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Maximal *CP* violation in the six-quark model

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Consideration is given to some experimental tests of the Kobayashi-Maskawa model of weak interactions for the case in which the *CP*-violating effects could be expected to be maximal, that is, $\delta=\pi/2$. A reanalysis of the model is carried out to verify that $\delta=\pi/2$ is consistent with existing data. In addition to the usual contributions of the high-energy quark box diagram, this analysis takes into account low-energy dispersive terms, which depend on the energy boundary μ. Double penguin diagrams are also shown to give a significant contribution to Δm, depending on μ. Fitting of the result for $\Delta m(\mu)$ to the measured mass difference leads to $\mu \approx 1$ GeV, if the top-quark mass $m_t \lesssim 45$ GeV. This places an upper limit on m_t since, for larger m_t, the calculated Δm is too high for all μ. The results of the analysis are applied to the dilepton asymmetry in B^0, $\bar{B}^0$ production by e^+e^- collisions and similar effects that can be obtained by comparing antineutrino with neutrino production of dileptons. When large asymmetries are found to occur, the rate is found to be hopelessly small because interference effects are suppressed. Large interference effects can occur if the parameters are chosen appropriately, but then the asymmetry is only 1 or 2%. It is also found that interference effects between T^0 and $\bar{T}^0$ are enormously suppressed for any choice of the parameters.

I. INTRODUCTION

The six-quark model of the weak interactions suggested by Kobayashi and Maskawa[1] places the burden for *CP* violation on a relative phase of terms in the weak current connecting quarks of different families. Although the *CP*-violating effects that have been observed are quite small, they do not involve the heavy quarks directly and it is conceivable that larger effects might be found in experiments on phenomena for which the heavy quarks play a direct role. If larger effects were to be found, they might provide additional opportunities to investigate the elusive origins of the *CP* violation, a possibility deserving close attention because of the fundamental nature of the *CP* and time-reversal symmetries.

In any model, the term in the effective Hamiltonian leading to *CP* or *T* violation is "imaginary" in the (Wigner) sense of time reversal.[2] That implies the introduction of a complex phase between terms in the Hamiltonian. For the Kobayashi-Maskawa (KM) model, the phase factor $e^{i\delta}$ is introduced in the quark currents as one of the parameters in the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix relating the quarks of different families. The three other parameters in the matrix are the Cabibbo-type angles θ_1, θ_2, θ_3.

Since *CP* violation is determined by the imaginary part of the effective Hamiltonian, it is clear that sine δ will provide a measure of the effects to be expected from the KM model. The largest effects then correspond to $\delta=\pi/2$, and we refer to this as the case of "maximal *CP* violation" within the context of the model.[3] Because it offers the "best" experimental situation that one may hope for within the context of the model, the case of maximal *CP* violation appears to us to warrant special attention. If the calculated effects for a particular experiment should turn out to be large the experiment will provide good opportunities for testing the KM model in some detail. On the other hand, if the estimated effects are small even for $\delta=\pi/2$, it is unlikely that the experiment can provide detailed information.

Before any such tests are considered, it is necessary to confirm that the assumption $\delta=\pi/2$ is consistent with existing information on the weak interactions. There have been several analyses[4] of the limitations imposed on the CKM matrix by experimental data under a variety of assumptions and approximations. Generally, these allow a range of values of θ_2, θ_3, and δ, including in some cases the possibility $\delta=\pi/2$, although much of the emphasis has been placed on small values of δ to account for the small value of $|\epsilon|$. However, there are many uncertainties in such analyses because exact calculations of hadronic effects are not available. We find that there are two qualitative aspects of the determination of the K_L-K_S mass difference Δm that have not been included in the analyses that should be taken into account if one is to have confidence in the main features of the results. One of these is a severe constraint that can be placed on the low-energy (long-distance) contributions to the dispersive term in the mass matrix, which has been neglected without adequate justification or treated[5] as an undetermined parameter in other analyses. The other is the effect of the high-energy (short-distance) contributions of double penguin diagrams,[6] which cannot be ignored if, as some authors have suggested,[7] the single penguin diagram makes a significant contribution to the K_S decay amplitude. Of course none of these contributions can be evaluated exactly, but it is important that each be estimated at about the same level of plausibility.

The purpose of this work is to reconsider these analyses of the parameters in the CKM matrix for the interesting special case $\delta=\pi/2$, taking comparable account of each of the contributions, and to apply the results to obtain an estimate of *CP* violation in several types of high-energy experiments. The determination of the parameters leads to two new results: (1) the lower limit ("infrared cutoff") of the region of asymptotic freedom is constrained to be about $\mu=1$ GeV by the condition imposed on the dispersive term, and (2) the introduction of the double penguin diagram leads to an upper limit on the mass of the t quark, $m_t \lesssim 45$ GeV.

A generic analysis of possible high-energy experiments on *CP* violation has been carried out by Pais and Treiman[8] and several authors[9] have considered specific tests on the *B*-meson or *T*-meson systems. The most promising of these appear to be measurements of the charge asymmetry in the production of like-charge dimuons. The earlier analyses have been repeated here using values of the parameters that emerge from our assumption of maximal *CP* violation.

