Relativistic corrections in quarkonium

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Relativistic corrections to quarkonium energy levels and electric dipole transition rates are calculated. We find a consistent description with both electric confinement and tensor quark-antiquark forces at distances beyond that at which perturbative QCD should be valid. The preliminary masses of the χ_b and χ'_b levels are described satisfactorily. The transitions $\Upsilon''\to\gamma\chi_b$, suppressed in some nonrelativistic models, are predicted to be observable here.

Quarkonium is an ideal system for the detailed study of the quark-antiquark force. The charmonium ($c\bar{c}$) and $\Upsilon(b\bar{b})$ systems have been subjected to nonrelativistic descriptions with some success.[1] One has learned, for example, that the quarkonium potential is flavor-independent (as expected in QCD), and that a short-distance Coulomb-type behavior (also as expected in QCD) is consistent with present data.

The recent study of electric dipole radiative transitions in the Υ system[2,3] has emphasized the relevance of a more precise understanding of relativistic and spin-dependent effects. The observed suppression of electric dipole rates in charmonium[4] with respect to nonrelativistic estimates[1] is one source of encouragement for efforts[5-9] to understand relativistic corrections. Much study has also been devoted to magnetic dipole transitions[9,10] and to fine- and hyperfine-structure effects.[5,6,11] Quantitative predictions in accord with experiment have been obtained for $\psi'\to\gamma\chi$ decays in models with a long-range scalar quark-antiquark interaction.[5,6]

Here we wish to present results of a straightforward analysis of relativistic corrections in a manner which allows the effects of initial assumptions to be seen most transparently. Since we do not *know* the precise form of the quarkonium interaction at long distances (beyond that where perturbative QCD should hold), we wish to see how various aspects of the data determine this form most effectively. For that reason, we adopt a first-order perturbation-theory treatment.[7]

We find that a crude (qualitative) understanding of relativistic effects can be obtained in charmonium, but that second-order corrections (which we do not estimate) are likely to be significant. The relevant expansion parameter here is v^2/c^2 (0.4 for charmonium). For the Υ system, where we find $v^2/c^2<0.1$, a first-order treatment is found to make sense. We find by comparing our results with preliminary data on the P-wave $b\bar{b}$ levels χ_b and χ'_b that there is evidence at large distances for (a) a long-range force which behaves as the fourth component of a four-vector[12] and (b) a tensor force, of somewhat longer range than the spin-orbit force. A tensor force beyond the range of perturbative QCD has been proposed on the basis of lattice calculations.[13]

A satisfactory description of the fine structure and relativistic effects in the $b\bar{b}$ system has also been given under the assumption that the long-range confining force is of Lorentz-scalar type.[6] We shall compare some of our results with those of Ref. 6.

We also present results for as yet unseen transitions and levels in the Υ system. In particular we find the following.

(i) The rates for the decays $\Upsilon''\to\gamma\chi_b$, suppressed in some nonrelativistic treatments,[14] are substantially raised by relativistic corrections. Weak evidence for these transitions actually exists.[2,15]

(ii) Tensor $({}^3D_1)$ $b\bar{b}$ mesons are expected to lie at 10.151 and 10.433 GeV/c^2. (We use the energy scale[15] whereby Υ, Υ', and Υ'' lie at 9.46, 10.02, and 10.35 GeV/c^2.) However, the predicted leptonic widths of these states are of order electron volts, so they should be hard to observe. Such states are of interest because they could initially be confused with Higgs bosons,[16] or with vibrational string excitations.[17]

We derive relativistic corrections to the bound-state equation by considering the covariant amplitude for the scattering of two spin-$\frac{1}{2}$ fermions:

$$M_{fi}=(\bar{u}'_1\gamma^\mu u_1)V(q^2)(\bar{u}'_2\gamma_\mu u_2)\ \text{(Lorentz vector)}$$
$$+(\bar{u}'_1u_1)S(q^2)(\bar{u}'_2u_2)+\cdots\ \text{(Lorentz scalar)}\,,$$
$$q\equiv p'_1-p_1\equiv p_2-p'_2\ .$$

