Physics of weak nonleptonic decays: $K \to \pi\pi$

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We evaluate long-distance effects in $K \to \pi\pi$ decays using a calculational scheme based on partial conservation of axial-vector current, current algebra, and suppression of exotic Regge meson trajectories. We find that the $\Delta I = \frac{1}{2}$ rule emerges naturally from these properties of hadronic physics. Moreover the calculated long-distance contributions are seen to play the dominant role, as opposed to short-distance effects.

INTRODUCTION

Weak nonleptonic decays of hadrons have since their discovery been one of the most complicated puzzles in elementary particle physics, due to the surprising difference observed between the $\Delta I = \frac{1}{2}$ and the $\Delta I = \frac{3}{2}$ amplitudes (typically a factor of 20).

After many attempts carried out in the 1960's, based mainly on PCAC (partial conservation of axial-vector current) and current-algebra, dynamical calculations by a few authors[1,2] suggested that the puzzle could be solved, at least for S-wave baryon decays, in terms of a peculiar cancellation mechanism holding for $\Delta I = \frac{3}{2}$ but not for $\Delta I = \frac{1}{2}$ amplitudes. Such calculations showed that long-distance effects ($\simeq 1$ fm) play a crucial role in determining both the suppression of $\Delta I = \frac{3}{2}$ amplitudes and the absolute magnitude of $\Delta I = \frac{1}{2}$ decays.

This fact may appear somewhat surprising since the nonleptonic Hamiltonian explores regions of very small distances, typically of the order of the W Compton wavelength $1/m_W$. The essential feature responsible for the above fact is the absence of the leading singularity ($\simeq 1/x^2$) in the product of two currents at short distances, which can be proved to hold in a theory where quarks are either free or asymptotically free at short distances (as in QCD).[2]

Starting from Wilson's suggestion[3] that, due to possible anomalous dimensions, one could get some enhancement of the $\Delta = \frac{1}{2}$ amplitude from short ($\sim 1/m_W$) distances, the problem received in the mid-1970's extra impetus.[4] It was shown that in asymptotically free QCD Wilson's suggestion was indeed correct and several attempts were made to obtain quantitative predictions for weak nonleptonic decays. However, even though this type of analysis in QCD has been pushed up to the two-loop approximation,[5] with the currently accepted values of $\Lambda_{\rm QCD}$ the conclusion is unavoidable that short-distance enhancement cannot alone explain the difference between $\Delta I = \frac{1}{2}$ and $\frac{3}{2}$ amplitudes. Thus, even accepting the QCD implementation of Wilson's suggestion, long-distance enhancement and/or suppression mechanisms must play a major role.

The latter conclusion, which is not in disagreement with analyses based on "penguin diagrams,"[6] is now accepted by many authors,[7] who have attempted to corroborate this scenario by considering different mechanisms.

The aim of this paper is to develop ideas about long-distance dominance of the dynamics of $K \to \pi\pi$ weak nonleptonic amplitudes, based on some well-established facts of hadronic physics, i.e., PCAC, current algebra, dispersion relations, and asymptotic Regge behavior. We shall see that a crucial role in determining the correct description of $\Delta I = \frac{1}{2}$ and $\frac{3}{2}$ amplitudes is played by the suppression of "exotic" $\Delta I = \frac{3}{2}$ meson Regge trajectories with respect to $\Delta I = \frac{1}{2}$ exchanges.[8] A similar circumstance is seen to occur in the old calculation of the π^+-π^0 electromagnetic mass difference which, being dominated by an exotic ($\Delta I = 2$) exchange, turns out to be calculable in terms of a few low-lying states only.[9,10]

Some of the ideas and results reported in this paper have already been published in a Letter.[11] In this paper we shall describe the details of our calculational scheme, as well as refine our analysis by considering also the contribution from vector-meson intermediate states in dispersion relations. We shall also make a more detailed analysis of the separation between short- and long-distance physics by employing a smoother cutoff procedure, suggested by precocious light-cone behavior.

The plan of the paper is as follows. In Sec. II we calculate disconnected diagrams and describe the calculational scheme; in Sec. III general properties

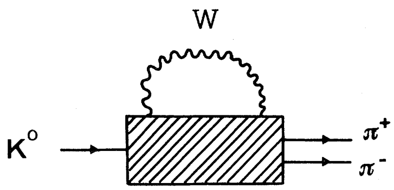

FIG. 1. The diagrammatic picture of the matrix element $\langle \pi^+\pi^- | H_{\rm NL} | K^0 \rangle$.

of connected amplitudes are described, while Sec. IV is devoted to the calculation of pole and continuum contributions to the amplitudes. We present our results and a discussion of long-distance versus short-distance effects in Sec. V. Some concluding remarks are made in Sec. VI, while the relevant matrix elements used in the paper are collected in the Appendix.

II. THE CALCULATIONAL SCHEME: DISCONNECTED AND CONNECTED GRAPHS

Let us consider the amplitude for the weak nonleptonic, strangeness-changing decay

$$K^0 \to \pi^+\pi^-$$

which can be only mediated, in the standard model, by the intermediate W boson.[12,13] According to Fig. 1, we write ($H_{\rm NL}$ is the weak nonleptonic Hamiltonian)

$$\begin{aligned} a(K^0\to\pi^+\pi^-) &= \langle \pi^+\pi^- | H_{\rm NL} | K^0\rangle \\ &= -\frac{i}{2}\frac{g^2}{8}\sin\theta\cos\theta \int d^4x\,\Delta^{\mu\nu}(x,m_W{}^2) \\ &\qquad \times \langle \pi^+\pi^- | T[J_\mu^{1+i2}(x)J_\nu^{4-i5}(0)+J_\mu^{4+i5}(x)J_\nu^{1-i2}(0)+\text{H.c.}] | K^0\rangle \\ &= \frac{i}{2}\frac{g^2}{8}\sin\theta\cos\theta \int \frac{d^4q}{(2\pi)^4}\frac{T(q)}{m_W{}^2-q^2-i\epsilon}\ , \end{aligned} \tag{2.1}$$

where

$$\begin{aligned} T(q) = -i\int d^4x \exp(iq\cdot x)(g^{\mu\nu}-q^\mu q^\nu/m_W{}^2) \\ \times \langle \pi^+\pi^- | T[J_\mu^{1+i2}(x)J_\nu^{4-i5}(0)+J_\mu^{4+i5}(x)J_\nu^{1-i2}(0)+\text{H.c.}] | K^0\rangle\ , \end{aligned} \tag{2.2}$$

$g^2/8m_W{}^2 = G/\sqrt{2}$, and $\Delta^{\mu\nu}(x,m_W{}^2)$ is the W-boson propagator, which, as is well-known, dies off exponentially for large values of $|x|$. This feature is the starting point for the short-distance expansion of the two-currents product JJ in (2.1) and (2.2). However, before considering operator-product expansions, we must stress a remarkable difference between two classes of diagrams contributing to the amplitude $T(q)$. We may in fact write

$$T(q) = T_{\rm disc}(q) + T_{\rm conn}(q)\ , \tag{2.3}$$

where $T_{\rm disc}(q)$ is the contribution of the disconnected graph of Fig. 2(a) and $T_{\rm conn}(q)$ the contribution of the connected graph of Fig. 2(b). Only the fully connected Green's function $T_{\rm conn}(q)$ must be considered in the renormalization-group equations giving the logarithmic deviations from the canonical quark-parton-model expectations through the anomalous dimensions of QCD; on the other hand the disconnected amplitude $T_{\rm disc}(q)$ gets no renormalization from gluon exchanges and it must be treated separately.

