretical assumptions, has been recently improved[8] with respect to previous determinations, mainly based on CP-nonconserving effects.[9] B decays (more generally, heavy-quark decays) are expected to give restrictive bounds on the elements of the mixing matrix in the standard six-quark model.

Even if presented as distinct points, the previous arguments are more or less intimately connnected. Mainly from a phenomenological point of view, the analysis of B decays would require distinguishing between the different effects, and this will be possible only when a larger amount of experimental data will be available. At present, however, the possibility of finding specific theoretical relations between the different parameters enables a better understanding of the relevance of the involved phenomena, although within the limits proper of specific theoretical assumptions.

In this paper, the consequences of a well defined, and reasonable, structure of the quark mixing, as can be deduced on the basis of a recently proposed approach to the problems of natural flavor conservation (NFC) and CP violation in the framework of left-right-symmetric models,[10,11] are analyzed. The dependence of the mixing-matrix elements on the t-quark mass is outlined, and the consequences on B decays are considered. As far as B decays are concerned, the possible, and expected, effects related to strong interactions (QCD) are taken into account in the leading-logarithmic limit. Actual experimental limits seem capable of providing a well defined range of values for the t-quark mass. Obviously, future experiments, in particular, an estimate of the t-quark mass, would represent a consistency check of the approach presented here.

The paper is organized as follows. Section II contains a brief review of the model, together with an estimate of the quark-mixing parameters in terms of the quark masses. In Sec. III, B decays are analyzed with a specific attention to the QCD effects and by taking into account in a proper way the phase-space effects in the different decay mechanisms. In Sec. IV, the conclusions are drawn.

II. AN APPROACH TO GENERALIZED CABIBBO MIXING

The framework of the approach followed here is well known,[12] and is based on the left-right-symmetric group $SU(2)_L \times SU(2)_R \times U(1)$, which appears at present to be one of the most serious candidates to an extension of the standard $SU(2)_L \times U(1)$ theory. In fact, it reproduces the present experimental "low-energy" data, it explains at a speculative level the $V-A$ character of the observed electroweak interactions, and it introduces in a natural way a mass hierarchy based on the ratio M_{W_L}/M_{W_R}. It is characteristic of this approach however, that flavor-changing neutral currents cannot be avoided. But it has been recently shown[10] that there exists the possibility of naturally obtaining their suppression as a consequence of the spontaneous breakdown of the gauge symmetry, once a suitable discrete symmetry is introduced, the degree of suppression being the same which characterizes right-handed currents. NFC is ensured without spoiling the meaning of the Cabibbo mixing: Conversely, the imposed left-right symmetry severely restricts the possible forms of the generalized Cabibbo matrix.

In the spirit of attributing CP violation to the spontaneous-symmetry-breaking mechanism (it can be shown that in the three-generation case by assuming a suitable Higgs-boson content it is possible to satisfy the strong CP requirements and obtain a Higgs-boson-induced superweak CP violation[11]), the specific form induced by the discrete symmetry to the most general Yukawa coupling ensures that the mass matrices are real and symmetric independently of the number of quark generations. Their diagonalization can then be obtained through biorthogonal transformations, i.e.,

$$O_u^T M_u O_u = D_u, \quad O_d^T M_d O_d = D_d \;, \tag{1}$$

where M_u, M_d are the mass matrices in the "weak" basis, D_u, D_d their diagonal counterparts, and O_u, O_d suitable orthogonal transformations. The Cabibbo matrix is then orthogonal and given by

$$O_c = O_u^T O_d \;. \tag{2}$$

All physics is contained in the specific structure of M_u, M_d. It will be assumed that to lowest order the masses of lighter quarks originate only from mixing with the quarks of the successive generation. How this can be "derived" is an open problem, even though different attitudes can be assumed:

(a) It can be taken as an ansatz, i.e., a reasonable assumption which is consistent with the present experimental evidence.[10]

(b) More ambitiously, it can be ascribed to a well-defined, but not even understood, mechanism of perturbative mass generation: Accordingly, only the third generation may get a mass at the tree level, the first and second generations remaining massless at this level because of some "forbidding" mechanism (symmetry?). In this scheme, the second generation gets mass "naturally" at the one-loop level, the first one through two-loop diagrams. Mechanisms for generating fermion masses in perturbation theory have been examined in the literature[13] even if basic conclusions have not been obtained so far.

We are led here to adhere to the point of view (a), even though an approach of type (b) is more appealing. It would require, however, at least an enlargement of the minimal Higgs-boson content adopted here (the same as in Refs. 10 and 11) and probably also of the gauge group.

Let us then assume the following form for M_u:

$$M_u = \begin{bmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{bmatrix} \tag{3}$$

and analogously for M_d. The diagonalization problem can be exactly solved for M_u and M_d, separately. It is then an easy matter to write down the Cabibbo matrix in terms of the quark masses. With the usual notations,

$$\begin{aligned} O_{ud} = \frac{1}{M} & [(m_t - m_u)(m_c + m_u)(m_b - m_d)(m_s + m_d)]^{-1/2} \\ & \times \{ M[m_u(m_t - m_c)m_d(m_b - m_s)]^{1/2} + [m_c m_t (m_t - m_c) m_s m_b (m_b - m_s)]^{1/2} \\ & \quad + [m_u(m_t + m_u)(m_c - m_u) m_d (m_b + m_d)(m_s - m_d)]^{1/2} \} \;, \end{aligned} \tag{4}$$

$$\begin{aligned} O_{us} = \frac{1}{M} & [(m_t - m_u)(m_c + m_u)(m_s + m_d)(m_b + m_s)]^{-1/2} \\ & \times \{ M[m_u(m_t - m_c)m_s(m_b + m_d)]^{1/2} - [m_c m_t (m_t - m_c) m_d m_b (m_b + m_d)]^{1/2} \\ & \quad + [m_u(m_t + m_u)(m_c - m_u) m_s (m_b - m_s)(m_s - m_d)]^{1/2} \} \;, \end{aligned} \tag{5}$$

$$O_{ub}=\frac{1}{M}[(m_t-m_u)(m_c+m_u)(m_b-m_d)(m_b+m_s)]^{-1/2}$$
$$\times\{M[m_u(m_t-m_c)m_b(m_s-m_d)]^{1/2}+[m_c m_t(m_t-m_c)m_d m_s(m_s-m_d)]^{1/2}$$
$$-[m_u(m_t+m_u)(m_c-m_u)m_b(m_b-m_s)(m_b+m_d)]^{1/2}\}\ , \tag{6}$$

$$O_{cd}=\frac{1}{M}[(m_t+m_c)(m_c+m_u)(m_b-m_d)(m_s+m_d)]^{-1/2}$$
$$\times\{M[m_c(m_t+m_u)m_d(m_b-m_s)]^{1/2}-[m_u m_t(m_t+m_u)m_s m_b(m_b-m_s)]^{1/2}$$
$$+[m_c(m_t-m_c)(m_c-m_u)m_d(m_b+m_d)(m_s-m_d)]^{1/2}\}\ , \tag{7}$$

$$O_{cs}=\frac{1}{M}[(m_t+m_c)(m_c+m_u)(m_b+m_s)(m_s+m_d)]^{-1/2}$$
$$\times\{M[m_c(m_t+m_u)m_s(m_b+m_d)]^{1/2}+[m_u m_t(m_t+m_u)m_d m_b(m_b+m_d)]^{1/2}$$
$$+[m_c(m_t-m_c)(m_c-m_u)m_s(m_b-m_s)(m_s-m_d)]^{1/2}\}\ , \tag{8}$$

$$O_{cb}=\frac{1}{M}[(m_t+m_c)(m_c+m_u)(m_b-m_d)(m_b+m_s)]^{-1/2}$$
$$\times\{M[m_c(m_t+m_u)m_b(m_s-m_d)]^{1/2}-[m_u m_t(m_t+m_u)m_d m_s(m_s-m_d)]^{1/2}$$
$$-[m_c(m_t-m_c)(m_c-m_u)m_b(m_b-m_s)(m_b+m_d)]^{1/2}\}\ , \tag{9}$$

$$O_{td}=\frac{1}{M}[(m_t-m_u)(m_t+m_c)(m_b-m_d)(m_s+m_d)]^{-1/2}$$
$$\times\{M[m_t(m_c-m_u)m_d(m_b-m_s)]^{1/2}+[m_u m_c(m_c-m_u)m_s m_b(m_b-m_s)]^{1/2}$$
$$-[m_t(m_t-m_c)(m_t+m_u)m_d(m_b+m_d)(m_s-m_d)]^{1/2}\}\ , \tag{10}$$