Expand this covariant expression in powers of $1/c$, taking

$$u=\sqrt{2m}\begin{bmatrix}(1-\vec{p}^{\,2}/8m^2c^2)w\\ \vec{\sigma}\cdot\vec{p}w/2mc\end{bmatrix}, \tag{2}$$

where w is a Schrödinger spinor. Expand $V,S\cdots$ in a power series in $1/c^2$. Write Dirac matrices in terms of Pauli matrices. Then (1) may be cast in the form of a nonrelativistic, first-order Born approximation:

$$M_{fi}\approx-(2m_1)(2m_2)w'^*_2w'^*_1U(\vec{p}_1,\vec{p}_2,\vec{q})w_1w_2\ . \tag{3}$$

Go to the c.m. frame; set masses equal; Fourier transform the potential $U(\vec{p}_1,\vec{p}_2,\vec{q})$ to obtain a coordinate space potential $U(r)$; extract relativistic corrections. The result is an expression containing spin-independent, spin-orbit, spin-spin, and tensor terms. The last three, for example, lead to predictions of the fine structure of P-wave quarkonium levels.

Now, the concept of a potential having a definite Lorentz structure is meaningless for quarks, which are not free. (They cannot recede to infinity.) Therefore, we replace (1) and its consequences by the following ansatz for the quark-antiquark interaction:

Short distances: $\gamma^\mu\gamma_\mu$ (1-gluon) (4)

Long distances: $\gamma^0\gamma^0$(confinement by longitudinal color electric field[12])

and long-range tensor force (*ad hoc*) . (5)

We may implement this ansatz by writing $\gamma^\mu\gamma_\mu=\gamma^0\gamma^0-\vec{\gamma}\cdot\vec{\gamma}$ and damping out the transverse degrees of freedom in $\vec{\gamma}\cdot\vec{\gamma}$ using a cutoff function (except for the tensor term). A Gaussian $f_c(r)=\exp[-(r/r_c)^2]$ is used. Our result is that the effective Schrödinger Hamiltonian may be written

$$H=\frac{\vec{p}^{\,2}}{2m}+V(r)+V_1+V_2+V_3\ , \tag{6}$$

where

$$V_1=\frac{f_c(r)}{8m^2c^2}\left[rV'''+5V''+\frac{4V'}{r}\right]+\frac{if_c(r)}{2m^2c^2}\left[V''+\frac{2V'}{r}\right]\vec{r}\cdot\vec{p}+\frac{f_c(r)V\vec{p}^{\,2}}{2m^2c^2}-\frac{f_c(r)V'\vec{r}\cdot(\vec{r}\cdot\vec{p})\vec{p}}{2m^2c^2r}+\frac{\nabla^2V}{4m^2c^2}-\frac{\vec{p}^{\,4}}{4m^3c^2}\ , \tag{7}$$

$$V_2=\frac{V'(r)}{2m^2c^2r}[2f_c(r)+1]\,\vec{l}\cdot\vec{S}\ , \tag{8}$$

$$V_3=\frac{(2\vec{S}^{\,2}-3)}{6m^2c^2}f_c(r)\left[V''+\frac{2V'}{r}\right]-\frac{1}{6m^2c^2}\left[V''-\frac{V'}{r}\right]\left[\frac{3(\vec{S}\cdot\vec{r})(\vec{S}\cdot\vec{r})}{r^2}-\vec{S}^{\,2}\right]. \tag{9}$$

For the potential $V(r)$ we use a modification of the form suggested by Richardson,[18] and Carlitz and Creamer[19]:

$$V(r)=kr+\frac{8\pi\Lambda}{33-2n_f}\left[-\frac{\mathscr{F}(\Lambda r)}{\Lambda r}\right]. \tag{10}$$

The parameters k and Λ are arbitrary; n_f is the number of light flavors (which we take equal to 3). $\mathscr{F}(\Lambda r)$ is a specific slowly varying function which is defined such that Eq. (10) would reduce to the Richardson form for a particular choice of k. For small r,[20]

$$V\approx\{8\pi/[33-2n_f]r\ln(e^\gamma\Lambda r)\}\ ,\quad \gamma=0.5772\ldots\ . \tag{11}$$

