PHYSICAL REVIEW D VOLUME 27, NUMBER 3 1 FEBRUARY 1983

Three-preon models of quarks and leptons and the generation problem

Y. Tosa and R. E. Marshak
Department of Physics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061
(Received 27 August 1982)

We have carried out a search for three-fermion preon models that predict at least three generations of quarks and leptons. The conditions imposed are the following: (1) The preons are (massless) Weyl spinors and belong to low-dimensional chiral representations of the gauged symmetry group $G(\mathrm{MC})\times G(\mathrm{CF})$, where MC stands for metacolor and CF for color-flavor. (2) G(MC) is an asymptotically free simple group while G(CF) is a grand-unification-theory (GUT) or partial-unification-theory (PUT) group. (3) The Pauli principle holds when generalized to the MC degree of freedom. (4) No anomalies exist in the MC and CF sectors. (5) The composite quarks and leptons are massless on the MC scale. (6) There are no low-representation exotics and no mirror fermions. The only GUT preon models satisfying these six conditions are SU(3)(MC)$\times$SO(10)(CF) with four generations and F_4(MC)$\times$SO(10)(CF) with three generations; however, asymptotic freedom is marginal for the two GUT models. The only permissible PUT preon model is $E_6(\mathrm{MC})\times \mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ with three generations, and satisfactory asymptotic behavior. The PUT preon model is therefore the most promising and further implications are discussed.

I. INTRODUCTION

With the use of three generations of quarks and leptons, the standard gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)$ has been highly successful in explaining the strong, electromagnetic, and weak interactions of hadrons and the electromagnetic and weak interactions of leptons. The unbroken non-Abelian color group $\mathrm{SU}(3)_C$, with its property of asymptotic freedom, has not encountered any conceptual difficulties so far in accounting for the hadronic strong interaction (QCD theory). The same cannot be said of the spontaneously broken electroweak group $\mathrm{SU}(2)_L \times \mathrm{U}(1)$.[1]

In our view, there are a number of conceptual difficulties with the standard electroweak gauge group $\mathrm{SU}(2)_L \times \mathrm{U}(1)$. First, the acceptance of the purely left-handed group $\mathrm{SU}(2)_L$ implies a massless neutrino and *a fortiori* the surrender of the full quark-lepton correspondence for weak interactions (since there is no neutrino analog to the up quark in the right-handed representation). One of the striking developments in particle physics during the past decade has been the identification of parallel generations of quark and lepton doublets and we are reluctant to give up any facet of this correspondence. A second difficulty is that the weak-hypercharge generator associated with U(1) has no clear-cut physical meaning. This deficiency is reflected in the fact that the cancellation of the triangle anomalies associated with $\mathrm{SU}(2)_L \times \mathrm{U}(1)$, required for renormalizability, seems highly accidental.[2] A third difficulty, perhaps not unrelated to the second, is that when one writes down the Lagrangian for the standard electroweak theory, two conserved global quantum numbers emerge, namely, baryon number (B) and lepton number (L). However, as 't Hooft has proved,[3] instanton effects destroy the conservation of global B and global L separately, but do not affect the conservation of global $B-L$. It seems likely that there is a more natural explanation of this cancellation of instanton effects and that the role of $B-L$ is deeper than allowed by the standard electroweak theory.

It has been shown that one can overcome the conceptual difficulties enumerated above by enlarging the standard electroweak gauge group to $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$.[4] A right-handed neutrino joins the charged lepton of each generation in doublet representation of $\mathrm{SU}(2)_R$ and the full quark-lepton correspondence is restored. Furthermore, the weak-hypercharge generator becomes $B-L$, independent of the left- or right-handed character of the particle representation for quarks and leptons. It is then easy to prove that the cancellation of triangle anomalies and the cancellation of instanton effects for $B-L$ follow in a straightforward (and essentially trivial) way. The price paid for these desirable features of the left-right-symmetric (LRS) electroweak gauge group is the introduction of another triplet of weak bosons—the right-handed weak bosons—that must be more massive than the left-handed weak bosons. The new physical effects predicted by the LRS group must still be found[5] but, on balance, it appears that the LRS group is a serious alternative to the standard electroweak group (with the understanding that the former must break down to the latter at sufficiently low energies). In sum, for the purposes of this paper, we treat $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ on a par with $SU(3)_C \times SU(2)_L \times U(1)$ as a subgroup of any larger group that purports to unify the strong, electromagnetic, and weak interactions of all quarks and leptons.

Many larger unification groups—of which either $SU(3)_C \times SU(2)_L \times U(1)$ or $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ or both are subgroups—have been considered. If the larger group is characterized by a single coupling constant, we speak of grand unification theory (GUT); if by more than one, we speak of partial unification theory (PUT). Here, we briefly mention SU(5) and SO(10) GUT's (Ref. 6) and $SU(4)_C \times SU(2)_L \times SU(2)_R$ PUT (Ref. 7) and take note of some properties that will be useful for our later discussion. SU(5) GUT has $SU(3)_C \times SU(2)_L \times U(1)$ as its maximal subgroup,[6] and the left-handed quarks, leptons, and their antiparticles, all of a single generation, are fitted into the $\underline{5}^*$ and $\underline{10}$ representations. It has no room for neutrino mass, does not have $B-L$ as a generator, but it does predict global $B-L$ conservation.[8] If the neutrino turns out to be massive, some proponents of SU(5) GUT will accept SO(10) GUT passing through $SU(5) \times U(1)$.

Interestingly, SO(10) GUT can break down directly to the $SU(4)_C \times SU(2)_L \times SU(2)_R$ PUT group (which is its maximal semisimple subgroup), which in turn breaks down to the LRS group. The basic representation of SO(10) is $\underline{16}$ and this single representation accommodates all left-handed particles and antiparticles—including the left-handed antineutrino—of a single generation. Moreover, $B-L$ is now a generator of SO(10) and this property holds at the PUT level as well as at the electroweak level (LRS group). Since the GUT jump to SO(10) still leads to very high values of the unification mass, we leave our options open by considering PUT as well as GUT groups in our study of preon models.

Whatever be the merits of SU(5) GUT or SO(10) GUT or the Pati-Salam PUT group—or any other group larger than $SU(3)_C \times SU(2)_L \times U(1)$ or $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$—not one of these groups has given a credible clue to a solution of the generation problem for quarks and leptons. Whether one adjoins a discrete or continuous horizontal group[9] to the GUT or PUT group to deal with the "superfluous replication of generations,"[10] progress is difficult without some degree of "naturalness," freedom from anomalies, and absence of color or weak exotics belonging to low-dimensional representations. The composite (preon) model of quarks and leptons is another approach to the generation problem[11]; it has been argued that the proliferation of families of quark and lepton doublets is produced by radial or orbital excitations of the preon composites, *or* by adding preon pairs to a basic three-preon composite, *or* by some form of discrete symmetry: $M \sim \alpha^2\Lambda, \alpha\Lambda, \Lambda$ where α denotes the strength of the color interaction and Λ is the preon-binding mass scale.

We like the preon-model approach to the generation problem but fail to see how any of the above-cited arguments can predict the number of quark and lepton generations. It seems to us that group theory is a more promising approach to the preon model. Here one begins with a small number of chiral preon representations and constructs preon composites that can be identified with the quarks and leptons of at least three generations. If such a group-theoretic preon model can be found, dynamical calculations would then be justified.

We started such a program by searching for composite models of quarks and leptons with preons having the strong $SU(3)_C$ quantum numbers and either the electroweak $SU(2)_L \times U(1)$ *or* $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ quantum numbers.[12] The preon models considered were the simplest: *FFF* and *FB* (*F* is a spin-$\frac{1}{2}$ fermion and *B* is a spin-0 boson). Following the analogy of the SU(3) color force (supposed to bind quarks together into hadrons) not disturbing the flavor group, we assumed that the metacolor force, which is hypothesized to bind preons together into quarks and leptons, does not disturb the color nor the flavor group. Without any more explicit statement concerning the metacolor group, we imposed the following constraints on these models: (1) the absence of color or weak exotics belonging to low-dimensional representations, (2) cancellation of anomalies in the color and electroweak sectors, and (3) an equal number of quark and lepton generations. Under these conditions, we find a "no-go" result for all possible *FFF* and *FB* models.

In view of the negative result for composite models of quarks and leptons with preons being assigned the strong and electroweak quantum numbers, we have extended our search to larger color-flavor groups and have taken explicit cognizance of the metacolor degree of freedom.[13] We have limited our investigation to *FFF* models since *FB* models seem less attractive. More explicitly, we have tried to understand the existence of at least three generations of quark and lepton doublets by means of a three-fermion preon model under the following conditions: (1) the preons are (massless) Weyl spinors and belong to at most two low-dimensional representations of a gauged symmetry group; (2) the gauged symmetry group is $G(\mathrm{MC})\times G(\mathrm{CF})$ where G(MC) is the metacolor group (that produces a metacolor singlet out of three preons) and G(CF) is the color-flavor group; (3) only simple groups are considered for G(MC) which must be asymptotically free; (4) G(CF) is any one of the GUT groups with complex representations or the PUT group $\mathrm{SU(n)}_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ ($n=4$ gives the Pati-Salam group); (5) no anomalies exist in the metacolor and color-flavor sectors; (6) composites satisfy the generalization of the Pauli principle to the metacolor degree of freedom; (7) quarks and leptons are massless on the metacolor scale Λ_{MC} without the use of the 't Hooft condition[14]; and (8) there are no low-dimensional representation exotics and no mirror fermions.

We report our results in this paper. Having assigned Weyl spinors to the preon fields, we develop in Sec. II the notation for the decomposition rule for three Weyl spinor composites, using the Lorentz group $L_+^\uparrow \sim \mathrm{SU}(2)\text{-}L \times \mathrm{SU}(2)\text{-}R$. [One should not confuse this group with electroweak $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry.] In Sec. III we examine various possibilities for the metacolor group G(MC). If we limit ourselves to the basic representations of simple groups, then G(MC) can only be one of the following: SU(3), E_6, E_8, F_4, or G_2.[15] We also study the asymptotic freedom condition for these possible choices of G(MC).

We next turn to the color-flavor group G(CF). As we have remarked, we cannot use $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)$ nor $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ as G(CF). Therefore, our next step is to use larger groups. In Sec. IV we study the case where G(CF) is one of the GUT groups. In Sec. V, we examine the PUT group $\mathrm{SU}(n)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. In each case, we present concrete models which make definite predictions for the number of quark and lepton families. We employ the generalized Pauli principle to predict the number of families. Although the masslessness of the preon composites is guaranteed by the color-flavor group G(CF) which is used, it can also be assured by the discrete symmetry of preons, as we will see. Thus, the full chiral symmetry for masslessness of composites is not needed.[14]

The final section is devoted to conclusions and further questions to be resolved.

II. SPINS OF PREON COMPOSITES

In the case of the quark model of hadrons, the nonrelativistic picture, such as SU(6), worked well, although the use of the Schrödinger equation with a central potential reveals that it is applicable only for heavy quarks (charm, bottom, etc.). In the case of composite models of quarks and leptons, we must decide on one of two possibilities: one is that preons are massless and thus the use of Weyl spinors is most appropriate. The other is that preons are massive and the use of Dirac spinors is suitable. Here, we assume that preons are massless and are represented by Weyl spinors.

As is well known, the Compton wavelength of quarks and leptons is much larger than the present experimental limit on the size of quarks and leptons (10^{-16} cm).[16] The crude picture of hadrons based on relativistic dynamics, the bag model, does not work for composite models of quarks and leptons. Because of this lack of knowledge of the dynamics of composite models of quarks and leptons, we examine the spins of composites in the following simplified but relativistic way: we regard fermionic objects as spin $\frac{1}{2}$ if they transform as either $(\frac{1}{2},0)$ or $(0,\frac{1}{2})$ representations of the Lorentz group $L_+^\uparrow \sim \mathrm{SU}(2)\text{-}L \times \mathrm{SU}(2)\text{-}R$. Left-handed Weyl spinors transform as $(\frac{1}{2},0)$, while right-handed ones transform as $(0,\frac{1}{2})$.