$$O_{ts}=\frac{1}{M}[(m_t-m_u)(m_t+m_c)(m_b+m_s)(m_s+m_d)]^{-1/2}$$
$$\times\{M[m_t(m_c-m_u)m_s(m_b+m_d)]^{1/2}-[m_u m_c(m_c-m_u)m_d m_b(m_b+m_d)]^{1/2}$$
$$-[m_t(m_t-m_c)(m_t+m_u)m_s(m_b-m_s)(m_s-m_d)]^{1/2}\}\ , \tag{11}$$

$$O_{tb}=\frac{1}{M}[(m_t-m_u)(m_t+m_c)(m_b-m_d)(m_b+m_s)]^{-1/2}$$
$$\times\{M[m_t(m_c-m_u)m_b(m_s-m_d)]^{1/2}+[m_u m_c(m_c-m_u)m_d m_s(m_s-m_d)]^{1/2}$$
$$+[m_t(m_t-m_c)(m_t+m_u)m_b(m_b-m_s)(m_b+m_d)]^{1/2}\}\ , \tag{12}$$

where $M^2=(m_t-m_c+m_u)(m_b-m_s+m_d)$.

Even if matrix elements depend in a rather intricate way on the six-quark masses, it is easily seen that in the limit of very large masses of the third quark generation (with respect to those of the first two generations), the submatrix describing the mixing between the first two generations decouples, and asymptotically

$$O_C\rightarrow\begin{pmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{pmatrix}\ . \tag{13}$$

This gives an insight into the so-called Cabibbo universality, and *a posteriori* tends to corroborate the assumed form (3) for the mass matrix in the weak basis: Cabibbo universality is essentially a consequence of the large mass difference between quarks of different generations once the form (3) is taken. In the asymptotic form (13),

$$\theta_C = \arctan\frac{(m_c m_d)^{1/2} - (m_u m_s)^{1/2}}{(m_c m_s)^{1/2} + (m_u m_d)^{1/2}} , \quad (14)$$

which is called to reproduce the usual Cabibbo angle, apart from small corrections due to the effect of the third generation. In other words, the mixing of the first two generations is essentially regulated by the masses of the corresponding quarks, with a minimal influence of the third generation (this is helpful in fixing their masses). The mixing between the first two generations and the third one is, on the contrary, rather strongly dependent on the heaviest quark masses. Let us assume, accordingly, with usual estimates

$$m_s = 0.15 \text{ GeV}, \quad m_b = 4.5 \text{ GeV} ,$$
$$m_c = 1.50 \text{ GeV}, \quad m_t > 20 \text{ GeV} . \quad (15)$$

m_u, m_d are to be taken of the order of a few MeV in order to approximately reproduce O_{ud}, O_{us}, as they are derived from nuclear β decay and νN interactions. Even though some theoretical uncertainties affect these derivations, let us assume, with Ref. 8,

$$O_{ud} = 0.9737 \pm 0.0025, \quad O_{us} = 0.027 \pm 0.016 . \quad (16)$$

Once the choice (15) is made, an analysis of Eqs. (4) and (5) shows that O_{ud}, O_{us} depend crucially on m_u, m_d. By assuming

$$m_u = 5 \text{ MeV}, \quad m_d = 13 \text{ MeV} , \quad (17)$$

one obtains independently of m_b in the range $4.0 \le m_b \le 5.5$ GeV and with $m_t = 25$ GeV,

$$|O_{ud}| = 0.9739, \quad |O_{us}| = 0.2269 . \quad (18)$$

The above values are seen to change by less than 0.01% when m_t varies from 20 to 150 GeV. Moreover, they are reproduced with negligible error by the asymptotic form (14).

O_{cd}, O_{cs} are found to be only slightly dependent on m_t, being

$$\left.\begin{array}{l} O_{cd} = 0.2255 - 0.2268 \\ O_{cs} = 0.9697 - 0.9740 \end{array}\right\} \text{ for } 20 \le m_t \le 150 \text{ GeV} , \quad (19)$$

the lowest (highest) value being reached at $m_t \simeq 20$ GeV ($m_t \simeq 45$ GeV). The values (19) are in evident agreement with the theoretical estimates deduced by analyzing the production rate of opposite-sign dileptons from the valence quarks in neutrino scattering on isoscalar target, or by considering both neutrino and antineutrino charm production.[8] The latter approach, followed by Paschos and Türke, gives[8]

$$|O_{cd}| = 0.25 \pm 0.04, \quad |O_{cs}| > 0.81 \quad (20)$$

with $|O_{cd}|$ slightly decreasing if in the estimate the integrated quark distribution is substituted by the experimentally measured charged-current cross sections.

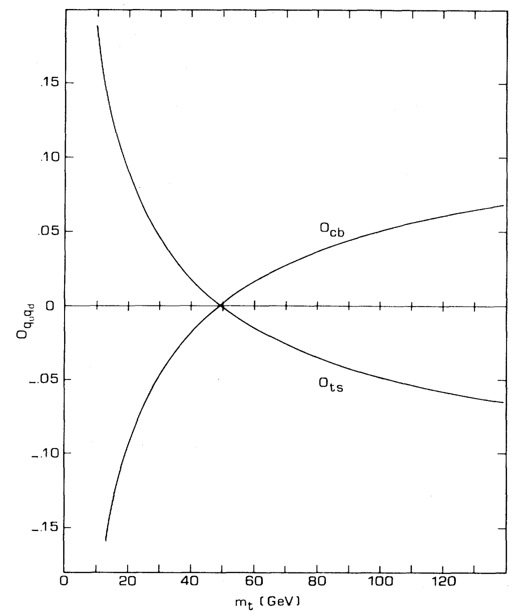


FIG. 1. O_{cb} and O_{ts} versus m_t, as derived from Eqs. (9) and (11). Quark masses are chosen according with Eqs. (15) and (17).

The quantities which are seen to depend in a stringent way on the t-quark mass are the remaining elements of the generalized Cabibbo matrix. In Fig. 1, O_{cb}, O_{ts} in terms of m_t are reported: they verify with a very good approximation the symmetry $|O_{cb}| \simeq |O_{ts}|$. The matrix elements O_{ub}, O_{td} (Fig. 2) are rather small for all the m_t values above 30 GeV with a marked departure from the approximate symmetry which characterizes the generalized Ca-

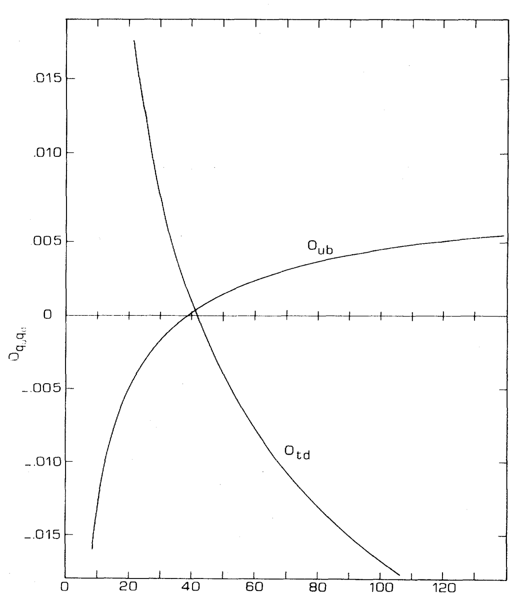


FIG. 2. O_{ub} and O_{td} versus m_t, derived from Eqs. (6) and (10). Quark masses are chosen according with Eqs. (15) and (17).

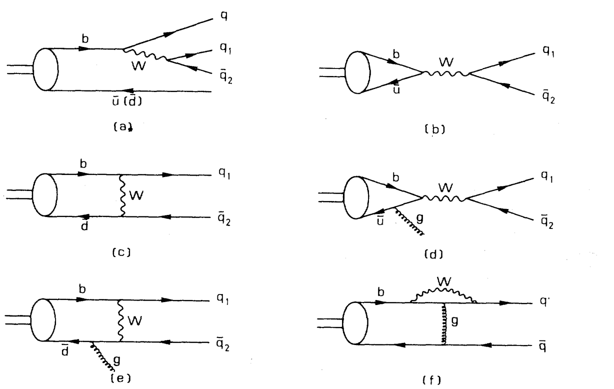


FIG. 3. Graphs contributing to lowest B decays: (a) spectator, (b) and (c) annihilation, (d) and (e) gluon emission, (f) penguin diagrams.

bibbo matrix. Finally, O_{tb} is very near to 1 in all the range of m_t values above 20 GeV.