The experiments usually considered for this purpose would make use of e^+e^- collisions to produce the dimuons. We call attention to the fact that dimuons produced by neutrinos and antineutrinos on a heavy target may also be used for this purpose. The ratio of the dimuon charge asymmetries for the neutrino and antineutrino experiments provides a measure of essentially the same parameters as those obtainable from the e^+e^- experiment.

II. THE DISPERSIVE TERM IN THE MASS MATRIX

The essential experimental constraint on δ is the experimental value[10]

$$|\epsilon|=2.28\times10^{-3} \tag{1}$$

of the parameter measuring the *CP*-violating admixture of K^0 and $\bar{K}^0$ states in K_L. An additional constraint is imposed by the measured value of the K_L-K_S mass difference

$$\Delta m=0.48\Gamma_S\ , \tag{2}$$

where Γ_S is the total decay rate of K_S. Use is also made of the result

$$\arg\epsilon\approx\phi\ , \tag{3}$$

where

$$\phi=\arctan(2\Delta m/\Gamma_S)\approx\frac{\pi}{4}\ . \tag{4}$$

The quantities ϵ and Δm are directly related to the mass matrix $\mathcal{M}$ which is to be expressed in terms of the parameters of the CKM matrix. If we introduce the notation $|K^0\rangle=|1\rangle$, $|\bar{K}^0\rangle=|2\rangle$, then

$$\epsilon=(\mathcal{M}_{12}-\mathcal{M}_{21})/2(Z_S-Z_L) \tag{5}$$

and

$$-\Delta m=m_S-m_L=M_{12}+M_{21}\ , \tag{6}$$

when corrections of order ϵ are neglected. Here, Z_S and Z_L are the eigenvalues of $\mathcal{M}$ associated with the K_S and K_L states:

$$\begin{aligned} Z_S&=m_S-\tfrac{1}{2}i\Gamma_S\ ,\\ Z_L&=m_L-\tfrac{1}{2}i\Gamma_L\ , \end{aligned} \tag{7}$$

and

$$M=\tfrac{1}{2}(\mathcal{M}+\mathcal{M}^*)\ , \tag{8}$$

where $\mathcal{M}^*$ is the Hermitian conjugate of $\mathcal{M}$.

The phenomenological mass matrix can always be expressed in terms of an effective weak Hamiltonian H_w by means of the Weisskopf-Wigner perturbation theory.[11] In general, H_w is made up of two terms, a $|\Delta S|=1$ contribution $H_w^{(1)}$ and a $|\Delta S|=2$ contribution $H_w^{(2)}$, where S is the strangeness. The contribution of $H_w^{(1)}$ to the mass matrix arises through the virtual decay amplitudes

$$A_c^1(E)=\langle c,\ \text{out}\,|H_w^{(1)}|\,1\rangle[2\pi\rho_c(E)]^{1/2}e^{-i\delta_c(E)} \tag{9a}$$

and

$$A_c^2(E)=\langle c,\ \text{out}\,|H_w^{(1)}|\,2\rangle[2\pi\rho_c(E)]^{1/2}e^{-i\delta_c(E)}\ , \tag{9b}$$

where E is the energy in the decay channel $|c\rangle$, $\rho_c(E)$ is the density of energy states in that channel, and $2\delta_c(E)$ is the eigenphase of the strong-interaction S matrix for channel c. Then, if we let

$\alpha,\beta=1$ or 2, the mass matrix has the form

$$\mathcal{M}_{\alpha\beta}=M_{\alpha\beta}-\tfrac{1}{2}i\Gamma_{\alpha\beta} \tag{10}$$

with

$$M_{\alpha\beta}=-\frac{1}{2\pi}\sum_c \mathrm{P}\int dE\frac{A_c^{\alpha *}(E)A_c^{\beta}(E)}{E-m_K} + \langle \alpha \,|\, H_w^{(2)} \,|\, \beta\rangle \;, \tag{11}$$

where P denotes Cauchy principal value, and

$$\Gamma_{\alpha\beta}=\sum_c A_c^{\alpha *}(m_K)A_c^{\beta}(m_K) \;. \tag{12}$$

From Eq. (6) it is evident that Δm depends on a dispersive term as well as a $\Delta S=2$ term. The dispersion integral depends on the virtual decay amplitudes $A_c^{\alpha}(E)$ to all channels at all energies and early attempts[12] to calculate it were impaired by the apparent divergence at high energy of contributions arising from channels with increasing numbers of particles. However, for the effective Hamiltonian produced by the six-quark model, the Glashow-Iliopoulos-Maiani (GIM) mechanism[13] guarantees the rapid convergence of these amplitudes and the dispersion integral will contribute to Δm only over a range of energies below the mass of the heaviest quark. Therefore the dispersive contributions at energies above μ may be obtained by using the usual effective $\Delta S=2$ Hamiltonian, while the low-energy part can be estimated reliably by inserting the measured decay amplitudes in the integral. This procedure yields a reasonably unambiguous dispersive term depending only on the choice of the cutoff energy.

The dispersive contribution, which turns out to be of order $\frac{1}{2}\Delta m$, *must* be taken into account because it is the outcome of the most general principles of quantum mechanics (or, of course, quantum field theory[14]) for any decaying system. Although others have included it as an adjustable parameter, as suggested by Wolfenstein,[4] in their evaluations of the CKM parameters, we find that there is little room for adjustment. The fact that its value is appreciable for a cutoff as low as $\mu\approx 1$ GeV acts as an important constraint on the CKM parameters.