The contribution from the diagram of Fig. 2(a) is calculable by using our knowledge of semileptonic physics. Considering the $\Delta I = \frac{3}{2}$ and $\Delta I = \frac{1}{2}$ amplitudes separately we have

$$\begin{aligned} a_{\rm disc}^{3/2}(K^0\to\pi^+\pi^-) &= \frac{1}{3}\frac{G}{\sqrt{2}}\cos\theta\sin\theta \langle \pi^+ | J_\mu^{1+i2}(0) | 0\rangle \langle \pi^- | J^{\mu 4-i5}(0) | K^0\rangle\ , \\ a_{\rm disc}^{1/2}(K^0\to\pi^+\pi^-) &= 2a_{\rm disc}^{3/2}(K^0\to\pi^+\pi^-)\ . \end{aligned} \tag{2.4}$$

The relevant matrix elements are

$$\langle 0 | A_\mu^j(0) | \pi^k(P)\rangle = i\frac{f_\pi}{\sqrt{2}}P^\mu\delta_{jk}\ , \tag{2.5}$$

$$\langle \pi^k(P') | V_\mu^j(0) | K^j(P)\rangle = if_{ijk}[f_+(q^2)(P+P')_\mu + f_-(q^2)(P-P')_\mu]\ , \tag{2.6}$$

where $f_\pi = 132$ MeV, $q^2 = (P'-P)^2$, and the explicit form of $f_\pm(q^2)$ is given in the Appendix.

From Eqs. (2.4)–(2.6) we get

$$ia^{3/2}_{\rm disc}(K^0\rightarrow\pi^+\pi^-)=\frac{G}{\sqrt{2}}\cos\theta\,\sin\theta f_\pi\,\tfrac{1}{3}f_+(m_\pi{}^2)(m_K{}^2-m_\pi{}^2)=1.88\times10^{-8}\ \text{GeV}\ , \tag{2.7}$$

$$ia^{1/2}_{\rm disc}(K^0\rightarrow\pi^+\pi^-)=3.76\times10^{-8}\ \text{GeV}\ . \tag{2.8}$$

From Eqs. (2.7) and (2.8) we see that the disconnected graphs, as already recognized by Feynman,[14] are of the same order of magnitude and show no $\Delta I=\frac{1}{2}$ enhancement. Comparing them with the experimental numbers[15]

$$|\,a_{3/2}(K^0\rightarrow\pi^+\pi^-)\,|=0.86\times10^{-8}\ \text{GeV}\ , \tag{2.9a}$$

$$|\,a_{1/2}(K^0\rightarrow\pi^+\pi^-)\,|=2.71\times10^{-7}\ \text{GeV} \tag{2.9b}$$

we see that the contributions to $\Delta I=\frac{3}{2}$ and $\Delta I=\frac{1}{2}$ amplitudes from the disconnected diagrams are a factor of about 2 larger and a factor of about 7 lower than the experimental numbers (2.9a) and (2.9b), respectively. So our conclusion is that other contributions, different from those of Fig. 2(a) are necessary, and we are led to consider the connected amplitude $T_{\rm conn}(q)$.

We now come back to Eqs. (2.1) and (2.2). Wilson's[3] argument, as applied to the time-ordered product of the two currents in (2.2), is that due to the strong damping in x^2 of the W-boson propagator, the x integral takes relevant contributions only from the region $|\,x\,|\sim1/m_W$; in this region we perform an operator-product expansion,

$$T[J(x)J(0)]\underset{x^2\rightarrow0}{\sim}\sum_n C_n(x^2)O_n(0)\ , \tag{2.10}$$

where

$$C_n(x^2)=(-x^2)^{-\alpha_n/2}f_n(x^2)\ ,\qquad \alpha_n=6-d_n\ , \tag{2.11}$$

d_n is the dimension of the local operator O_n, and the f_n are dimensionless functions. It is well known[13] that, both in the free-quark model and in QCD, the leading operators in (2.10) are those with $\alpha_n=0$ and $d_n=6$, i.e., operators which are quartic in quark fields. In particular the $1/x^2$ singularity can be rotated away[2] for the present case.[16]

The local operators, quartic in quark fields, can be written as Wick products of currents, and in QCD the short-distance expansion tells us that[4]

$$\langle\pi^+\pi^-\,|\,T[J^{1+i2}_\mu(x)J^{4-i5\mu}(0)]\,|\,K^0\rangle\underset{-x^2\sim1/m_W{}^2}{\longrightarrow}\langle\pi^+\pi^-\,|\,[C_-(1/m_W)O_-(0)+C_+(1/m_W)O_+(0)]\,|\,K^0\rangle\ , \tag{2.12}$$

where

$$O_\pm(0)=\tfrac{1}{2}{:}\{J^{1+i2\mu}(0)J^{4-i5}_\mu(0)\pm[J^3_\mu(0)+(\tfrac{1}{3})^{1/2}J^8_\mu(0)+(\tfrac{2}{3})^{1/2}J^0_\mu(0)]J^{6-i7\mu}(0)\}{:}\ , \tag{2.13}$$

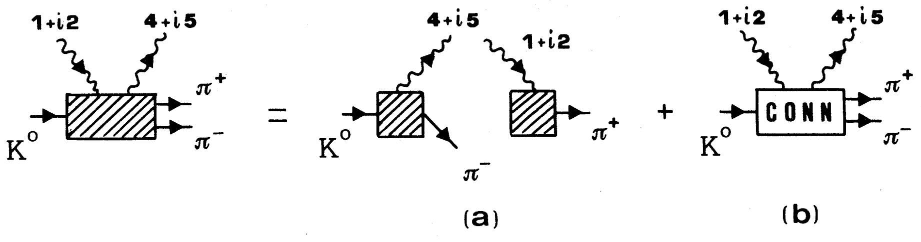

FIG. 2. The two classes of diagrams contributing to the amplitude $T(q)$ of Eq. (2.2): (a) the disconnected graph, (b) the connected graph.

and $C_{\pm}(1/m_W)$ are Wilson coefficients which are equal to 1 in the free-quark field theory, where dimensions remain canonical.

In QCD, on the other hand, one obtains[4] ($b=33-2n_F$, n_F is the number of flavors)

$$C_-(1/m_W)=\left[1-\frac{bg_s{}^2(\mu)}{24\pi^2}\ln(\mu/m_W)\right]^{0.48}, \tag{2.14a}$$

$$C_+(1/m_W)=\left[1-\frac{bg_s{}^2(\mu)}{24\pi^2}\ln(\mu/m_W)\right]^{-0.24} \tag{2.14b}$$

with

$$g_s{}^2(\mu)=4\pi/[1+(b/6\pi)\ln(\mu/\Lambda)]\ . \tag{2.15}$$

$C_{\pm}$ are normalized in such a way that $C_{\pm}=1$ for $\mu=m_W$ and μ is a mass scale (typically of order ~ 1 GeV) which represents the onset of the scaling.

Notice that the decomposition (2.13) is such that O_+ contains a mixture of $I=\frac{1}{2}$ and $I=\frac{3}{2}$ operators, while O_- is a pure $I=\frac{1}{2}$ operator. Thus the conclusion can be drawn[4] that, since $C_-(1/m_W)\geq 1$ and $C_+(1/m_W)\leq 1$, the connected $\Delta I=\frac{3}{2}$ amplitude can only be depressed by short-distance physics, whereas the $\Delta I=\frac{1}{2}$ amplitude is likely to be enhanced.

However, even if one considers two-loop corrections,[5] no quantitative conclusion can be drawn from this analysis. In order to do this we must in fact be able to calculate the matrix elements of the operators $O_{\pm}$. As it stands up to this point the short-distance effect alone is too small to be able to explain the observed $\Delta I=\frac{1}{2}$ enhancement in $K\to\pi\pi$ decays. The next section shall be devoted to the study of these matrix elements and their dependence on the mass scale μ.