A few remarks about the numerical determinations given above. They are obviously related to the choices (15) and (17). The former, concerning the masses of the second and third quark generations, is rather usual, and a slight modification does not change in an appreciable way the subsequent numerical estimates. Conversely, choice (17) is crucial. The first general indication is the necessity of making use of the so-called current quark masses: this emerges,[14] on the other hand, also in the totally different context of the analysis of the leptonic τ decay,[15] where $B_L(\tau)$ agrees with the QCD-corrected predictions[16] only if current quark masses are used. Once this is assumed, then choice $m_s=150$ MeV in (15) agrees with the standard estimates.[17] The specific values (17) are fixed in such a way as to reproduce O_{ud}, O_{us} [Eq. (18)] near to the phenomenological values (16). It is worth noting that they agree with the recent estimates of the lightest quark masses through QCD sum rules,[18] once the QCD running masses in the modified minimal-subtraction ($\overline{\text{MS}}$) scheme, $\hat{m}_u, \hat{m}_d$, are rescaled[19] with reasonable values of $\Lambda_{\overline{\text{MS}}}$, and agree also with the estimates obtained in the completely different approach of the lattice-gauge-theory numerical calculations.[20]

III. *B*-MESON DECAYS

The best laboratory in which the approach to the generalized Cabibbo mixing of Sec. II can be tested is certainly represented by B-meson decays: they allow in principle the measurement of O_{ub}, O_{cb}, which have been predicted in terms of the t-quark mass. But the situation is a bit more confused, in view of the following.

(a) From the experimental point of view the present knowledge of B-meson decays is rather poor, although rapidly growing.[4,21–23]

(b) From a theoretical point of view the mechanisms of the nonleptonic weak decays are not fully understood and several experimental findings in weak decays of charmed particles do not agree in a satisfying way with the theoretical predictions.[4,14] This seems to indicate that strong-interaction effects must be included and weak decays of heavy hadrons proceed via some kind of interplay of weak and strong interactions.

There are, however, theoretical reasons to believe that in B-meson decays strong-interaction effects are less important than in strange- or charmed-particle decays. It is then reasonable to calculate all the contributions coming from spectator and nonspectator diagrams, by taking the strong-interaction effects to lowest order, and then compared with the available experimental data, by assuming generalized Cabibbo mixing, as provided by the approach followed in Sec. II. Data are not able at present to prove or disprove the approach. However, consistency with the data can be interpreted as an indication of reliability of both the descriptions (generalized Cabibbo mixing and B-meson decay mechanisms), waiting for the strongest constraints coming from experiments.

Even though weak interactions are described by the left-right-symmetric group $SU(2)_L \times SU(2)_R \times U(1)$, flavor-changing neutral currents, Higgs-boson-exchange effects, and right-handed currents, are all strongly suppressed,[10,11] and weak effects are dominated by left-handed charged currents, in the same way as in the standard model.

The spectator contribution to B-meson decays is represented in Fig. 3 (a): In the usual units of $\Gamma_0 = G_F{}^2 m_b{}^5/192\pi^3$, it corresponds to

$$\Gamma_{\text{sp}}(B \to qX) = \sum_{qq_1\bar{q}_2(l\nu_l)} c_{si}^{(\text{sp})} O_{qb}{}^2 \rho_{\text{sp}}(x, x_1, x_2) \;. \tag{21}$$

$c_{si}^{(\text{sp})}$ is the strong enhancement factor, which takes into account that quarks come in three colors and includes the effects of the QCD corrections induced by gluon exchange in the leading-logarithmic approximation.[24] It is

$$c_{si}^{(\text{sp})} = \begin{cases} 1 \text{ if } b \to q l \bar{\nu}_l \quad (\text{semileptonic decay}) \\ 2f_+{}^2 + f_-{}^2 \text{ if } b \to q q_1 \bar{q}_2 \quad (\text{nonleptonic decay}) \,, \end{cases} \tag{22}$$

f_+, f_- being the coefficients appearing in the nonleptonic weak Hamiltonian after rearrangement due to QCD effects, given by[24]

$$f_- = \frac{1}{f_+{}^2} = \left[\frac{\alpha_s(m_b)}{\alpha_s(M_W)} \right]^{4/b} , \tag{23}$$

where (n = number of active flavors)

$$b = 11 - \tfrac{2}{3} n, \quad \alpha_s(m) = \frac{2\pi}{b \ln(m/\Lambda)} \;. \tag{24}$$

If QCD effects are neglected ($\alpha_s \to 0$) $f_+ = f_1 = 1$ and $c_{si}^{(\text{sp})}$ reduces to the simple factor 3 due to color. In Eq. (21), O_{qb} corresponds to the generalized Cabibbo mixing and $\rho_{\text{sp}}(x, x_1, x_2)$ describes the effect of the phase-space factor, depending on $x = m_q/m_b$, $x_i = m_{q_i}/m_b$ ($q_1, \bar{q}_2$ must be substituted by $l, \bar{\nu}_l$ in the semileptonic case). The explicit form of ρ_{sp}, together with the numerical details, is given in the Appendix. Finally, $\sum$ indicates that a summation on all possible final states must be performed. The next to the leading-logarithmic correction, which seems to enforce the effect of enhancement of the nonleptonic rate,[25] will not be included.

Nonspectator contributions can be thought to arise from W-exchange graphs in the s or t channel (annihilation graphs), depending on the charge state of the B meson.[26] These graphs, reported in Figs. 3(b) and 3(c), are however strongly suppressed (helicity suppression) when light particles appear in the final state: it is easily found for the s-channel annihilation (Γ_0 units),

$$\Gamma_{\text{ann}}[b\bar{q}\to q_1\bar{q}_2]_s = 24\pi^2 c_{\text{si}}^{(\text{ann})} O_{qb}{}^2 O_{q_1q_2}{}^2 \left[\frac{f_B}{m_b}\right]^2 \times \left[\frac{M_B}{m_b}\right]^3 \rho_{\text{ann}}\left[\frac{m_1}{M_B},\frac{m_2}{M_B}\right], \tag{25}$$

the same expression being valid for the t-channel W-exchange graph (which can contribute to nonleptonic decay alone) with the replacements $O_{qb}\to O_{q_1b}$, $O_{q_1q_2}\to O_{qq_2}$. In Eq. (25), f_B represents the pure leptonic decay constant of the B meson, in a nonrelativistic picture related to the probability of finding the two quarks at the origin (M_B is the B-meson mass)

$$f_B{}^2 = 12\frac{|\psi(O)|^2}{M_B}. \tag{26}$$

ρ_{ann}, through its dependence on m_1, m_2 (see the Appendix), induces the helicity suppression: at least one heavy quark in the final state is required in order to have a contribution from the annihilation graph. Finally, $c_{si}^{(\text{ann})}$ is the strong-interaction enhancement factor appearing in nonleptonic decays,

$$c_{si}^{(\text{ann})} = \begin{cases} \frac{1}{3}(2f_+ + f_-)^2 \underset{\alpha_s\to 0}{\longrightarrow} 3 \ (s\text{-channel exchange}), \\ \frac{1}{3}(2f_+ - f_-)^2 \underset{\alpha_s\to 0}{\longrightarrow} \frac{1}{3} \ (t\text{-channel exchange}). \end{cases} \tag{27}$$

Because of the helicity suppression, it is hard to suppose annihilation graphs are responsible for the observed difference in the lifetime of charged and neutral D-mesons (in the context in which they have been proposed). Several authors have suggested a mechanism to avoid helicity suppression,[27–29] in which gluon emission from a color-neutral state allows the quark-antiquark system to be in a vector state and annihilate without helicity suppression effects. Even though gluon radiation must be considered a convincing mechanism enhancing nonspectator contributions, a quite reliable calculation of its contribution is impossible, since nonperturbative effects are probably significant. If it is assumed that lowest-order perturbation theory can be applied, then the dominant contribution comes[27] from one-gluon emission off initial quark lines [Figs. 3(d) and 3(e)], the corresponding decay rate being given by (Γ_0 units, summation implied)

$$\Gamma_g = \frac{8\pi}{27}\alpha_s(M_B) c_{si}^{(g)} O_{qb}{}^2 O_{q_1q_2}{}^2 \left[\frac{f_B}{m_q}\right]^2 \times \left[\frac{M_B}{m_b}\right]^5 \rho_g\left[\frac{m_1}{M_B},\frac{m_2}{M_B}\right], \tag{28}$$

when the s-channel exchange is considered, the t-channel exchange being obtained by the same expression with the replacements $O_{qb}\to O_{q_1b}$, $O_{q_1q_2}\to O_{qq_2}$. The strong-enhancement factor is

$$c_{si}^{(g)} = \begin{cases} \frac{1}{4}(f_+ - f_-)^2 \ (s\text{-channel exchange}), \\ \frac{1}{4}(f_+ + f_-)^2 \ (t\text{-channel exchange}). \end{cases} \tag{29}$$

The phase-space factor ρ_g is given in the Appendix.