If objects are composites of preons, we use the naive decomposition rule for products of representations of preons to find out the Lorentz properties of composites. For example,

$$LLL = (\tfrac{1}{2},0)\times(\tfrac{1}{2},0)\times(\tfrac{1}{2},0) = (\tfrac{3}{2},0)+2(\tfrac{1}{2},0)\,, \tag{2.1}$$

$$LRR = (\tfrac{1}{2},0)\times(0,\tfrac{1}{2})\times(0,\tfrac{1}{2}) = (\tfrac{1}{2},0)+(\tfrac{1}{2},1)\,. \tag{2.2}$$

Hence, $(LLL)_S$ or $L(RR)_S$, where S denotes symmetrization, are not spin-$\frac{1}{2}$ objects. Note that $(LLL)_M$ (M= mixed symmetry) is a left-handed Weyl spinor, although the nonrelativistic picture yields only spin $\frac{3}{2}$ for (LLL), because the helicity sum is $\sim\frac{3}{2}$. In the argument by 't Hooft on composite models,[14] the handedness of composites is fixed by the consistency condition (i.e., positive indices mean left-handed, negative ones right-handed). Thus, in his case $L(RR)_A$ (A= antisymmetrization) could be right-handed, while in our case $L(RR)_A$ is fixed as left-handed.

For later convenience, we list the left-handed spin-$\frac{1}{2}$ combinations for three left-handed Weyl spinors:

$$(LLL)_M,\ \ L(\bar{L}\bar{L})_A \tag{2.3}$$

since $\bar{L}=R^tC$ where C denotes the charge-conjugation matrix.

Our point of view can be regarded as the relativistic version of nonrelativistic SU(6). Since we have fixed how to combine spins, we can discuss the generalized Pauli principle, including spins of composites.

III. METACOLOR GROUP

A. Possible metacolor groups

We investigate the possible metacolor groups. Cvitanović[15] some time ago discussed the possibilities for the color group, using the defining (basic) representation R. Here, we discuss the problem of the metacolor group[17] from the viewpoint of the necessary condition, using congruence number.[18] Although necessity does not imply sufficiency, we can eliminate a great number of representations.

We assume that metacolor singlets consist of *three* fermions with one *single irreducible representation R* of the metacolor group G. (It is simple to extend our method to five-fermion singlets, etc.) Group-theoretically, we look for a *simple group G* which has an *irreducible representation R* that satisfies $R\times R\times R\supset 1$. In terms of the congruence number, we must have

$$3C(R)=0\ (\mathrm{mod}\ N)\ , \qquad (3.1)$$

where N depends on the group G. This is because the congruence number $C(R)$ satisfies the relation[18]:

$$C(R_1\times R_2)=C(R_1)+C(R_2)$$
$$=C(R_3)=C(R_4)=\cdots\ , \qquad (3.2)$$

where $R_1\times R_2=R_3+R_4\cdots$ and R_j denotes an irreducible representation of G. Thus, *the necessary condition that G is a metacolor group with an irreducible representation R is that there exists an irreducible representation R of G which satisfies* $3C(R)=0$ *(mod N)*. One immediate consequence is that any representation R with $C(R)=0$ (mod N) for any simple group G can be a candidate for $R\times R\times R\supset 1$. However, are there any other representations with $C(R)\neq 0$ (mod N) that still satisfy $R\times R\times R\supset 1$? We investigate each simple group.

For SU(n) ($n\geq 2$) ($=A_{n-1}$), the congruence number[18] is defined by

$$C(R)=\sum_{k=1}^{n-1} km_k\ (\mathrm{mod}\ n)\ , \qquad (3.3)$$

where $R=\sum m_j\lambda_j$ and λ_j are the fundamental weights. Then, the necessary condition becomes

$$3k=mn\ (<3n)\ , \qquad (3.4)$$

where $C(R)=k$ ($<n$) and m is an integer. Since 3 is prime, we have either $m=3l'$ or $n=3l$ where l' and l are integers. However, the case where $m=3l'$ contradicts the fact that $k<n$, except $k=0$. Thus, the necessary condition is either G=SU($3l$) with $C(R)=ml$ ($m=0,1,2$), or G=SU(n) with $C(R)=0$. If we limit ourselves to the basic representation, $R=\lambda_1$ [$C(R)=1$], then only SU(3) with $\underline{3}$ survives and we have a singlet in the totally antisymmetric part of $3\times 3\times 3$.

For SO($2n+1$) ($n\geq 3$) ($=B_n$), the congruence number is

$$C(R)=m_n\ (\mathrm{mod}\ 2)\ , \qquad (3.5)$$

where $R=\sum m_j\lambda_j$. Thus, the necessary condition is

$$C(R)=0\ (\mathrm{mod}\ 2)\ . \qquad (3.6)$$

The basic representation $R=\lambda_1$ (vector) satisfies this condition since $C(\lambda_1)=0$. However, this is not sufficient and we show that $R\times R\times R$ with $R=\lambda_1$ does not contain a singlet, by explicitly calculating the decomposition:

$$(100\cdots)\times(100\cdots)\times(100\cdots)=\begin{cases}(300\cdots)+2(110\cdots)+3(100\cdots)+(0010\cdots) & \text{for } B_n\ (n\geq 4),\\ (300)+2(110)+3(100)+(002) & \text{for } B_3\ ,\end{cases} \qquad (3.7)$$

where we have used the Dynkin notation,[19] i.e., $(m_1,m_2,m_3,\ldots)$ denotes the representation $R=\sum m_j\lambda_j$. Thus, the basic representation of SO($2n+1$) cannot do the job.

For Sp($2n$) ($n\geq 2$) ($=C_n$), the congruence number is

$$C(R)=m_1+m_3+m_5+\cdots\ (\mathrm{mod}\ 2)\ , \qquad (3.8)$$

where $R=\sum m_j\lambda_j$. Thus, the necessary condition is

$$C(R)=0\ (\mathrm{mod}\ 2)\ . \qquad (3.9)$$

Since the basic representation $R=\lambda_1$ has $C(R)=1$, this cannot do the job.

For SO($2n$) ($n\geq 4$) ($=D_n$), the congruence number is defined by

$$C(R)=[m_{n-1}+m_n,\ 2m_1+2m_3+\cdots+2m_{n-3}+(n-2)m_{n-1}+nm_n]\quad \text{for even } n\ , \qquad (3.10)$$

$$C(R)=[m_{n-1}+m_n,\ 2m_1+2m_3+\cdots+2m_{n-2}+(n-2)m_{n-1}+nm_n] \quad \text{for odd } n\ , \tag{3.11}$$

where the first term in the bracket is calculated mod 2 and the second term is done mod 4. Thus, we have

$$C(R)=(0,0),(0,2),(1,0),(1,2) \quad \text{for even } n\ , \tag{3.12}$$

$$C(R)=(0,0),(0,2),(1,1),(1,3) \quad \text{for odd } n\ . \tag{3.13}$$

In both cases, the basic representation $R=\lambda_1$ (vector) cannot do the job, since $C(R)=(0,2)$. The spinor representation $R=\lambda_n$ also cannot do the job, since $C(R)=(1,n)$. Only R with $C(R)=(0,0)$ can be a candidate.

For E_6, the congruence number is

$$C(R)=m_1-m_2+m_4-m_5 \pmod 3\ . \tag{3.14}$$

Thus, the necessary condition is satisfied by any representation. We can check that the basic representation $R=\lambda_5$ actually satisfies the condition $R\times R\times R\supset 1$ (Ref. 20):

$$\begin{aligned}
&(000010)\times(000010)=(000020)+(000100)+(100000)\ ,\\
&(100000)\times(000010)=(000000)+(000001)+(100010)\ ,\\
&(000100)\times(000010)=(000001)+(100010)+(001000)+(000110)\ ,\\
&(000020)\times(000010)=(000030)+(000110)+(100010)\ ,
\end{aligned} \tag{3.15}$$

where the dimensions of the representations are

$$\begin{aligned}
&(000010)=27,\ (100000)=\overline{27},\ (000001)=78,\ (000020)=351,\ (000100)=351\ ,\\
&(100010)=650,\ (001000)=2925,\ (000110)=5824,\ (000030)=3003\ .
\end{aligned}$$

The reason why $R=\lambda_5$ satisfies $R\times R\times R\supset 1$ is that E_6 has one "primitive invariant tensor"[15] d_{abc}, which is totally symmetric, in addition to primitives $d^{ab\cdots c}$, $f^{ab\cdots c}$, δ_a^b, $\epsilon^{ab\cdots c}$, which are totally symmetric, totally antisymmetric, δ, and Levi-Civita tensor, respectively. Thus, we have a singlet in the *totally symmetric part* of $27\times 27\times 27$ of E_6.

For E_7, the congruence number is

$$C(R)=m_4+m_6+m_7 \pmod 2\ . \tag{3.16}$$

The basic representation $R=\lambda_6$ does not satisfy $C(R)=0$. Hence, the basic representation cannot do the job.

For E_8, F_4, and G_2, the congruence number is zero for any representation. Note that one only has to show $R\times R\supset R$ for these groups, since $R\sim\bar{R}$ and $R\times\bar{R}\supset 1$. For the basic representations of E_8, F_4, G_2, we have

$$\begin{aligned}
&248\times 248=1+248+3875+30380+27000 \quad \text{for } E_8\ ,\\
&26\times 26=1+26+52+273+324 \quad \text{for } F_4\ ,\\
&7\times 7=1+7+14+27 \quad \text{for } G_2\ .
\end{aligned} \tag{3.17}$$

Therefore, all the basic representations of E_8, F_4, and G_2 satisfy $R\times R\times R\supset 1$. This is related to the fact[15] that E_8, F_4, and G_2 have primitives C_{abc}, d_{abc}, and f_{abc}, respectively. That is, while F_4 has a singlet in the totally symmetric part of $R\times R\times R$, E_8 and G_2 have singlets in the totally antisymmetric part of $R\times R\times R$.

We summarize our results as follows: among the basic representations of various metacolor groups, only SU(3), E_8, E_6, and G_2 can satisfy the relation $R\times R\times R\supset 1$, where for SU(3), E_8, and G_2, a singlet is contained in the *totally antisymmetric part of the product representation,* while for E_6 and F_4 a singlet is contained in the *totally symmetric part of the product representation.*

Because of its mathematical interest we give in Appendix A the metacolor groups that satisfy the condition $R\times R\times R\supset 1$, when R is *not* a basic representation.

At this stage we have no reason to choose $G(\mathrm{MC})=\mathrm{SU}(3)_{\mathrm{MC}}$ from among the five possible metacolor groups enumerated above. In the case of the color group, arguments for SU(3) come from several directions in addition to the R ratio in $e\bar{e}$ reactions and $\pi^0\to 2\gamma$ decay.[21] If we limit ourselves to basic representations as for metacolor, we would have the same five color-group candidates as for metacolor. The success of SU(6) reduces these five possibilities to three: SU(3), E_8, and G_2. The possibility of having E_8 or G_2 as the color group can be eliminated as follows. Since E_8 and G_2 do not have complex representations, we would have color-singlet states $qq\bar{q}$ and $q\bar{q}\,\bar{q}$, in addition to qqq, where

q denotes a quark. These new color singlets are fractionally charged objects. Another reason is given by Okubo.[22] He has found a G-parity-like operation for simple groups and, using this, the nonexactness of the Okubo-Zweig-Iizuka rule allows only SU(3) among SU(3), E_8, and G_2. [Okubo's argument by itself permits, among simple groups, only SU(n) ($n \geq 3$), SO($4n+2$) ($n \geq 2$), or E_6.] A completely different approach, deriving from the conditions of grand unification, also leads to the SU(3) color group.[23] However, none of these arguments appears to be applicable to the metacolor group and so we must keep an open mind about the precise nature of this group—including the possibility that it may not be vector—assuming that such a group exists and that quarks and leptons are metacolor singlets.[24]