In the KM model, contributions to $\mathcal{M}$ from quark energies above μ, the lower bound on the domain of asymptotic freedom, can be estimated by the method of Gaillard and Lee,[15] which makes use of the box diagram, Fig. 1, to obtain, in effect, a $|\Delta S|=2$ weak Hamiltonian. Strong interaction corrections to this $H_w^{(2)}$ have been considered in the limit of asymptotic freedom by Gilman and Wise,[16] who have shown that its magnitude is markedly reduced

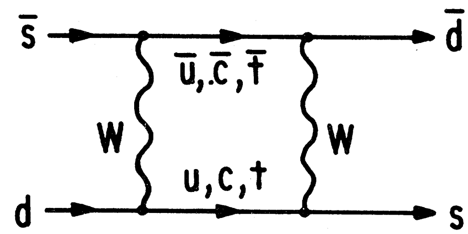

FIG. 1. Standard box diagram for the $K^0\rightarrow\bar{K}^0$ mass matrix element $\mathcal{M}_{21}$.

in the leading-logarithm approximation. A complete description of the strong-interaction corrections would take account of all possible quark-loop insertions inside the box of Fig. 1 and, in part, they include confined quarks which are outside the asymptotic freedom limit and correspond to virtual hadron states which should be included in the dispersive term in Eq. (11). See, for example, Fig. 2.

We shall make the assumption that the unadorned box diagram includes both the "pure" $|\Delta S|=2$ contribution (the reduced value given by Gilman and Wise) and the high-energy contributions to the dispersive term. In essence, we are conjecturing a sum rule to the effect that the result of making all quark-loop insertions in the box and introducing all gluon corrections and confinement effects is about the same as that obtained from the skeleton. Since the conjecture applies only to the high-energy contributions, the error in our final result should not be large both because we will find that the resulting contribution to the mass matrix is only a fraction of the low-energy contribution and because the GIM mechanism sharply limits the contributions of all of these high-energy diagrams.

Our procedure, then, is to take over the work of others on the simple box diagram to obtain contributions to $\mathcal{M}$ from the range of energy above the "infrared" cutoff μ and to carry the dispersion integral up to energy μ. The appropriate choice of μ is

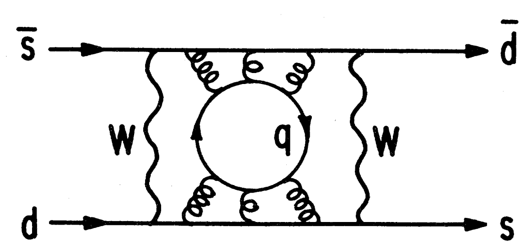

FIG. 2. Schematic of a box diagram modified by a quark-loop insertion. In the nonperturbative (infrared) limit such diagrams become entangled with two-meson dispersive terms.

determined by joining the two contributions (and a third from the double penguin diagram) smoothly and fitting the result to the measured value of Δm.

To evaluate the amplitudes appearing in the dispersion integral we note that the effect of confinement presumably dominates any calculation of the low-energy contributions of the quark currents. Therefore, we should be able to estimate low-energy decay amplitudes by treating the quark field operators as interpolating fields for constituent quarks, in the spirit of a bag model.[17] At small values of E, the matrix elements appearing in the amplitudes Eq. (9) can be expected to be nearly independent of energy because they depend primarily on geometrical factors resulting from the overlap of the constituent quark states. Hence, for $0<E<\mu$, where μ is the limit of "low energy," $A_c(E)$ can be replaced by

$$A_c(E)\approx A_c(m_K)[\rho_c(E)/\rho_c(m_K)]^{1/2}\,, \quad 0<E<\mu\,, \tag{13}$$

in which $A_c(m_K)$ is the observable decay amplitude and $\rho_c(E)$ the density of states. The corresponding contribution to the dispersive term in the mass matrix can thereby be obtained with a reasonable degree of reliability.

Since it will turn out that $\mu\approx 1$ GeV, the only amplitudes that contribute to the dispersive integral are the one-particle amplitudes A_{π^0} and A_η, the two-pion amplitudes A_I, where $I=0,2$ is the isotopic spin, and possibly, the dominant part of the three-pion amplitude $A_{3\pi}$.

Since the 2π decay mode is dominant, we first consider the contribution of the $I=0$ and $I=2$ amplitudes to the dispersive term in $\mathscr{M}_{12}$ and $\mathscr{M}_{21}$, to terms of order ϵ. From the *CPT* theorem it follows[18] that $A_I^2=A_I^{1*}$ and then, from Eq. (13) we find that this dispersive term is

$$D_{2\pi}=-\frac{1}{2\pi}\frac{\Delta m}{\rho_{2\pi}(m_K)}\mathrm{P}\int_0^\mu \frac{dE}{E-m_K}\rho_{2\pi}(E)\,, \tag{14}$$

where use has been made of the experimental results, Eq. (2), in the approximate form

$$\Delta m\approx\tfrac{1}{2}\Gamma_S=A_0{}^2+A_2{}^2\,. \tag{15}$$

Since

$$\rho_{2\pi}(E)\sim E(E^2-4m_\pi{}^2)^{1/2}\,, \tag{16}$$

the integral in Eq. (14) can be evaluated in closed form. The result to leading order in $\mu/2m_\pi$, when inserted into Eq. (6), yields the contribution of the 2π amplitude to Δm

$$\Delta m_{2\pi}(\mu)=\frac{1}{\pi}\Delta m\frac{\frac{1}{2}(\mu/2m_\pi)^2+(m_K/2m_\pi)(\mu/2m_\pi)}{(m_K/2m_\pi)[(m_K/2m_\pi)^2-1]^{1/2}} \tag{17}$$