A further nonspectator contribution must be added in principle, coming from the so-called penguin diagram [Fig. 3(f)], not suppressed by the Cabibbo mixing (as happens in charmed-meson decays). However, it can be estimated[30] that penguin contribution in B decays is at most a few percent with respect to the spectator contribution, so that it is not considered further.

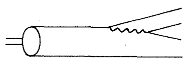
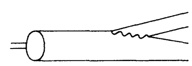
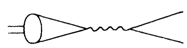
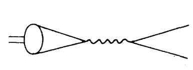
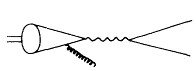
The different contributions coming from spectator, annihilation, and gluon-emission graphs [Eqs. (21), (25), and (28), respectively] in the analysis of the decays of the lightest B mesons, B_u and B_d, are succinctly summarized in Tables I and II, respectively, by considering all possible final states and separating the contributions depending on $O_{ub}{}^2$ from those $\sim O_{cb}{}^2$.

The details of the numerical calculations are given in the Appendix. It is worth noting that unfortunately several numerical inputs are characterized by a rather large indetermination. Apart from the problem of meson and quark masses, whose values can be slightly different from those adopted here, but with negligible effects on the final results, there are the following problems.

(i) The QCD effects are less or more marked, depending on the QCD parameter Λ: the value $\Lambda=0.25$ GeV is assumed (see, for example, Ref. 25), but lower or higher values are quite compatible with the literature. The effect on the spectator rate is relatively important: the nonleptonic piece is enhanced by $\sim 12\%$ for $\Lambda=0.25$ GeV, and grows a bit further if $\Lambda=0.50$ is assumed. More marked is the effect on annihilation and gluon-emission graphs [the latter proportional to $\alpha(M_B)$], where, however, more serious doubts about the numerical inputs come from f_B.

(ii) As stated above, all the nonspectator contributions are proportional to f_B [in the annihilation to $(f_B/m_b)^2$, in the gluon emission to $(f_B/m_u)^2$]. Theoretical estimates based on potential models,[31] QCD sum rules,[32] and bag models[33] can be found (see also Ref. 14): models agree with the general property of f decreasing when the meson mass increases. Consistently, it is reasonable to assume $f_B\sim 0.10$—0.15 GeV. But higher values cannot be excluded: to be safe, an upper limit of $f_B=0.5$ GeV will be adopted. Nonspectator contributions are then calculated in the two cases (a) $f_B=0.10$ GeV, and (b) $f_B=0.50$ GeV. Note that in the gluon-emission rate, as far as m_u (m_d) is concerned, it is usual to make use[27] of the constituent

TABLE I. B_u decay parameters, by separating different channels and different decay mechanisms. Numerically, the partial contributions $\sim O_{ub}{}^2$ and $O_{cb}{}^2$ are indicated, in $\Gamma_0 = G_F{}^2 m_b{}^5/192\pi^3$ units. The two entries (a) and (b) refer to the choices indicated in Eq. (30).

Decay rates and graphs		c_{si}	Contributions $\sim \lvert O_{ub} \rvert^2$ $q_1\bar{q}_2(l\bar{\nu}_l)$	Phase space	Numerically	Contributions $\sim \lvert O_{cb} \rvert^2$ $q_1\bar{q}_2(l\bar{\nu}_l)$	Phase space	Numerically
$\Gamma_{\rm sp}^{(nl)}$		$2f_+{}^2 + f_-{}^2$	$d\bar{u},s\bar{u}$	$\rho_{\rm sp}(0,0,0)$	3.377	$d\bar{u},s\bar{u}$	$\rho_{\rm sp}\left[\frac{m_c}{m_b},0,0\right]$	1.511
			$s\bar{c},d\bar{c}$	$\rho_{\rm sp}\left[0,\frac{m_c}{m_b},0\right]$	1.511	$s\bar{c},d\bar{c}$	$\rho_{\rm sp}\left[\frac{m_c}{m_b},0,\frac{m_c}{m_b}\right]$	0.400
$\Gamma_{\rm sp}^{(sl)}$		1	$e\bar{\nu}_e,\mu\bar{\nu}_\mu$	$\rho_{\rm sp}(0,0,0)$	2.000	$e\bar{\nu}_e,\mu\bar{\nu}_\mu$	$\rho_{\rm sp}\left[\frac{m_c}{m_b},0,0\right]$	0.895
			$\tau\bar{\nu}_\tau$	$\rho_{\rm sp}\left[0,\frac{m_\tau}{m_b},0\right]$	0.323	$\tau\bar{\nu}_\tau$	$\rho_{\rm sp}\left[\frac{m_c}{m_b},\frac{m_\tau}{m_b},0\right]$	0.065
$\Gamma_{\rm ann}^{(nl)}$		$\frac{1}{3}(2f_+ + f_-)^2$	$d\bar{u},s\bar{u}$	$\rho_{\rm ann}(0,0)$				
			$d\bar{c},s\bar{c}$	$\rho_{\rm ann}\left[\frac{m_c}{M_B},0\right]$	0.040 (a) 1.002 (b)			
$\Gamma_{\rm ann}^{(sl)}$		1	$e\bar{\nu},\mu\bar{\nu}_\mu$	$\rho_{\rm ann}(0,0)$				
			$\tau\bar{\nu}_\tau$	$\rho_{\rm ann}\left[\frac{m_\tau}{M_B},0\right]$	0.017 (a) 0.412 (b)			
$\Gamma_g^{(nl)}$		$\frac{1}{4}(f_+ - f_-)^2$	$d\bar{u},s\bar{u}$	$\rho_g(0,0)$	0.004 (a) 0.099 (b)			
			$d\bar{c},s\bar{c}$	$\rho_g\left[\frac{m_c}{M_B},0\right]$	0.003 (a) 0.067 (b)			

masses, $m_u = 0.3$ GeV. This is rather contradictory with the tendency of using extensively current quark masses. More properly then, in the gluon-emission rate, the two following cases are examined ($m_u = m_d$):

$$\text{(a)}\,\frac{f_B}{m_u} = 0.33, \quad \text{(b)}\,\frac{f_B}{m_u} = 1.66\,, \tag{30}$$

which are then assumed as lower and upper bounds of this somewhat mysterious ratio.

From the decay rates given in Tables I and II, it follows at a glance that all nonspectator rates are quite negligible in the case (a), i.e., when the lowest value of f_B is taken, this irrespective of the specific QCD corrections. In case (a), then, the total width is essentially the same for both charged and neutral mesons, and coincides with the spectator rate,

$$\Gamma(B_u) \simeq \Gamma(B_d) \simeq \Gamma_{\rm sp}^{(\rm tot)} = 7.211 O_{ub}{}^2 + 2.871 O_{cb}{}^2\,. \tag{31}$$

If, conversely, the upper value of f_B is used, then the nonspectator rate becomes rather important and contributes in different ways to neutral and to charged mesons: in case (b) for B_u it is

$$\Gamma_{\rm ann}(B_u) = 1.414 O_{ub}{}^2\,, \tag{32}$$

$$\Gamma_g(B_u) = 0.166 O_{ub}{}^2\,, \tag{33}$$

and then

$$\Gamma_{\rm nsp}(B_u) = 1.580 O_{ub}{}^2\,, \tag{34}$$

$$\Gamma_{\rm tot}(B_u) = 8.791 O_{ub}{}^2 + 2.871 O_{cb}{}^2\,, \tag{35}$$

whereas for B_d,

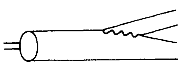
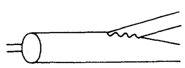
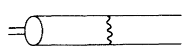
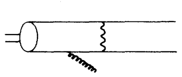
$$\Gamma_{\rm ann}(B_d) = 0.001 O_{ub}{}^2 + 0.010 O_{cb}{}^2\,, \tag{36}$$

$$\Gamma_g(B_d) = 1.633 O_{ub}{}^2 + 1.105 O_{cb}{}^2\,, \tag{37}$$

so that

$$\Gamma_{\rm nsp}(B_d) = 1.634 O_{ub}{}^2 + 1.115 O_{cb}{}^2\,, \tag{38}$$

TABLE II. As in Table I, for B_d decay.

