PHYSICAL REVIEW D VOLUME 27, NUMBER 3 1 FEBRUARY 1983

Buchmüller-Grunberg-Tye-type potential consistent with smaller values of $\Lambda_{\overline{MS}}$

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(Received 16 September 1982)

The Buchmüller-Grunberg-Tye potential for heavy quarkonia is modified so that it is consistent with the smaller values of the scale parameter $\Lambda_{\overline{MS}}$, 50–100 MeV, as suggested recently. ($\overline{MS}$ denotes the modified minimal-subtraction scheme.) The potential has a practically asymptotic region above 2–3 GeV in momentum space, or at distances shorter than 0.05–0.1 fm. This is consistent with the experimentally measured spectra of ψ and Υ families and their leptonic decay widths. It is shown that the spectra of $t\bar{t}$ states would provide a nice way to determine $\Lambda_{\overline{MS}}$ in the range quoted above.

A number of models for the interquark potential have emerged so far[1–5] and they are equally successful in reproducing observed mass spectra and leptonic widths for heavy quarkonia. Among them, the potential presented by Buchmüller, Grunberg, and Tye[4,5] (BGT) is the most interesting since it is consistent with QCD, the only candidate for the strong interaction. The BGT potential is constructed in the following way. The Fourier transform of the potential defines an effective coupling constant $\rho(Q^2)$ (for momentum transfer $\vec{Q}$, $Q^2 \geq 0$). The Q^2 dependence of ρ can be determined by $Q^2\partial\rho/\partial Q^2=\beta(\rho)$, the renormalization-group equation (RGE), while its behavior at small and large Q^2 are, respectively, characterized by the string constant K and the QCD scale parameter Λ. If $\beta(\rho)$ is known, the potential can be determined completely (up to an integration constant) by integrating the RGE. Buchmüller and others[4] assumed intuitively a special form with a single parameter, say l, for the β function. Since Λ and K are two alternates of the integration constant, the BGT potential has only two independent parameters l and Λ, or, equivalently, Λ and $K=(2\pi\alpha')^{-1}$, where α' is the Regge slope measured in light-hadron spectroscopy. With $\alpha'=1.0$ GeV^{-2} ($\sqrt{K}=400$ MeV) BGT found that $\Lambda_{\overline{MS}}$ should be around 500 MeV to reproduce experimental data on quarkonia. ($\overline{MS}$ denotes the modified minimial-subtraction scheme.) They also claimed that the calculated spectra are very sensitive to the β function and the correct β function will be numerically very close to theirs.

The suggested values of $\Lambda_{\overline{MS}}$ in other analyses are, however, much smaller than 500 MeV. The Monte Carlo simulation by Creutz[6] gives a relation between K and Λ_L, where Λ_L is the scale parameter defined on a lattice. When $\alpha'=1.0$ GeV^{-2}, Λ_L becomes 2.0 MeV for SU(3), which corresponds to $\Lambda_{\overline{MS}}$ of 60 ± 20 MeV in the pure gauge theory.[7] The analyses[8] of the ratio of gluonic to leptonic widths in ψ and Υ decays lead to the values of 100 ± 30 MeV for $\Lambda_{\overline{MS}}$. In the analyses of deep-inelastic scattering and of three-jets cross sections most of the authors found small values of $\Lambda_{\overline{MS}}$, 160 ± 100 MeV.[9,10]

Therefore, it is important to investigate whether the BGT-type potential can be constructed which is compatible with the smaller values of $\Lambda_{\overline{MS}}$ but still consistent with the data on quarkonia. In this Brief Report we will show that a simple modification of the BGT β function is sufficient to solve the problem.