A similar estimate of the 3π contribution to Δm can be made by making use of the measured 3π decay rate of K_L. Although the rate is very much smaller than Γ_S because of the small density-of-states factor, $\rho_{3\pi}(E)$ increases with E much more rapidly than $\rho_{2\pi}(E)$ so that the ratio $\Delta m_{3\pi}(\mu)$ to $\Delta m_{2\pi}(\mu)$ grows rapidly with μ. However, it turns out that at $\mu=1$ GeV the ratio is still only about 2%, and this contribution can be neglected.

The one-particle contributions have been estimated by Itzykson, Jacob, and Mahoux[12] in terms of Γ_S by making use of soft-pion theorems and (to include the η channel) SU(3) mixing coefficients of the π and η states. We take over their result:

$$\Delta m_{\pi^0}+\Delta m_\eta=-1.44\Delta m\,. \tag{18}$$

The sum of the terms of Eqs. (17) and (18) is the dispersive contribution expressed in terms of the measured mass difference Δm.

III. HIGH-ENERGY CONTRIBUTIONS TO THE MASS MATRIX

The short-distance contributions, i.e., those for $E>\mu$, to $\mathscr{M}_{12}$ lead to an effective $|\Delta S|=2$ Hamiltonian $H_w^{(2)}$ that arises from quark currents governed by the CKM unitary matrix

$$V=\begin{bmatrix} V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{bmatrix}, \tag{19}$$

where the subscripts refer to quark flavors up, down, etc. In the standard KM notation (with $\delta=\pi/2$)

$$\begin{aligned} &V_{ud}=c_1,\quad V_{us}=s_1c_3,\quad V_{ub}=s_1s_3\,,\\ &V_{cd}=-s_1c_2,\quad V_{cs}=c_1c_2c_3+is_2s_3\,,\\ &V_{cb}=c_1c_2s_3-is_2c_3\,,\quad V_{td}=-s_1s_2\,,\\ &V_{ts}=c_1s_2c_3-ic_2s_3\,,\\ &V_{tb}=c_1s_2s_3+ic_2c_3\,,\end{aligned} \tag{20}$$

where $c_i=\cos\theta_i, s_i=\sin\theta_i$. Because of our assumption of "maximal" *CP* violation, the KM phase δ has been set equal to $\pi/2$.

The original treatment of the box diagram, Fig. (1), by Ellis, Gaillard, and Nanopoulos[4] led to an effective Hamiltonian $H_{\rm box}^{(2)}=C\mathscr{O}_{JJ}$ (in the notation of

Shrock and Treiman[4]) where $\mathscr{O}_{JJ}$ is an operator on quark fields and

$$C=\frac{G_F{}^2}{16\pi^2}\left[m_c{}^2\lambda_c{}^2+m_t{}^2\lambda_t{}^2 + \frac{2\lambda_c\lambda_t m_c{}^2}{1-m_c{}^2/m_t{}^2}\ln\left[\frac{m_t{}^2}{m_c{}^2}\right]\right] \tag{21}$$

with

$$\lambda_u=V_{ud}V_{us}^*,\ \lambda_c=V_{cd}V_{cs}^*,\ \lambda_t=V_{td}V_{ts}^*\ , \quad \lambda_u+\lambda_c+\lambda_t=0\ . \tag{22}$$

Note that the term depending on the "infrared cutoff" μ is so small as to be neglected for the range of parameters considered here. This is an excellent approximation because m_t/m_u is so large.

We adopt the result of the bag-model[17] calculation of the matrix element by Shrock and Treiman[4] to obtain, in round numbers,

$$M_{21}^{\rm box}\approx\tfrac{2}{3}f_K{}^2m_KC\ , \tag{23}$$

where f_K is the kaon decay constant $f_K\approx1.23m_\pi$.

The possible importance of contributions arising from the double penguin diagram, Fig. 3, which is topologically distinct from the box diagram, to an analysis of the mass matrix has been suggested by Hill[6] on the basis of the argument[7] that penguin diagrams may make a significant contribution to the two-pion decay amplitude. He considered the low-energy contributions of this type of diagram, treating them as contributions to the dispersive term just as has been done here.

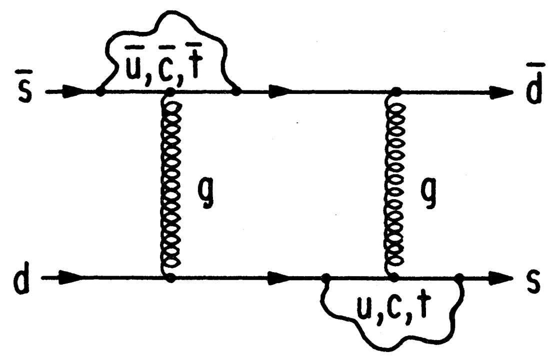

FIG. 3. Double penguin diagram for $\mathscr{M}_{21}$.