B. Asymptotically free condition for the metacolor group

Asymptotic freedom for metacolor is required to give meaning to the composite preon model. We investigate the asymptotically free condition for the metacolor groups SU(3), E_6, E_8, F_4 and G_2. As is well known,[25] the β function for left-handed Weyl spinors of irreducible representation R is given by

$$\beta = -\frac{g^3}{16\pi^2} B , \tag{3.18}$$

where

$$B = \tfrac{1}{3}[11 l(R_{\text{adj}}) - 2\Sigma l(R)] , \tag{3.19}$$

$$l(R) = \frac{d(R)}{d(R_{\text{adj}})} I_2(R) \tag{3.20}$$

with $d(R)$ and $I_2(R)$, respectively, the dimension and the eigenvalue of the second-order Casimir invariant for an irreducible representation R. The summation over $l(R)$ is over all preon degrees of freedom n. The values of $l(R_{\text{adj}})$ and $l(R)$ are as follows:

$$\begin{aligned}
&\text{SU(3)}:\ l(R_{\text{adj}}) = 3,\ \ l(R) = \tfrac{1}{2} \text{ with } R = \underline{3} ,\\
&E_6:\ l(R_{\text{adj}}) = 12,\ \ l(R) = 3 \text{ with } R = \underline{27} ,\\
&E_8:\ l(R_{\text{adj}}) = l(R) = 30 \text{ with } R = \underline{248} ,\\
&F_4:\ l(R_{\text{adj}}) = 18,\ \ l(R) = 6 \text{ with } R = \underline{26} ,\\
&G_2:\ l(R_{\text{adj}}) = 12,\ l(R) = 3 \text{ with } R = \underline{7} .
\end{aligned} \tag{3.21}$$

From the condition for asymptotic freedom, we find the condition

$$\begin{aligned}
n &< 33 \ \text{ for SU(3)} ,\\
&< 22 \ \text{ for } E_6 ,\\
&< \tfrac{11}{2} \ \text{ for } E_8 ,\\
&< \tfrac{33}{2} \ \text{ for } F_4 ,\\
&< 22 \ \text{ for } G_2 .
\end{aligned} \tag{3.22}$$

Since Weyl spinors have quantum numbers in both G(MC) and G(CF), the condition for asymptotic freedom is a very stringent condition, as we will see in the next section.

It is amusing to note that all asymptotically free representations of the exceptional groups listed above, i.e., $\underline{27}$ and $\underline{78}$ of E_6, $\underline{248}$ of E_8, $\underline{26}$ and $\underline{52}$ of F_4, $\underline{7}$, $\underline{14}$, and $\underline{27}$ of G_2, satisfy the relation $R \times R \times R \supset 1$. Also note that they are the lowest-dimensional and the adjoint representations of these exceptional groups, except $\underline{27}$ of G_2.

IV. GUT PREONS

A. Quantum numbers of preons

We discuss the case where G(CF) is one of the GUT groups. Since we want to have complex representations for quarks and leptons, G(CF) is SU(n) ($n \geq 3$), E_6, or SO($4n+2$) ($n \geq 2$). We take the simplest choice for preon representations of G(CF): only R and/or $\bar{R}$ of G(CF). Thus, the possible preon quantum numbers are

$$\begin{aligned}
&T_+ = (r,R)_L ,\ \ T_- = (r,\bar{R})_L ,\\
&V_+ = (\bar{r},R)_L ,\ \ V_- = (\bar{r},\bar{R})_L ,
\end{aligned} \tag{4.1}$$

where r denotes the metacolor representation. While we allow four low-dimensional preon chiral representations at the start, we shall soon show that we end up with at most two preon chiral representations—consistent with condition (1) in Sec. I—depending on the metacolor group.

The first observation is that if G(MC) is E_8, F_4, or G_2, then the possible preon representations are only

$$T = (r,R)_L ,\ \ V = (r,\bar{R})_L \tag{4.2}$$

since $r \sim \bar{r}$ for these metacolor groups. We assume the absence of $\bar{R}_L$ of G(CF) at the composite level since the existence of $\bar{R}_L$ implies the existence of parity doublets for quarks and leptons, or R_L would be eaten by $\bar{R}_L$ to become heavy.[26] This condition immediately excludes T from consideration since, if $T \neq 0$, the preon composite $T\bar{T}\bar{T} = (R \times \bar{R} \times \bar{R})$ would contain the unacceptable $\bar{R}_L$ [remember that $r \times \bar{r} \times \bar{r}$ contains a singlet for $G(\text{MC}) = E_8$, F_4, or G_2]. Thus, the only composites are

$$VVV=(\bar{R}\times\bar{R}\times\bar{R})_L$$
and (4.3)
$$V\bar{V}\bar{V}=(\bar{R}\times R\times R)_L \; .$$

The absence of $\bar{R}_L$ of G(CF) at the composite level also reduces the number of possible preon representations for $G(\text{MC})=\text{SU}(3)$ or E_6. We then have

$$T_+T_-=0,\;\; V_+V_-=0\;, \qquad T_-V_+V_-=0,\;\; V_-T_+T_-=0 \tag{4.4}$$

since $T_+T_-T_-$, $V_+V_-V_-$, $T_+\bar{V}_+\bar{V}_+$, $V_+\bar{T}_+\bar{T}_+$, $T_-\bar{V}_+\bar{V}_-$, and $V_-\bar{T}_+\bar{T}_-$ all have $R\times\bar{R}\times\bar{R}$, which contains $\bar{R}_L$. The solutions of Eq. (4.4) are

Case 1: $T_+=(r,R)$, $V_-=(\bar{r},\bar{R})$,

Case 2: $T_-=(r,\bar{R})$, $V_+=(\bar{r},R)$,

Case 3: $T_-=(r,\bar{R})$, $V_-=(\bar{r},\bar{R})$.

For case 1, we have composites as follows:

$$T_+T_+T_+=(R\times R\times R),\;\; T_+\bar{V}_-\bar{V}_-=(R\times R\times R)\;,$$
$$V_-\bar{T}_+\bar{T}_+=(\bar{R}\times\bar{R}\times\bar{R}),\;\; V_-V_-V_-=(\bar{R}\times\bar{R}\times\bar{R})\;.$$

We have a real representation as a whole. Case 2 yields a similar result. Consequently, cases 1 and 2 are ruled out. For case 3, we have

$$T_-T_-T_-=(\bar{R}\times\bar{R}\times\bar{R}),\;\; T_-\bar{V}_-\bar{V}_-=(\bar{R}\times R\times R)\;, \tag{4.5}$$
$$V_-\bar{T}_-\bar{T}_-=(\bar{R}\times R\times R),\;\; V_-V_-V_-=(\bar{R}\times\bar{R}\times\bar{R})\;,$$

which are complex as a whole; therefore case 3 is a candidate for a permissible preon model satisfying our conditions.

What is the representation R of G(CF)? Since $\bar{R}\times R\times R$ always contains R, we must take the $\underline{27}$ representation for $G(\text{CF})=E_6$, the spinor representation for $G(\text{CF})=\text{SO}(4n+2)$ $(n\geq 2)$, and the totally antisymmetric representation for $G(\text{CF})=\text{SU}(n)$. However, the use of a single totally antisymmetric representation for $G(\text{CF})=\text{SU}(n)$ yields the anomaly for preons. Hence, SU(n) GUT preons are not allowed.[27] In particular, SU(5) GUT cannot be the color-flavor group of a preon model satisfying our conditions. Indeed, we shall see that every GUT or PUT preon model satisfying our conditions contains $\text{SU}(2)_L\times\text{SU}(2)_R\times\text{U}(1)_{B-L}$ as a subgroup.

Does $\bar{R}\times\bar{R}\times\bar{R}$ contain R? For E_6, it does not, since the congruence numbers of $\overline{27}$ and $\underline{27}$ are $+1$ (mod 3) and -1 (mod 3), respectively. For $\text{SO}(4n+2)$ $(n\geq 2)$, it does. This is because the congruence numbers for antispinor $\bar{R}$ and spinor R are $(1,2n-1)$ and $(1,2n+1)$ (mod 2, mod 4). Thus, $C(\bar{R}\times\bar{R}\times\bar{R})=(3,6n-3)=(1,2(n-1)+1)=C(R)$. However, congruence-number mismatch only reduces the predicted number of quark and lepton generations but does not rule out the group (see below).

Summarizing, we obtain:

Lemma 1. For SU(3) or E_6 metacolor, the quantum numbers for GUT preons can only be

$$T=(r,\bar{R})_L,\;\; V=(\bar{r},\bar{R})_L\;, \tag{4.6}$$

where the color-flavor group with representation R is either E_6 or $\text{SO}(4n+2)$ $(n\geq 2)$. For E_8, F_4, or G_2 metacolor, the quantum numbers for GUT preons can only be

$$V=(r,\bar{R})_L\;, \tag{4.7}$$

where the color-flavor group is the same as above.

Now we look at the color-flavor quantum numbers of preon composites. For concreteness, we consider two cases: $G(\text{CF})=\text{SO}(10)$ with $R=\underline{16}$ and $G(\text{CF})=E_6$ with $R=\underline{27}$. We have the following decompositions:

for SO(10),

$$\overline{16}\times\overline{16}\times\overline{16}=2(16)+3(144)+560+672+2(1200)\;,$$
$$\overline{16}\times16\times16=3(16)+2(144)+2(560)+1200+1440\;; \tag{4.8}$$

for E_6,

$$\overline{27}\times\overline{27}\times\overline{27}=1+2(78)+3(650)+2925+2(5824)+3003\;,$$
$$\overline{27}\times27\times27=3(27)+2(351)+351'+2(1728)+7371+7722\;. \tag{4.9}$$

Therefore, for SU(3) or E_6 metacolor, we have at most ten families for $G(\text{CF})=\text{SO}(10)$ and six families for $G(\text{CF})=E_6$, if we regard $\underline{16}$ of SO(10) and $\underline{27}$ of E_6 as one family. For E_8, F_4, or G_2 metacolor, we have at most five families for $G(\text{CF})=\text{SO}(10)$ and four for $G(\text{CF})=E_6$. We will see in the next subsection that these numbers are reduced considerably by the imposition of the Pauli principle (generalized to include the metacolor degree of freedom).

B. Generalized Pauli principle and generations

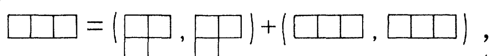
We recall the connection between SU(3) flavor symmetry and the Pauli principle. Without the Pauli principle, we have

$$3\times 3\times 3=1+8+8+10\ , \tag{4.10}$$

i.e., four possible baryon multiplets out of three-quark composites. Since we assume that quarks transform as (3,3) in (SU(3)-color, SU(3)-flavor) and baryons must be color singlets [totally antisymmetric (A) in color indices], the Pauli principle (generalized to the color degree of freedom) requires that composite states must be totally symmetric (S) under the symmetry [SU(3)-flavor $\times$ spin]. Hence, they transform as (8,2) or (10,4) under (SU(3)-flavor, spin). We end up having two octets with spin $\frac{1}{2}$ and one decuplet with spin $\frac{3}{2}$. However, experimentally we have only one octet and one decuplet. We need more restrictions. As is well known, the SU(6) group combining SU(3)-flavor and -spin was invented[28] to do the job. Under SU(6), the flavor and spin are embedded as

$$6=(3,2)\ \text{of (SU(3)-flavor, spin)}\ . \tag{4.11}$$

The Pauli principle for the SU(6) group then yields



(4.12)

i.e., one octet with spin $\frac{1}{2}$ and one decuplet with spin $\frac{3}{2}$.

We propose to apply a similar strategy to preon composites taking into account the metacolor degree of freedom. We first discuss the case where SU(3) or E_6 is the metacolor group. For composites TTT or VVV, we have either S [since SU(3)-metacolor is A] or A (since E_6-metacolor is S) for the symmetry G(CF)$\times$ spin. Then, we have (S,S) or (M,M) for SU(3)-metacolor and (A,S) or (M,M) for E_6-metacolor, where (S,S), (M,M), or (A,S) now indicates the symmetry of (G(CF), spin). Since S for spin implies spin $\frac{3}{2}$, we must have *M for $G(CF)$ independent of whether $SU(3)$ or E_6 is the metacolor.* Consequently, TTT has only one $\underline{16}$ (not two) for G(CF)=SO(10) with $R=\underline{16}$.[29] For G(CF)=E_6, TTT does not have any $\underline{27}$.