We set Λ and k (and the b-quark mass m_b) by fitting Υ ($1S,2S,3S$) exactly,[21] and choose r_c so that satisfactory P-state centers of gravity in χ_b and χ_b' are obtained. That this could be achieved via a single parameter choice was not obvious *a priori*. We choose m_c to obtain a satisfactory average description of charmonium. The results are

$$m_b=4.903\ \mathrm{GeV}/c^2,\quad \Lambda=0.455\ \mathrm{GeV}\ ,$$
$$k=0.148\ \mathrm{GeV}^2\ , \tag{12}$$
$$m_c=1.5\ \mathrm{GeV}/c^2,\quad r_c=0.3\ \mathrm{GeV}^{-1}\ .$$

Larger values of r_c are almost as satisfactory, but raise the predicted center of gravity of the χ_b and χ_b' states somewhat. Examination of the corresponding potential and cutoff function shows that the scale at which transverse degrees of freedom are damped is the same as that at which the perturbative part of V becomes invalid.

We emphasize that the cutoff function was inserted primarily to obtain a good fit to the spin-averaged spectrum (i.e., to control the spin-independent relativistic terms). We have arbitrarily applied the cutoff to the spin-orbit force and not to the tensor force so as to crudely reproduce the observed fine structure.

We do not pretend that the resulting description of the spin-orbit and tensor forces is precisely realistic. These forces are not sufficiently constrained by the data at present. When the data are refined, it may be possible to map these forces out in greater detail.

The spectra for $c\bar{c}$ and $b\bar{b}$ in this potential are given in Table I. The relativistic correction leads to a charmonium $2S$-$1S$ spacing which is 50 MeV too small, but higher-order effects are estimated to be around 50 MeV in this system. The fine structure in charmonium is characterized by a ratio

$$r\equiv\frac{M(^3P_2)-(^3P_1)}{M(^3P_1)-M(^3P_0)} \tag{13}$$

TABLE I. Masses in GeV/c^2.

		Theory [Eq. (10)]		Experiment (Refs. 2,3,15,22)
$c\bar{c}S$	ψ	3.125		3.0969±0.0001
	η_c	3.047		2.981±0.006
	ψ'	3.664		3.686±0.0001
	η_c'	3.615		3.592±0.005
	$\psi(4030)$	4.042		4.030±0.005
P	χ_2 } $\bar{\chi}=3.504$	3.527[a] }	$\bar{\chi}=3.525$	3.5558±0.0006
	χ_1 } $\psi'-\bar{\chi}=0.160$	3.494[a] }	$\psi'-\bar{\chi}=0.161$	3.5100±0.0006
	χ_0 }	3.416[a] }		3.415 ±0.001
	$^1P_1$	3.504[a]		
$^3D_1$	$\psi(3770)$	3.737		3.770±0.003
$b\bar{b}S$	Υ	9.460	(input)	9.46
	η_b	9.403		
	Υ'	10.020	(input)	10.02
	Υ''	10.350	(input)	10.35
	Υ'''	10.607		10.57
P	χ_{b2}	9.914	105[b]	108.2±0.3±2[b]
	χ_{b1}	9.903	116[b]	128.1±0.4±3[b]
	χ_{b0}	9.877	142[b]	149.4±0.7±5[b]
	$^1P_1$	9.906		
	χ_{b2}'	10.264	86[c]	84.2±0.3±2[c]
	χ_{b1}'	10.256	93[c]	101.4±0.3±3[c]
	χ_{b0}'	10.237	112[c]	122.1±0.7±5[c]
	$^1P_1$	10.257		
D	$^3D_{1,2,3}$	10.151, 10.161, 10.168		
	$^3D_{1,2,3}'$	10.433, 10.441, 10.447		

[a]Reference 6 predicts 3.525, 3.488, 3.382, and 3.502 GeV/c^2, for ψ at 3.097 GeV/c^2.
[b]E_γ, $\Upsilon'\to\gamma\chi_b$, in MeV (Ref. 6 predicts 82, 103, and 152 MeV).
[c]E_γ, $\Upsilon''\to\gamma\chi_b'$, in MeV (Ref. 6 predicts 75, 94, and 133 MeV.

which is (0.43,0.35,0.48) in (present theory, theory of Ref. 6, experiment). If $f_c(r)$ were inserted into the tensor-force expression [the second term in V_3 in Eq. (9)], the predicted value for this ratio would rise to 1.9. Our tensor force may be overestimated at large distances, but cannot be cut off as drastically as the $\langle\vec{L}\cdot\vec{S}\rangle$ force.