Strong-interaction effects on the high-energy contributions to the "bare" single penguin diagram have been considered in some detail, especially by Gilman and Wise, and by Guberina and Peccei.[19] In addition to corrections of this kind to the double penguin of Fig. 3, there are quark-loop corrections, such as those associated with Fig. 4 and these clearly include dispersive terms associated with a great variety of virtual states. Again, in the spirit of our treatment of the box diagram, we assume that the effective $|\Delta S|=2$ Hamiltonian obtained from the high-energy contributions to the bare double penguin diagram, Fig. 3, includes both the (high-energy) dispersive term and the actual $|\Delta S|=2$ term.

In contrast to the case of the box diagram, the large contributions to the integral over the quark-gluon loop arise from the low-energy end of the range, and the result depends sensitively on the infrared cutoff μ. The contribution to the effective $|\Delta S|=2$ Hamiltonian is

$$H_{\rm peng}^{(2)}=D\int d^3x\,\bar{s}(x)\gamma_\mu(1-\gamma_5)T^aT^bd(x)\bar{s}(x)\gamma^\mu(1-\gamma_5)T^bT^ad(x)+{\rm H.c.}\ , \tag{24}$$

where[20]

$$D=\frac{G_F{}^2}{18(4\pi)^4}m_t{}^2\left[\lambda_u\ln\left[\frac{m_c{}^2}{\mu^2}\right]-\lambda_t\ln\left[\frac{m_t{}^2}{m_c{}^2}\right]\right]^2 . \tag{25}$$

The T^a are the eight 3×3 matrices representing the generators of color SU(3).

Since the matrix element is taken between color-singlet states, only the color-blind part of the operator, having the same color structure as $H_{\rm box}^{(2)}$, will contribute and it can be expressed in terms of $M_{21}^{\rm box}$:

$$M_{21}^{\rm peng}/D=\tfrac{73}{9}M_{21}^{\rm box}/C\ . \tag{26}$$

Then from Eq. (23) based on the bag-model calcula-

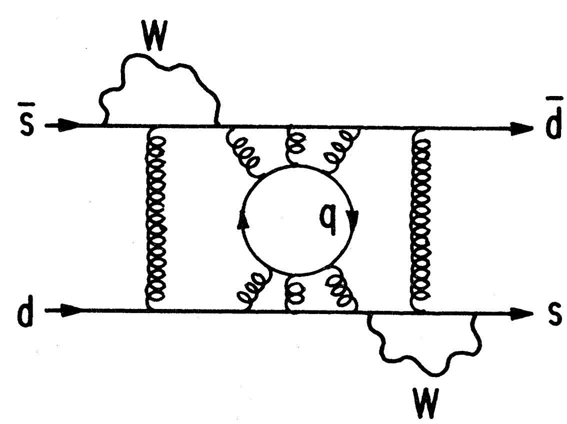

FIG. 4. A quark-loop schematic correction to the double penguin diagram corresponding, as in Fig. 2, to a two-meson dispersive term in the nonperturbative limit.

tion of Shrock and Treiman,[4] we find, again in round numbers,

$$M_{21}^{\rm peng} \approx \tfrac{16}{3} f_K{}^2 m_K D \ . \tag{27}$$

There is no contribution to Γ_{12} from either the box or the double penguin diagrams. Therefore Eqs. (23) and (27) provide the required information on the high-energy part of the mass matrix.

IV. EVALUATION OF CKM MATRIX ELEMENTS.

The parameters in the CKM matrix are evaluated by fitting the theoretical expressions Eqs. (5) and (6) for ϵ and Δm to the experimental values, Eq. (1), (2), and (3) under the assumption $\delta=\pi/2$, that is, using Eq. (20). First we note that, since $M_{\alpha\beta}$ and $\Gamma_{\alpha\beta}$ are Hermitian, Eq. (5) may be rewritten as[21]

$$\epsilon=[i\ {\rm Im}M_{21}+\tfrac{1}{2}{\rm Im}\Gamma_{21}]/[\Delta m+\tfrac{1}{2}i(\Gamma_S-\Gamma_L)] \ . \tag{28}$$

From the experimental result, Eq. (3), it can be concluded that

$$|\,{\rm Im}\Gamma_{21}\,| \ll |\,{\rm Im}M_{21}\,| \ . \tag{29}$$

But our analysis of the dispersive term shows that the long-distance contributions to M_{21} are of the same order as Γ_{21}. Therefore, we conclude from Eq. (29) that, in calculating ${\rm Im}M_{21}$, the *long-distance* dispersive contribution to ${\rm Im}M_{21}$ is negligible, and only the terms arising from the box and double penguin diagrams need be included.

As a starting point for determining the parameters, we take the Cabibbo fit of Shrock and Wang[22]:

$$|\,c_1\,| = 0.974 \pm 0.003 \tag{30}$$

and

$$|\,s_1 c_3\,| = 0.22 \pm 0.002 \ ,$$

from which we take the value of s_1 to be

$$s_1 = 0.23 \tag{31}$$

and the upper limit on s_3 to be

$$s_3 \lesssim 0.42 \tag{32}$$

with a central value $s_3 \approx 0.28$.