For $T\bar{V}\bar{V}$ and $V\bar{T}\bar{T}$, we apply the Pauli principle for ($\bar{V}\bar{V}$) and ($\bar{T}\bar{T}$) pairs. Since ($\bar{V}\bar{V}$) and ($\bar{T}\bar{T}$) must have A for spin [$\bar{T},\bar{V}\sim(0,\frac{1}{2})$], they have A in G(CF) for SU(3)-metacolor and S in G(CF) for E_6-metacolor. Using the decompositions

for SO(10),

$$\begin{aligned}&16\times 16=126(S)+120(A)+10(S)\ ,\\&126(S)\times\overline{16}=16+560+1440\ ,\\&120(A)\times\overline{16}=16+144+560+1200\ ,\\&10(S)\times\overline{16}=16+144\ ,\end{aligned} \tag{4.13}$$

for E_6,

$$\begin{aligned}&27\times 27=\overline{351}(S)+\overline{351}'(A)+\overline{27}(S)\ ,\\&\overline{351}(S)\times\overline{27}=27+1728+7722\ ,\\&\overline{351}'(A)\times\overline{27}=27+351+1728+7371\ ,\\&\overline{27}(S)\times\overline{27}=27+351+351'\ ,\end{aligned} \tag{4.14}$$

we see that $T\bar{V}\bar{V}$ has one family for SU(3)-metacolor, or two families for E_6-metacolor. Summarizing, after applying the Pauli principle, we have the following number of families:

$$4=1+1+1+1\quad\text{for}\quad G(\text{CF})=\text{SO}(10),\ \ G(\text{MC})=\text{SU}(3)\ , \tag{4.15}$$

$$6=1+2+2+1\quad\text{for}\quad G(\text{CF})=\text{SO}(10),\ \ G(\text{MC})=E_6\ , \tag{4.16}$$

$$2=0+1+1+0\quad\text{for}\quad G(\text{CF})=E_6,\ \ G(\text{MC})=\text{SU}(3)\ , \tag{4.17}$$

$$4=0+2+2+0\quad\text{for}\quad G(\text{CF})=E_6,\ \ G(\text{MC})=E_6\ , \tag{4.18}$$

where the first, second, third, and fourth terms indicate the contributions from TTT, $T\bar{V}\bar{V}$, $V\bar{T}\bar{T}$, and VVV, respectively.

Clearly, the choice G(MC)=SU(3), G(CF)=E_6 is ruled out because it predicts only two families. Other possibilities can be ruled out if we impose the condition of asymptotic freedom on the metacolor sector. Then SU(3)(MC)$\times$SO(10)(CF) is the unique choice from among G(MC)=SU(3), E_6 and G(CF)=E_6, SO($4n+2$) ($n\geq 2$). The reason is that for G(CF)=E_6 with $R=\underline{27}$, we have $n=54$ and this is too large for G(MC)=E_6 [see Eq. (3.22)]. Among G(CF)=SO($4n+2$) ($n\geq 2$), SO(10) with $R=\underline{16}$ has the lowest value of n, namely, 32, and G(MC)=E_6 is therefore ruled out; however, SO(10) for the color-flavor group and SU(3) for the metacolor group has a positive but exceedingly small value for B ($\frac{1}{3}$—this is to be compared to $B=9$ for QCD for u,d,s quarks).

For the case where E_8, F_4, or G_2 is the metacolor group, we have

$$2=1+1 \text{ for } G(\mathrm{CF})=\mathrm{SO}(10)\,, \quad G(\mathrm{MC})=\mathrm{E}_8,\mathrm{G}_2\,, \tag{4.19}$$

$$3=1+2 \text{ for } G(\mathrm{CF})=\mathrm{SO}(10)\,, \quad G(\mathrm{MC})=F_4\,, \tag{4.20}$$

$$1=0+1 \text{ for } G(\mathrm{CF})=\mathrm{E}_6\,, \quad G(\mathrm{MC})=\mathrm{E}_8,\mathrm{G}_2\,, \tag{4.21}$$

$$2=0+2 \text{ for } G(\mathrm{CF})=\mathrm{E}_6\,, \quad G(\mathrm{MC})=\mathrm{F}_4\,. \tag{4.22}$$

Only $G(\mathrm{CF})=\mathrm{SO}(10)$ with $G(\mathrm{MC})=\mathrm{F}_4$ has three generations. It is also asymptotically free—by a much better margin ($B=2$) than $\mathrm{SU}(3)_{\mathrm{MC}}\times\mathrm{SO}(10)_{\mathrm{CF}}$ but still small compared to QCD.

We are thus left with two GUT preon models: one is $G(\mathrm{MC})\times G(\mathrm{CF})=\mathrm{SU}(3)\times\mathrm{SO}(10)$ with two preon representations: $T=(3,\overline{16})_L$ and $V=(\overline{3},\overline{16})_L$. The left-handed composites are TTT, $T\overline{V}\overline{V}$, $V\overline{T}\overline{T}$, and VVV and four quark and lepton generations are predicted. The other GUT preon model is $G(\mathrm{MC})\times G(\mathrm{CF})=\mathrm{F}_4\times\mathrm{SO}(10)$ with one preon representation: $V=(26,16)_L$. The left-handed composites are VVV and $V\overline{V}\overline{V}$ and three families of quarks and leptons are predicted. In both cases, the rate of decrease of the metacolor coupling constant is sufficiently slow that the asymptotic energy region for metacolor should be higher than the Planck mass. Therefore, we do not think that these two models are plausible.

V. PUT PREONS

A. Quantum numbers of preons

Here, we consider the possibility of preons having the quantum numbers of $\mathrm{SU}(n)_C\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$,[30] in addition to metacolor quantum numbers. The group $\mathrm{SU}(n)_C$ contains the Pati-Salam $\mathrm{SU}(4)_C$.[7] We show that the preons are only in the $(\bar{n},2,1)_L$ and $(n,1,2)_L$ representations of $\mathrm{SU}(n)_C\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$. To prove this, we assume that no superheavy objects, composed of preons, can exist at the $\mathrm{SU}(n)_C\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ level. In group-theoretical terms, this implies that no real composite representations exist at this level. Furthermore, in accordance with the conditions stated in Sec. I, no anomalies exist at the $\mathrm{SU}(n)_C\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ level. Finally, we assume that the preons have at most the following quantum numbers of $\mathrm{SU}(n)_C\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ (it appears that we are contradicting one of our conditions—that at most two chiral representations will be allowed for the preons—but the point is to show that four of the six chiral representations, including the two "color" singlets, are ruled out):

$$\begin{aligned} &T_+(n,2,1)_L\,,\quad V_+(n,1,2)_L\,,\\ &T_0(1,2,1)_L\,,\quad V_0(1,1,2)_L\,,\\ &T_-(\bar{n},2,1)_L\,,\quad V_-(\bar{n},1,2)_L\,. \end{aligned} \tag{5.1}$$

We proceed with the proof. We look at the FFF combinations. Permitting at first all possible FFF combinations to be realized, we argue that T_0 and V_0 are not allowed. The reason is that they produce real representations: $T_0T_0T_0\supset(1,2,1)_L$ and $V_0V_0V_0\supset(1,1,2)_L$. Next, we show that T_+ and V_- are not allowed. Since $T_+T_-T_-\supset(\bar{n},2,1)_L$, $V_+V_+V_-\supset(n,1,2)_L$, $T_+V_-V_-\supset(\bar{n},2,1)_L$, while $T_-V_+V_+\supset(n,2,1)_L$, $T_-T_-V_+\supset(\bar{n},1,2)_L$, it is possible for $(\bar{n},2,1)_L$ and $(n,2,1)_L$ to combine and become superheavy. Thus, we must have

$$T_+T_-=0,\quad T_+V_-=0, \text{ and } V_+V_-=0 \tag{5.2}$$

since we must kill $(\bar{n},2,1)_L$ [remember that $(\bar{n},2,1)_L$ contains $(\bar{4},2,1)_L$]. If $T_+\neq 0$, then we would have (T_+,V_+) which is not anomaly free in the $\mathrm{SU}(n)_C$ sector. Hence, the surviving preon representations are T_- and V_+.

If we include the metacolor representation, we can say that FFF composites can only be constructed out of

$$T=(r,\bar{n},2,1)_L\,,\quad V=(r,n,1,2)_L\,, \tag{5.3}$$

where the metacolor representation is r and the metacolor group is SU(3), E_6, E_8, F_4, or G_2. However, it is evident that if SU(3) is the metacolor group, the preon set (5.3) is not anomaly free in the metacolor sector. This objection can be overcome by simply changing r to $\bar{r}$ in V so that we have

$$T=(r,\bar{n},2,1)_L\,,\quad V=(\bar{r},n,1,2)_L \tag{5.4}$$

for $G(\mathrm{MC})=\mathrm{SU}(3)$. This leads us to the following.

Lemma 2. The representations of PUT $[\mathrm{SU}(n)_C\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R]$ preons must be chosen from among

$$\begin{aligned} &(r,\bar{n},2,1)_L\,,\quad (r,n,1,2)_L\,,\\ &(\bar{r},\bar{n},2,1)_L\,,\quad (\bar{r},n,1,2)_L\,, \end{aligned}$$

where r is the representation of the metacolor group SU(3), E_6, E_8, F_4, or G_2.

It is not necessary to have all four types of preon representations in order to have an anomaly-free set. If we limit ourselves to two representations, there are two alternatives[13,31]:

$$T=(r,\bar{n},2,1)_L\,,\quad V=(\bar{r},n,1,2)_L \tag{5.5}$$

or

$$T=(r,\bar{n},2,1)_L\,,\quad V=(r,n,1,2)_L\,, \tag{5.6}$$

where in the first set, Eq. (5.5), the metacolor group can be, in principle, SU(3), E_6 E_8, F_4, or G_2, while in the second set, Eq. (5.6), the metacolor group cannot be SU(3), because of the anomaly.

Actually, for the metacolor groups E_8, F_4, or G_2, there is only one possibility, since these groups have only real representations ($r\sim\bar{r}$). Furthermore, all possible left-handed combinations of three preons are metacolor singlets, since all of $r\times r\times r$, $r\times r\times\bar{r}$, $r\times\bar{r}\times\bar{r}$ contain singlets for these groups. Thus, we have composites as follows: TTT, TTV, TVV, VVV, $T\bar{T}\bar{T}$, $T\bar{T}\bar{V}$, $T\bar{V}\bar{V}$, $V\bar{T}\bar{T}$, $V\bar{T}\bar{V}$, and $V\bar{V}\bar{V}$. Among them, $T\bar{V}\bar{V}$ and $V\bar{T}\bar{T}$ contain the forbidden composite representations $(\bar{n},2,1)_L$ and $(n,1,2)_L$. Therefore, $G(\mathrm{MC})=E_8$, F_4, or G_2 is excluded. In other words, the possible metacolor group for Eq. (5.5) is either SU(3) or E_6, while the possible metacolor group for Eq. (5.6) is only E_6.