III. GENERAL PROPERTIES OF CONNECTED DIAGRAMS

If we insert the short-distance expansion (2.12) in the connected part of the amplitude (2.1) we get

$$a_{\text{conn}}(K^0\to\pi^+\pi^-)=(G/\sqrt{2})\sin\theta\cos\theta\langle\pi^+\pi^-\,|\,[C_-(1/m_W)O_-(0)+C_+(1/m_W)O_+(0)+\text{H.c.}]\,|\,K^0\rangle \tag{3.1}$$

and our aim is to calculate matrix elements of the type

$$\langle\pi^+\pi^-\,|\,O_{\pm}(0)\,|\,K^0\rangle\ . \tag{3.2}$$

Now the PCAC hypothesis enables us to reduce (3.2) to matrix elements between single-particle states as follows:

$$\begin{aligned}a_{\text{conn}}(K^0\to\pi^+\pi^-)\underset{\text{SPL}}{\simeq}\frac{-i}{2f_\pi}\frac{G}{\sqrt{2}}\sin\theta\cos\theta\{&\langle\pi^+\,|\,[Q_5^+,C_-(1/m_W)O_-(0)+C_+(1/m_W)O_+(0)]\,|\,K^0\rangle+\text{H.c.}\\&+\langle\pi^-\,|\,[Q_5^-,C_-(1/m_W)O_-(0)+C_+(1/m_W)O_+(0)]\,|\,K^0\rangle+\text{H.c.}\}\ ,\end{aligned} \tag{3.3}$$

where

$$Q_5^{\pm}=\int d^3\vec{x}\,A_0^{1\pm i2}(\vec{x},0) \tag{3.4}$$

are the axial-vector charges, and SPL denotes the soft-pion limit.

Some remarks are now in order.

(1) In the SU_2 limit the two-pion final state in the decay $K^0\to\pi^+\pi^-$ is symmetric under the interchange $P_{\pi^+}\leftrightarrow P_{\pi^-}$ so that, in applying the SPL we have treated the pions symmetrically.[17]

(2) In the SPL $P\to 0$, we can drop terms proportional to P^λ because there are no poles in the integral which one obtains from (3.1) after the reduction formula has been employed and integration by parts has been performed. In baryon nonleptonic decays[18] the situation is different.

(3) The expressions (3.1) and (3.3) do not contain any explicit dependence of the mesons' momenta.[19–21] This is equivalent, up to the factor $(1-m_\pi^2/m_K^2)$ which is very close to 1, to the continuation connecting the SPL with the physical amplitude used in Ref. 20, provided that the two pions are treated symmetrically.

Going back to Eq. (3.3) we see that commutators can be evaluated by current-algebra commutation relations; by separating the $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ amplitudes we get

$$a^{3/2}_{\rm conn}(K^0\to\pi^+\pi^-)=-ia^{3/2}_{+}C_+(1/m_W)\,, \tag{3.5}$$

$$a^{1/2}_{\rm conn}(K^0\to\pi^+\pi^-)=-i[a^{1/2}_{+}C_+(1/m_W)+a^{1/2}_{-}C_-(1/m_W)]\,, \tag{3.6}$$

where

$$a^{3/2}_{+}=\frac{1}{2f_\pi}\frac{G}{\sqrt{2}}\sin\theta\cos\theta[\langle\pi^+|:J^{1+i2}_{\mu}(0)J^{6-i7\mu}(0):|K^0\rangle + \tfrac{2}{3}\langle\pi^-|:2J^{3}_{\mu}(0)J^{4-i5\mu}(0)-J^{1-i2}_{\mu}(0)J^{6-i7\mu}(0):|K^0\rangle]\,, \tag{3.7}$$

$$a^{1/2}_{+}=\frac{1}{2f_\pi}\frac{G}{\sqrt{2}}\sin\theta\cos\theta\langle\pi^-|:\tfrac{1}{6}J^{3}_{\mu}(0)J^{4-i5\mu}(0)+\tfrac{1}{6}J^{1-i2}_{\mu}(0)J^{6-i7\mu}(0) + \frac{1}{2\sqrt{3}}J^{8}_{\mu}(0)J^{4-i5\mu}(0)+\frac{1}{\sqrt{6}}J^{0}_{\mu}(0)J^{4-i5\mu}(0):|K^0\rangle\,, \tag{3.8}$$

$$a^{1/2}_{-}=\frac{1}{2f_\pi}\frac{G}{\sqrt{2}}\sin\theta\cos\theta\langle\pi^-|:\tfrac{1}{2}J^{3}_{\mu}(0)J^{4-i5\mu}(0)+\tfrac{1}{2}J^{1-i2}_{\mu}(0)J^{6-i7\mu}(0) - \frac{1}{2\sqrt{3}}J^{8}_{\mu}(0)J^{4-i5\mu}(0)-\frac{1}{\sqrt{6}}J^{0}_{\mu}(0)J^{4-i5\mu}(0):|K^0\rangle\,, \tag{3.9}$$

and $J_\mu J^\mu = V_\mu V^\mu + A_\mu A^\mu$.

From Eqs. (3.5)—(3.9) we see that our problem consists in evaluating matrix elements of the type

$$\langle\pi|:J^{\alpha}_{\mu}(0)J^{\beta\mu}(0):|K\rangle\,. \tag{3.10}$$

We shall do that by saturating the product of the currents by a complete set of intermediate states. But before performing this calculation, let us make the following remark. Owing to the precocity of short-distance behavior we expect that the light-cone expansion (2.12) will be valid approximately (within 10%) for $|x|\lesssim 1/\mu$, where μ, describing the onset of scaling, is of the order of 1 GeV. Thus we can write the matrix elements (3.10) as follows $[x_0=(0,\vec{n}/\mu)$ and $|\vec{n}|=1]$:

$$\langle\pi|:J^{\alpha}_{\mu}(0)J^{\beta\mu}(0):|K^0\rangle \simeq \frac{1}{2}\frac{\int d\vec{n}\,\delta(|\vec{n}|-1)\langle\pi|T[J^{\alpha}_{\mu}(x_0/2)J^{\beta\mu}(-x_0/2)+(\alpha\leftrightarrow\beta)]|K^0\rangle}{\int d\vec{n}\,\delta(|\vec{n}|-1)}$$

$$=\frac{\int d\vec{n}\,\delta(|\vec{n}|-1)\int\frac{d^4q}{(2\pi)^4}\exp(-iqx_0)T^{\alpha\beta}(q)}{\int d\vec{n}\,\delta(|\vec{n}|-1)}$$

$$=\int\frac{d^4q}{(2\pi)^4}\frac{\sin(|\vec{q}|/\mu)}{(|\vec{q}|/\mu)}T^{\alpha\beta}(q)\,, \tag{3.11}$$

where

$$T^{\alpha\beta}(q)=\tfrac{1}{2}\int d^4x\,\exp(iq\cdot x)\langle\pi|T[J^{\alpha}_{\mu}(x/2)J^{\beta\mu}(-x/2)+(\alpha\leftrightarrow\beta)]|K^0\rangle\,. \tag{3.12}$$

Note that the function

$$f_\mu(|\vec{q}|)=\frac{\mu}{|\vec{q}|}\sin(|\vec{q}|/\mu) \tag{3.13}$$

cuts off the integral (3.11) for large values of $|\vec{q}|$, in such a way that the integration is over a finite range.

This feature is especially useful for practical purposes, because it allows us to use expressions for form factors, suggested by dispersion relations, which are only valid over finite ranges of the momentum transfer q^2. As discussed above we can demonstrate the absence of singularities in light-cone expansion and hence the convergence of the integral (2.1) only in the case that energy is conserved between the initial (K^0) and the final ($\pi^+\pi^-$) states. This feature is lost in the soft-pion limit, and there can appear terms which behave for instance as

$(P_f - P_i)^2/x^2$. In any case the function (3.13), which cuts off very small distances, ensures the convergence of integrals like (3.11), in such a way that possible spurious singularities, introduced by the PCAC approximation, are seen to play no role in final results.