The first step is to determine s_2 by fitting Eq. (28), which reduces to

$$|\,\epsilon\,| = \frac{\sqrt{2}}{3}\frac{f_K{}^2 m_K}{\Delta m}\,|\,{\rm Im}C + 8\,{\rm Im}D\,| \ , \tag{33}$$

to Eq. (1). For this purpose we take $m_c=1.5$ GeV and evaluate s_2 as a function of m_t. Since ${\rm Im}C$ is independent of the infrared cutoff μ, and ${\rm Im}D$ has only a logarithmic dependence, the values of s_2 are not very sensitive to the value of μ. A reasonable guess is $\mu=1$ GeV, and that will be used here. Actually we will determine μ from the fit to Δm and it will be seen that this is a good guess. ${\rm Im}C$ and ${\rm Im}D$ may be obtained from Eqs. (21), (22), and (33) to yield a cubic equation for s_2. Solutions are presented in Table I for the central value $s_3=0.28$ and, for comparison, $s_3=10^{-2}$.

By making use of these values of s_2, we may now express Δm as a function of m_t and μ by adding together $\Delta m_{\rm box}$, $\Delta m_{\rm peng}$, and the dispersive term. The expression for $\Delta m_{\rm box}$ given by Eqs. (6) and (23) is

$$\Delta m_{\rm box} = \tfrac{4}{3} f_K{}^2 m_k {\rm Re}C \ , \tag{34}$$

which is independent of μ and essentially independent of m_t over the range of interest because of the very small values of $s_2{}^2$. We find, for $10 \le m_t/m_c \le 30$,

$$\Delta m_{\rm box} = 0.314\Delta m \ . \tag{35}$$

According to Eq. (18) the value of $\Delta m_{\pi 0}+\Delta m_\eta$ is also independent of m_t and μ. Therefore,

$$\Delta m(\mu) = \Delta m_{2\pi}(\mu) + \Delta m_{\rm peng}(\mu) - 1.24\Delta m \ . \tag{36}$$

The expression for $\Delta m_{2\pi}(\mu)$ is Eq. (17) and that for $\Delta m_{\rm peng}(\mu)$ obtained from Eqs. (6) and (27) is

$$\Delta m_{\rm peng}(\mu) \approx \tfrac{32}{3} f_K{}^2 m_K {\rm Re}D \ , \tag{37}$$

where D is given by Eq. (25). The function $\Delta m(\mu)$ given by Eq. (36) is shown in units of the experimental value of Δm as the set of curves in Fig. 5 for the values of m_t, s_2, and s_3 given in Table I. The only values of μ that are consistent with the measured Δm are those corresponding to the intersections of the curves with the horizontal line corresponding to $\Delta m = 1$.

The horizontal band is shown in Fig. 5 to give some indication of the sensitivity of the results to

TABLE I. Values of $s_2=\sin\theta_2$ determined from $|\,\epsilon\,|$ for various values of s_3 and m_t.

s_3	m_t (GeV)	s_2
0.28	15	0.6×10^{-2}
	30	0.18×10^{-2}
	45	0.15×10^{-2}
10^{-2}	15	0.13
	30	0.67×10^{-1}
	45	0.39×10^{-1}

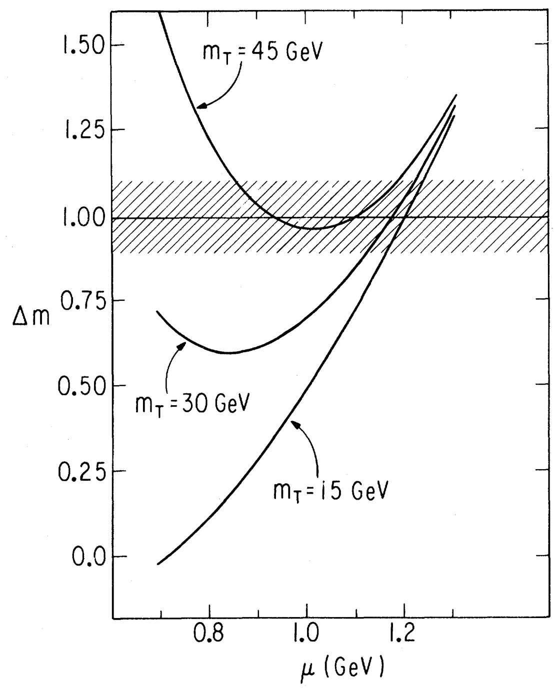

FIG. 5. Sum of contributions to Δm, Eq. (36), in units of the measured value, given as a function of the infrared cutoff μ. The shaded band corresponds to an uncertainty in the calculation of $\pm 10\%$.

corrections. The width corresponds to a variation of $\pm 10\%$ in Δm. The curves are shown for $s_3 = 0.28$ but the shift in the curves for the alternate choice $s_3 = 10^{-2}$, is insignificant.

All of the intersections shown correspond to $\mu \approx 1$ GeV, in good agreement with the usual estimate of the limits of the small-distance approximation. The stability of the result is particularly gratifying.