The first set with $G(\mathrm{MC})=\mathrm{SU}(3)$ and $n=4$ was taken by Achiman.[31] The preon composites are as follows:

$$\begin{aligned} TTT&=(1,\bar{n}\times\bar{n}\times\bar{n},2\times2\times2,1)\,,\\ T\bar{V}\bar{V}&=(1,\bar{n}\times\bar{n}\times\bar{n},2,2\times2)\,,\\ V\bar{T}\bar{T}&=(1,n\times n\times n,2\times2,2)\,,\\ VVV&=(1,n\times n\times n,1,2\times2\times2)\,. \end{aligned} \tag{5.7}$$

The second choice was taken by us[13]; the preon composites are

$$\begin{aligned} TTT&=(1,\bar{n}\times\bar{n}\times\bar{n},2\times2\times2,1)\,,\\ TVV&=(1,\bar{n}\times n\times n,2,2\times2)\,,\\ TTV&=(1,\bar{n}\times\bar{n}\times n,2\times2,2)\,,\\ VVV&=(1,n\times n\times n,1,2\times2\times2)\,. \end{aligned} \tag{5.8}$$

B. Generalized Pauli principle and generations

In this subsection, we formulate the generalized Pauli principle for our PUT preons and consider its effect on the number of predicted generations. We count the number of generations (families) at the $\mathrm{SU}(4)_C\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ level, after applying the "survival hypothesis."[26] For this purpose, we list the decompositions of the three SU(n) representations into SU(4) representations as follows:

$$\begin{smallmatrix}\square\\ \square\\ \square\end{smallmatrix}\rightarrow\underline{\bar{4}}+(n-4)\underline{6}+\tfrac{1}{2}(n-4)(n-5)\underline{4}+\tfrac{1}{6}(n-4)(n-5)(n-6)\underline{1}\,, \tag{5.9}$$

$$\square\square\square\rightarrow\underline{20}'+(n-4)\underline{10}+\tfrac{1}{2}(n-3)(n-4)\underline{4}+\tfrac{1}{6}(n-2)(n-3)(n-4)\underline{1}\,, \tag{5.10}$$

$$\begin{smallmatrix}\square\square\\ \square\;\;\end{smallmatrix}\rightarrow\underline{20}+(n-4)\underline{10}+(n-4)\underline{6}+(n-4)^2\underline{4}+\tfrac{1}{3}(n-3)(n-4)(n-5)\underline{1}\,. \tag{5.11}$$

We are now in a position to investigate the preon model defined by Eq. (5.5). For TTT, we have

$$\left(\square\square\square+2\begin{smallmatrix}\square\square\\ \square\;\;\end{smallmatrix}+\begin{smallmatrix}\square\\ \square\\ \square\end{smallmatrix},\ \square\square\square+2\begin{smallmatrix}\square\square\\ \square\;\;\end{smallmatrix},\ 1\right)_L\,, \tag{5.12}$$

where the quantum numbers are indicated in the order $\mathrm{SU}(n)_C$, $\mathrm{SU}(2)_L$, $\mathrm{SU}(2)_R$, spin. We examine which of these satisfy the Pauli principle. We follow the SU(6) approach: consider the largest symmetry and apply the Pauli principle. The symmetry in this case is SU($4n$) with the embedding of $(\overline{4n})=(\bar{n},2,2)$ of $\mathrm{SU}(n)_C\times\mathrm{SU}(2)_L\times$ spin. The symmetry of TTT with respect to SU($4n$) is A or S, depending on whether the metacolor part is S or A. For SU($4n$)$\rightarrow$SU($2n$)$\times$ spin, the spin-$\frac{1}{2}$ part requires SU($2n$) mixed symmetry (M), independent of the metacolor symmetry, since $S=(S,S)+(M,M)$ and $A=(A,S)+(M,M)$ where the symbols in parentheses indicate the symmetry of (SU($2n$), spin). With the use of the decomposition

$$\overline{\begin{smallmatrix}\square\square\\ \square\;\;\end{smallmatrix}}\rightarrow\left(\overline{\square\square\square},\begin{smallmatrix}\square\square\\ \square\;\;\end{smallmatrix}\right)+\left(\overline{\begin{smallmatrix}\square\square\\ \square\;\;\end{smallmatrix}},\square\square\square\right)+\left(\overline{\begin{smallmatrix}\square\\ \square\\ \square\end{smallmatrix}},\begin{smallmatrix}\square\square\\ \square\;\;\end{smallmatrix}\right)+\left(\overline{\begin{smallmatrix}\square\square\\ \square\;\;\end{smallmatrix}},\begin{smallmatrix}\square\square\\ \square\;\;\end{smallmatrix}\right)\ :\ \mathrm{SU}(2n)\rightarrow\mathrm{SU}(n)\times\mathrm{SU}(2)\,, \tag{5.13}$$

the left-handed spin-$\frac{1}{2}$ composites are

$$(\overline{\ydiagram{3}},2,1),\ (\overline{\ydiagram{1,1,1}},2,1),\ (\overline{\ydiagram{2,1}},2,1),\ \text{and}\ (\overline{\ydiagram{2,1}},4,1)\ , \tag{5.14}$$

where the symbols in parentheses indicate the $\mathrm{SU}(n)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ quantum numbers. We count the number of $(4,2,1)_L$ and $(\bar{4},2,1)_L$ representations in the decomposition, using Eqs. (5.9)–(5.11). We find

$$\text{No. of } (4,2,1)_L = 1, \quad \text{No. of } (\bar{4},2,1)_L = 2(n-4)^2\ . \tag{5.15}$$

For $T\bar{V}\,\bar{V}$, we apply the Pauli principle for $(\bar{V}\bar{V})$. This factor must be A for SU(2), A for spin. Hence, the SU(n) symmetry is S or A, depending on whether the metacolor part is A or S. Thus, the surviving composites of type $T\bar{V}\bar{V}$ are

$$\begin{aligned} &(\overline{\ydiagram{2,1}},2,1)_L,\ (\overline{\ydiagram{3}},2,1)_L && \text{for } A \text{ in metacolor}\ ,\\ &(\overline{\ydiagram{2,1}},2,1)_L,\ (\overline{\ydiagram{1,1,1}},2,1)_L && \text{for } S \text{ in metacolor}\ . \end{aligned} \tag{5.16}$$

They contain

$$\begin{aligned} &\text{No. of } (4,2,1)_L = 0, \quad \text{No. of } (\bar{4},2,1)_L = (n-4)^2 + \tfrac{1}{2}(n-3)(n-4) \quad \text{for } A \text{ in metacolor}\ ,\\ &\text{No. of } (4,2,1) = 1\ , \quad \text{No. of } (\bar{4},2,1)_L = (n-4)^2 + \tfrac{1}{2}(n-4)(n-5) \quad \text{for } S \text{ in metacolor}\ . \end{aligned} \tag{5.17}$$

Combining Eq. (5.17) with Eq. (5.15), we obtain the number of surviving $(4,2,1)_L$ (i.e., the number of generations), which is given by

$$\begin{aligned} &\text{No. of generations} = 1 - 3(n-4)^2 - \tfrac{1}{2}(n-3)(n-4) \quad \text{for } A \text{ in metacolor}\ ,\\ &\text{No. of generations} = 2 - 3(n-4)^2 - \tfrac{1}{2}(n-4)(n-5) \quad \text{for } S \text{ in metacolor}\ . \end{aligned} \tag{5.18}$$

Therefore, in the Achiman model,[31] the number of generations is at most 2, for $n=4$, i.e., $\mathrm{SU}(4)_C$. Thus, we do not discuss this model further.

Now, we discuss our model.[13] The preon quantum numbers are given in Eq. (5.6) with E_6-metacolor. For FFF where $F=T$ or V, we can use SU($8n$) symmetry, which branches into the following:

$$\begin{aligned} &\mathrm{SU}(8n) \to \mathrm{SU}(4n) \times \text{ spin} \quad \text{with } 8n = (4n,2)\ ,\\ &\mathrm{SU}(4n) \to \mathrm{SU}(2n)\text{-}V \times \mathrm{SU}(2n)\text{-}T \quad \text{with } 4n = (2n,1) + (1,\overline{2n})\ ,\\ &\mathrm{SU}(2n)\text{-}V \to \mathrm{SU}(n) \times \mathrm{SU}(2)_R \quad \text{with } 2n = (n,2)\ ,\\ &\mathrm{SU}(2n)\text{-}T \to \mathrm{SU}(n) \times \mathrm{SU}(2)_L \quad \text{with } \overline{2n} = (\bar{n},2)\ , \end{aligned} \tag{5.19}$$

where we have specified the embedding. While the metacolor symmetry is S (E_6-metacolor), the SU($4n$) symmetry associated with the spin-$\frac{1}{2}$ part is fixed as M, since $A = (A,S) + (M,M)$ where the symbols in parentheses indicate the symmetry of (SU($4n$), spin). Note the following decomposition:

$$\ydiagram{2,1} \to (\ydiagram{2,1},1) + (1,\overline{\ydiagram{2,1}}) + (\ydiagram{1,1},\overline{\ydiagram{1}}) + (\ydiagram{2},\overline{\ydiagram{1}}) + (\ydiagram{1},\overline{\ydiagram{2}}) + (\ydiagram{1},\overline{\ydiagram{1,1}}):\quad \mathrm{SU}(4n) \to \mathrm{SU}(2n) - V \times \mathrm{SU}(2n) - T\ , \tag{5.20}$$

$$\ydiagram{2,1} \to (\ydiagram{2,1},\ydiagram{2,1}) + (\ydiagram{3},\ydiagram{2,1}) + (\ydiagram{1,1,1},\ydiagram{2,1}) + (\ydiagram{2,1},\ydiagram{3}):\quad \mathrm{SU}(2n) \to \mathrm{SU}(n) \times \mathrm{SU}(2)\ .$$

Therefore, we obtain

$$TTT = (\overline{\ydiagram{2,1}} + \overline{\ydiagram{3}} + \overline{\ydiagram{1,1,1}},2,1)_L + (\overline{\ydiagram{2,1}},4,1)_L\ , \tag{5.21}$$

$$TVV = (\overline{\ydiagram{1}} \times \ydiagram{1,1},2,1)_L + (\overline{\ydiagram{1}} \times \ydiagram{2},2,1)_L + (\overline{\ydiagram{1}} \times \ydiagram{1,1},2,3)_L + (\overline{\ydiagram{1}} \times \ydiagram{2},2,3)_L\ , \tag{5.22}$$

where we have indicated the $\mathrm{SU}(n)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times$ spin quantum numbers.

Using the decomposition of SU(n) into SU(4) such as

$$\square\square=\underline{10}+(n-4)\underline{4}+\tfrac{1}{2}(n-3((n-4)\underline{1}\ ,$$

$$\begin{array}{c}\square\\[-4pt]\square\end{array}=\underline{6}+(n-4)\underline{4}+1/2(n-4)(n-5)\underline{1}\ ,$$

$$\square=\underline{4}+(n-4)\underline{1}\ ,$$

$$\overline{\square}\times\square\square=[1+(n-4)^2]\underline{4}+\tfrac{1}{2}(n-3)(n-4)\underline{4}+\underline{36}+(n-4)\underline{15}+(n-4)\underline{10}$$
$$+[(n-4)+\tfrac{1}{2}(n-3)(n-4)^2]\underline{1}\ ,$$

$$\overline{\square}\times\begin{array}{c}\square\\[-4pt]\square\end{array}=[1+(n-4)^2]\underline{4}+\tfrac{1}{2}(n-4)(n-5)\underline{\bar{4}}+\underline{20}+(n-4)\underline{15}+(n-4)\underline{6}$$
$$+[(n-4)+\tfrac{1}{2}(n-4)^2(n-5)]\underline{1}\ ,$$

we count for TVV and TTT

$$\text{No. of } (4,2,1)_L=3+2(n-4)^2\ ,$$

$$\text{No. of } (\bar{4},2,1)_L=3-(n-4)^2\ ,$$

Thus, we obtain

$$\text{No. of generations}=3(n-4)^2\ . \tag{5.23}$$

Our model predicts three generations of quarks and leptons for $SU(4)_C\times SU(2)_L\times SU(2)_R$. (Without the generalized Pauli principle, the prediction would be four generations.[13]) The metacolor group is E_6 and the metacolor part is asymptotically free by a wide margin [$B=12$—see Eq. (3.22)]. It appears therefore that the PUT preon group

$$G(\mathrm{MC})\times G(\mathrm{CF})=E_6(\mathrm{MC})\times SU(4)_C\times SU(2)_L\times SU(2)_R$$

gives the most promising three-fermion preon model of quarks and leptons. It predicts precisely three generations and its degree of asymptotic freedom makes physical sense. However, this model—like all preon models—must pass some other important tests, of which the chief one is that the masses of all three generations of quarks and leptons must be small on the metacolor scale Λ_{MC}. (This is different from QCD where the hadronic masses are of the order of Λ_{QCD}.) We must therefore show that our model can guarantee the masslessness of the quarks and leptons in the unbroken symmetry limit. We turn to this question in the next subsection.

C. Masslessness of quarks and leptons

The problem of ensuring the masslessness of quarks and leptons in composite models has been examined in detail by 't Hooft.[14] He derived the well-known 't Hooft condition, requiring the "conservation" of anomalies in going from the preon to the composite level—in order to protect the composite quarks and leptons from acquiring masses. The 't Hooft anomaly condition is a very severe constraint[32] and, indeed, it is easy to show that our PUT preon model does not obey it (see below). Fortunately, 't Hooft's anomaly condition is only a sufficient condition and there are other ways to ensure the masslessness of quarks and leptons in composite models. Our gauged PUT preon group can guarantee masslessness in two other ways. The first guarantee follows from the fact that our preon group contains the gauged $SU(2)_L\times SU(2)_R$ chiral group which manifestly prevents the composite quarks and leptons from acquiring mass on the metacolor scale.