In the Υ system, the predicted $4S$ level is too high by 30 MeV; this behavior is familiar[18] and may reflect threshold effects. The predicted ratios r are

$$\chi_b,\chi_b':r=\begin{cases}0.4,\ 0.4\ [\text{no } f_c(r) \text{ in tensor term}]\ ,\\ 1.6,\ 1.5\ [f_c(r) \text{ in tensor term}]\ ,\\ 0.93,\ 0.85\ (\text{expt})\ .\end{cases} \tag{14}$$

Present experimental data on χ_b and χ_b' levels are not sufficiently precise to confirm the presence or absence of a cutoff term $f_c(r)$. Reference 6 finds $r=(0.45,0.48)$ for the (χ_b,χ_b'). Reference 12 finds $r\approx 1$ for (χ_c,χ_b,χ_b').

Other approaches[12,13] in fact have a tensor term extending beyond the validity of perturbation theory, though not of the form suggested here, by virtue of assuming a Coulomb-type interaction all the way out to $r=\infty$. Our long-range tensor force is more important at large distances and thus leads to a smaller value of r than these other approaches. (In a pure Coulomb potential, $r=0.8$.) There is no preference for one form over the other in view of the present quality of the data; the agreement of our form with experiment for charmonium may be fortuitous in view of the crude nature of our description elsewhere for $c\bar{c}$ systems. What both we and the authors of Refs. 12, 13, and 23 are saying is that there must be *some* form of tensor interaction *beyond* about 0.5 GeV^{-1}=0.1 fm, which is the largest distance at which perturbative QCD can be trusted.[24]

The specific description based on a long-range scalar interaction adopted in Ref. 6 agrees with experiment about as well as ours for charmonium, but has fine-structure

TABLE II. Electric dipole decay rates.

	Decay	$\langle r\rangle_{NR}$ (GeV^{-1})	$[\langle r\rangle/\langle r\rangle_{NR}]^2$	Predicted rate keV[a,b]	Experimental rate keV
$c\bar{c}$ $S\to P$	$\psi'\to\chi_2\gamma$	2.65	1.04	41	17 ± 5[c]
	$\chi_1\gamma$		0.94	48	19 ± 5[c]
	$\chi_0\gamma$		0.68	37	21 ± 6[c]
$P\to S$	$\chi_2\to\psi\gamma$	2.08	1.23	609	490 ± 330[c]
	$\chi_1\to\psi\gamma$		1.25	460	<700[c]
	$\chi_0\to\psi\gamma$		1.30	226	97 ± 38[c]
$b\bar{b}$ $S\to P$	$\Upsilon'\to\chi_{b2}\gamma$	1.64	1.06	2.2 (Total 5.3)	Total 4.9 ± 1.8
	$\chi_{b1}\gamma$		1.01	2.1	
	$\chi_{b0}\gamma$		0.90	1.0	
	$\Upsilon''\to\chi'_{b2}\gamma$	2.68	1.06	2.7 (Total 6.9)	Total 8.4 ± 1.4
	$\chi'_{b1}\gamma$		1.01	2.8	
	$\chi'_{b0}\gamma$		0.91	1.4	
	$\Upsilon''\to\chi_{b2}\gamma$	$-.024$	5.4	0.15	Seen?
	$\chi_{b1}\gamma$		1.4	0.025	
	$\chi_{b0}\gamma$		3.6	0.025	
$P\to S$	$\chi_{b2}\to\Upsilon\gamma$	1.08	1.05	38	Seen
	$\chi_{b1}\to\Upsilon\gamma$		1.06	36	Seen
	$\chi_{b0}\to\Upsilon\gamma$		1.09	31	
	$\chi'_{b2}\to\Upsilon'\gamma$	1.89	0.90	16	Seen
	$\chi'_{b1}\to\Upsilon'\gamma$		0.94	14	Seen
	$\chi'_{b0}\to\Upsilon'\gamma$		1.04	12	
	$\chi'_{b2}\to\Upsilon\gamma$	0.26	1.29	14	Seen
	$\chi'_{b1}\to\Upsilon\gamma$		1.13	12	Seen
	$\chi'_{b0}\to\Upsilon\gamma$		0.77	7.5	