According to BGT, the Fourier transform $\tilde{V}(Q^2)$ of the static potential between a heavy-quark–antiquark pair in the color-singlet state is given in terms of the effective coupling constant $\rho(Q^2)$ by

$$\tilde{V}(Q^2)=-\frac{64\pi^2\rho(Q^2)}{3Q^2}\ , \qquad (1)$$

where $\rho(Q^2)$ satisfies the RGE mentioned earlier. The β function and hence ρ are given in the following way in this paper. First we assume that we are in an asymptotic-free region at least for Q larger than $Q_0 \simeq 10$ GeV, the production threshold of a $b\bar{b}$ pair. This implies that the β is well approximated by

$$\beta(\rho)=-\rho^2(B_0+B_1\rho) \quad \text{for } Q \geq Q_0\ , \qquad (2)$$

where $B_0=\frac{25}{3}$ and $B_1=\frac{154}{3}$ are the renormalization-scheme-independent coefficients of β for QCD with four flavors. ρ is then given by

$$\ln\left[\frac{Q^2}{\Lambda^2}\right]=\frac{1}{B_0\rho}+\frac{B_1}{B_0^2}\ln\left[\frac{B_0\rho}{1+(B_0/B_1)\rho}\right] , \quad (3)$$

where $\Lambda=1.424\Lambda_{\overline{\rm MS}}$ is the scale parameter appropriate to the potential.

For large values of ρ corresponding to Q much smaller than Q_0, the β is assumed to be very close to that of BGT, i.e.,

$$\frac{b_0}{\beta(\rho)}\simeq\frac{b_0}{\beta_{\rm BGT}}=-\frac{1}{\rho^2}\left[1-\exp\left[-\frac{1}{b_0\rho}\right]\right]^{-1}+\frac{b_1}{b_0\rho}\exp(-l\rho) , \quad (4)$$

where $b_0=9$ and $b_1=64$ are the coefficients of the β for three flavors. The parameter l has been fixed as 24 in Refs. 4 and 5 for the best fit; however, it will be readjusted to reproduce the experimental data on the quarkonia in a new β given below. In the transitional region, $Q_0/10\leq Q\leq Q_0$, β must deviate appreciably from $\beta_{\rm BGT}$ to make ρ with $\sqrt{K}=400$ MeV continuous to that given in (3) when $\Lambda_{\overline{\rm MS}}$ is much smaller than 500 MeV. Therefore, we put

$$\frac{1}{\beta(\rho)}=\frac{1}{\beta_{\rm BGT}(\rho)}+\lambda f(\rho) \text{ for } Q\leq Q_0 , \quad (5)$$

where $f(\rho)$ is a Gaussian-type function which is normalized as $\int_0^\infty f(x)\,dx=1$ and has appreciable magnitude only in the transition region. As an illustration, we consider

$$f(x)=\tfrac{1}{3}G^4x^7\exp(-Gx^2) , \quad (6)$$

which has a peak at $x=\sqrt{7/2G}$ with half width $\Delta x\simeq 1.1/\sqrt{G}$. With G in the range 7000–20 000, ρ at the peak corresponds to Q falling in the range 1–10 GeV when $\Lambda_{\overline{\rm MS}}=50$–100 MeV (see Fig. 1). The parameter λ in (5) is not free but, in fact, is fixed when K, $\Lambda_{\overline{\rm MS}}$, l, and G are settled, as will be discussed later.

Integrating the RGE with β given in (5), we obtain

$$\ln\left[\frac{K}{Q^2}\right]=\ln\left[\frac{8\pi\rho}{3}\right]+\int_\rho^\infty\left[\frac{1}{x}+\frac{1}{\beta(x)}\right]dx=-\ln\left\{\frac{3b_0}{8\pi}\left[\exp\left[\frac{1}{b_0\rho}\right]-1\right]\right\}+\frac{b_1}{b_0^2}E_i(l\rho)+\lambda h(\rho) \text{ for } Q\leq Q_0 , \quad (7)$$