The second way to ensure massless quarks and leptons is more complicated and requires a slightly modifed version of another argument by 't Hooft. 't Hooft has shown[3] that instantons can break the U(1) axial symmetry implicit in gauged groups and reduce it to a discrete symmetry. Since the U(1) axial charge is equal to the difference between the number of left-handed particles and the number of right-handed particles, we can reinterpret his results as follows: for a massless *left-handed* Weyl spinor, which belongs to a representation R of a gauged symmetry G, instantons of G induce

$$\Delta n(L)=2\frac{d(R)I_2(R)}{d(R_{\mathrm{adj}})}\nu=2l(R)\nu\ , \tag{5.24}$$

where $\Delta n(L)$ denotes the change of the number of left-handed spinors and $I_2(R)$ and $d(R)$ denote, respectively, the eigenvalue of the second-order Casimir invariant and the dimension of the representation R of G. This is because we have

$$\partial^\mu J^5_\mu = \frac{g^2}{16\pi^2}\mathrm{Tr}(T_a T_b) G^a_{\mu\nu} G^{b\mu\nu}$$
$$= 4\frac{d(R)I_2(R)}{d(R_{\rm adj})}\frac{g^2}{64\pi^2} g_{ab} G^a_{\mu\nu} G^{b\mu\nu} , \quad (5.25)$$

where

$$\nu = \frac{g^2}{64\pi^2}\int d^4x\, g_{ab} G^a_{\mu\nu} G^{b\mu\nu} \quad \text{(winding number)}, \quad (5.26)$$

and left-handed fermions contribute only $\frac{1}{2}$ to the right-hand side. For a basic representation of SU(n), we have $d(R)=\underline{n}$, $I_2(R)=(n^2-1)/(2n)$, $d(R_{\rm adj})=n^2-1$, and thus

$$\Delta n(L) = \nu . \quad (5.27)$$

We can use Eq. (5.1) to examine the masslessness of composites if the discrete symmetry is not broken spontaneously. There is a possibility that no chiral symmetry is preserved, but still no mass terms are generated, because of the discrete symmetry.[33]

In our case, the global symmetry of our model is

$$\mathrm{SU}(8)_T \times \mathrm{SU}(8)_V \times \mathrm{U}(1)_T \times \mathrm{U}(1)_V \quad (5.28)$$

which is broken by metacolor instantons into

$$\mathrm{SU}(8)_T \times \mathrm{SU}(8)_V \times \mathrm{U}(1)_{T\text{-}V} \times Z_T \times Z_V , \quad (5.29)$$

where Z denotes the discrete symmetry. The existence of $\mathrm{U}(1)_{T\text{-}V}$ is obvious, since

$$\Delta n(T) = 16\, l(r), \quad \Delta n(V) = 16\, l(R) . \quad (5.30)$$

The $\mathrm{SU}(8)_T \times \mathrm{SU}(8)_V$ anomalies for composites do not match those for preons[34] so that the 't Hooft anomaly condition is not satisfied. However, the $\mathrm{U}(1)_{T\text{-}V}$ anomaly trivially matches, since (A stands for anomaly):

$$A(\text{preons}) = 1^3 + (-1)^3 ,$$
$$A(\text{composites}) = 3^3 + 2(-1)^3 + 2\cdot 1^3 + (-3)^3 .$$

Therefore, the $\mathrm{U}(1)_{T\text{-}V}$ symmetry can be an unbroken symmetry. Glancing at Table I, we find that this symmetry forbids part of the mass terms, $(TTT)^t C(TTV)$ and $(TVV)^t C(VVV)$. Furthermore, if the discrete symmetries, Eq. (5.30), are not broken, then all the mass terms are forbidden, as can be seen from the table. That is, masslessness of the composites is guaranteed by the discrete symmetries. Thus, we have shown that masslessness of our quark and lepton composites is guaranteed by two mechanisms; gauged chiral symmetry and discrete symmetries.

TABLE I. Mass terms for PUT preons.

	$\Delta n(T)$	$\Delta n(V)$	$\Delta n(T)$-$\Delta n(V)$
$(TTT)^t C(VVV)$	3	3	0
$(TTT)^t C(TTV)$	5	1	4
$(TVV)^t C(TVV)$	1	5	-4
$(TVV)^t C(TTV)$	3	3	0

D. Concluding remarks concerning PUT preons

Having demonstrated that the PUT preon model mandates the masslessness of the composite quarks and leptons on the metacolor scale $\Lambda_{\rm MC}$, we must now ask how the small—but nevertheless real—quark and lepton masses are generated within our model. In the usual theory, the masses of quarks and leptons are generated by the doublet Higgs scalar responsible for the spontaneous breaking of the gauged electroweak group [whether it be $\mathrm{SU}(2)_L \times \mathrm{U}(1)$ or $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B\text{-}L}$]. Under plausible assumptions about the effective Yukawa couplings, the vacuum expectation values (VEV's) of the Higgs scalars turn out to be small on the metacolor scale. It would therefore be disconcerting if the preon model allowed two-preon scalar condensates to give masses since the natural values of VEV of scalar condensates would be of the order of $\Lambda_{\rm MC}$. We shall see that both two and four-preon scalar condensates cannot give masses to quarks and leptons in the PUT model while six-preon scalar condensates can do so.

Let us consider the possibility of giving masses to fermion composites (e.g., quarks or leptons) via the mechanism of scalar preon condensates. Scalar preon condensates must have quantum numbers (1,2,2) in $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. Thus, scalar preon condensates consist of *odd numbers of T preons and odd number of V preons.* For two-preon scalar composites we have two candidates $T^t CV$ and $\bar{T}\not{D}V$, where C (D_μ) denotes the charge-conjugation operator (covariant derivative). However, $T^t CV$ is not a metacolor singlet, since both T and V transform as $\underline{27}$ of E_6. The combination $\bar{T}\not{D}V$ transforms as 4×4 of $\mathrm{SU}(4)_C$. Hence two-preon scalar condensates cannot give masses to fermion composites. Hereafter, we omit and C and $\not{D}$.

For four-preon scalar composites, we have either three T's and one V or three V's and one T. However, $TTTV$ is not a metacolor singlet. Neither the combination $\bar{T}\bar{T}TV$ nor $TT\bar{T}\bar{V}$ transforms as a singlet of $\mathrm{SU}(4)_C$. Hence, four-preon scalar condensates cannot give masses to fermions either.

It is obvious that six-preon scalar condensates can give masses, since quarks and leptons are three-preon composites. This desirable feature of the PUT preon model may solve one small hierarchy problem of the left-right symmetric electroweak model. The problem is to understand why the right-handed weak gauge bosons are much heavier than the left-handed ones. An explanation may lie in the fact that while two- or four-preon scalar condensates cannot give masses to quarks and leptons, they can give masses to the right-handed weak gauge bosons. Indeed, one may expect a difference in the values of VEV between, say, the Higgs triplet Δ_R responsible for the mass of W_R and the Higgs doublet ϕ responsible for the mass of W_L in the LRS model.[5] This follows from the observation that Δ_R can be two- or four-preon condensate in the PUT model (since now an even number of T's and V's can contribute) whereas ϕ can only be a six-preon condensate.

Finally, we comment on the exotic fermion composites predicted by the PUT preon model. Although we eliminated low-dimensional exotics, i.e., $(V+A)$ counterparts of quarks and leptons that belong to the same congruence classes, we end up with some higher-dimensional exotics that belong to the same congruence classes as quarks and leptons. These exotics are the following [see Eqs. (5.21) and (5.22)]:

from TTT,

$$(\overline{\ydiagram{2,1}},2,1)_L=(\overline{20},2,1)_L\,,\quad(\overline{\ydiagram{3}},2,1)_L=(\overline{20'},2,1)_L\,,$$
$$(\overline{\ydiagram{2,1}},4,1)_L=(\overline{20},4,1)_L\,; \tag{5.31}$$

from TVV,

$$(\ydiagram{1},2,3)_L=(4,2,3)_L\,,\quad(\overline{\ydiagram{2,1}},2,3)_L=(\overline{20},2,3)_L\,,$$
$$(\ydiagram{3,1,1},2,3)_L=(36,2,3)_L\,;\quad(\overline{\ydiagram{2,1}},2,1)_L=(\overline{20},2,1)_L\,,$$
$$(\ydiagram{3,1,1},2,1)_L=(36,2,1)_L\,. \tag{5.32}$$

All the fermion composites in Eqs. (5.31) and (5.32) are massless on the Λ_{MC} scale—like the quark-lepton composites—and are therefore a class of exotics that were not eliminated *ab initio* from the theory since they belong to the same congruence class of quarks and leptons. It should be emphasized that there exist no exotics which belong to other congruence classes, e.g., that of the mirror fermions.

We argue that the $SU(4)_C$ exotics can be made heavy, using an old argument of Nambu and Han to explain why the higher-dimensional representations of color SU(3) give larger masses than the low-dimensional representation.[35] An analogous argument can readily be extended to $SU(4)_C$. Consider that the fermion composites must ultimately acquire mass through some symmetry-breaking process. These particles would then have to become Dirac particles and they would have mass terms which take the form

$$M=M_0+AI_2(R)\,, \tag{5.33}$$

where $I_2(R)$ denotes the eigenvalue of the second-order Casimir invariant for an irreducible representation R of $SU(4)_C$. As shown in Appendix B, the higher-dimensional representations of $SU(4)_C$ have larger eigenvalues for $I_2(R)$ than does the basic representation. A similar argument cannot be applied to the weak exotic $(4,2,3)_L$ and this is still an open question.

VI. GENERAL CONCLUSIONS

The popular grand unified groups—such as SU(5) and SO(10)—achieve unification of the strong, electromagnetic, and weak interactions at a mass of the order of $\Lambda_{GUT}\sim10^{15}$ GeV. Unfortunately, these GUT's do not explain the proliferation of quark and lepton generations.[6] We also know that quarks and leptons exhibit no structure down to 10^{-16} cm ($\sim10^2$ GeV). The region between 10^2 and 10^{15} GeV is therefore *terra incognita* (desert or otherwise) and may just accommodate a composite preon model that both gives structure to the quarks and leptons and predicts the right number of families (at least three).

In this paper, we have examined the group-theoretic consequences of the preon model under a number of plausible assumptions spelled out in Sec. I. Despite our very restrictive assumptions, we do find two GUT preon models and one PUT preon model satisfying all our conditions. We manage to do so because we can ensure the masslessness of quarks and leptons on the metacolor scale ($\Lambda_{MC}\gg10^2$ GeV) via a mechanism (gauged chiral subgroup or discrete symmetry) other than consistency with 't Hooft's anomaly condition.[14]

The two permissible GUT preon models are SU(3)(MC)×SO(10)(CF) and F_4(MC)×SO(10)(CF). The first model works with two basic preon chiral representations: $T=(3,\overline{16})_L$ and $V=(\overline{3},\overline{16})_L$ and the composite three-fermionic objects defining quarks and leptons are $(TTT)_L$, $(T\overline{V}\,\overline{V})_L$, $(V\overline{T}\,\overline{T})_L$, and $(VVV)_L$. With the use of the Pauli principle generalized to the metacolor degree of freedom, the number of massless quark and lepton generations is four. The second model only requires one basic preon chiral representation $V=(26,16)_L$. There are three generations of quarks and leptons arising from $(VVV)_L$ and $(V\overline{V}\,\overline{V})_L$. In both GUT preon models (particularly the first), the rate of decrease of the metacolor coupling constant is so slow that the asymptotic energy region for metacolor would be higher than the Planck mass ($\sim 10^{19}$ GeV) and hence both GUT models are highly unlikely.