Decay rates and graphs	c_{si}	Contributions $\sim \lvert O_{ub} \rvert^2$ $q_1\bar{q}_2(l\bar{\nu}_l)$	Phase space	Numerically	Contributions $\sim \lvert O_{cb} \rvert^2$ $q_1\bar{q}_2(l\bar{\nu}_l)$	Phase space	Numerically
$\Gamma_{sp}^{(nl)}$ 	$2f_+^2+f_-^2$	$d\bar{u},s\bar{u}$	$\rho_{sp}(0,0,0)$	3.377	$d\bar{u},s\bar{u}$	$\rho_{sp}\left[\frac{m_c}{m_b},0,0\right]$	1.511
		$s\bar{c},d\bar{c}$	$\rho_{sp}\left[0,\frac{m_c}{m_b},0\right]$	1.511	$s\bar{c},d\bar{c}$	$\rho_{sp}\left[\frac{m_c}{m_b},0,\frac{m_c}{m_b}\right]$	0.400
$\Gamma_{sp}^{(sl)}$ 	1	$e\bar{\nu}_e,\mu\bar{\nu}_\mu$	$\rho_{sp}(0,0,0)$	2.000	$e\bar{\nu}_e,\mu\bar{\nu}_\mu$	$\rho_{sp}\left[\frac{m_c}{m_b},0,0\right]$	0.895
		$\tau\bar{\nu}_\tau$	$\rho_{sp}\left[0,\frac{m_\tau}{m_b},0\right]$	0.323	$\tau\bar{\nu}_\tau$	$\rho_{sp}\left[\frac{m_c}{m_b},\frac{m_\tau}{m_b},0\right]$	0.065
$\Gamma_{ann}^{(nl)}$ 	$\frac{1}{3}(2f_+-f_-)^2$	$u\bar{u}$	$\rho_{ann}(0,0)$		$c\bar{u}$	$\rho_{ann}\left[\frac{m_c}{M_B},0\right]$	(a) 0.009 (b)
		$u\bar{c}$	$\rho_{ann}\left[\frac{m_c}{M_B},0\right]$	(a) 0.001 (b)	$c\bar{c}$	$\rho_{ann}\left[\frac{m_c}{M_B},\frac{m_c}{M_B}\right]$	(a) 0.001 (b)
$\Gamma_{g}^{(nl)}$ 	$\frac{1}{4}(f_++f_-)^2$	$u\bar{u}$	$\rho_g(0,0)$	0.063 (a) 1.575 (b)	$c\bar{u}$	$\rho_g\left[\frac{m_c}{M_B},0\right]$	0.043 (a) 1.072 (b)
		$u\bar{c}$	$\rho_g\left[\frac{m_c}{M_B},0\right]$	0.001 (a) 0.058 (b)	$c\bar{c}$	$\rho_g\left[\frac{m_c}{M_B},\frac{m_c}{M_B}\right]$	0.003 (b) 0.033 (b)

$$\Gamma_{tot}(B_d)=8.845 O_{ub}^2+3.896 O_{cb}^2 \ . \qquad (39)$$

It follows that (i) $\Gamma_{nsp}(B_u)$ is dominated by the annihilation contribution and contributes only with terms proportional to O_{ub}^2, and (ii) $\Gamma_{nsp}(B_d)$ depends essentially on the gluon-emission effects and contributes with both the kinds of terms $\sim O_{ub}^2$ and $\sim O_{cb}^2$. It is worth noting that even though $f_B=0.5$ GeV may appear a too large estimate, however the dependence on f_B/m_q ($q=u,d$), typical of the gluon emission would make significant the corresponding rates [essentially $\Gamma_g(B_d)$, Eq. (37)] also for smaller f_B if at the same time m_q is taken smaller than the usual constituent mass. Conversely, the annihilation rates would be suppressed.

Let us now assume O_{ub},O_{cb} given in terms of the t-quark mass as provided by the approach of Sec. II: it is possible then to represent the lifetime of the lightest B mesons in terms of m_t. In Fig. 4, $\tau(B_u)$ and $\tau(B_d)$ versus m_t are drawn, starting from Eqs. (35) and (39) (full lines): Because of the small values of O_{ub}, $\tau(B_u)$ is essentially indistinguishable from the purely spectator contribution (31), whereas in Fig. 4, $\tau(B_d)$ represents a lower limit of the B_d lifetime, since it takes into account the nonspectator contribution in case (b).

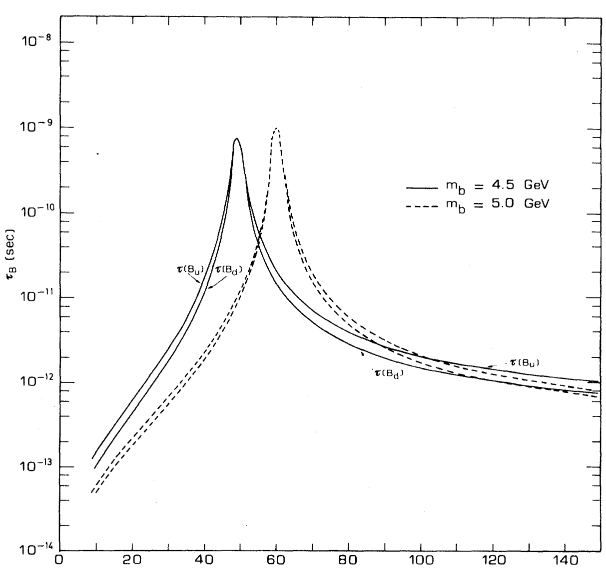


FIG. 4. Theoretical estimate of the lifetime of the lightest B mesons in terms of m_t. Solid curves refer to $m_b=4.5$ GeV, dashed curves to $m_b=5$ GeV.

What is experimentally measured is the decay of a $B\bar{B}$ system with an average of charged and neutral mesons. It must be compared with something of intermediate between the two curves drawn in Fig. 4, nearer to $\tau(B_u)$ as far as the nonspectator contribution is suppressed. The following is easily seen.

(i) By comparison with the present experimental limits,

$$\tau_B < 1.4\times 10^{-12} \text{ sec } (95\% \text{ C.L.}) \quad \text{JADE collaboration (Ref. 21)}, \qquad (40)$$
$$\tau_B < 3.7\times 10^{-12} \text{ sec } (95\% \text{ C.L.}) \quad \text{MAC collaboration (Ref. 22)},$$

a large interval of m_t values is ruled out: on the basis of Fig. 4, (solid curves), it appears reasonable to assume $m_t < 30$ GeV (or even less).

(ii) Very large m_t values cannot be excluded: $m_t > 110-120$ GeV is also compatible with the present experimental limits, even though so high values deserve dangerous consequences when $\theta_{\text{QFD}}^{\text{1-loop}}$ is calculated,[11] and more generally we are venturing toward m_t values near to the bounds imposed by renormalization effects on the ratios[34] m_b/m_τ and on the ratio of neutral to charged currents.[35]

(iii) The previous estimate is rather largely dependent on m_b: strongly dependent on m_b is, in fact, $\Gamma_0 = G_F{}^2 m_b{}^5/192\ \pi^3$, whereas the effect on O_{ub}, O_{cb} and on phase-space factors is relatively modest. All these effects contribute to a decrease of both $\tau(B_u)$ and $\tau(B_d)$ (with a smaller relative influence of the spectator rate), thus restricting the interval of forbidden m_t values. This can be seen in Fig. 4, where dashed curves represent $\tau(B_u)$ and $\tau(B_d)$ when $m_b = 5$ GeV is assumed. An upper limit $m_t < 37$ GeV follows if very large m_t values are excluded.

