PHYSICAL REVIEW D VOLUME 27, NUMBER 4 15 FEBRUARY 1983

Angular momentum of the cosmic background radiation

C. J. Goebel
Physics Department, University of Wisconsin—Madison, Madison, Wisconsin 53706
(Received 5 July 1979; revised manuscript received 20 October 1981)

It is argued that, contrary to a recent claim, the observed deviations of the cosmic background radiation from a blackbody spectrum are not explained by its "angular momentum."

A recent Physical Review Letter[1] of the above title has attempted to derive an improved fit to the observed spectrum of the cosmic background radiation by finding "the modified Planck law when the total angular momentum is significant." We show here that their derivation is faulty and that they have given no physical basis for their spectrum.

There is nothing wrong with their Section (1), Eqs. (1)–(4) (see Ref. 1) except for being rather unnecessary. The Planck distribution with additional conserved quantities is well known; for example, conservation of a particle number gives a chemical potential term in the exponent.

The trouble begins in section (2), with consideration of angular momentum as the additional conserved quantity. Firstly, although the total angular momentum $M=\sum M_A$ of a system of interacting particles is conserved, and hence likewise its magnitude squared, $M^2=(\sum M_A)^2$, the same is not true of the quantity supposed conserved by JKS, namely, the sum of the squares of the single-particle angular momenta $\sum M_A{}^2$. Footnote 4 of JKS argues that "isotropic and stochastic considerations" result in the conservation of $\sum M_A{}^2$. But this is begging the question. If the distribution differs from Planck, due to the supposed conservation of $\sum M_A{}^2$, then it is *not* isotropic, and the argument fails. Explicitly, the single-particle thermal distribution resulting from the supposed conservation of $\sum M_A{}^2$ is

$$\frac{d^6 f}{dr^3\,dp^3}=C\left[\exp\left(\frac{cp}{kT}+\gamma L^2\right)-1\right]^{-1}, \qquad (1)$$

where

$$P=(\vec{p}^{\,2})^{1/2},\quad L^2=(\vec{r}\times\vec{p})^2\equiv r^2 p_\perp{}^2=r^2p^2\sin^2\theta\ .$$

[This is the integrand of JKS Eq. (6).] The spin of the photon has been neglected (as in JKS), so $M^2=L^2$. The distribution has been written in phase space $(\vec{r},\vec{p})$; this is, of course, meaningless on a fine scale, but yields correct "coarse grained" results, which depend only on the distribution averaged over phase-space volumes larger than $\hbar^3$. If $\gamma\neq 0$, the distribution (1) is anisotropic in momentum space for all points $r\neq 0$, the anisotropy growing with r. Interaction of the quanta (scattering) will make the momentum distribution more isotropic, thus contradicting the assumption that it was a thermal (equilibrium) distribution and that $L_A{}^2$ is conserved. This can also be seen directly: The term γL^2 in (1) has the effect of suppressing the occurrence of large values of L^2 (just as the term cp/kT suppresses larger values of p), hence particles at large r have limited transverse momentum and are moving nearly radially; any scattering of two such particles will on the average increase the L^2 of both. Of course, if the particles are essentially noninteracting, which is certainly true now for the cosmic Planck photons, then each will conserve its L^2; but this is totally irrelevant for the *setting up* of the distribution. There is no way for Eq. (1), with $\gamma\neq 0$, to arise as a thermal equilibrium distribution, because $\sum L_A{}^2$ is not a conserved quantity in the presence of interactions.

Of course, it would be very strange to find the cosmic photons obeying Eq. (1) because this distribution is not translationally invariant: The origin $\vec{r}=0$ is a preferred point. But that is the way it is with angular momentum—there is a preferred point where the angular momentum of any particle vanishes. How do JKS avoid this? Formally, they do so by integrating the distribution Eq. (1) over $l(=L)$ in their Eqs. (5) and (6). But L^2, $\vec{r}$, and $\vec{p}$ are related [by $L^2=(\vec{r}\times\vec{p})^2$]. Since it is the energy spectrum at one point $\vec{r}$ in space which is desired, one cannot freely integrate over L^2. [It is clear that the energy spectrum which results from Eq. (1) by integrating over the direction of $\vec{p}$ is Planck at $\vec{r}=0$; at $r\neq 0$ it differs from Planck but noticeably so only if the distribution in $\vec{p}$ was noticeably anisotropic.[2]]

A clue to their mysterious integration over L^2 is given in the first paragraph on p. 1789 of JKS: "[the total angular momentum squared M^2] is strictly nonvanishing in every photon state, though possibly unobservably small." And again in their last paragraph: "direct measurements of the angular momentum of the [cosmic blackbody radiation] should be of interest." These statements are of course meaningless if JKS mean by "angular momentum" what everyone else does, in view of the dependence of angular momentum on the choice of origin.

Perhaps a final comment should be made on the situation in a spherical three-space S^3, used by JKS, rather than a flat three-space. It is no different because S^3 has no more symmetry, and no less, than flat space; the two spaces are equally homogeneous and isotropic, and have the same number of conserved quantities. In any case, it is hard to see how a global consideration could affect a thermal equilibrium argument.

---

[1]H. P. Jakobsen, Mark Kon, and I. E. Segal, Phys. Rev. Lett. 42, 1788 (1979). We refer to this paper as JKS. For reference, I have numbered their equations, (1)–(7). There are two obvious misprints: In (3), the first "-1" should not be a superscript; in footnote 4, last line, $\vec{M}_1'$ and $\vec{M}_2'$ should be squared.

[2]See also Edward L. Wright, Phys. Rev. D 22, 2361 (1980).