In order to calculate (3.11) and (3.12) we follow a strategy similar to the one employed by Cottingham[22] in its attempts to calculate the proton-neutron mass difference. By inserting a complete set of intermediate states between the two currents in (3.12), one sees immediately that $T^{\alpha\beta}(q_0,\vec{q})$ is, for fixed $\vec{q}$, an analytic function of q_0 except for poles and cuts just below the positive real axis and just above the negative real axis as shown in Fig. 3. In order to show how the Wick rotation of the contour works, let us introduce some kinematical definitions: $q=(k_1+k_2)/2$, $P_1=P_k$, $P_2=P_\pi$, and $P_1+k_1=P_2+k_2$ (momentum conservation). We work in the reference frame in which

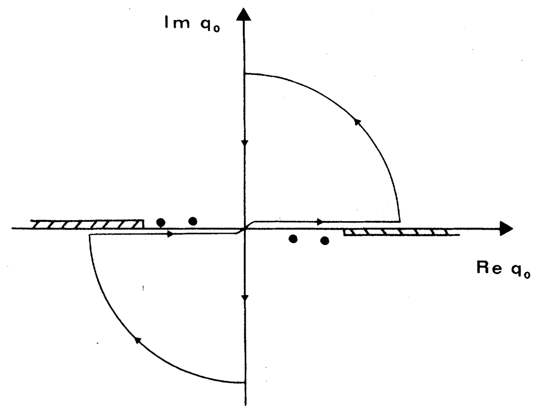

FIG. 3. The q_0 plane with the singularities (poles and cuts) of $T^{\alpha\beta}$ and the Wick-rotated contour.

$$P=(P_1+P_2)/2=(E,\vec{0}) ,$$
$$q=(k_1+k_2)/2=(q_0, |\vec{q}|\sin\theta\cos\phi, |\vec{q}|\sin\theta\sin\phi, |\vec{q}|\cos\theta) . \tag{3.14}$$

Note that, since $P_1{}^2=m_K^2$, $P_2{}^2=m_\pi^2$, and $t=(P_1-P_2)^2=m_\pi{}^2$ are fixed, we have

$$E=(2m_K{}^2+m_\pi{}^2)^{1/2}/2 . \tag{3.15}$$

The positions of poles of Fig. 3 are

$$q_0=-E+(\vec{q}^{\,2}+m_n{}^2)^{1/2}-i\epsilon , \quad q_0=-E-(\vec{q}^{\,2}+m_n{}^2)^{1/2}+i\epsilon , \tag{3.16}$$

where m_n is the mass of the single-meson intermediate state. From (3.16) we see that, except for the special case in which $m_n=m_\pi$ and $|\vec{q}|\le(2m_K{}^2-m_\pi{}^2)^{1/2}/2=0.33$ GeV (we shall return to this point later) the contour can be rotated as shown in Fig. 3, so that the q_0 integration is transferred to the imaginary axis[23,24]; by changing variables in the manner of Cottingham[22] in $Q^2=-q^2$ and $k_0=-iq_0$, we get the following formula:

$$\begin{aligned}\langle\pi(P_2)|{:}J_\mu^\alpha(0)J^{\mu\beta}(0){:}|K(P_1)\rangle &\simeq \int\frac{d^4q}{(2\pi)^4}f_\mu(|\vec{q}|)T^{\alpha\beta}(q)\\ &=\frac{i}{2}\frac{1}{(2\pi)^4}\int_0^\infty dQ^2\int_{-(Q^2)^{1/2}}^{+(Q^2)^{1/2}}dk_0(Q^2-k_0{}^2)^{1/2}\int_{-1}^{+1}dz\int_0^{2\pi}d\Phi\, T^{\alpha\beta}(-Q^2,ik_0,z=\cos\theta,\Phi)f_\mu((Q^2-k_0{}^2)^{1/2}) ,\end{aligned} \tag{3.17}$$

where $T^{\alpha\beta}$ is given by (3.12) and $f_\mu(|\vec{q}|)$ by (3.13). Equation (3.17) is the starting point in calculations of the relevant matrix elements; in order to see how (3.17) is calculable, we introduce invariant amplitudes in a way similar to the one used in nonforward Compton scattering. We write $T^{\alpha\beta}(q)$ of Eq. (3.12) as follows (see Fig. 4):

$$T^{\alpha\beta}(q)=\tfrac{1}{2}g^{\mu\nu}[T^{\alpha\beta}_{VV,\nu\mu}(q)+T^{\alpha\beta}_{AA,\nu\mu}(q)+(\alpha\leftrightarrow\beta)] , \tag{3.18}$$

where

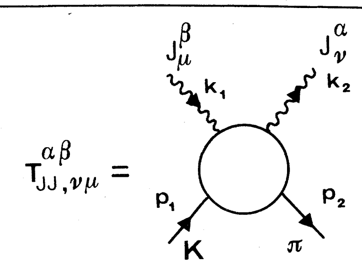

FIG. 4. The amplitude $T^{\alpha\beta}_{JJ,\nu\mu}$ with the relevant kinematics. JJ can be vector-vector currents (VV) or axial-vector—axial-vector currents (AA). α and β are SU(3) indices.

$$T^{\alpha\beta}_{VV,\nu\mu}=\int d^4x\, e^{+iqx}\times\langle\pi(P_2)\,|\,T\left[V^\alpha_\nu\left[\frac{x}{2}\right]V^\beta_\mu\left[-\frac{x}{2}\right]\right]\times|K(P_1)\rangle\ ,$$
$$T^{\alpha\beta}_{AA,\nu\mu}=\int d^4x\, e^{+iqx}\times\langle\pi(P_2)\,|\,T\left[A^\alpha_\nu\left[\frac{x}{2}\right]A^\beta_\mu\left[-\frac{x}{2}\right]\right]\times|K(P_1)\rangle\ . \tag{3.19}$$

As is well known the quantities given in Eqs. (3.19) are not tensor, but they can be made Lorentz invariant by adding the Schwinger terms $S_{\mu\nu}$ which are polynomials in q_0 (Ref. 10); in this way we obtain two tensors $\widetilde{T}^{\alpha\beta}_{VV,\nu\mu}$ and $\widetilde{T}^{\alpha\beta}_{AA,\nu\mu}$ which can be written in terms of invariant amplitudes as follows[25]:

$$\begin{aligned}\widetilde{T}^{\alpha\beta}_{JJ,\nu\mu}=&A^{\alpha\beta}_{JJ}g_{\nu\mu}+B^{\alpha\beta}_{JJ}P_\nu P_\mu+C^{\alpha\beta}_{1JJ}P_\nu k_{2\mu}\\&+C^{\alpha\beta}_{2JJ}k_{1\nu}P_\mu+D^{\alpha\beta}_{JJ}k_{1\nu}k_{2\mu}+E^{\alpha\beta}_{1JJ}P_\nu k_{1\mu}\\&+E^{\alpha\beta}_{2JJ}k_{2\nu}P_\mu+F^{\alpha\beta}_{JJ}k_{2\nu}k_{1\mu}+G^{\alpha\beta}_{JJ}k_{1\nu}k_{1\mu}\\&+H^{\alpha\beta}_{JJ}k_{2\nu}k_{2\mu}\ ,\end{aligned}\tag{3.20}$$

where $P=(P_1+P_2)/2$ and $JJ=VV$ or AA. Let us assume, conventionally, that the currents J^α have $\Delta Y=0$ so that Ward identities give [as a consequence of CVC (conserved vector current)]

$$k_2^\nu\widetilde{T}^{\alpha\beta}_{VV,\nu\mu}=0\ ,\quad k_1^\mu\widetilde{T}^{\alpha\beta}_{VV,\mu\nu}=0\ , \tag{3.21}$$

