Moving bag and baryon magnetic moments

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It is shown that the recoil effect contributes about 30% to baryon magnetic moments calculated using the static bag model. Boosting of a stationary system with substructure seems to be an important ingredient in this particular example. Connections with other quark models are also clarified.

The ratios of baryon magnetic moments in the static MIT bag model follow the well-known pattern based on SU(6).[1,2] However, their absolute values seem to be too small if the model radius is fixed in such a way as to give a correct mass spectrum.[2] This discrepancy is caused, to some extent, by the stationary character of the model; it can be partially removed by the application of a suitable boost.[3-5] Such an effect was studied in Ref. 5, where an increase in proton magnetic moment was mentioned. In this Brief Report we want to give a detailed calculation of this contribution. We have found that the value of the proton magnetic moment $\mu(p)$ is increased by $\frac{3}{5}\mu_N$ ($1\mu_N = e/2M$, where M is the proton mass). This 30% increase makes the calculated value much closer to the experimental one.

Let us start with a general decomposition of the electromagnetic-current matrix element, such as

$$\langle \vec{p}_2 | J_\mu(0) | \vec{p}_1 \rangle = \bar{U}(\vec{p}_2)[\gamma_\mu F_1(q^2) + i(\sigma_{\mu\nu}q^\nu/2M)F_2(q^2)]U(\vec{p}_1)\ , \quad q = p_2 - p_1\ , \tag{1a}$$

which, in the Breit frame of reference, turns into

$$\left\langle \frac{\vec{q}}{2} \middle| (J_0, \vec{J}) \middle| -\frac{\vec{q}}{2} \right\rangle = \chi^\dagger \left(\frac{M}{E} G_E(\vec{q}^{\,2}),\ i\frac{\vec{\sigma}\times\vec{q}}{2E} G_M(\vec{q}^{\,2}) \right) \chi\ , \quad G_E = F_1 - \frac{\vec{q}^{\,2}}{4M^2}F_2\ , \quad G_M = F_1 + F_2\ . \tag{1b}$$

Here p, E, and M are the momenta, energies, and masses of baryons, respectively. The magnetic-moment contribution G_M is multiplied by the baryon momentum transfer $\vec{q}$. One expects that boosting baryon bags should give contributions to their magnetic moments because of the motion of the bag as a whole. The internal motion inside the bag is superimposed on the recoil motion and this superposition results in the final baryon magnetic moment. As indicated by Eqs. (1), the recoil or more precisely the momentum transfer q appears in any measurement of G_M. This is best visualized in the limit in which the internal motion of quarks is neglected. This means that each quark should be simply described by a two-component spinor χ. Boosting a two-component spinor produces a four-component spinor U. The interplay of the large (s-wave) component of this spinor with the small (p-wave) component leads to a result which is proportional to $\vec{\sigma}\times\vec{q}$. In the interaction with the electromagnetic potential $\vec{A}$, each quark acquires a magnetic moment which is a fraction (Q_i) of the magnetic moment of $1\mu_N$:

$$Q_i \frac{e}{2M}(i\,\vec{\sigma}_i \times \vec{q})\vec{A} \rightarrow \frac{Q_i e}{2M}\vec{\sigma}_i \cdot \vec{B}\ . \tag{2}$$

In nonrelativistic quark models, the factor $Q_i e/2M$ is replaced by either the anomalous magnetic moment $\mu_i(q)$ (Ref. 6) or the factor $Q_i e/2m_i$ (Refs. 7,8). The choice of $\mu_i(q)$ or m_i is made to fit experimental baryon magnetic moments. Combination of quark contributions (2) gives the proton a magnetic moment of $1\mu_N$, as one would naively expect for a charged particle with spin $\frac{1}{2}$. However, the corresponding neutron magnetic moment is not zero but $-\frac{2}{3}\mu_N$.

Boost of the bag-model wave function which already contains small (p-wave) components $\nu\,\vec{\sigma}\cdot\vec{r}_0$ [see Eq. (3)] leads to a complicated interplay of the internal motion and the recoil.

The preceding considerations can be easily demonstrated using the standard bag-model wave functions ψ,[2]

$$\psi=\begin{pmatrix} iu(r) \\ v(r)\vec{\sigma}\cdot\vec{r}_0 \end{pmatrix} = \frac{N(\omega)}{\sqrt{4\pi}} \begin{pmatrix} ij_0(pr) \\ -\left(\frac{\omega-mR}{\omega+mR}\right)^{1/2} j_l(pr)\vec{\sigma}\cdot\vec{r}_0 \end{pmatrix}, \tag{3}$$

and employing the boost operator B,

$$B(\vec{p})=\left(\frac{E+M}{2E}\right)^{1/2}\begin{pmatrix} 1 & \frac{\vec{\sigma}\cdot\vec{p}}{E+M} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M} & 1 \end{pmatrix}. \tag{4}$$

For the matrix element defined by (1) one finds

$$\left\langle \frac{\vec{q}}{2}\left|\vec{J}(0)\right|-\frac{\vec{q}}{2}\right\rangle = \sum_{i=l,s}\eta_i \int_{\text{bag}} d^3r\, e^{i\vec{q}\cdot\vec{r}}\psi_i^\dagger(r)B^\dagger\left(\frac{\vec{q}}{2}\right)\vec{\alpha}B\left(-\frac{\vec{q}}{2}\right)\psi_i(r). \tag{5}$$