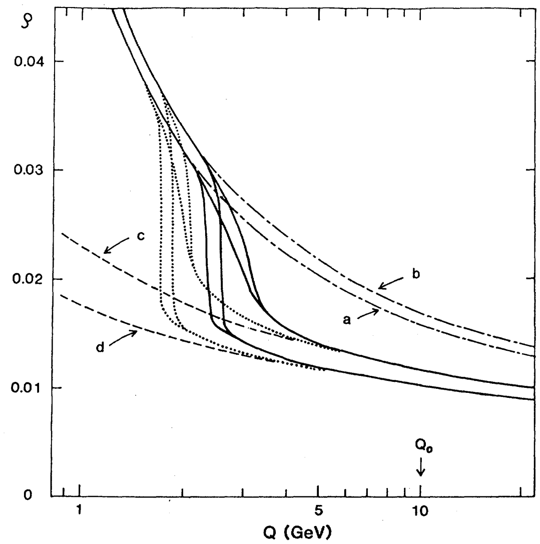

FIG. 1. ρ vs Q in the transitional region. The solid curves represent the effective coupling constants with $G=10^4$ and the dotted curves denote those with $G=7\times10^3$. The dot-dashed curves express ρ in BGT with (a) $l=24$ and (b) $l=48$, respectively. The dashed curves show the extrapolated values of ρ given by (3) with (c) $\Lambda_{\overline{\rm MS}}=100$ MeV and (d) 50 MeV, respectively. $\sqrt{K}$ is fixed at 400 MeV throughout.

with

$$h(\rho)=\left[1+z+\frac{z^2}{2}+\frac{z^3}{6}\right]\exp(-z)\Big|_{z=G\rho^2} ,$$

where E_i is the exponential integral. Since both of ρ's calculated by (3) and (7) coincide with each other at $Q=Q_0$, we have

$$\lambda=\frac{1}{h(\rho_0)}\ln\left\{\frac{3b_0K}{8\pi Q_0^2}\left[\exp\left[\frac{1}{b_0\rho_0}-1\right]\right]-\frac{b_1}{b_0^2}E_i(l\rho_0)\right\} , \quad (8)$$

where ρ_0 is the value of ρ estimated at $Q=Q_0$ by (3).

It is easy to obtain numerically the effective coupling constant $\rho(Q^2)$ through (3) or (7). The results are shown in Fig. 1 when $\Lambda_{\overline{\rm MS}}=50$ and 100 MeV. Other parameters are chosen as $l=24$ or 48, $G=7000$ or 10 000, and $\sqrt{K}=400$ MeV. The dashed curves in the figure are the values of ρ which are extrapolated from the asymptotic form given by (3). A region of Q may be regarded as practically asymptotic where ρ can be approximated by its asymptotic extrapolation within a few precent. The lower boundary Q_c of the region is estimated in Fig. 1 as 2.5–3.5 GeV for $\Lambda_{\overline{\rm MS}}=100$ MeV and 1.8–2.5 GeV for $\Lambda_{\overline{\rm MS}}=50$ MeV, when G is in the range (7×10^3)–10^4 and l is 24–48. The point of interest is whether we can find Q_c less than Q_0 without contradiction to the data of quarkonia, or how small it could be if it is found anyway.

The potential $V(r)$ in coordinate space is given by

$$V(r)=\frac{1}{(2\pi)^3}\int d^3Q\, e^{i\vec{Q}\cdot\vec{r}}\,\tilde{V}(Q^2)$$

$$=Kr-\frac{32}{3r}\int_0^\infty dQ\frac{\sin Qr}{Q}\left[\rho(Q^2)-\frac{3K}{8\pi Q^2}\right] . \qquad (9)$$

The results by numerical computation for $\Lambda_{\overline{\rm MS}}=50$ or 100 MeV are shown in Fig. 2 with $G=10^4$ and $l=48$, the best choice of the parameters, besides the one given by BGT, for comparison. We have also confirmed that the potential with $G\geqslant 10^4$ is very close to $V_{\rm BGT}$, the potential given by BGT, for $r\geqslant 0.1$ fm when $l\simeq 50$, and that the potential with $G\leqslant 7000$ deviates from $V_{\rm BGT}$ appreciably for $r\leqslant 0.2$ fm even wtih the careful choice of l in the reasonable range.