The only PUT preon model satisfying our conditions is $E_6(MC)\times SU(4)_C\times SU(2)_L\times SU(2)_R$ with $T=(27,\bar{4},2,1)_L$ and $V=(27,4,1,2)_L$. Precisely three generations of quarks and leptons are predicted from $(TTT)_L$, $(TVV)_L$, $(VTT)_L$, and $(VVV)_L$. The generations are exact copies in the sense of the G(CF) representation, but differ in their compositions, thereby opening up the possibility of different masses for the three generations when the original preon symmetry is broken. The asymptotic behavior of the metacolor constant in the PUT model is much faster (even faster than in QCD) than in the two GUT models. For this reason alone, the PUT preon model is the most promising of all three models.

Several interesting consequences of the PUT preon model—with regard to scalar preon condensates, higher-dimensional exotics, etc.—were pointed out in Sec. V D. Here we note some features of the model which are especially challenging. (1) The metacolor group G(MC) is E_6 and only its symmetric part [the reverse is true when G(MC) is SU(3)] contributes to the total antisymmetrization (generalized Pauli principle) of the metacolor, color-favor, and spin degrees of freedom; this implies that the PUT group $SU(4)_C\times SU(2)_L\times SU(2)_R$ (with the metacolor state now singlet) for the quark-lepton three-preon composites automatically obeys the Pauli principle for the color-flavor and spin degrees of freedom. (2) The quark-lepton composites belong to representations (albeit different) of the *same* color-flavor group as do the preon representations. Perhaps this is an indication that there will be no need for pre-preons, etc. (3) A PUT preon model exists that satisfies all of our conditions except the limitation to a maximum of two chiral preon representations. It is $SU(3)(MC)\times SU(4)_C\times SU(2)_L\times SU(2)_R$ with the *three* chiral preon representations $(3,\bar{4},2,1)_L$, $(3,4,1,2)_L$, and $(\bar{3},1,1,1)_L$. The last representation (there are 16 of them) is needed to cancel anomalies on the metacolor level when SU(3)(MC) is used. However, the number of preonic degrees of freedom is increased to such an extent that asymptotic freedom is barely achieved ($B=\frac{1}{3}$) as in the GUT preon model $SU(3)\times SO(10)$, and the objection is the same. A small number of chiral preon representations is mandated by the condition of asymptotic freedom for the metacolor group. (4) $B-L$ is a generator of the PUT preon group (the two candidate GUT preon groups share this property) and therefore can be a broken local symmetry.[36] (5) Color-flavor asymptotic freedom does not hold—in contrast to metacolor asymptotic freedom—for the preons in the PUT model. This may be connected with the fact that it is not possible to have a "meta-GUT" group, i.e., to unify E_6(MC) and $SU(4)_C\times SU(2)_L\times SU(2)_R$ because an exceptional group of sufficient rank does not exist. The color and flavor coupling constants could then become large at the preon level and new physics would be indicated. Of course, color and flavor asymptotic freedom still holds for the preon composites which comprise the quarks and leptons. (6) Crucial experimental tests of many of these ideas would be the detection of neutron oscillations and the observation of deviations from the structureless character of quarks and leptons.

ACKNOWLEDGMENTS

We would like to thank S. Okubo and P. Xue for discussions and A. Salam for correspondence. One of us (Y.T.) is also indebted to Professor J. Iizuka for encouragement. This work was supported in part by the U.S. Department of Energy under Contract No. DE-AS05-80ER10713.

APPENDIX A: NONBASIC REPRESENTATIONS OF METACOLOR GROUPS SATISFYING $R\times R\times R\supset 1$

If we do not limit ourselves to basic representations, then any metacolor group can have a representation R which satisfies $R\times R\times R\supset 1$. Candidates are those with $C(R)$ as follows:

$$
\begin{aligned}
&C(R)=ml\ (m=0,1,2)\ (\text{mod } 3l) \quad \text{for SU}(3l),\\
&C(R)=0\ (\text{mod } n) \quad \text{for SU}(n)\ (n\geq 2),\\
&C(R)=0\ (\text{mod } 2) \quad \text{for SO}(2n+1)\ (n\geq 3),\\
&C(R)=0\ (\text{mod } 2) \quad \text{for Sp}(2n)\ (n\geq 2),\\
&C(R)=(0,0)\ (\text{mod } 2,\ \text{mod } 4) \quad \text{for SO}(2n)\ (n\geq 4),\\
&C(R)=0\ (\text{mod}2) \quad \text{for } E_7,\\
&C(R)=\text{any} \quad \text{for } E_6,\ E_8,\ F_4,\ G_2\ .
\end{aligned}
\tag{A1}
$$

Note that the condition is only necessary. For groups SO($2n+1$). SO($4n$), Sp($2n$), E_7, E_8, F_4, and G_2 and real or pseudoreal representations of SU(n), E_6, and SO($4n+2$), it is enough to look for R which satisfies the relation that $R\times R\supset R$. For example,

$$\begin{aligned}&SO(7) \text{ with } 21,27,\ldots,\\ &So(8) \text{ with } 28,\ldots,\\ &SO(9) \text{ with } 36,\ldots,\\ &SO(10) \text{ with } 45,54,210,\ldots,\\ &Sp(4) \text{ with } 10,\ldots,\\ &E_7 \text{ with } 133,\ldots .\end{aligned} \tag{A2}$$

APPENDIX B: EIGENVALUES OF $I_2(R)$ FOR ARBITRARY R OF SU(N)

We express the eigenvalues of the second-order Casimir invariants of SU(N) in terms of those of SU(M) ($M<N$). Thus, we can calculate all eigenvalues of the second-order Casimir invariants of SU(N) in terms of the SU(2) values, which are easy to calculate. Although the general expression for eigenvalues of the second-order Casimir invariants exist,[19] the method below is greatly simplified.

Lemma. The eigenvalue of the second-order Casimir invariant for an irreducible representation R of SU(N) can be expressed in terms of those of SU(M) ($M<N$), using the branching of $R(\text{SU}(N))$ into $\oplus_j R_j'(\text{SU}(M))$ as follows:

$$I_2^{(N)}(R)=\sum_j I_2^{(M)}(R_j')\frac{d(R_j')}{d(R)}\frac{d(R_{\text{adj}})}{d(R'_{\text{adj}})}\ , \tag{B1}$$

where $d(R)$ is the dimension of R and $I_2(R)$ is the eigenvalue of the second-order Casimir invariant.

Proof. We denote generators of an irreducible representation R, of SU(N) by X_μ $[\mu=1,2,\ldots,(N^2-1)]$. Then we have

$$\text{Tr}X_\mu X_\nu=\frac{d(R)}{d(R_{\text{adj}})}I_2^{(N)(}(R)g_{\mu\nu}\ , \tag{B2}$$

where $g_{\mu\nu}$ is the Killing form[19] and $I_2^{(N)}(R)=g_{\mu\nu}X^\mu X^\nu$. We embed SU($M$) in SU($N$) ($M<N$) such that X_α $[\alpha=1,2,\ldots,(M^2-1)]$ become generators of a *reducible* representation of SU(M). Then, Eq. (A2) holds for SU(M) also, if we limit the indices to those of SU(M). Multiplying by $g^{\alpha\beta}$ $[\alpha,\beta=1,2,\ldots,(M^2-1)]$, we have

$$g^{\alpha\beta}\text{Tr}X_\alpha X_\beta=\frac{d(R)}{d(R_{\text{adj}})}I_2^{(N)}(R)g^{\alpha\beta}g_{\alpha\beta}\ . \tag{B3}$$

Since R branches into $\oplus_j R_j'$, where R_j' are irreducible representations of SU(M), we have

$$g^{\alpha\beta}\text{Tr}X_\alpha X_\beta=\sum_j I_2^{(M)}(R_j')d(R_j')\ . \tag{B4}$$

Using the fact that

$$g^{\alpha\beta}g_{\alpha\beta}=d(R'_{\text{adj}})\ \ [\alpha,\beta=1,2,\ldots,(M^2-1)]$$

we have

$$I_2^{(N)}(R)=\sum_j I_2^{(M)}(R_j')\frac{d(R_j')}{d(R)}\frac{d(R_{\text{adj}})}{d(R'_{\text{adj}})}\ .$$

Q.E.D.

Hereafter, we use the normalization of $I_2^{(2)}(R)$ of SU(2) as

$$I_2^{(2)}(\square)=\tfrac{1}{2}(\tfrac{1}{2}+1)=\tfrac{3}{4}\ . \tag{B5}$$

Then we have

$$I_2^{(2)}(\square\square)=2\ ,\quad I_2^{(2)}(\square\square\square)=\tfrac{15}{4}\ . \tag{B6}$$

To express the SU(4) eigenvalues in terms of the SU(2) quantities, we use

$$I_2^{(4)}(R)=5\sum_j I_2^{(2)}(R_j')d(R_j')/d(R) \tag{B7}$$

since $d(R_{\text{adj}})=15$ and $d(R'_{\text{adj}})=3$. We calculate the eigenvalues of the second-order Casimir invariant of SU(4) for the representations we use in the text. The branchings of these representations are

$$\begin{aligned}
&\square \to \square + 2\,\underline{1}\ ,\\
&\square\square \to \square\square + 2\,\square + 3\,\underline{1}\ ,\\
&\square\square\square \to \square\square\square + 2\,\square\square + 3\,\square + 4\,\underline{1}\ ,\\
&\begin{smallmatrix}\square\square\\ \square\end{smallmatrix} \to 2\,\square\square + 5\,\square + 4\,\underline{1}\ ,\\
&\begin{smallmatrix}\square\square\square\\ \square\\ \square\end{smallmatrix} \to \square\square\square + 4\,\square\square + 7\,\square + 6\,\underline{1}\ .
\end{aligned} \tag{B8}$$

Therefore, we obtain

$$\begin{aligned}&I_2^{(4)}(\square)=\tfrac{15}{8}\ ,\ I_2^{(4)}(\square\square)=\tfrac{9}{2}\ ,\ I_2^{(4)}(\square\square\square)=\tfrac{63}{8}\ ,\\ &I_2^{(4)}\left(\begin{smallmatrix}\square\square\\ \square\end{smallmatrix}\right)=\tfrac{39}{8}\ ,\ I_2^{(4)}\left(\begin{smallmatrix}\square\square\square\\ \square\\ \square\end{smallmatrix}\right)=\tfrac{55}{8}\ .\end{aligned} \tag{B9}$$

Evidently, the lowest value corresponds to the basic representation.

APPENDIX C: ANOMALY COEFFICIENTS $A^{(N)}(R)$ FOR ARBITRARY R of SU(N)

We show how to calculate the anomaly coefficients $A^{(N)}(R)$ for arbitrary R of SU(N) in terms of those of SU(3). Although the general expression for $A^{(N)}(R)$ exists,[37] the method below is easy for quick calculation.

We define the anomaly coefficient $A^{(N)}(R)$ for an irreducible representation R of SU(N) as follows:

$$\mathrm{Tr}[X_\alpha\{X_\beta,X_\gamma\}]=d_{\alpha\beta\gamma}A(R)\ , \tag{C1}$$

where $d_{\alpha\beta\gamma}$ is totally symmetric and X_α is an element of the SU(N) Lie algebra in the representation R. From the definition, it is easy to derive the following:

$$A(\underline{1})=0, \tag{C2}$$

$$A(\bar{R})=-A(R), \tag{C3}$$

$$A\left[\sum_j R_j\right]=\sum_j A(R_j), \tag{C4}$$

$$A(R_1\times R_2)=A(R_1)d(R_2)+A(R_2)d(R_1)\ . \tag{C5}$$

Using these formulas, we can calculate all anomaly coefficients of SU(3), starting from the basic representation.

The calculation of the anomaly coefficients of SU(N) can be reduced to that of SU(3), using the branching of R (SU(N)) into $\oplus_j R_j'$ (SU(3)). We obtain

$$A^{(N)}(R)=\sum_j A^{(3)}(R_j') \tag{C6}$$

since we use the embedding of SU(3) into SU(N) such that X_α $(\alpha=1,2,\ldots 8)$ out of X_α $[\alpha=1,2,\ldots(N^2-1)]$ become generators of a reducible representation of SU(3). From Eq. (C4), we immediately obtain

$$A^{(N)}(\bar{N})=A^{(3)}(\bar{3}) \tag{C7}$$

since $\underline{N}=\underline{3}+(N-3)\underline{1}$.