[a]Includes a finite-size correction as described in text.
[b]Based on observed photon energies.
[c]First work in Ref. 4. Broader limits are quoted for $\chi\to\psi\gamma$ rates in second work in Ref. 4.

splitting which is somewhat larger than experiment, and considerably larger than what we predict, for $b\bar{b}$ states. This can be traced to the different sign of $\langle\vec{L}\cdot\vec{S}\rangle$ contributions for the long-range interaction in the two approaches, and may provide a means of eventually distinguishing between them. Such a distinction is impossible so far because systematic errors in photon energies can easily be of the order of 10 MeV in the present data.[2,3]

Electric dipole matrix elements are evaluated nonrelativistically ($\langle r\rangle_{NR}$) and using first-order perturbed wave functions calculated in the presence of the relativistic corrections (7)—(9).[25] In addition, the lowest-order (nonrelativistic) dipole matrix element is calculated in the presence of finite-size corrections, with $\langle f\mid r\mid i\rangle$ replaced by

$$\langle f\mid r\mid i\rangle_{\text{finite size}}=E_\gamma^{-1}\left[1+\frac{E_\gamma}{4m}\right]\left\langle f\left|3j_1\left[\frac{E_\gamma r}{2}\right]\right|i\right\rangle . \tag{15}$$

We present results for electric dipole rates in Table II. The corrections in charmonium are in the right direction, but they are nowhere as large as in Refs. 6 and 11. We ascribe this in part to our strictly first-order treatment. Parts of higher-order contributions are contained in these other approaches. We cannot tolerate larger first-order corrections without even more serious distortions to the charmonium spectrum than already appear in Table I. Another source of suppression of $E1$ transitions in many other treatments (Eichten *et al.*,[1] and Ref. 6) is the larger quark mass ($m_c>1.8$ GeV, $m_b>5.1$ GeV), which leads to spatially more compact systems. This effect is particularly striking for charmonium. Quark masses larger than those we use can be accommodated at the price of a negative additive constant in the potential. In the absence of any other physical motivation for such a constant, we do not introduce it here.

The corrections in the Υ system to $E1$ rates are typically tens of percent or less in the present model. They are compared with experiment and with the predictions of a scalar confinement model[6] in Table II. The scalar confinement model predicted larger corrections for charmonium (in accord with experiment) and predicts them for $b\bar{b}$.

TABLE III. Predicted and observed ratios of $\Gamma_{E1}/E_\gamma^3(2J_f+1)$ for $\Upsilon'\to\gamma\chi_b$ and $\chi''\to\gamma\chi_b'$.

Transition		$J=2$	$J=1$	$J=0$
$\Upsilon'\to\gamma\chi_{bJ}$	Predicted	1.05		0.88
	Predicted (Ref. 6)	1.13	1 (definition)	0.76
	Experimental (Ref. 3)	1.04 ± 0.3		1.13 ± 0.5
$\Upsilon''\to\gamma\chi_{bJ}'$	Predicted	1.04		0.90
	Predicted (Ref. 6)	1.17	1 (definition)	0.73
	Experimental (Refs. 2,3)	0.86 ± 0.4		0.84 ± 0.5

Present $b\bar{b}$ data on relative branching ratios are uncertain to about $\pm30\%$ (Ref. 3), so no experimental distinction is possible yet between the two schemes.