The mass spectra of ψ and Υ families calculated through the potentials given in Fig. 2 are tabulated in Tables I and II. It is found that the predicted spectra are as satisfactory as those of BGT, or even better. With $G=7000$, however, the computed masses of the Υ family coincide with the experimentally measured ones only within about 40 MeV even for the best choice of $l\simeq 50$. This implies that the lowest possible value of Q_c is about 2.0 or 3.0 GeV, respectively, for $\Lambda_{\overline{\rm MS}}=50$ or 100 MeV.

In the framework of the potential model the lep-

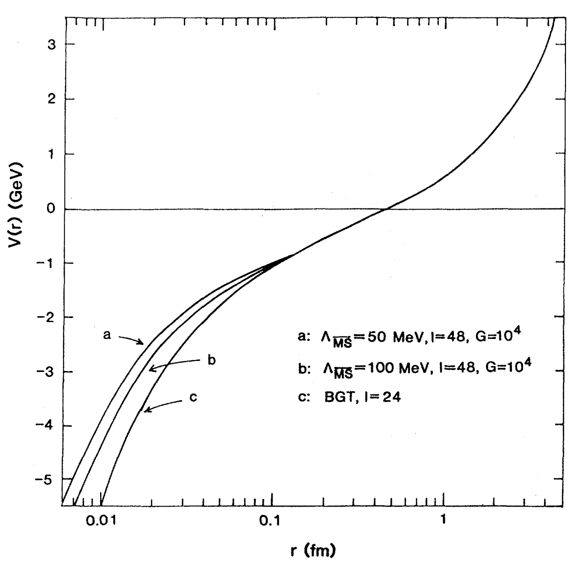

FIG. 2. Potentials corresponding to Eq. (9) and the potential of BGT. The values of the parameters are shown in the figure.

TABLE I. The ψ spectrum obtained from Eq. (9) with $G=10^4$ and $l=48$. The masses are given in GeV. The results quoted as BGT which are recalculated coincide with those that appeared in Refs. 4 and 5. In parentheses, the values of the wave functions at the origin are given in $\mathrm{GeV}^{3/2}$. The experimental results of Ref. 11 are also summarized.

	Present model		BGT	Expt.
m_c (GeV)	1.42		1.48	
$\Lambda_{\overline{\rm MS}}$ (MeV)	50	100	509	
$1S$	3.10 (0.233)	3.10 (0.239)	3.10 (0.246)	3.097 ±0.001
$1P$	3.52	3.52	3.52	3.521 ±0.003
$2S$	3.70 (0.187)	3.70 (0.193)	3.70 (0.200)	3.685 ±0.003
$1D$	3.82	3.82	3.81	3.768 ±0.003
$2P$	3.97	3.97	3.97	
$3S$	4.13 (0.173)	4.13 (0.178)	4.12 (0.186)	4.030 ±0.005
$2D$	4.20	4.20	4.19	4.159 ±0.020
$3P$	4.34	4.34	4.33	
$4S$	4.47 (0.168)	4.47 (0.172)	4.48 (0.178)	4.415 ±0.006

TABLE II. The Υ spectrum obtained from Eq. (9) with $G=10^4$ and $l=48$. The masses are given in GeV. The results quoted as BGT which are recalculated coincide with those that appeared in Refs. 4 and 5. In parentheses, the values of the wave functions at the origin are given in $\mathrm{GeV}^{3/2}$. The experimental results of Ref. 11 are also summarized.