The assumed form for generalized Cabibbo mixing then provides a rather strong limit on the t-quark mass, with $20 < m_t < 37$ GeV, to a large extent irrespective of specific choices concerning quark masses and with a relative influence of the role of nonspectator contributions in B decays. A refinement of the present experimental limit would allow a more stringent determination of m_t (with a possible definitive exclusion of very large m_t values). It is worth noting that the allowed interval of values includes the present predictions of m_t (those survived to the refinement of the experimental limit), both consistent with $m_t \sim 25$ GeV.[36,37]

In Fig. 5, the ratio $|O_{ub}/O_{cb}|$ is reported. In the range of small m_t (those compatible with the present limit on τ_B) this ratio is widely expected to be smaller than the present experimental limit, a limit deduced in a model-dependent way from the lepton energy spectrum in semileptonic decays. From electron spectra the CUSB collaboration obtains

$$\left.\frac{O_{ub}}{O_{cb}}\right|_{\text{exp}} \leq 0.21\ , \qquad (41)$$

similar limits being obtained by the CLEO collaboration in the muon analysis.[23] In the intermediate region (where τ_B would assume values incompatible with the present experimental limit), the ratio changes very rapidly from zero to infinity since both O_{ub}, O_{cb} are very small and cross the zero successively. Finally, the very-high-m_t case ($m_t \gtrsim 120$ GeV) corresponds to a ratio still compatible with the experimental limit (41), even if larger than that expected for small m_t.

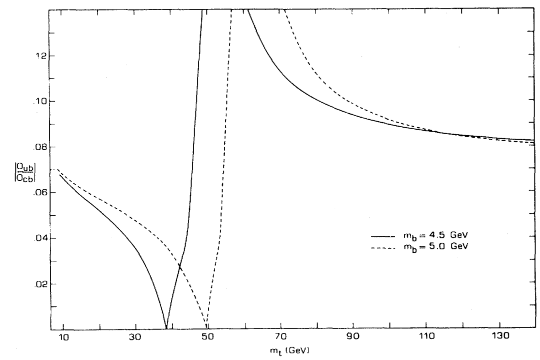


FIG. 5. O_{ub}/O_{cb} in terms of m_t.

Let us now list some of the phenomenological consequences of the analysis performed above.

(i) Ratio $\tau(B_u)/\tau(B_d)$. This depends on the nonspectator rate which contributes in different way to $\tau(B_u), \tau(B_d)$. Then $\tau(B_u)/\tau(B_d) \simeq 1$ if the nonspectator rate is suppressed [case (a) in Eq. (30)]. If, conversely, it is enhanced [case (b)], then, independently of m_t in the given range,

$$\frac{\tau(B_u)}{\tau(B_d)} = \begin{cases} 1.21 & (m_b = 4.5 \text{ GeV}), \\ 1.39 & (m_b = 5.0 \text{ GeV}), \end{cases} \qquad (42)$$

with a weak dependence on the b-quark mass.

(ii) Semileptonic branching ratio $B(B \to X l \nu_l)$ (with $l = e, \mu$). It depends on the nonspectator rate which modifies (enhances) $\Gamma_{\text{tot}}(B\bar{B})$. Independently of m_t in the interval under analysis, one finds in case (a) (i.e. with the minimal nonspectator effect)

$$\text{(a) } B(B \to X l \nu_l) = \begin{cases} 0.156 & (m_b = 4.5 \text{ GeV}), \\ 0.147 & (m_b = 5.0 \text{ GeV}), \end{cases} \qquad (43)$$

whereas in case (b) (nonspectator rate enhanced)

$$\text{(b) } B(B \to X l \nu_l) = \begin{cases} 0.130 & (m_b = 4.5 \text{ GeV}), \\ 0.133 & (m_b = 5.0 \text{ GeV}), \end{cases} \qquad (44)$$

with a very weak influence of m_b. Since O_{ub}/O_{cb} is very small, only the terms $\sim O_{cb}{}^2$ contribute in practice. By comparing with the available experimental results[4,23,38]

$$B(B\to Xe\nu_e)=\begin{cases}0.127\pm0.017\pm0.013 \ \text{(CLEO)}\ ,\\ 0.131\pm0.012\pm0.020 \ \text{(CUSB)}\ ,\\ 0.11\pm0.03\pm0.02 \ \text{(MARK II)}\ ,\\ 0.136\pm0.05\pm0.04 \ \text{(TASSO)}\ ,\end{cases} \tag{45}$$

$$B(B\to X\mu\nu_\mu)=\begin{cases}0.122\pm0.017\pm0.031 \ \text{(CLEO)}\ ,\\ 0.15\ \pm0.035\pm0.035 \ \text{(CUSB)}\ ,\\ 0.093\pm0.029\pm0.020 \ \text{(CLEO)}\ ,\end{cases}$$

one may be induced to regard case (b) as the most reliable (but care must be taken and a more convincing experimental evidence must be expected). As far as semileptonic decays are concerned, the shape of the differential lepton-energy spectrum allows us to obtain an indication about M_X: data are consistent with D,D^* production ($M_X\simeq2$ GeV), which leads to the (somewhat weak) limit (41).

(iii) Number of $K^0,K^\pm$/hadronic event. In a model-dependent way this allows the comparison of O_{ub} with O_{cb}. Both CLEO and CUSB collaborations[23] present data favoring O_{cb} over O_{ub}, but without a strong limit.

It is worth noting that if predictions given in Fig. 5 are taken to be true, then it appears very difficult (if not impossible) to derive O_{ub} from the above analyses, the O_{ub} contribution being masked by the unavoidable theoretical uncertainty which characterizes the coefficient of O_{cb}: only some limit can be derived, rather far from the predicted value. The above analyses can, conversely, be very useful in determining the relative relevance of the nonspectator rate.

Very interesting in order to find further information on the parameters entering B decays is[39] the

(iv) Specific decay $B\to\tau\nu_\tau$. This rate

$$\Gamma(B\to\tau\nu_\tau)=\frac{G_F{}^2}{8\pi}O_{ub}{}^2f_B{}^2m_\tau{}^2M_B\left[1-\frac{m_\tau{}^2}{M_B{}^2}\right]^2 \tag{46}$$

depends uniquely on O_{ub} and f_B. The mean life is then a measurement of $(O_{ub}f_B)^2$: in the framework of the approach proposed here it depends essentially on f_B and weakly on the quark masses (Fig. 6). The predicted mean life ranges between 10^{-9} and 10^{-7} sec, rather larger than that predicted in Ref. 39. The corresponding branching ratio $B(B\to\tau\nu_\tau)$ is therefore very small (see Fig. 7), going from 10^{-5} to 10^{-4}: apart from the obvious experimental difficulties, a measurement of B lifetimes would represent the best way of verifying at the same time f_B, the ratio O_{ub}/O_{cb} and the relative influence of the strong-enhancement effects.

IV. CONCLUSION

An approach to the generalized Cabibbo mixing has been described, starting from a left-right-symmetric model of weak interactions in which[10,11] (i) natural flavor conservation can be ensured through the introduction of a suitable discrete symmetry, (ii) *CP* violation can be attributed to the Higgs-boson-exchange mechanism, through an enlargement of the Higgs-boson content of the model, and (iii) at low energy all the typical features of the standard

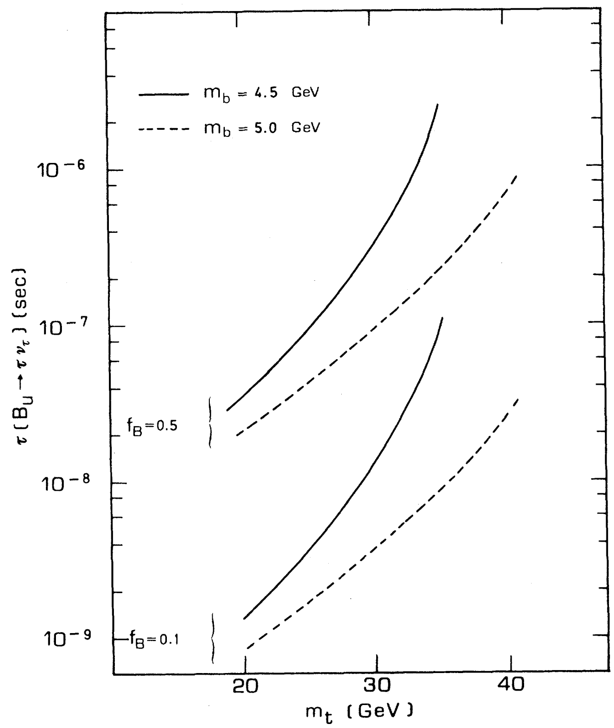


FIG. 6. Theoretical estimate of the lifetime of the decay $B_u\to\tau\nu_\tau$. Different possibilities about f_B and m_b are considered.

