Soft-pion emission in $\bar{p}p$ annihilations

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Previous soft-pion studies of proton-antiproton annihilations into kaons and pions are extended to annihilations that produce exclusively pions. It is argued that the expressions derived earlier can be used to compute the values of branching ratios for processes such as $\bar{p}p \rightarrow \pi^+\pi^-\pi^0+2\pi^0$. The results are compared with the available experimental data.

Within the past decade, the methods of current algebra and the partially conserved axial-vector current (PCAC) hypothesis for pions have been fairly successful in dealing with various $\bar{p}p$ annihilation processes.[1–7] The several final states including those with additional pions admitted by the $\bar{p}p$ annihilation even at rest make the processes particularly suitable for the application of the formalism. In most of the reactions considered, $\bar{p}p \rightarrow KK$ is related to the ones with one or more additional soft pions. This kind of relationship between two reactions is a general feature of current-algebra applications. Another reason for studying the $\bar{p}p$ annihilation processes is that the annihilation at rest occurs in S state and in the past current-algebra predictions for S waves were in good agreement with experiment.

$\bar{p}p$ annihilations can produce hyperons, kaons, and pions, and pions alone. Over 90% of the time, annihilation at rest produces only pions.[8] Treating these all-pion final states within the current-algebra—PCAC formalism creates certain difficulties. Ideally all the pions must be treated as soft. But when the pion number is four or more it is nearly impossible to maneuver the derivatives through time-ordered products. Also in a reaction such as $\bar{p}p \rightarrow 4\pi$, if all the pions are considered soft, the problem of relating the reaction with a suitable one arises. One can, however, treat two of the pions as soft and relate it to $\bar{p}p \rightarrow 2\pi$. This would of course mean that the pions are given an unsymmetrical treatment.

In the $\bar{p}p$ annihilations studied previously, the central interaction has usually the form

$$\mathfrak{M}=A+B\gamma\cdot Q \quad , \tag{1}$$

where $Q=q_1-q_2$, the difference in kaon momenta. That is, in processes like $\bar{p}p \rightarrow KK+n\pi$, where n is the number of pions, the kaons form part of the external final state and only pions are treated through PCAC as soft. It is interesting to note that the form of $\mathfrak{M}$ does not change even when the kaons are replaced with pions.

If i and f are hadronic states, the matrix element for the process

$$i \rightarrow f+\pi \tag{2}$$

appears in the well-known reduction formula

$$ik_\mu\langle f|A_\mu^i|i\rangle=\langle f|\partial_\mu A_\mu^i|i\rangle \quad , \tag{3}$$

where $i=1,2,3$ are the isospin indices of the axial-vector current operator A_μ. According to PCAC for pions,

$$\partial_\mu A_\mu^i=(1/\sqrt{2})C_\pi\phi_\pi^i \quad , \tag{4}$$

with

$$C_\pi=\sqrt{2}G_A M_N\mu^2/g_r(0) \quad ; \tag{5}$$

$G_A \simeq 1.18$, g_r is the rationalized, renormalized pion-nucleon coupling constant ($g_r^2/4\pi \simeq 14.6$), ϕ_π is the renormalized pion-field operator, and M_N and μ are the nucleon and pion masses. Introducing Klein-Gordon operators in (3), we have

$$ik_\mu\langle f|A_\mu^i|i\rangle=\frac{M_N G_A\mu^2}{g_r(\mu^2+k^2)}\langle f|(\mu^2-\Box)\phi_\pi^i|i\rangle \quad . \tag{6}$$

One has to investigate this equation in the limit $k\rightarrow 0$. As $k\rightarrow 0$, the right-hand side approaches $M_N G_A/g_r$ times the matrix element for the emission of a zero-four-momentum pion and the left-hand side vanishes unless it has pole terms. For reactions of the type (2) considered, the matrix element is of zeroth order in pion momenta.[9] So we look for pole terms that go as k^{-1} and these arise when the axial-vector current is attached to the external line that does not terminate.[10] Insertion of A_μ^i into a pseudoscalar-meson line is forbidden by parity. If $i=\bar{p}p$ and $f=KK$, then one has to consider insertions in the baryon-antibaryon lines only. The situation does not change if the kaons are replaced with pions.