Here the summation runs over various quark flavors weighted by the factor η_i according to their abundances in a particular hyperon. In deriving Eq. (5), we imposed translational invariance, as done in Ref. 5 where one can find a lengthly derivation of an analogous formula. For a general momentum transfer $\vec{q}$, the integration due to the Lorentz contraction goes over the ellipsoidal region. In the static case ($\vec{q}\to 0$), the region of integration is spherical.[9] The old value of the magnetic moment[1,2] is obtained by neglecting the boost, i.e.,

$$\langle 0|\vec{J}|0\rangle(\vec{q})=\sum_i \eta_i \int_{\text{bag}} d^3r\, e^{i\vec{q}\cdot\vec{r}}\psi_i^\dagger(r)\vec{\alpha}\psi_i(r). \tag{6}$$

For the proton, one finds

$$\langle 0|\vec{J}|0\rangle_p=-\left(\chi^\dagger i\frac{\vec{\sigma}\times\vec{q}}{2M}\chi\right)4M\lim_{\vec{q}\to 0}\int_{\text{bag}} d^3r\, uv\frac{j_1(qr)}{q}$$
$$=\chi^\dagger i\frac{\vec{\sigma}\times\vec{q}}{2M}\chi G_{M,S}(0), \tag{7}$$
$$G_{M,S}(0)=\mu_S(p)=-\tfrac{4}{3}M\int_{\text{bag}} d^3r\, ruv.$$

By applying the boost, one obtains the proton magnetic moment

$$G_M(0)=\mu(p)=\int_{\text{bag}} d^3r\left[-\tfrac{4}{3}Mruv+(u^2-\tfrac{1}{3}v^2)\right]. \tag{8}$$

Expression (8) also contains the contribution (2), which is due to the recoil of the bag as a whole. This can be easily deduced by replacing the wave function ψ in Eq. (5) by a spinor χ. The spatial integration and the exponential factor should also be omitted. This leads to the following replacements in Eq. (8): $v\to 0$, $\int d^3r\, u^2\to 1$, i.e., $\mu(p)=1\mu_N$ and $\mu(n)\to -\frac{2}{3}\mu_N$. The piece $(u^2-\frac{1}{3}v^2)$ in (8) is obtained by omitting the exponential in (5). Since baryon states retain SU(6) symmetry, the ratio of neutron to proton magnetic moment does not change even when the full expression (5) is used. The other hyperon magnetic moments deviate slightly from SU(6) predictions because of flavor-symmetry breaking.

The boost (3) is closely connected with the operator U defined in Ref. 4. Thus, the whole discussion presented in this reference applies to our consideration. In our approach, some modifications are needed which would take into account both the small components coming from the internal motion of quarks and the small components coming from the boost.

In Table I we compare the predicted and experimental values for baryon magnetic moments. One has to take into account differences between baryon masses and bag radii. This is obvious when the general formula (1) is combined with the particular expression (8). For a hyperon Y, the magnetic moment is

$$\mu(Y)=\frac{M}{M_Y}\sum_{i=l,s}\kappa_i(Y)\int_{\text{bag}(Y)} d^3r\left[-\tfrac{4}{3}M_Y r u_i v_i + (u_i^2-\tfrac{1}{3}v_i^2)\right]. \tag{9}$$

Here κ_i weights the contribution of light ($l=u,d$) and strange (s) quarks in a given baryon (see Table I). The calculated values are in reasonable overall qualitative agreement with experimental data, particularly with the recent value for $\mu(\Xi^-)$.[11] The agreement is the poorest for $\mu(\Sigma^+_-)$. A small change in bag-model proton radius, namely, an increase from $R=5$ GeV^{-1}, used by Ref. 2, to $R=5.6$ GeV^{-1}, can produce a correct value for $\mu(p)$. One cannot attach too much significance to such a fit, because the model is still rather crude. A more sophisticated approach should also include additional effects, such as pion-cloud contributions.[12]

TABLE I. Magnetic moments of baryons (in units of μ_N).

Baryon	(κ_l, κ_s)	μ	Expt.	Ref.
p	(1,0)	2.5511	2.793	10
n	$(-\frac{2}{3},0)$	−1.7009	−1.913	10
Λ	$(0,-\frac{1}{3})$	−0.7031	−0.614 ± 0.005	10
Σ^+	$(\frac{8}{9},\frac{1}{9})$	2.3584	2.33 ± 0.13	11
Σ^-	$(-\frac{4}{9},\frac{1}{9})$	−0.8336	−1.48 ± 0.37	11
Ξ^0	$(-\frac{2}{9},-\frac{4}{9})$	−1.4077	−1.253±0.014	11
Ξ^-	$(\frac{1}{9},-\frac{4}{9})$	−0.6292	−0.75±0.07	11

It is important to mention that the zero-momentum-transfer value of the axial-vector form factor G_A is not influenced by boosting. This is to be expected because, in the Breit frame of reference, the factor G_A contributes even in the $q \to 0$ limit, i.e.,

$$\langle \vec{p}_2 | A_\mu | \vec{p}_1 \rangle \to \chi^\dagger \left[\vec{\sigma} G_A(\vec{q}^2) - q(\vec{\sigma}\cdot\vec{q}) \frac{1}{2E} \left(\frac{G_A(\vec{q}^2)}{2(E+M)} + H_A(\vec{q}^2) \right) \right] \chi \ . \qquad (10)$$

Thus, with the inclusion of boost or with no boost included,[2,13] we obtain

$$G_A(0) = g_A = \frac{5}{3} \int_{\text{bag}} d^3r \, (u^2 - \tfrac{1}{3} v^2) \ . \qquad (11)$$

There is an interesting relation between $\mu(p)$ and g_A which can be extracted from Eq. (8) for the proton as follows:

$$\mu(p) = \mu_S(p) + \tfrac{3}{5} g_A \ . \qquad (12)$$

This sum rule shows the importance of boosting or of recoil corrections.

In concluding this paper, we wish to say that boosting helps produce form factors with a reasonable $\vec{q}^2$ dependence[5] and contributes a very useful increase to magnetic moments. It does not spoil other bag-model results.

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