	Present model		BGT	Expt.
m_b (GeV)	4.90	4.91	4.88	
$\Lambda_{\overline{\rm MS}}$ (MeV)	50	100	509	
$1S$	9.44 (0.578)	9.44 (0.615)	9.46 (0.684)	9.4332 ±0.0002
$1P$	9.84	9.85	9.89	
$2S$	9.99 (0.420)	9.99 (0.441)	10.02 (0.486)	9.9932 ±0.0003
$1D$	10.09	10.11	10.14	
$2P$	10.20	10.22	10.25	
$3S$	10.32 (0.383)	10.33 (0.394)	10.35 (0.427)	10.3235 ±0.0004
$2D$	10.38	10.40	10.43	
$3P$	10.48	10.50	10.53	
$4S$	10.58 (0.356)	10.59 (0.369)	10.62 (0.398)	10.5462 ±0.001

tonic widths of the S states are given by

$$\Gamma_{ee}=\frac{16\pi e_q{}^2\alpha^2}{M_n{}^2(q\bar{q})}|\psi_n(0)|^2[1-\tfrac{64}{3}\rho((2m_q)^2)] \ , \qquad (10)$$

with obvious notations.[12] For Υ in the ground state, the width Γ_{ee} is not so suppressed in spite of the decrease of $\psi(0)$ in comparison with $\psi(0)$ of BGT (cf. Table II), since the radiative correction is also reduced due to the decrease of ρ at $Q=9.5$ GeV (cf. Fig. 1). Thus we estimate the values of Γ_{ee} as

$$\Gamma_{ee}(\Upsilon)=\begin{cases}0.91 \text{ keV for } \Lambda_{\overline{\rm MS}}=50 \text{ MeV} \ ,\\ 0.98 \text{ keV for } \Lambda_{\overline{\rm MS}}=100 \text{ MeV} \ ,\end{cases}$$

which are both consistent with current experiments.[8] For ψ we predict $\Gamma_{ee}(\psi)=5.2$ or 5.0 keV for $\Lambda_{\overline{\rm MS}}=50$ or 100 MeV, respectively. However, these estimates depend on the choice of G because $\rho(Q^2)$ is sensitive to the value of G near $Q=3.1$ GeV (cf. Fig. 1).

It is expected that the analyses of the spectra of quarkonia heavier than Υ allow us to select the best among the potentials with different asymptotes. As an illustration, we have tabulated in Table III the computed mass spectra of the $t\bar{t}$ states, assuming $m_t=25$ GeV. We see that any of the excitation energies increases by an amount of 70–100 MeV when $\Lambda_{\overline{\rm MS}}$ changes from 50 to 100 MeV, which provides a good way to determine the scale parameter.

In conclusion, we have constructed a BGT-type potential, which is in harmony with the small value of $\Lambda_{\overline{\rm MS}}$ in the range 50–100 MeV but still consistent with the data on ψ and Υ, and with the Regge slope of 1.0 GeV^{-2}. The potential has the asymptotic region in Q space above $Q_c=2.5-3.5$ GeV or at distances shorter than $r_c=Q_c^{-1}=0.06$–0.08 fm, while it deviates abruptly from its asymptote below Q_c. The reason for such an abrupt "perturbative-to-nonperturbative transition is an open question. Anyhow, the investigation of $t\bar{t}$ spectra would provide an accurate determination of $\Lambda_{\overline{\rm MS}}$ in the range mentioned above.

TABLE III. The mass differences of $t\bar{t}$ states obtained from Eq. (9). The mass of the top quark is assumed to be 25 GeV. The results quoted as BGT which are recalculated coincide with those that appeared in Refs. 4 and 5. In parentheses, the values of the wave functions at the origin are given in GeV$^{3/2}$.

Present model		$m_t=25$ GeV	BGT
$\Lambda_{\overline{\rm MS}}$ (MeV)	50	100	509
$2m_t-1S$	0.81 (2.38)	0.91 (2.77)	1.06 (3.66)
$1P-1S$	0.39	0.46	0.62
$2S-1S$	0.51 (1.72)	0.58 (1.86)	0.62 (2.07)
$1D-1S$	0.62	0.72	0.90
$2P-1S$	0.73	0.82	0.97
$3S-1S$	0.81 (1.42)	0.90 (1.48)	1.04 (1.63)
$2D-1S$	0.89	0.98	1.14
$3P-1S$	0.96	1.05	1.20
$4S-1S$	1.02 (1.22)	1.11 (1.29)	1.26 (1.42)

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