where we have not introduced the matrix element of the currents commutator[25] whic cancels when the symmetrization $\alpha\leftrightarrow\beta$ in Eq. (3.18) is performed; moreover the usual hypothesis[25] of cancellation between Schwinger and contact terms in the Ward identity has been used in order to write (3.21). By using (3.18)—(3.21) one finally gets the following expression:

$$\begin{aligned}T^{\alpha\beta}(q)=\frac{1}{2}\Bigg\{&\left[E^2-\frac{(k_2\cdot P)(k_1\cdot P)}{(k_1\cdot k_2)}\right](B^{\alpha\beta}_{VV}+B^{\beta\alpha}_{VV})-3(k_2\cdot P)C^{\alpha\beta}_{1VV}-3(k_1\cdot P)C^{\beta\alpha}_{2VV}-3(k_1\cdot k_2)(D^{\alpha\beta}_{VV}+D^{\beta\alpha}_{VV})\\&+\left[(k_1\cdot P)-\frac{k_1{}^2(k_2\cdot P)}{(k_1\cdot k_2)}\right](E^{\alpha\beta}_{1VV}+E^{\beta\alpha}_{1VV})+\left[(k_2\cdot P)-k_2{}^2\frac{(k_1\cdot P)}{(k_1\cdot k_2)}\right](E^{\alpha\beta}_{2VV}+E^{\beta\alpha}_{2VV})\\&+\left[(k_1\cdot k_2)-\frac{k_1{}^2k_2{}^2}{(k_1\cdot k_2)}\right](F^{\alpha\beta}_{VV}+F^{\beta\alpha}_{VV})-3k_1{}^2G^{\beta\alpha}_{VV}-3k_2{}^2H^{\alpha\beta}_{VV}\\&+4(A^{\alpha\beta}_{AA}+A^{\beta\alpha}_{AA})+E^2(B^{\alpha\beta}_{AA}+B^{\beta\alpha}_{AA})+(k_2\cdot P)(C^{\alpha\beta}_{1AA}+C^{\beta\alpha}_{1AA})\\&+(k_1\cdot P)(C^{\alpha\beta}_{2AA}+C^{\beta\alpha}_{2AA})+(k_1\cdot k_2)(D^{\alpha\beta}_{AA}+D^{\beta\alpha}_{AA})+(k_1\cdot P)(E^{\alpha\beta}_{1AA}+E^{\beta\alpha}_{1AA})+(k_2\cdot P)(E^{\alpha\beta}_{2AA}+E^{\beta\alpha}_{2AA})\\&+(k_1\cdot k_2)(F^{\alpha\beta}_{AA}+F^{\beta\alpha}_{AA})+k_1{}^2(G^{\alpha\beta}_{AA}+G^{\beta\alpha}_{AA})+k_2{}^2(H^{\alpha\beta}_{AA}+H^{\beta\alpha}_{AA})\Bigg\}\ .\end{aligned}\tag{3.22}$$

This is the expression we must insert in Eq. (3.17). The values of the various invariant amplitudes will be discussed in what follows. Before closing this section we briefly discuss the contribution coming from the pion in the Wick rotation of the contour. This term can be easily evaluated to be given by the following expression:

$$T^{\alpha\beta}_\pi=\xi^{\alpha\beta}_\pi\int_0^{\bar q}d\,|\vec q\,|\ \,|\vec q\,|^2/(2\pi)^2\frac{f_\mu(\,|\vec q\,|\,)}{2E_n}\times\int_{-1}^{1}dz\{f_+(k_1{}^2)F_\pi(k_2{}^2)[(P+q)^2+2P\cdot(P+q)+m_K{}^2/2]\}_{q_0=-E+E_n}\ , \tag{3.23}$$

where $\xi_{\pi}^{\alpha\beta}$ is a Clebsch-Gordan coefficient, $\tilde{q}=(2m_K{}^2-3m_\pi{}^2)^{1/2}/2$, and $E_n=(\vec{q}^{\,2}+m_\pi{}^2)^{1/2}$.

The expression (3.23) is almost independent on μ (for μ ranging between 1 and 2 GeV) and it amounts to about 10% of the total pseudoscalar pole contribution to (3.22).

IV. POLE AND CONTINUUM CONTRIBUTIONS TO $\langle \pi \mid JJ \mid K^0 \rangle$ MATRIX ELEMENTS

The various invariant amplitudes in (3.22) obey (fixed-mass) dispersion relations in the s and u channels similar to the ones encountered in the calculation of $\pi^+-\pi^0$ mass difference.[9,10] In general, dispersion relations for the amplitudes will consist of (1) Born contributions (π,k poles), (2) resonances contributions, (3) absorptive integrals over the cuts shown in Fig. 3, and (4) subtraction constants. Let us isolate the first two contributions, Born and resonances terms, and write

$$T^{\alpha\beta}=T^{\alpha\beta}_{\text{pole}}+T^{\alpha\beta}_{C}\,, \tag{4.1}$$

where $T^{\alpha\beta}_{C}$, which hereafter we will refer to as "continuum" contributions, is comprehensive of subtraction constants and absorptive integrals, and the pole part of the matrix element $\langle \pi \mid JJ \mid K^0 \rangle$ ($T^{\alpha\beta}_{\text{pole}}$)is shown schematically in Fig. 5. We have included in $T^{\alpha\beta}_{\text{pole}}$ the Born terms (π,K poles) and the nearest vector-meson resonances (ρ,ω,K^*). The calculation of pole contributions to the different amplitudes in Eq. (3.22) is a straightforward exercise once the relevant current–single-meson states (which we list in the Appendix) are used. Born terms contribute only to the VV-type amplitudes, while vector mesons contribute to both VV and AA-type amplitudes. The results of the calculations are given in Table I, which shows the contributions of the Born term (0^- octet) and of the resonances belonging to the 1^- octet.

Vector resonances have been treated in the zero-width approximation. We present the results for the different amplitudes $a_+^{3/2}$ and $a_\pm^{1/2}$ defined in Eqs. (3.5)–(3.9) for different values of the mass parame-

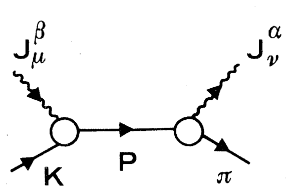

FIG. 5. Pole contributions to the invariant amplitudes of Eq. (3.22). P can be a pseudoscalar 0^- meson (π,K) or a vector 1^- meson (ρ,ω,K^*).

TABLE I. The contributions of different poles (0^- and 1^- octets) to the $\Delta I=\frac{3}{2}$ amplitude $a_+^{3/2}$ and to the $\Delta I=\frac{1}{2}$ amplitudes $a_\pm^{1/2}$ defined in Eqs. (3.5)–(3.9). Results are given for different values of the parameter μ which represents the onset of scaling. Units are 10^{-8} GeV.

	μ (GeV)	VV 0^-	VV 1^-	AA	Total
$a_+^{3/2}$	0.7	−2.6	−0.46	2.3	−0.73
	1.0	−3.8	−0.55	3.5	−0.85
	1.3	−4.9	−0.61	4.4	−1.1
	1.5	−5.4	−0.62	4.9	−1.1
$a_-^{1/2}$	0.7	6.2	−0.11	−5.4	0.69
	1.0	8.8	−0.12	−7.9	0.78
	1.3	11.3	−0.13	−10.0	1.2
	1.5	12.5	−0.13	−11.0	1.4
$a_+^{1/2}$	0.7	0.89	−0.23	−0.58	0.08
	1.0	1.0	−0.28	−0.67	0.05
	1.3	1.5	−0.30	−1.1	0.1
	1.5	1.8	−0.31	−1.2	0.3

ter μ, which represents as we have already stressed in Sec. III, the onset of scaling. In the 0^- column we have included the small contribution coming from Eq. (3.23).