The argument extends to any number of axial-vector-current insertions which correspond to the soft pions in a reaction of the type (2). Therefore we can use the matrix elements derived earlier for various

TABLE I. Values of branching ratios for $R_1 = w(\bar{p}p \to \pi^+\pi^-\pi^0 + 2\pi^0(\text{soft}))/w(\bar{p}p \to \pi^+\pi^-\pi^0)$, $R_2 = w(\bar{p}p \to \pi^+\pi^- + 3\pi^0(\text{soft}))/w(\bar{p}p \to \pi^+\pi^-)$, and $R_3 = w(\bar{p}p \to \pi^+\pi^- + 4\pi^0(\text{soft}))/w(\bar{p}p \to \pi^+\pi^-)$.

$\bar{p}$ laboratory momentum (GeV/c)	Center-of-mass energy (GeV)	R_1	R_2	R_3
0.0	1.876	0.337	2.686	
0.5	1.938	0.414	3.541	
0.686	1.985		4.338	
0.7	1.989	0.488		0.619
1.0	2.082	0.645	6.705	1.684
1.5	2.254	1.034	17.179	
1.61	2.293			2.785
2.0	2.430	1.589	32.936	4.340
2.5	2.602	2.316	52.872	6.640
3.0	2.767	3.462	85.063	9.770
3.5	2.926	4.838		13.860
4.0	3.077	6.395	173.800	19.010

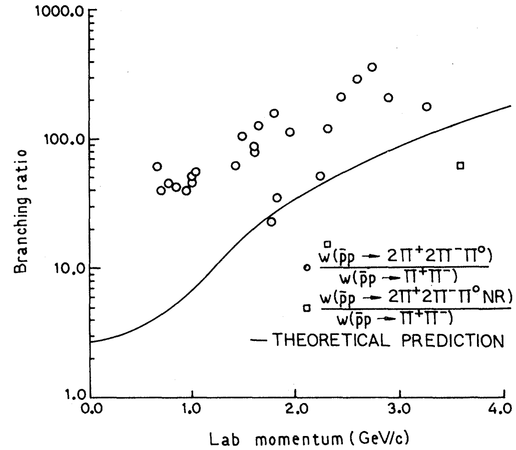

FIG. 2. Comparison of the theoretical prediction of the branching ratio, $R_2 = w(\bar{p}p \to \pi^+\pi^- + 3\pi^0(\text{soft}))/w(\bar{p}p \to \pi^+\pi^-)$, with experiment at various incident antiproton laboratory momenta. (NR indicates nonresonant, i.e., identified resonances are subtracted.)

soft-pion processes to find branching ratios such as

$$w(\bar{p}p \to n\pi)/w(\bar{p}p \to 2\pi) \ . \tag{7}$$

However, one has to keep in mind the various selection rules that operate for $\bar{p}p$ annihilation into various channels.

The spin and parity of the S states for a proton-antiproton system are 0^- and 1^- since fermion and antifermion possess opposite intrinsic parity. If we consider two π^0 mesons in the final state, they must have even angular momentum and therefore even parity. So $\bar{p}p \to 2\pi^0$ is forbidden. We, however, have $\bar{p}p \to \pi^+\pi^-$ and $\bar{p}p \to \pi^+\pi^-\pi^0$. Therefore we relate these processes to the ones with additional soft pions. The differential rates for the processes

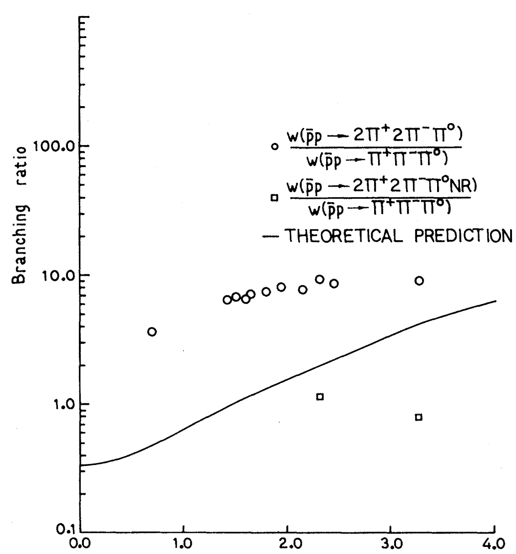

FIG. 1. Comparison of the theoretical prediction of the branching ratio, $R_1 = w(\bar{p}p \to \pi^+\pi^-\pi^0 + 2\pi^0(\text{soft}))/w(\bar{p}p \to \pi^+\pi^-\pi^0)$, with experiment at various incident antiproton laboratory momenta. (NR indicates nonresonant, i.e., identified resonances are subtracted.)