We have calculated ratios of leptonic widths (in lowest order only, neglecting corrections[26]) as a check of our nonrelativistic potential. The results are

$$\begin{aligned}(2S/1S)_{c\bar{c}}&=0.44 \text{ (expt } 0.45\pm0.06)\,,\\ (2S/1S)_{b\bar{b}}&=0.41 \text{ (expt } 0.45\pm0.02)\,,^{27}\\ (3S/1S)_{b\bar{b}}&=0.29 \text{ (expt } 0.31\pm0.02)\,,^{27}\\ (4S/1S)_{b\bar{b}}&=0.24 \text{ (expt } 0.24\pm0.03)\,.^{27}\end{aligned} \tag{16}$$

We can calculate leptonic widths of $^3D_1$ states as a combination of two effects: (a) mixing with $^3S_1$ states via the tensor force and (b) direct coupling to $R_D''(0)$. These effects are found to be of comparable magnitude. We find that with[28]

$$\Gamma(^3D_1\to e^+e^-)=\frac{200}{(2m_Q)^6}\alpha^2 e_Q{}^2\left|R_D''(0)+\frac{2\sqrt{2}m_Q{}^2}{5}R_s(0)\right|^2, \tag{17}$$

and with the potential noted here, values of

$$\Gamma(^3D_1,{}^3D_1'(b\bar{b})\to e^+e^-)=(1.5,2.7)\text{ eV}$$

are obtained. A similar approach gives $\Gamma(\psi''\to e^+e^-)=217$ eV, to be compared with the experimental value of 275 ± 60 eV.[22] The approach is certainly incomplete for charmonium, possibly because the ψ'' (lying above threshold) experiences important mixing effects via open decay channels. These effects may be smaller for the lowest two $^3D_1$ states in the Υ family. If so, these states will be hard to see. The predicted masses (10.151 and 10.433 GeV) are expected to be reliable to 10 MeV, however, in view of the success of predictions for the P states.

To summarize, we have shown that a model based on electric confinement (long-range $\gamma^0\gamma^0$ interaction[12]) adequately describes both the fine-structure and the center-of-gravity positions of the spin-triplet ground and first-excited $b\bar{b}$ P-wave states. We have shown that more precise data on fine structure and $E1$ rates in the $b\bar{b}$ system may be able to distinguish between models based on a scalar confining potential[5,6,10] and the present electric-confinement model. The $^3D_1$ $b\bar{b}$ leptonic widths are predicted to be small (electron volts).

A more stringent test of models for relativistic corrections in quarkonium awaits the discovery of newer, heavier quarks; for all but the most deeply bound $\zeta\equiv t\bar{t}$ systems, the relativistic effects are characterized by $v^2/c^2\approx0.5$ GeV$/m_t$, and should be extremely tiny for $m_t>20$ GeV. The form of the nonrelativistic quarkonium interaction should then be quite manifest, allowing us to check back to the Υ system to see whether the present picture is valid.

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[23]See also Nathan Isgur and Gabriel Karl, Phys. Lett. 72B, 109 (1977); and H. J. Schnitzer, Nucl. Phys. B207, 131 (1982), for a discussion of the ranges of different components of the interquark force.

[24]The short-range expression (11) for the potential actually turns over and begins to decrease again as r is increased beyond 0.5 GeV^{-1}, for the parameters given here.

[25]A. J. F. Siegert, Phys. Rev. 52, 787 (1937); R. G. Sachs and N. Austern, *ibid.* 81, 705 (1951); R. G. Sachs, *Nuclear Theory* (Addison-Wesley, Reading, Mass., 1953), pp. 232—245. The applicability of Siegert's theorem to the present relativistic situation was first demonstrated via explicit calculation by McClary (Ref. 7). We thank Nina Byers for a discussion of this work.

[26]Wai-Yee Keung and I. J. Muzinich, Phys. Rev. D 27, 1518 (1983). We use the standard Van Royen—Weisskopf expression whereby $\Gamma_{ee}\sim|\Psi(0)|^2/(\text{vector-meson mass})^2$.

[27]P. Franzini and J. Lee-Franzini, Ann. Rev. Nucl. Part. Sci. 33, 1 (1983).

[28]V. A. Novikov *et al.*, Phys. Rep. 41C, 1 (1978).