Theoretical uncertainties in the results of Table I come essentially from the imperfect knowledge of meson form factors, (see Appendix for more details). We estimate, perhaps somewhat optimistically, that the theoretical errors in the last column of Table I will be of the order of 20%. A glance at the entries of Table I shows immediately the following facts:

(i) The pole contribution to the $a_+^{3/2}$ amplitude has for reasonable values of μ ($\mu\simeq 1$ GeV) the correct sign and, roughly speaking, the right absolute value in order that, when summed to the contribution coming from the disconnected graph [Eq. (2.7)], it will be able to reproduce the experimental number (2.9a)

(ii) The pole contributions to the $\Delta I=\frac{1}{2}$ amplitudes are unable to reproduce satisfactorily the experimental quantity (2.9b) even in the presence of the short-distance enhancement of Eq. (2.14a).

We must now stress a remarkable difference between $\Delta I=\frac{3}{2}$ and $\Delta I=\frac{1}{2}$ amplitudes, which can overcome the latter difficulty. By assuming that the amplitudes (4.1) are Regge behaved for large values of $\nu=Eq_0$, one sees immediately that whereas the $\Delta I=\frac{1}{2}$ amplitudes will be dominated in the Regge region by K^{**} exchange [with $\alpha_{K^{**}}(0)\simeq 0.25$], the $\Delta I=\frac{3}{2}$ amplitudes will behave, for large ν's, as ν^{α_R}, where $\alpha_R<0$. The latter behavior is due to the fact that Regge meson trajectories with $\Delta I=\frac{3}{2}$ are "exotic."[26] This striking difference, which is a typical consequence of the dynamics of hadronic interactions, results in the fact that the "continuum" contribution defined in (4.1), which is present (and, as we will show soon, also calculable) in the $\Delta I = \frac{1}{2}$ case, is on the other hand absent in the $\Delta I = \frac{3}{2}$ amplitude.[8] It is for this reason that one can assume for it unsubtracted dispersion relations and approximate the value of the amplitude with low-lying meson states only. A similar situation occurs also in the successful calculation of $\pi^+ - \pi^0$ electromagnetic mass difference,[9,10] where the $\Delta I = 2$ invariant amplitudes t_1, t_2 are calculated by keeping only the nearest poles contributions (π, ω), due to the suppression of $\Delta I = 2$ exotic Regge meson trajectories.

Concerning the calculation of the continuum contribution to $a_{\pm}^{1/2}$,

$$a_{C\pm}^{1/2} = \frac{1}{2f_\pi} \frac{G}{\sqrt{2}} \sin\theta \cos\theta \frac{i}{2} \frac{1}{(2\pi)^4} \times \int_0^\infty dQ^2 \int_{-(Q^2)^{1/2}}^{(Q^2)^{1/2}} dk_0 \, \mu \sin((Q^2 - k_0{}^2)^{1/2}/\mu) \int_{-1}^{1} dz \int_0^{2\pi} d\phi \, T_{C\pm}^{1/2}(-Q^2, \, q_0 = ik_0 \, , \, z = \cos\theta \, , \, \phi) \, , \tag{4.2}$$

we make the following remarks:

(1) $T_{C\pm}^{1/2}(q)$ has cuts on the real axis of the q_0 plane as shown in Fig. 3.

(2) $T_{C\pm}^{1/2}(q)$ is even for change $q_0 \to -q_0$.

(3) $T_{C\pm}^{1/2}(q)$ is assumed to be Regge behaved in the limit of large $|\nu| = E|q_0|$.

According to point (3) above, we can write, by employing an ansatz suggested by the high-energy limit of a Veneziano amplitude,[27]

$$iT_{C\pm}^{1/2}(q^2,\nu) \underset{\text{large}\,|\nu|}{\longrightarrow} 2\xi_\pm \beta_M [\beta^{VV}(q^2) + \beta^{AA}(q^2)] \Gamma(l_e - \alpha(0))/2[(\alpha' s)^{\alpha(0)} + (-1)^{s_e}(-\alpha' s)^{\alpha(0)}](\alpha')^{1-l_e} \, , \tag{4.3}$$

where $\alpha(0)$ is the intercept of the leading Regge meson trajectory K^{**} $[\alpha(0) = \alpha_{K^*}(0) - 0.1 \simeq 0.25$ (Ref. 27)], β_M is the residue of the $K^{**} - K\pi$ vertex, $s_e = 2$, $l_e = 1$, $\alpha' \simeq 0.9$, and $\nu = Eq_0$. Moreover $\beta^{VV}(q^2)$ $[\beta^{AA}(q^2)]$ is the residue of the vertex $K^{**} - VV$ $[K^{**} - AA]$, and we have made the approximation $t \simeq 0$. As for the Clebsch-Gordan coefficients $\xi_\pm$, they are simply given by

$$\xi_\pm = \tfrac{1}{2} \, . \tag{4.4}$$

The Regge residues can be calculated as follows. By assuming the vector-meson-dominance model we have

$$\beta^{VV}(q^2) = \frac{\beta^{VV}(0)}{(1 - q^2/m_\rho{}^2)(1 - q^2/m_{K^*}{}^2)} \, , \qquad \beta^{AA}(q^2) = \frac{\beta^{AA}(0)}{(1 - q^2/m_{A_1}{}^2)(1 - q^2/m_{Q_1}{}^2)} \, . \tag{4.5}$$

Moreover, from the Weinberg sum rule,[18] we derive

$$\beta^{VV}(0) = \beta^{AA}(0) = \beta^{JJ}(0) \, . \tag{4.6}$$

Finally the values of β_M and $\beta^{JJ}(0)$ can be derived in a straightforward way, by making the usual hypothesis of factorization, SU(3) symmetry and exchange degeneracy for the residues, from the knowledge of the high-energy antiparticle-particle-reaction differences[28] and from the difference[29]

$$\sigma_T(\gamma p) - \sigma_T(\gamma n) \, . \tag{4.7}$$

One gets in this way[30]

$$\beta_M = 3.3 \sqrt{\text{mb}} \, , \qquad \beta^{JJ} = 0.12 \sqrt{\text{mb}} \, . \tag{4.8}$$

Now we assume a very simple extrapolation for the amplitude $T_{C\pm}^{1/2}(q^2,\nu)$, which satisfies requirements (1)–(3) and reduces to (4.3) in the Regge limit; that is, we write

$$iT_{C\pm}^{1/2}(q^2,\nu) = 2\xi_\pm \beta_M [\beta^{VV}(q^2) + \beta^{AA}(q^2)](\alpha')^{\alpha(0)} \Gamma(l_e - \alpha(0)) \tfrac{1}{2} \times [(m_0{}^2 - s)^{\alpha(0)} + (-1)^{s_e}(m_0{}^2 - u)^{\alpha(0)}](\alpha')^{1-l_e} \, , \tag{4.9}$$

where $s=2\nu+q^2+E^2$, $u=-2\nu+q^2+E^2$, $m_0{}^2$ is an effective threshold whose reasonable range is between 0.5 and 1 GeV (Ref. 31) and the other symbols are as in Eq. (4.3). The amplitude defined by (4.9) appears to be a good approximation of the continuum contribution for the following reasons. First, it meets the requirements (1)—(3). Second, if one calculates (4.2) by using the cutoff function $f_\mu(|\vec{q}\,|)$ replaced by a sharp cutoff $\theta(Q_0{}^2-Q^2)$, as in Ref. 11, one obtains results very close to the ones found by calculating the continuum contribution by superconvergence relations and finite-energy sum rules.[11]

With these in mind we calculate $a_{C\pm}^{1/2}$ as follows:

$$a_{C\pm}^{1/2}=\frac{1}{2f_\pi}\frac{G}{\sqrt{2}}\sin\theta\cos\theta\frac{\beta_M(\alpha')^{\alpha(0)}\Gamma(\frac{3}{4})}{4\pi^3}$$
$$\times\int_0^\infty dQ^2(Q^2)^{1/2}[\beta^{VV}(-Q^2)+\beta^{AA}(-Q^2)]\mu$$
$$\times\int_0^1 dx\,\sin\left[\frac{(Q^2)^{1/2}}{\mu}(1-x^2)^{1/2}\right]\cos[\alpha(0)\phi][\lambda^2(Q^2)+4E^2Q^2x^2]^{\alpha(0)/2}\,, \tag{4.10}$$

where

$$\lambda(Q^2)=m_0{}^2+Q^2-E^2\,,$$
$$\tan\phi=2Ex(Q^2)^{1/2}\lambda(Q^2)\,. \tag{4.11}$$

The results of the calculation are reported in Table II; let us comment on them briefly.