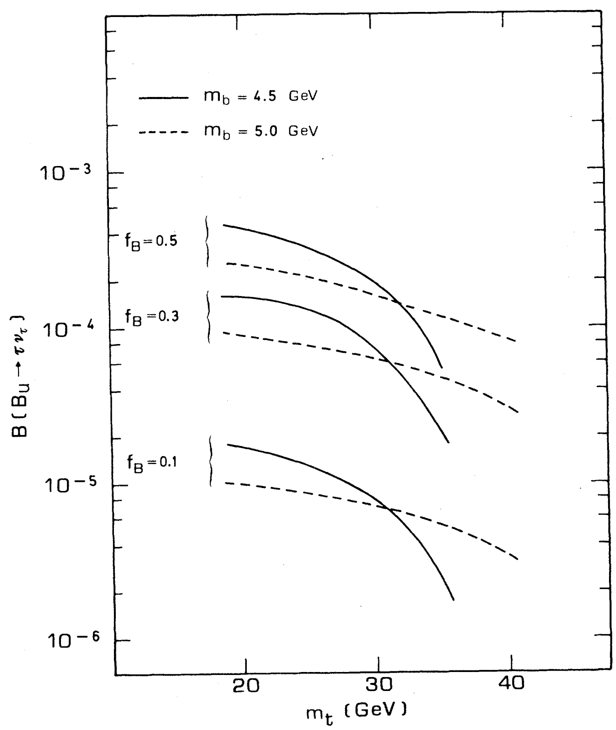


FIG. 7. Theoretical estimate of the branching ratio of the decay $B_u\to\tau\nu_\tau$. Its dependence on f_B and m_0 is indicated.

model can be reproduced.

By assuming in the above model the usual three-generation picture and by supposing that to lowest order the masses of the lighter quarks originate only from mixing with the quarks of the successive generation, the following holds.

(i) All the elements of the orthogonal mixing matrix can be explicitly derived in terms of the quark masses.

(ii) The general structure of the mixing matrix is obtained with the typical enhancement of the diagonal elements.

(iii) By introducing specific, and currently accepted, values for the masses of the "known" quarks, the "known" matrix elements are reproduced. In particular, as far as the lightest quark masses are concerned, they appear to be in good agreement with recent attempts at calculation.[18-20]

(iv) The usual Cabibbo angle of a four-quark model is approximately reproduced: it appears independent of the specific values assumed for the heaviest quark masses. In particular, it is stable with respect to m_t in all the range of values allowed by the experiments($m_t \geq 20$ GeV) and by theoretical arguments. Cabibbo universality is then ensured by the specific structure of generalized Cabibbo mixing.

The previous analysis of generalized Cabibbo mixing can be used in B-meson decays. It agrees with our present knowledge and allows us to make predictions about future experimental results. B-meson decays are considered by including (i) QCD effects to the leading-logarithmic approximation, (ii) phase-space effects due to heavy quarks and leptons, and (iii) nonspectator effects through annihilation and one-gluon-emission graphs.

From the comparison with the available experimental data a severe restriction on the possible values of m_t follows, coming from the present limit of $\tau(B\bar{B})$.[21,22] The t-quark mass is constrained in the interval $20 \leq m_t \leq 37$ GeV, the upper limit depending on the specific values assumed for m_b and, weakly, on the influence of the nonspectator rate. The present experimental limit, however, does not exclude an (unlikely) very large value ($m_t \gtrsim 120$ GeV).

Once m_t is constrained in the above interval, predictions can be obtained concerning B-meson decays: O_{ub}/O_{cb}, $\tau(B_u/\tau(B_d)$, semileptonic branching ratios, $B \to \tau\nu_\tau$ lifetime, and the corresponding branching ratio.

ACKNOWLEDGMENTS

S. Pakvasa is acknowledged for several useful conversations. The author is grateful for the kind hospitality of the TH Division at CERN, where this work was initiated.

APPENDIX

Here phase-space factors and numerical details are explicitly worked out by making it easy to repeat the calculations with different numerical inputs.

As far as phase-space factors are concerned, all lowest masses are assumed as being negligible, the only nonzero masses being

$$
\begin{aligned}
&m_c = 1.50 \text{ GeV} , \\
&m_b = 4.50 \text{ GeV}, \quad M_{B_u} = M_{B_d} = M_B = 5.25 \text{ GeV} . \\
&m_\tau = 1.78 \text{ GeV} .
\end{aligned}
\tag{A1}
$$

With this assumption, the only expression of $\rho_{sp}(x,x_1,x_2)$ relevant to our purposes involve no more than two mass ratios different from zero. It is easily found [$\lambda(a,b,c) = a^2+b^2+c^2-2ab-2ac-2bc$ is the usual triangular function]

$$
\begin{aligned}
&\rho_{sp}(0,0,0) = 1 , \\
&\rho_{sp}(x,0,0) = \rho_{sp}(0,x,0) = \rho_{sp}(0,0,x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x , \\
&\rho_{sp}(x,x,0) = \rho_{sp}(x,0,x) = \rho_{sp}(0,x,x) \\
&\qquad = (1 - 14x^2 - 2x^4 - 12x^6)(1-4x^2)^{1/2} + 24x^4(1-x^4) \ln \frac{1+(1-4x^2)^{1/2}}{1-(1-4x^2)^{1/2}} , \\
&\rho_{sp}(x,y,0) = \rho_{sp}(x,0,y) = \rho_{sp}(0,x,y) \\
&\qquad = [1 - 7(x^2+y^2) - (x^4+y^4) - 6(x^2-y^2)^2 + (x^2+y^2)(x^2-y^2)^2 - 6x^2y^2(x^2+y^2)]\lambda^{1/2}(1,x^2,y^2) \\
&\qquad\quad + 12 \left[x^4(1-y^4) \ln \frac{1+x^2-y^2+\lambda^{1/2}(1,x^2,y^2)}{1+x^2-y^2-\lambda^{1/2}(1,x^2,y^2)} + (x \leftrightarrow y) \right] ,
\end{aligned}
\tag{A2}
$$

where the symmetry properties are explicitly shown. Numerically

$$
\begin{aligned}
&\rho_{sp}\left[\frac{m_c}{m_b},0,0\right] = 0.447, \quad \rho_{sp}\left[\frac{m_c}{m_b},\frac{m_c}{m_b},0\right] = 0.119 , \\
&\rho_{sp}\left[\frac{m_\tau}{m_b},0,0\right] = 0.323, \quad \rho_{sp}\left[\frac{m_c}{m_b},\frac{m_\tau}{m_b},0\right] = 0.065 .
\end{aligned}
\tag{A3}
$$

The general expression of ρ_{ann} is given by

$$\rho_{\rm ann}(x_1,x_2)=[x_1{}^2+x_2{}^2-(x_1{}^2-x_2{}^2)^2]\lambda^{1/2}(1,x_1{}^2,x_2{}^2)\ . \tag{A4}$$

Numerically,

$$\rho_{\rm ann}(0,0)=0,\ \ \rho_{\rm ann}\left[\frac{m_c}{M_B},0\right]=0.070,\ \ \rho_{\rm ann}\left[\frac{m_\tau}{M_B},0\right]=0.091,\ \ \rho_{\rm ann}\left[\frac{m_c}{M_B},\frac{m_c}{M_B}\right]=0.136\ . \tag{A5}$$

In the annihilation graphs the two extreme values are assumed to be

$$\text{(a)}\ \frac{f_B}{m_b}=0.022,\ \ \text{(b)}\ \frac{f_B}{m_b}=0.111\ . \tag{A6}$$

The general expression of the phase-space factor in the gluon-emission case is given by the symmetric expression

$$\begin{aligned}\rho_g(x,y)=&[1-\tfrac{7}{2}(x^2+y^2)+\tfrac{3}{2}(x^2+y^2)^2+4(x^2-y^2)^2]\lambda^{1/2}(1,x^2,y^2)\\&+3\left\{x^2[y^2(x^2+y^2)-(x^2-y^2)^2]\ln\frac{1+x^2-y^2+\lambda^{1/2}(1,x^2,y^2)}{1+x^2-y^2-\lambda^2(1,x^2,y^2)}+(x\leftrightarrow y)\right\}\ .\end{aligned} \tag{A7}$$

As specific cases,

$$\begin{aligned}&\rho_g(0,0)=1\ ,\\&\rho_g(x,0)=(1-\tfrac{7}{2}x^2+\tfrac{11}{2}x^4)(1-x^2)+3x^6\ln x^2\ ,\\&\rho_g(x,x)=(1-7x^2+6x^4)(1-4x^2)^{1/2}+12x^6\ln\frac{1+(1-4x^2)^{1/2}}{1-(1-4x^2)^{1/2}}\ .\end{aligned} \tag{A8}$$

Numerically,

$$\rho_g\left[\frac{m_c}{M_B},0\right]=0.680,\ \ \rho_g\left[\frac{m_c}{M_B},\frac{m_c}{M_B}\right]=0.391\ . \tag{A9}$$

The two extreme values of f_B/m_q, adopted in Eq. (28), and corresponding to the cases (a) and (b) reported in Tables I and II, are given in (30).