Because of the μ dependence of $\Delta m_{\rm peng}$, there is a minimum in each of these curves (note that the origin of the μ scale is not shown) and the position of this minimum depends strongly on m_t, because of the strong m_t dependence of $\Delta m_{\rm peng}$. As a result, for $m_t \gtrsim 45$ GeV there is no intersection. Therefore, within the context of the model, an upper limit can be placed on m_t of

$$m_t \lesssim 45 \text{ GeV}. \tag{38}$$

This limit also depends on m_c, which has been taken to be $m_c = 1.5$ GeV, but it does not scale with m_c as might have been expected because of the important contribution from the low-energy terms.[23]

The important consequence of assuming maximal CP violation is the way it affects the form of the CKM matrix V. Since s_1, s_2, and s_3 are small we can expose the important qualitative features of V by setting $c_1 = c_2 = c_3 = 1$. Then we find the matrix (unitary to first order in s_i)

$$V \approx \begin{bmatrix} 1 & s_1 & s_1 s_3 \\ -s_1 & 1 & s_3 - i s_2 \\ -s_1 s_2 & s_2 - i s_3 & i \end{bmatrix}, \tag{39}$$

where we have also set $s_2 s_3 = 0$ since, according to Table I,

$$s_2 s_3 \lesssim 1.8 \times 10^{-3}. \tag{40}$$

Of particular interest is the fact that, aside from corrections of second order in s_2 and s_3, the diagonal element V_{tb} is purely imaginary. Also, V_{ts} is nearly imaginary if s_3 is close to its maximum and V_{cb} is nearly imaginary if s_3 is very small. These results, taken along with the small size of V_{td} and V_{ub} suggest that the Cabibbo-type mixing of top and bottom quarks with the other flavors is rather small but that, for some reason, the unmixed top and/or bottom states are out of phase with states carrying the other flavors. Confirmation of this characteristic would surely have important implications concerning the origin of CP violation.

V. IMPLICATIONS FOR SOME EXPERIMENTS

Experiments that might be expected to manifest substantial CP-violating effects on the basis of the CKM matrix Eq. (39) are those involving b and t quarks directly. Such experiments also would provide a direct determination of the parameters s_2 and s_3 for comparison with the entries in Table I. One measurement bearing on the value of s_2/s_3 would be the determination of the probability of $B^0,\bar{B}^0$ mixing, which is governed by a mass matrix $\mathcal{M}^B$ analogous to $\mathcal{M}$ for the $K^0,\bar{K}^0$ system. Okun, Zakharov, and Pontecorvo[26] have noted that a measure of the degree of mixing of $D^0,\bar{D}^0$ is given by the ratio of the number of like-charge dilepton (l,l) events to the total number of dilepton events in the production of $D^0,\bar{D}^0$ pairs in e^+e^- collisions and the same can be said of the dilepton events in $B^0,\bar{B}^0$ production. It should be remarked that dilepton events resulting from the production of $\bar{B}^0 + l^-$ by neutrinos or $B^0 + l^+$ by antineutrinos may be used in the same way.

The ratio of like- to unlike-charge dileptons is roughly proportional to $(\Delta m_B/\Gamma_B)^2$ which can be estimated by means of the calculation of the box diagram by Hagelin.[27] We find that for a CKM matrix of the form Eq. (39)

$$\frac{\Delta m_B}{\Gamma_B} \approx \frac{1}{2\pi}\left[\frac{m_t^2}{m_B^2} + \ln\frac{m_t^2}{m_B^2} + \frac{1}{3}\right]\frac{s_1^2 s_2^2}{s_2^2 + s_3^2}, \tag{41}$$

which is very sensitive to s_2/s_3, varying from infinitesimal values at $s_2/s_3 \ll 1$ to values of the order of $\frac{1}{2}(m_t/10m_B)^4$ for $s_2/s_3 > 1$. Thus for our solutions in Table I, the case $s_3=0.28$ would correspond to no detectable like-charge dilepton production. On the other hand, the solution for $s_3=10^{-2}$ corresponds to a like-charge dilepton rate ranging between 0.5 and 50 %, depending on the value of m_t/m_B.

The direct test of our assumption of maximal *CP* violation is, of course, a measurement of *CP* violation. Pais and Trieman[8] have carried out a generic analysis of possible tests making use of charmed particles. Several authors have considered specific tests on the *B*-meson system.[9] In particular, Carter and Sanda, and Hagelin have emphasized that the absorptive contribution to the mass matrix makes the like-charge asymmetry

$$a=\frac{N(l^+l^+)-N(l^-l^-)}{N(l^+l^+)+N(l^-l^-)} \tag{42}$$

in the dipleton production through $B^0,\bar{B}^0$ pairs serve as an especially sensitive measure of the phase in the CKM matrix. If one writes

$$\mathcal{M}^B_{\alpha\beta}=M^B_{\alpha\beta}-\frac{i}{2}\Gamma^B_{\alpha\beta} \tag{43}$$

in analogy with Eq. (10), it is not difficult to show that[28]

$$a=\frac{\mathrm{Re}M^B_{12}\mathrm{Im}\Gamma^B_{12}-\mathrm{Im}M^B_{12}\mathrm{Re}\Gamma^B_{12}}{|M^B_{12}|^2+\left|\frac{1}{2}\Gamma^B_{12}\right|^2} \tag{44}$$