First, we observe that our evaluation of the continuum contribution leads to a large effect in $\Delta I=\frac{1}{2}$ enhancement as a consequence mainly of long-distance physics (we shall return to this point in the next section). Second, Table II shows a strong dependence on μ; this unpleasant feature is a consequence of the approximation in using PCAC: we can adopt the philosophy of choosing μ in such a way that the $\Delta I=\frac{3}{2}$ amplitude is fitted [this fixes $\mu\simeq 1$ GeV (Ref. 32)] and then calculating the $\Delta I=\frac{1}{2}$ amplitude. Finally we remark that the main sources of theoretical uncertainties in the calculation of the numbers in Table II are due to the approximation in the Regge formulas [SU(3) symmetry, factorization, . . .]. Thus we expect the results of Table II to be valid within 30%.

V. RESULTS

By collecting the numbers given in Tables I and II we get numerical estimates for the connected amplitudes $a_+^{3/2}$ and $a_\pm^{1/2}$. These results, together with the short-distance effects expressed by Eqs. (2.14), (3.5), and (3.6) have been summed to disconnected amplitudes (2.7) and (2.8) to give the entries of Table III, where we compare our predictions for $|a^{1/2}(K^0\to\pi^+\pi^-)|$ and $|a^{3/2}(K^0\to\pi^+\pi^-)|$, calculated for $\mu=1$ GeV and for different values of the QCD parameter $\Lambda_{\rm QCD}$. From e^+e^- and deep-inelastic-scattering physics $\Lambda_{\rm QCD}$ is known to be of the order of a few hundred MeV.

Our results, although affected by the theoretical uncertainties we discussed in Sec. IV, strongly suggest that QCD short-distance effects play a minor role in determining the experimental $\Delta I=\frac{1}{2}$ enhancement, whereas long-distance effects ($\simeq 1$ fm), which can be understood in terms of well-known properties of hadronic matter, are dominant. In particular we stress, once again, the role played

TABLE II. Continuum contributions to $\Delta I=\frac{1}{2}$ amplitudes for different values of the parameter μ. Units are 10^{-8} GeV.

μ (GeV)	$a_{C+}^{1/2}=a_{C-}^{1/2}$
0.7	5.6
1.0	12.0
1.3	19.0
1.5	24.0

TABLE III. Calculated values of $|a^{1/2}(K^0\to\pi^+\pi^-)|$ and $|a^{3/2}(K^0\to\pi^+\pi^-)|$ for $\mu=1$ GeV and for different values of the QCD parameter $\Lambda_{\rm QCD}$. The value $\Lambda=0$ corresponds to "canonical" short-distance behavior. Units are 10^{-8} GeV.

$\Lambda_{\rm QCD}$ (GeV)	$\|a^{1/2}(K^0\to\pi^+\pi^-)\|$	$\|a^{3/2}(K^0\to\pi^+\pi^-)\|$
0.4	36.2	1.26
0.2	34.3	1.22
0.1	33.1	1.19
0	28.6	1.03
Expt.	27.1	0.86

by the suppression of exotic Regge trajectories versus nonexotic ones, which is, in the present calculation, the main source of $\Delta I=\frac{1}{2}$ enhancement.

Although our conclusions rely on a definite mechanism, different from those employed by other authors, it is not useless to remark that the dominance of long-distance effects have been already recognized[7]; in principle this dominance could be explored in different ways and calculated by using other hypotheses. We believe that the reliability of our calculation is higher because it rests on some simple assumptions deriving from hadronic physics and its well known properties.

VI. CONCLUSIONS

Our conclusions are based on Table III, which shows that the weak nonleptonic decay $K\to\pi\pi$ and the puzzle of the $\Delta I=\frac{1}{2}$ rule can be not only understood in term of properties of hadronic physics, but also calculated. Moreover our calculation shows a dominance of long-range effects over short-distance enhancement and/or suppression.

In order to confirm our conclusions, further study on this subject is needed. In particular we expect mechanisms similar to the ones explored in this paper to be at work on related processes such as baryon nonleptonic decays, charm-particle nonleptonic decays, and weak radiative decays. We shall consider this very interesting physics in future publications.

APPENDIX: MATRIX ELEMENTS $\langle M\,|\,J_\mu\,|\,M'\rangle$

In this appendix we list the relevant matrix elements

$$\langle M'\,|\,J_\mu(0)\,|\,M\rangle\ , \tag{A1}$$

where M and M' are single-meson states.

1. $M,M'=$ pseudoscalar mesons (0^-), $J^\mu=$vector current

We write the matrix elements in general as

$$\langle M'^k(p')\,|\,V^J_\mu(0)\,|\,M^i(p)\rangle$$
$$=if_{ijk}[f_{MM'}(q^2)(p+p')_\mu+\tilde f_{MM'}(q^2)(p-p')_\mu]\ ,$$

where $q^2=(p-p')^2$.

In the case where M and M' are π's, $\tilde f_{\pi\pi}$ vanishes due to CVC and we have

$$f_{\pi\pi}(q^2)=F_\pi(q^2)=1/(1-q^2/m_A{}^2)\ , \tag{A2}$$

where the pion form-factor mass m_A is given experimentally by

$$m_A{}^2=0.47\ \mathrm{GeV}^2\ . \tag{A3}$$

In the case where M and M' are K's, we have, again, $\tilde f_{KK}=0$, and it can be assumed that

$$f_{KK}(q^2)=F_K(q^2)=1/(1-q^2/m_A{}^2)\ ,$$

where $m_A{}^2$ is given by (A3).

Finally we have to consider the case $M=K$, $M'=\pi$. The corresponding matrix element is known from K_{l3} decay experiments,[33] $f_{K\pi}(q^2)$ is given by

$$f_{K\pi}(q^2)=f_+(q^2)=\frac{1}{1-q^2/m_+{}^2} \tag{A4}$$

with $m_+{}^2=0.65$ GeV^2, whereas little is known experimentally about $\tilde f_{K\pi}(q^2)=f_-(q^2)$. As a matter of fact the ratio $\xi(0)=f_-(0)/f_+(0)$ ranges experimentally in the interval $(0,-0.5)$; we choose $\xi(0)=0$ for simplicity, although final results, as given in Sec. V, are not sensitive to variations of $\xi(0)$ in the aforementioned interval.

2. $M=$pseudoscalar meson (0^-), $M'=$vector meson (1^-), $J^\mu=$vector current

The matrix elements are given by

$$\langle M'^k(P')\,|\,V^j_\mu(0)\,|\,M^i(P)\rangle$$
$$=d_{kji}g_V(q^2)\epsilon_{\mu\nu\alpha\beta}P'^\nu P^\alpha\epsilon^\beta(P',\lambda)\ , \tag{A5}$$

where $q^2=(P-P')^2$ and $\epsilon^\beta(P',\lambda)$ is the polarization vector of the meson M'.