For example, we calculate $A^{(N)}(\underline{\tfrac{1}{2}N(N+1)})$, using Eqs. (C2)—(C7). Using the branching

$$\underline{\tfrac{1}{2}N(N+1)}=\underline{6}+(N-3)\underline{3}+\tfrac{1}{2}(N-2)(N-3)\underline{1}$$

we obtain

$$A^{(N)}(\underline{\tfrac{1}{2}N(N+1)})=A^{(3)}(\underline{6})+(N-3)A^{(3)}(\underline{3})\ . \tag{C8}$$

However, we have

$$A^{(3)}(\underline{3}\times\underline{3})=A^{(3)}\underline{6}+A^{3)}(\bar{\underline{3}})$$
$$=A^{(3)}(\underline{6})-A^{(3)}(\underline{3})$$

while

$$A^{(3)}(\underline{3}\times\underline{3})=2A^{(3)}(\underline{3})d(\underline{3})\ .$$

Thus,

$$A^{(3)}(\underline{6})=[2d(\underline{3})+1]A^{(3)}(\underline{3})$$
$$=7A^{(3)}(\underline{3})\ . \tag{C9}$$

Combining Eqs. (C7)—(C9), we have

$$A^{(N)}(\underline{\tfrac{1}{2}N(N+1)})=(N-3+7)A^{(3)}(\underline{3})$$
$$=(N+4)A^{(N)}(\underline{3})\ .$$

[1]S. L. Glashow, Nucl. Phys. 22, 579 (1961); S. Weinberg, Phys. Rev. Lett. 27, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8),* edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968).

[2]Cf. R. E. Marshak, Proc. Nat. Acad. Sci. 79, 3371 (1982).

[3]G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976); Phys. Rev. D 14, 3432 (1976); 18, (1978). See also N. H. Christ, *ibid.* 21, 1591 (1980); S. Dimopoulos and L. Susskind, *ibid.* 18, 4500 (1978); R. J. Noble, *ibid.* 25, 825 (1982).

[4]The original motivations for enlarging $SU(2)_L\times U(1)$ to the left-right-symmetric group was to restore parity as a high-energy symmetry—cf. J. C. Pati, and A. Salam, Phys. Rev. D 10, 275 (1974); R. N. Mohapatra and J. C. Pati, *ibid.* 11, 566 (1975); G. Senjanovic and R. N. Mohapartra, *ibid.* 12, 1502 (1975). Later, it was shown that the generator of U(1) should be identified with $B-L$ local symmetry—cf. R. E. Marshak and R. N. Mohapatra, Phys. Lett. B91, 222 (1980); Trans. N. Y. Acad. Sci. 40, 124 (1980).

[5]Cf. R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. 44, 1316 (1980); Riazuddin, R. E. Marshak, and R. N. Mohapatra, Phys. Rev. D 24, 1310 (1981).

[6]Cf. P. Langacker, Phys. Rep. 72, 186 (1981). For reviews on group theory of grand unification, see R. Slansky, Phys. Rep. 79, 1 (1981); Y. Tosa, Ph.D. thesis, University of Rochester, 1981 (unpublished).

[7]This is the Pati-Salam group [J. C. Pati and A. Salam, Phys. Rev. D 10, 275 (1974)] except that these authors regarded L as the fourth color instead of $B-L$, as we do (see Ref. 5). We shall consider the more general PUT group $SU(n)_C\times SU(2)_L\times SU(2)_R$ below.

[8]F. Wilczek and A. Zee, Phys. Lett. 88B, 311 (1979).

[9]Cf. K. C. Wali, in *Weak Interactions as Probes of Unification,* proceedings of the Workshop of the Virginia Polytechnic Institute, 1980, edited by G. B. Collins, L. N. Chang, and J. R. Ficenec (AIP, New York, 1981).

[10]S. L. Glashow, in *Quarks and Leptons, Cargèse, 1979,* edited by M. Lévy *et al.* (Plenum, New York, 1980).

[11]Cf. H. Harari, Canadian Summer School Lectures, Banff, 1981, Report No. WIS-82/1/Jan-ph (unpublished). Also M. Peskin, in *Proceedings of the 1981 International Symposium on Lepton and Photon Interactions*

at High Energies, Bonn, edited by W. Pfeil (Universität Bonn, Bonn, 1981); S. Dimopoulos, S. Raby, and L. Susskind, Nucl. Phys. B173, 208 (1980); T. Banks, S. Yankielowicz, and A. Schwimmer, Phys. Lett. 96B, 67 (1980); Y. Frishman, S. Schwimmer, T. Banks, and S. Yankielowicz, Nucl. Phys. B177, 157 (1981); G. Farrar, Phys. Lett. 96B, 2731 (1980); R. Barbieri, L. Maiani, and R. Petronzio, *ibid.* 96B, 63 (1980); J. Preskill and S. Weinberg, Phys. Rev. D 24, 1059 (1981); I. Bars, Phys. Lett. 109B, 73 (1981); I. Bars and S. Yankielowicz, *ibid.* 101B, 159 (1981); R. Barbieri and R. N. Mohapatra, Phys. Rev. D 25, 2419 (1982); R. Barbieri, R. N. Mohapatra, and A. Masiero, Phys. Lett. 105B, 369 (1981); 107B, 455(E) (1981); C. H. Albright, Phys. Rev. D 24, 1969 (1981); S. Weinbreg, Phys. Lett. 102B, 401 (1981); C. H. Albright, B. Schrempp, and F. Schrempp, *ibid.* 108B, 291 (1982); 113B, 225 (1982); S. Coleman and B. Grossman, Nucl. Phys. B203, 205 (1982).

[12]Y. Tosa and R. E. Marshak, Phys. Rev. D 26, 303 (1982).

[13]A preliminary report on this work will be found in Y. Tosa and R. E. Marshak, Report No. VPI-HEP 81/10 (unpublished) and Proceedings of the Harvard Workshop on Neutron Oscillations, 1982 (unpublished).

[14]G. 't Hooft, in *Recent Developments in Gauge Theories,* proceedings of the NATO Advanced Study Institute, Cargése, 1979, edited by G. 't Hooft *et al.* (Plenum, New York, 1980).

[15]P. Cvitanovic, Phys. Rev. D 14, 1536 (1976); University of Oxford Report No. 40/77, 1977 (unpublished).

[16]Cf. J. G. Branson, in *Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn* (Ref. 11).

[17]We use the term metacolor for the confining preon group to avoid confusion with other new "colors" in the literature (technicolor, hypercolor, precolor, etc.).

[18]F. W. Lemire and J. Patera, J. Math. Phys. 21, 2026 (1980).

[19]For example, see R. Slansky, Phys. Rep. 79, 1 (1981); S. Helgason, *Differential Geometry, Lie Groups, and Symmetric Spaces* (Academic, New York, 1978); N. Jacobson, *Lie Algebras* (Dover, New York, 1979); H. Samelson, *Notes on Lie Algebras* (Van Nostrand—Reinhardt, New York, 1969).

[20]The validity of decomposition is checked by dimensions and second- and fourth-order indices. See how they work in W. G. McKay and J. Patera, *Tables of Dimensions, Indices, and Branching Rules for Representations of Simple Lie Algebras* (Marcel Dekker, New York, 1981).

[21]Cf. D. Cords, *High Energy Physics—1980,* proceedings of the XXth International Conference, Madison, Wisconsin, edited by L. Durand and L. G. Pondrom (AIP, New York, 1981), p. 590.

[22]S. Okubo, Phys. Rev. D 16, 3535 (1977).

[23]Y. Tosa and S. Okubo, Phys. Rev. D 23, 2468 (1981); 23, 3058 (1981). Y. Tosa, *ibid.* 25, 1150 (1982); 25, 1714 (1982); S. Okubo, Had. J. 5, 7 (1981); Phys. Rev. D 26, 2893 (1982); H. Georgi, Harvard Report No. HUTP-80/A054, 1980 (unpublished); A. Zee, Phys. Lett. 99B, 110 (1981).

[24]I. Bars, Phys. Lett. 114B, 118 (1982); Yale University Reports Nos. YTP-82-04 and YTP-82-12 (unpublished).

[25]D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); Phys. Rev. D 8, 3633 (1973); 9, 980 (1974); H. Georgi and H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973); Phys. Rev. D 9, 416 (1974).

[26]For example, H. Georgi, Nucl. Phys. B156, 126 (1977).

[27]R. Casalbuoni and R. Gatto, Phys. Lett. 93B, 47 (1980); F. Dong, T. Tu, and P. Xue, *ibid.* 101B, 67 (1981); A. A. Ansel'm, Zh. Eksp. Teor. Fiz. 80, 49 (1981) [Sov. Phys.—JETP 53, 23 (1981)].

[28]F. Gürsey and L. A. Radicati, Phys. Rev. Lett. 13, 173 (1964); A. Pais, *ibid.* 13, 174 (1964); B. Sakita, Phys. Rev. 136, B1756 (1964).

[29]S. Okbuo (private communication) has found a method of identifying the symmetry property of $R\times R\times R$, using dimensions and second-order Dynkin indices. In the case of SO(10) with $R=\underline{16}$, we have $S=\underline{672}+\underline{144}$, $A=\underline{560}$, $M=\underline{1200}+\underline{144}+\underline{16}$.

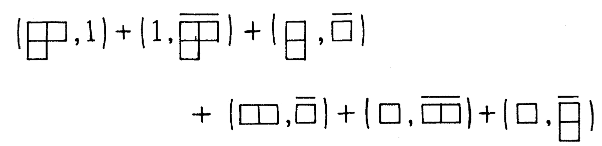
[30]P. Xue (private communication). An early version of the PUT preon section was done in collaboration with P. Xue [see Report No. VPI-HEP-82/2 (unpublished)], to whom we express our thanks.

[31]Y. Achiman, University of Wuppertal Report No. WU-B 81-13, 1981 (unpublished). Achiman found three generations of quarks and leptons with $G(\mathrm{MC})=\mathrm{SU}(3)$ and without the generalized Pauli principle; we shall see below that the Pauli principle reduces the number to one generation.

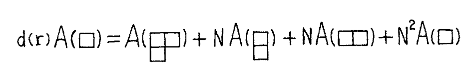
[32]I. Bars (Ref. 23) has shown that the 't Hooft conditions (anomaly plus decoupling) cannot be satisfied with two chiral preon representations. Even allowing for a larger number of chiral preon representations, Bars has not been able to find a viable preon model consistent with the 't Hooft conditions.

[33]E. Guadagnini and K. Konishi Nucl. Phys. B196, 165 (1982); S. Weinberg, Phys. Lett. 102B, 401 (1981); K. Yamawaki and T. Yokota, *ibid.* 133B, 293 (1982); S. Dimopoulos, S. Raby, and L. Susskind, Nucl. Phys. B173, 208 (1980).

[34]For $\mathrm{SU}(N)_V\times\mathrm{SU}(N)_T$ $(N=2n)$, our composites are [see Eq. (5.20)]



The 't Hooft anomaly-matching condition for SU(N)-V is (A stands for anomaly)



where $d(r)$ is the dimension of the metacolor group.

Using the fact that (see Appendix C)

$$A(\yng(2,1)) = (N^2-9)A(\yng(1)),\quad A(\yng(1,1)) = N(N-4)A(\yng(1)),$$

$$A(\yng(2)) = N(N+4)A(\yng(1)),\quad A(\yng(3)) = \tfrac{1}{2}(N+3)(N+6)A(\yng(1)),$$

$$A(\yng(1,1,1)) = \tfrac{1}{2}(N-3)(N-6)A(\yng(1)),$$

we obtain

$$d(r) = 4N^2 - 9 .$$

But $N=8$ for $SU(4)_C$ and $d(r)=27$ for E_6. Thus, the 't Hooft anomaly condition is not obeyed.

[35]Y. Nambu and M. Y. Han, Phys. Rev. D 10, 674 (1974).
[36]Y. Tosa, R. E. Marshak, and S. Okubo, Phys. Rev. D 27, 444 (1983).
[37]J. Banks and H. Georgi, Phys. Rev. D 14, 1159 (1976); S. Okubo, *ibid.* 16, 3528 (1977); J. Patera and R. T. Sharp, J. Math. Phys. 22, 2352 (1981); A. Baha Balantekin, *ibid.* 23, 486 (1982).