Another method for getting at this important interference effect is to make use of the production of dileptons through $\bar{B}^0$ or B^0 states by neutrinos or antineutrinos, respectively. The quantity of interest in this case is the ratio

$$R=\frac{[N(l^+l^-)-N(l^-l^-)]}{[N(l^+l^-)+N(l^-l^-)]}\frac{[\bar{N}(l^+l^-)+\bar{N}(l^+l^+)]}{[\bar{N}(l^+l^-)-\bar{N}(l^+l^+)]}\ , \tag{45}$$

where N and $\bar{N}$ are rates of dilepton production by neutrinos and antineutrinos, respectively. This ratio is given in terms of the phenomenological parameters by

$$R=\frac{(1+\alpha_B{}^2)+(1-\alpha_B{}^2)\chi}{(1+\alpha_B{}^2)-(1-\alpha_B{}^2)\chi}\ , \tag{46}$$

where α_B is the parameter measuring the amount of B^0-$\bar{B}^0$ interference in the notation of Pais and Treiman,[8]

$$|\alpha_B-1|\approx(\Delta m_B/\Gamma_B)^2\ , \tag{47}$$

and

$$\chi=\frac{\mathrm{Im}M^B_{12}\mathrm{Re}\Gamma^B_{12}-\mathrm{Re}M^B_{12}\mathrm{Im}\Gamma^B_{12}}{|\mathcal{M}^B_{12}\mathcal{M}^B_{21}|}\ . \tag{48}$$

We have already seen that for $s_3=0.28$ the interference effects are expected to be extremely small in the KM model. For the other case we have considered, $s_3=10^{-2}$, both the asymmetry a for the e^+e^- experiment and $R-1$ for the neutrino experiment turn out to be of the order of 1 or 2 %.

One might expect the situation to improve for $T^0,\bar{T}^0$ production because of the large imaginary matrix element V_{tb}. *CP*- and *T*-violating effects for this process have recently been considered by Cheng.[3] Unfortunately, just because the magnitude of V_{tb} is large, the decay rate Γ_T is very large compared to Δm, which is suppressed by a factor of $(m_b/m_T)^4$ because of GIM cancellation,[29] and by factors of s_i. Therefore tests of *CP* depending on T^0-$\bar{T}^0$ interference effects do not appear to be feasible within the context of the KM model.

VI. CONCLUSION

It has been found that the condition $\delta=\pi/2$ for maximal *CP* violation in the KM model is in good accord with existing experimental data. That this result is not at the outer edge of the range of acceptable values of the parameters in the CKM matrix is a direct consequence of our result that the long-distance (low-energy) dispersive contributions to the mass matrix of the K^0-$\bar{K}^0$ system calculated phenomenologically as a function of μ, the lower-energy bound of the quark field perturbation treatment, are large. The value $\mu\approx1$ GeV obtained by fitting $\Delta m(\mu)$ to the experimental value is eminently reasonable.

In making this fit, we have included a rough estimate of the contribution of the double penguin diagram with the result that an upper limit of $m_t \lesssim 45$ GeV is placed on the top-quark mass. Although the numerical value of this limit is specific to our model and to our approximations, the general features of the μ dependence of the double penguin and dispersive contributions indicate that any complete attempt to fit Δm within the context of the KM model will lead to some such limit.

Even with maximal violation, which introduces some almost imaginary matrix elements into the CKM matrix, the experimental *CP*-violating effects we have examined (charge asymmetries in dilepton production) are quite small. The principal difficulty is that they depend on the occurrence of vacuum regeneration in the B^0-$\bar{B}^0$ or T^0-$\bar{T}^0$ systems and, for those values of the parameters leading to large *CP* effects, the regeneration is essentially nonexistent.

On this basis, the observation of a large charge asymmetry in dilepton production through B^0 and $\bar{B}^0$ states would be a strong indication that the KM model is not sufficient to account for CP violation. However, a word of caution about these calculations is appropriate. They are based on a rough approximation[27] to the bare box diagram for the $B^0,\bar{B}^0$ system and therefore may be subject to substantial corrections. On the other hand, observation of like-charge prompt-dilepton production, or any other evidence of interference effects in $T^0,\bar{T}^0$ production would appear to be contrary to the KM model of weak interactions, because these results do not depend sensitively on the details of the model.[29]

It remains to be seen whether other experiments, including experiments on time-reversal violation, might offer more promising opportunities to manifest the existence of a large CP-violating phase in the CKM matrix. The discovery that there is, indeed, a phase difference of $\pi/2$ between the third family of quarks and other families would undoubtedly have important fundamental implications.

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[29]The off-diagonal elements of $\mathscr{M}^T$, which determine Δm_T take the form $\sum_{i,j}\chi_i\chi_j f(m_T{}^2,m_i{}^2,m_j{}^2)$ with $i,j=d,s,b$ where $\chi_i = V_{ti}^* V_{ui}$, in analogy to Eq. (22). Since $m_i \ll m_T$,

$$f \approx a + b(m_i{}^2+m_j{}^2)/m_T{}^2 + c\ (m_i{}^2+m_j{}^2)^2/m_T{}^4 .$$

Since V is unitary, $\sum_i \chi_i = 0$, which expresses the GIM condition, and the contributions of the zeroth- and first-order terms in f to the expression for the off-diagonal element vanish. On the other hand, the diagonal elements of $\mathscr{M}^T$, which determine Γ_T, are proportional to products of $|V_{ti}|^2$ and $|V_{uj}|^2$, each of which sums to unity (rather than zero) with the result that Γ is of zero order in $m_i{}^2/m_T{}^2$ while Δm_T is of second order.