PHYSICAL REVIEW D VOLUME 27, NUMBER 4 15 FEBRUARY 1983

Operator product and vacuum instability (addendum)

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(Received 1 November 1982)

This paper is an addendum to our previous paper of the same title [Phys. Rev. D 26, 499 (1982)]. We discuss an apparent scheme dependence of the results reported in a note added in proof. We find that, when the q^2 dependence of operators is carefully identified, the results are scheme independent. Thus, for any subtraction scheme, our conclusion is that the operator-product expansion gives different results at next-to-leading twist when made about the physical vacuum and when made about the unstable symmetric vacuum with nonvanishing vacuum expectation values allowed for nontrivial operators. We include also some errata for the original paper.

This Brief Report is an extension of the discussion of our previous paper of the same title[1] and is meant to be read in conjunction with that paper. We pointed out there that, using Bogolubov-Parasiuk-Hepp-Zimmermann (BPHZ) subtraction prescriptions, one obtains a discrepancy between the operator-product expansions for a scalar theory with a broken-symmetry vacuum if one expands about the broken vacuum or if one expands about the symmetric vacuum but allows nontrivial operators to have nonzero vacuum expectation values. (We refer the reader here to some errata for that paper[1] which are listed as an appendix to this report.) In a note added in proof we remarked on an apparent scheme dependence of this result.

The purpose of this report is to show that, when q^2 dependence of operators is carefully defined, the results are scheme independent, and that there is a discrepancy between the two operator-product expansions.

It has been pointed out by Ellwanger and subsequently also by Taylor and McClain[2] that, using any form of dimensional regularization and the equation

$$\gamma_i = -\mu \frac{\partial}{\partial \mu} \ln Z_i , \quad i = m, \phi^2, \ldots \tag{1}$$

gives, to leading order in λ,

$$\hat{\gamma}_m = \tfrac{1}{2} \hat{\gamma}_{\phi^2} = \gamma_m \quad . \tag{2}$$

Using this result one then finds no discrepancy between the two operator-product expansions at this order. However, using the BPHZ expansion of Feynman integrals we found

$$\hat{\gamma}_m = \hat{\gamma}_{\phi^2} = 0 \tag{3}$$

to leading order in λ. As commented in our note added in proof this apparent scheme dependence of the results is unsatisfactory and needs explanation.

The explanation lies in the fact that Eq. (1) is incorrect unless the subtraction scheme used is mass independent, which it is not in these theories. We define the quantities $\hat{\gamma}_m$ and $\hat{\gamma}_{\phi^2}$ by the equations

$$m(q^2) = m(q_0^2) \left(\frac{q^2}{q_0^2} \right)^{\hat{\gamma}_m} ;$$
$$\langle N_2(\phi^2) \rangle_{q^2} = \langle N_2(\phi^2) \rangle_{q_0^2} \left(\frac{q^2}{q_0^2} \right)^{\hat{\gamma}_{\phi^2}} . \tag{4}$$

Let us first study corrections to the mass to leading order in λ in the symmetric theory. The only diagram which contributes is Fig. 2(b) of Ref. 1, which clearly does not introduce any q^2 dependence. Hence regardless of the subtraction prescription used the correct result is $\gamma_m = 0$. This does not imply $\hat{Z}_m{}^2 - 1$ is nonzero; in fact, there is a contribution to $\hat{Z}_m{}^2 - 1$ from this diagram of the form

$$\hat{Z}_m{}^2 - 1 = \left(\frac{\lambda}{32\pi^2} \ln(m^2/\mu^2) + \text{constants} \right) . \tag{5}$$

Hence it is quite clear that Eqs. (1) and (4) are incompatible because of the mass dependence of Z_m.

For the shifted theory there is a q^2 dependence to the mass which arises from the second diagram of Fig. 2(a) (Ref. 1). This gives a contribution to $Z_m{}^2 - 1$ of the form (for $q^2 >> m^2$)

$$\tfrac{3}{2} \lambda \left(\frac{1}{16\pi^2} \ln \frac{q^2}{\mu^2} \right) + \text{constants} \tag{6}$$

while the remaining two diagrams give

$$(-\tfrac{3}{2}\lambda + \tfrac{1}{2}\lambda) \frac{1}{16\pi^2} \ln \frac{m^2}{\mu^2} + \text{constants} \ , \tag{7}$$

where the two terms in (6) come from the remaining diagrams of Figs. 2(a) and 2(b), respectively. Thus one sees that, although $\mu(\partial/\partial\mu)\ln Z_m$ is the same in the two theories to this order, the q^2 dependence, which is the subject of our paper, is different.

The situation is quite similar for $\hat{\gamma}_{\phi^2}$. The only diagram which contributes is shown in Fig. 1. Once again the integral requires subtraction but is q^2 dependent; thus again Eq. (1) is incompatible with the definition (4) of the quantity $\hat{\gamma}_{\phi^2}$ because of the dependence of the subtraction on other scales such as p^2 or m^2 even in the limit of very large q.

The discrepancy between Eqs. (1) and (4) is not peculiar to the unshifted theory. As can be seen from Eqs. (5) and (6) this discrepancy arises as well in the shifted theory. The result (2), which is obtained by using the incorrect equation (1) in both treatments, gives agreement between the two treatments but is not correct for either. Hence it must be regarded as a spurious result. It occurs because it must be true that the μ dependence of the Z's have this relationship, owing to the well-known property that the subtractions of the symmetric theory are sufficient to render finite the shifted theory, but it has no bearing on the q^2 dependence in question.

Thus we reiterate the conclusion reached in our paper, that the q^2 dependence of next-to-leading twist terms differs in the two procedures. This result is now understood to be subtraction scheme independent when the q^2 dependence is correctly calculated, without the use of Eq. (1), which is invalid because of the dependence of the counterterms on other mass and/or momentum scales.

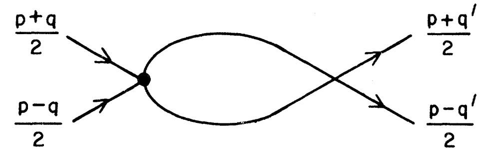

FIG. 1. Leading-order correction to $N_2(\phi^2)$ in the symmetric theory.

We would like to acknowledge the written comments of U. Ellwanger and discussions with C. Taylor and B. McClain which prompted us to clarify this point. This work was supported by the Department of Energy under Contract No. DE-AC03-76SF00515.

APPENDIX: ERRATA FOR PHYS. REV. D 26, 499 (1982)

(i) Equation (2.2) should read

$$\phi_0 = Z^{1/2}\phi, \quad M_0 = Z_m M, \quad \lambda_0 = Z_\lambda \lambda \ .$$

(ii) Immediately following Eq. (2.2) eliminate the phrase "and defining all Feynman integrals by dimensional continuation."

(iii) Replace Eq. (2.7) by the equation

$$m(q^2) = m(q_0^2)\left(\frac{q^2}{q_0^2}\right)^{\gamma_m} \ .$$

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[1]S. Gupta and H. R. Quinn, Phys. Rev. D 26, 499 (1982).

[2]C. Taylor and B. McClain (private communication); MIT Report No. CTP 1024, 1982 (unpublished).