Finally, by adopting $\Lambda=0.25$ GeV, it is easily found [see Eqs. (22) and (24)]

$$f_+=0.847,\ \ f_-=1.394\ , \tag{A10}$$

in terms of which all the strong-interaction factors are calculated.

[1]H. Krasemann, DORIS Workshop at DESY, Hamburg, 1981 (unpublished); P. Franzini and J. Lee-Franzini, Phys. Rep. 81, 239 (1982). The most recent experimental data coming from CUSB and CLEO Collaborations have been recently presented by J. Lee-Franzini, in *Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982,* edited by P. Petiau and M. Pornuef [J. Phys. (Paris) Colloq. 43, C3-92 (1982)].

[2]F. Halzen, in *Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982* (Ref. 1).

[3]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity,* edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[4]An updated description is given by G. Kalmus, in *Proceedings of the 21st International Conference on Physics, Paris (1982)* (Ref. 1) [J. Phys. (Paris) Colloq. 43, C3-431 (1982)].

[5]D. M. Scott, in *Proton-Antiproton Collider Physics—1981,* proceedings of the Workshop on Forward Collider Physics, Madison, Wisconsin, edited by V. Barger, D. Cline, and F. Halzen (AIP, New York, 1982).

[6]V. Barger and S. Pakvasa, Phys. Lett. 81B, 195 (1979); G. L.

Kane, University of Michigan Report No. UMHE 80-18, 1980 (unpublished); M. Gorn and E. A. Paschos, Phys. Lett. 101B, 255 (1981).

[7]A. Silverman, in *Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn,* edited by W. Pfeil (Physikalisches Institut, Universität Bonn, Bonn, 1981).

[8]S. Pakvasa, S. F. Tuan, and J. J. Sakurai, Phys. Rev. D 23, 2799 (1981); E. A. Paschos and U. Türke, Phys. Lett. 116B, 360 (1982); S. Pakvasa, in *Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982* (Ref. 1) [J. Phys. (Paris) Colloq. 43, C3-234 (1982)].

[9]V. Barger, W. F. Long, and S. Pakvasa, Phys. Rev. Lett. 42, 1585 (1979); R. Shrock, L. L. Wang, and S. B. Treiman, Phys. Rev. Lett. 42, 1589 (1979).

[10]P. Cea and G. L. Fogli, Phys. Lett. 102B, 22 (1981).

[11]G. L. Fogli, Phys. Lett. 123B, 57 (1983).

[12]R. N. Mohapatra and J. C. Pati, Phys. Rev. D 11, 566 (1975); 11, 2558 (1975); G. Senjanović and R. N. Mohapatra, *ibid.* 12, 1502 (1975); A. De Rújula, H. Georgi, and S. L. Glashow, Ann. Phys. (N.Y.) 109, 242 (1977); R. N. Mohapatra, F. E. Paige, and D. P. Sidhu, Phys. Rev. D 17, 2462 (1978); G. Senjanović, Nucl. Phys. B153, 334 (1979).

[13]D. V. Nanopoulos, in *High Energy Physics—1980,* proceedings of the XXth International Conference, Madison, Wisconsin, edited by L. Durand and L. G. Pondrom (AIP, New York, 1981); R. Barbieri and D. V. Nanopoulos, Phys. Lett. 91B, 369 (1980); P. Ramond, in *Unified Field Theories and Beyond,* proceedings of the 5th Johns Hopkins Workshop on Current Problems in Particle Theory, 1981, edited by G. Domokos and S. Kovesi-Domokos (Johns Hopkins University, Baltimore, 1981), p. 101.

[14]L. Maiani, in *Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982* (Ref. 1) [J. Phys. (Paris) Colloq. 43, C3-631 (1982)].

[15]C. A. Blocker *et al.*, Mark II Collaboration, Phys. Lett. 109B, 119 (1982).

[16]C. S. Lam and T. M. Yan, Phys. Rev. D 16, 703 (1977).

[17]H. Leutwyler, Nucl. Phys. B76, 413 (1974); Bern Report No. BUTP-16, 1981 (unpublished); S. Weinberg, in *Festschrift for I. I. Rabi,* edited by L. Motz (N.Y. Academy of Sciences, New York, 1977); P. Minkowski and A. Zepeda, Nucl. Phys. B164, 25 (1980).

[18]S. Narison, N. Paver, E. de Rafael, and D. Trelani, Nucl. Phys. B212, 365 (1983); T. N. Truong, Phys. Lett. 117B, 109 (1982).

[19]E. de Rafael, Marseille Report No. CPT-81/P-1344, 1981 (unpublished).

[20]F. Fucito, G. Martinelli, C. Omero, G. Parisi, R. Petronzio, and F. Rapuano, Nucl. Phys. B210, 407 (1982).

[21]JADE Collaboration, DESY Report No. 82-014, 1982 (unpublished).

[22]MAC collaboration, presented by D. Ritson, in *Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982* (Ref. 1) [J. Phys. (Paris) Colloq. 43, C3-52 (1982)].

[23]CLEO Collaboration, presented by B. Gittelman, in *Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982* (Ref. 1) [J. Phys. (Paris) Colloq. 43, C3-110 (1982)]; CUSB Collaboration, presented by P. Franzini, *ibid.* [43, C3-114 (1982)].

[24]M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. 52B, 351 (1974).

[25]G. Altarelli, G. Curci, G. Martinelli, and S. Petrarca, Phys. Lett. 99B, 141 (1981).

[26]I. Bigi and L. Stodolsky, Report No. SLAC-PUB-2410, 1979 (unpublished); S. P. Rosen, Phys. Rev. Lett. 44, 4 (1980); Phys. Lett. 93B, 492 (1980).

[27]M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. 44, 7 (1980); 44, 962(E) (1980).

[28]H. Fritzsch and P. Minkowski, Phys. Lett. 90B, 455 (1980); Nucl. Phys. B171, 413 (1980).

[29]V. Barger, J. P. Leveille, and P. M. Stevenson, Phys. Rev. D 22, 693 (1980); W. Bernreuther, O. Nachtmann, and B. Stech, Z. Phys. C 4, 257 (1980).

[30]J. L. Cortes, X. Y. Pham, and A. Tounsi, Phys. Rev. D 25, 188 (1982).

[31]H. Krasemann, Phys. Lett. 96B, 397 (1980).

[32]E. V. Shuryak, Nucl. Phys. B198, 83 (1982).

[33]E. Golowich, Phys. Lett. 91B, 271 (1980).

[34]D. V. Nanopoulos and D. A. Ross, Phys. Lett. 108B, 351 (1982).

[35]M. Veltman, M. Einhorn, and T. Jones, in *Weak Interactions as Probes of Unification,* proceedings of the Workshop, Virginia Polytechnic Institute, Blacksburg, edited by G. B. Collins, L. N. Chang, and J. R. Ficenec (AIP, New York, 1980).

[36]S. L. Glashow, Phys. Rev. Lett. 45, 1914 (1980).

[37]Y. Yamanaka, H. Sugawara, and S. Pakvasa, Phys. Rev. D 25, 1895 (1982); K. Kanaya, H. Sugawara, S. Pakvasa, and S. F. Tuan, Phys. Lett. 116B, 61 (1982); K. Kanaya, Phys. Rev. D 26, 1758 (1982).

[38]MARK J Collaboration, presented by J. D. Burger, in *Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982* (Ref. 1) [J. Phys. (Paris) Colloq. 43, C3-63 (1982)].

[39]V. Barger, J. P. Leveille, P. M. Stevenson, and R. J. N. Phyllips, Phys. Rev. Lett. 45, 83 (1980).