The value of the matrix element at $q^2=0$ can be obtained from the knowledge of decay $\omega\to\pi^0\gamma$, giving the information[34]

$$g_V(0)=2.59\ \mathrm{GeV}^{-1}\ . \tag{A6}$$

As for $g_V(q^2)/g_V(0)$, following Zucker,[35] we have assumed for simplicity

$$\frac{g_V(q^2)}{g_V(0)}=F_2^V(q^2)\ , \tag{A7}$$

where $F_2^V(q^2)$ is the magnetic nucleon form factor which is given by

$$F_2{}^V(q^2)=\frac{1}{[1-q^2/m_V{}^2]^2}\,\frac{1}{1-q^2/4m^2}\ , \tag{A8}$$

where $m_V{}^2=0.71$ GeV^2 and $m^2=0.88$ GeV^2.

3. $M=$pseudoscalar meson (0^-), $M'=$vector meson (1^-), $J^\mu=$axial-vector current

Following Segré and Walecka[36] we consider only two form factors:

$$\langle M'^k(P')\,|\,A^j_\mu(0)\,|\,M^i(P)\rangle$$
$$=\mathrm{i}f_{ijk}[f_1(q^2)\epsilon_\mu(P',\lambda)-f_3(q^2)\epsilon^\nu(P',\lambda)q_\nu q_\mu]\ , \tag{A9}$$

where $q_\mu=(P'-P)_\mu$ and $\epsilon^\mu(P',\lambda)$ is the polarization vector of the vector meson M'.

$f_1(m_\rho{}^2)$ can be obtained, through soft-pion theorems, from the electromagnetic coupling of the ρ meson:

$$f_1(m_\rho{}^2)=\frac{2}{f_\pi}f_\rho=1.8\ \text{GeV}\ , \tag{A10}$$

whereas its q^2 dependence is obtained, by assuming a simple pole-dominated dispersion relation, by the following approximated expression:

$$\frac{f_1(q^2)}{f_1(0)}=\frac{1}{1-q^2/m_B{}^2}\ , \tag{A11}$$

where $m_B{}^2=m_{A_1}{}^2$ $(m_{Q_1}{}^2)$ for $\Delta Y=0$ $(\Delta Y=1)$ currents.

As for $f_3(q^2)$, one obtains, from dispersion relations,

$$f_3(q^2)=\frac{f_1(q^2)}{q^2-m_P{}^2}\ , \tag{A12}$$

where $m_P{}^2=m_\pi{}^2$ $(m_K{}^2)$ if one is dealing with a $\Delta Y=0$ $(\Delta Y=1)$ current.

[1] Y. Hara, Prog. Theor. Phys. (Kyoto) 37, 710 (1967); Y. T. Chiu and J. Schechter, Phys. Rev. Lett. 16, 1022 (1966); Y. T. Chiu, J. Schechter, and Y. Ueda, Phys. Rev. 150, 1201 (1966); 157, 1317 (1967).

[2] S. Nussinov and G. Preparata, Phys. Rev. 175, 2180 (1968).

[3] K. Wilson, Phys. Rev. 179, 1499 (1969).

[4] M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. 52B, 351 (1974).

[5] G. Altarelli, G. Curci, G. Martinelli, and S. Petrarca, Nucl. Phys. B187, 461 (1981).

[6] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B120, 316 (1977); Zh. Eksp. Teor. Fiz. 72, 1275 (1977) [Sov. Phys.—JETP 45, 670 (1977)].

[7] C. Schmid, Phys. Lett. 66B, 353 (1977); M. D. Scadron, Rep. Prog. Phys. 44, 213 (1981).

[8] G. Preparata, Phys. Lett. 34B, 412 (1971).

[9] H. Harari, Phys. Rev. Lett. 17, 1303 (1966).

[10] M. Elitzur and H. Harari, Ann. Phys. (N.Y.) 56, 81 (1970).

[11] G. Nardulli and G. Preparata, Phys. Lett. 104B, 399 (1981).

[12] Z^0 exchanges are relevant only for $\Delta S=\Delta C=0$ processes; Higgs-boson exchanges and tadpoles can be neglected (see Ref. 13).

[13] G. Altarelli, in *New Phenomena in Subnuclear Physics,* proceedings of the International School of Subnuclear Physics, Erice, 1975, edited by A. Zichichi (Plenum, New York, 1977), p. 465.

[14] R. P. Feynman in *Symmetries in Elementary Particle Physics,* edited by A. Zichichi (Academic, New York, 1965), p. 111.

[15] T. J. Devlin and J. O. Dickey, Rev. Mod. Phys. 51, 237 (1979).

[16] A different situation occurs for the time-ordered product of two electromagnetic currents, in which energy conservation cannot be used to conclude that the $1/x^2$ singularity is absent; see Ref. 2.

[17] M. Suzuki, Phys. Rev. 144, 1154 (1966).

[18] R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1969).

[19] J. F. Donoghue, H. Golowich, W. R. Ponce, and B. R. Holstein, Phys. Rev. D 21, 186 (1980).

[20] M. Bonvin and C. Schmid, Institut für Theoretische Physik Eidgenössische Technische Hochschule, Zürich report (unpublished).

[21] M. Milosevic, D. Tadic, and J. Trampetic, Nucl. Phys. B187, 514 (1981); B. Guberina, D. Tadic, and J. Trampetic, *ibid.* B202, 317 (1982).

[22] W. N. Cottingham, Ann. Phys. (N.Y.) 25, 424 (1963).

[23] The contribution from the half circles in Fig. 3 is seen to vanish if one assumes Regge behavior at infinity, as shown in Ref. 24. This is due to the fact that, since the q^2 integration is over a finite range [see Eq. (3.13)], there are no limitations on the Regge-behavior hypothesis; see Ref. 10.

[24] A. Rabl, Phys. Rev. 176, 2034 (1968).

[25] V. De Alfaro, S. Fubini, G. Furlan, and G. Rossetti, *Currents in Hadron Physics* (North-Holland, Amsterdam, 1973).

[26] V. De Alfaro, S. Fubini, G. Rossetti, and G. Furlan, Phys. Lett. 21, 576 (1966).

[27] A. C. Irving and R. P. Worden, Phys. Rep. 34, 117 (1977).

[28] J. G. Rushbrooke and B. R. Webber, Phys. Rep. 44, 1 (1978).

[29] J. Ballam *et al.*, Phys. Rev. Lett. 21, 1544 (1968); 23, 498 (1969); H. Meyer *et al.*, in *The Fourth International Symposium on Electron and Photon Interactions at High Energies, Liverpool, 1969,* edited by D. W. Braben and R. E. Rand (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970), pp. 274 and 311; W. P. Heese *et al.*, Phys. Rev. Lett. 25, 613 (1970).

[30]Our definition of β_M differs by a factor $(\pi\alpha')^{-1/4}$ from that of Ref. 28.

[31]Final results vary within 5% by varying $m_0{}^2$ in this range. For the sake of definiteness we have chosen $m_0{}^2=0.5$ GeV^2.

[32]A smaller value of the cutoff is used by M. I. Vysotskii, Yad. Fiz. 31, 1535 (1980) [Sov. J. Nucl. Phys. 31, 797 (1980)] in the calculation of the $K^0-\bar{K}^0$ matrix element by means of an approach similar to the one employed in the present paper; however the quoted value for the onset of the scaling ($\simeq$200–500 MeV) seems to be too small.

[33]Particle Data Group, Rev. Mod. Phys. 52, S1 (1980).

[34]The information coming from $\rho\rightarrow\pi\gamma$ and $K^*\rightarrow K\gamma$ decays is more ambiguous due to large experimental errors.

[35]P. A. Zucker, Phys. Rev. D 4, 3350 (1971).

[36]G. Segré and J. D. Walecka, Ann. Phys. (N.Y.) 40, 337 (1966).