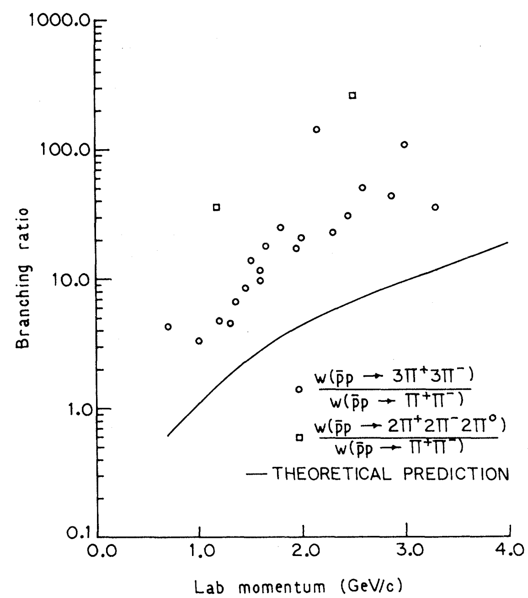

FIG. 3. Comparison of the theoretical prediction of the branching ratio, $R_3 = w(\bar{p}p \to \pi^+\pi^- + 4\pi^0(\text{soft}))/w(\bar{p}p \to \pi^+\pi^-)$, with experiment at various incident antiproton laboratory momenta.

$\bar{p}p \to K^+K^-\pi^0 + 2\pi^0$(soft), $\bar{p}p \to K^+K^- + 3\pi^0$(soft), and $\bar{p}p \to K^+K^- + 4\pi^0$(soft) recently studied by us[5–7] hold here with kaons replaced by pions. The limits of integration also are different. The various branching ratios thus obtained are given in Table I.

When compared with the available experimental results,[11] the values of branching ratios are small as shown in Figs. 1, 2, and 3. Through PCAC we assumed that the unphysical amplitude derived at zero-pion-momentum limit is slowly varying and valid for the physical "soft" pions. Even for $\bar{p}p$ annihilation at rest, the available center-of-mass energy is such that the pions produced are not necessarily soft. This is one of the sources of discrepancy between theoretical predictions and experiment. We have applied the current-algebra–PCAC formalism in a straightforward way and did not include any resonances such as ρ and ω. Also the experimental data is mostly for pions in charge states other than those considered here. A better comparison with the available experimental results would have ensued if, instead of the soft pions in neutral mode, charged pions were taken. Then the theoretical values of branching ratio would be enhanced due to the additional commutators present.[4]

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[1] R. A. Uritam, Phys. Rev. D 6, 3233 (1972).
[2] P. Nuthakki and R. A. Uritam, Phys. Rev. D 8, 3196 (1973).
[3] G. W. Intemann and G. K. Greenhut, Phys. Rev. D 10, 3653 (1974).
[4] G. K. Greenhut and G. W. Intemann, Phys. Rev. D 14, 764 (1976).
[5] R. Purushottamudu and P. Nuthakki, Phys. Rev. D 26, 2272 (1982).
[6] R. Purushottamudu and P. Nuthakki, J. Phys. G (to be published).
[7] R. Purushottamudu and P. Nuthakki, Z. Phys. C (to be published).
[8] C. Baltay *et al.*, Phys. Rev. 145, 1103 (1966).
[9] R. F. Dashen and M. Weinstein, Phys. Rev. 183, 1261 (1969).
[10] S. L. Adler, Phys. Rev. 139, B1638 (1965).
[11] R. Hamatsu *et al.*, Nucl. Phys. B123, 189 (1977); P. S. Eastman *et al.*, *ibid.* B51, 29 (1973); Y. Oren *et al.*, *ibid.* B71, 189 (1974); J. Debray *et al.*, *ibid.* B62, 13 (1973); J. Clayton *et al.*, *ibid.* B30, 605 (1971); R. R. Burns *et al.*, *ibid.* B85, 337 (1975); R. W. Green *et al.*, Phys. Rev. D 9, 90 (1974); T. C. Bacon *et al.*, *ibid.* 7, 577 (1973); N. Xuong *et al.*, Phys. Rev. 128, 1849 (1962); G. R. Lynch *et al.*, *ibid.* 131, 1276 (1963); C. K. Chen *et al.*, Phys. Rev. D 17, 42 (1978); T. Ferbel *et al.*, Phys. Rev. 143, 1096 (1966); 173, 1307 (1968); H. Nicholson *et al.*, Phys. Rev. D 7, 2572 (1973); C. Angelini *et al.*, Nuovo Cimento A32, 243 (1976); J. B. Gay *et al.*, *ibid.* A31, 593 (1976); J. A. Danysz *et al.*, *ibid.* A51, 801 (1967); High Energy Reactions Analysis Group, CERN Report No. HERA 79-03, 1979 (unpublished).