Radiative corrections to atomic parity violation

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Electroweak radiative corrections to the $SU(2)_L\times U(1)$ parity-violating electron-quark interaction are presented, including axial-vector isoscalar effects induced at the one-loop level. Implications of our results for parity-violation searches in ordinary hydrogen, deuterium, and heavy atoms are discussed.

Several experiments have already detected or are preparing to search for weak parity-violating effects in atomic transitions.[1–9] Of particular interest are precise experiments with ordinary hydrogen and deuterium,[6–9] since those simple atomic systems may provide a means of measuring higher-order quantum effects.[10] Motivated by that possibility, we have put together a complete $O(\alpha)$ calculation of the $SU(2)_L\times U(1)$ electroweak corrections to the electron-quark parity-violating interaction. In this paper we present our results, discuss strong-interaction effects, and comment on some experimental implications.

The electron-quark parity-violating Hamiltonian (at zero momentum transfer) is conventionally parametrized as follows[10–12]:

$$H_{\rm PV}=\frac{G_\mu}{\sqrt{2}}(C_{1u}\bar{e}\gamma_\mu\gamma_5 e\bar{u}\gamma^\mu u+C_{2u}\bar{e}\gamma_\mu e\bar{u}\gamma^\mu\gamma_5 u+C_{1d}\bar{e}\gamma_\mu\gamma_5 e\bar{d}\gamma^\mu d+C_{2d}\bar{e}\gamma_\mu e\bar{d}\gamma^\mu\gamma_5 d+\cdots)\,, \tag{1}$$

where the ellipsis represents heavy-quark terms $q=s,c,b,t$. (We do not illustrate those terms because they are not important for our analysis and differ from those presented only by heavy-quark-mass effects.[13]) In Eq. (1) we have factored out the muon decay constant

$$G_\mu=(1.166\,32\pm0.000\,02)\times10^{-5}\ {\rm GeV}^{-2}\,,$$

since that accurately measured parameter provides a convenient normalization for neutral-current amplitudes.[14] The $SU(2)_L\times U(1)$ model's predictions for the C_{iq}'s in Eq. (1) can be obtained from our previous work[14–16] when combined with some additional calculations which we subsequently describe. In total, one finds

$$C_{1u}=\tfrac{1}{2}\rho_{\rm PV}[1-\tfrac{8}{3}\kappa_{\rm PV}(0)\sin^2\hat{\theta}_W(m_W)]-\frac{\alpha}{4\pi}(1-\tfrac{8}{3}s^2)-\frac{\alpha}{9\pi}(1-4s^2)\left[\ln\frac{m_Z{}^2}{m_e{}^2}+\tfrac{1}{6}\right]$$
$$+\frac{\alpha}{2\pi}\left[\frac{1}{s^2}+(1-4s^2)\left[\ln\frac{m_Z{}^2}{M^2}+\tfrac{3}{2}\right]+\frac{3}{32s^2c^2}(1-\tfrac{8}{3}s^2)[1+(1-4s^2)^2]\right]\,, \tag{2a}$$

$$C_{1d}=-\tfrac{1}{2}\rho_{\rm PV}\left[1-\tfrac{4}{3}\kappa_{\rm PV}(0)\sin^2\hat{\theta}_W(m_W)\right]+\frac{\alpha}{4\pi}(1-\tfrac{4}{3}s^2)+\frac{\alpha}{18\pi}(1-4s^2)\left[\ln\frac{m_Z{}^2}{m_e{}^2}+\tfrac{1}{6}\right]$$
$$+\frac{\alpha}{2\pi}\left[-\frac{1}{4s^2}+\tfrac{1}{2}(1-4s^2)\left[\ln\frac{m_Z{}^2}{M^2}+\tfrac{3}{2}\right]+\frac{3}{32s^2c^2}(1-\tfrac{4}{3}s^2)[1+(1-4s^2)^2]\right]\,, \tag{2b}$$

$$C_{2u}=\tfrac{1}{2}\rho_{\rm PV}[1-4\kappa_{\rm PV}(0)\sin^2\hat{\theta}_W(m_W)]-\frac{\alpha}{9\pi}(1-4s^2)+\frac{\alpha}{9\pi}\left[\ln\frac{m_W{}^2}{m^2}+\tfrac{1}{6}\right]-\frac{\alpha}{9\pi}(1-\tfrac{8}{3}s^2)\left[\ln\frac{m_Z{}^2}{m^2}+\tfrac{1}{6}\right]$$
$$+\frac{\alpha}{2\pi}\left[\frac{1}{s^2}+(1-\tfrac{8}{3}s^2)\left[\ln\frac{m_Z{}^2}{M^2}+\tfrac{3}{2}\right]+\frac{3}{32s^2c^2}(1-4s^2)[1+(1-\tfrac{8}{3}s^2)^2]\right], \tag{2c}$$

$$C_{2d}=-\tfrac{1}{2}\rho_{\rm PV}[1-4\kappa_{\rm PV}(0)\sin^2\hat{\theta}_W(m_W)]+\frac{\alpha}{36\pi}(1-4s^2)-\frac{2\alpha}{9\pi}\left[\ln\frac{m_W{}^2}{m^2}+\tfrac{1}{6}\right]+\frac{\alpha}{18\pi}(1-\tfrac{4}{3}s^2)\left[\ln\frac{m_Z{}^2}{m^2}+\tfrac{1}{6}\right]$$
$$+\frac{\alpha}{2\pi}\left[-\frac{1}{4s^2}+\tfrac{1}{2}(1-\tfrac{4}{3}s^2)\left[\ln\frac{m_Z{}^2}{M^2}+\tfrac{3}{2}\right]+\frac{3}{32s^2c^2}(1-4s^2)[1+(1-\tfrac{4}{3}s^2)^2]\right], \tag{2d}$$

where

$$\rho_{\rm PV}=1+\frac{\alpha}{4\pi}\left[\frac{3}{4s^4}\ln c^2-\frac{7}{4s^2}+\frac{3}{4s^2}\frac{m_t{}^2}{m_W{}^2}+\frac{3}{4}\frac{\xi}{s^2}\left[\frac{\ln(c^2/\xi)}{c^2-\xi}+\frac{1}{c^2}\frac{\ln\xi}{1-\xi}\right]\right], \tag{3a}$$

$$\kappa_{\rm PV}(0)=1-\frac{\alpha}{2\pi s^2}\left[\frac{7}{9}-\frac{s^2}{3}+\frac{1}{6}\sum_i(C_{3i}Q_i-4s^2Q_i{}^2)\ln(m_i{}^2/m_W{}^2)\right]. \tag{3b}$$

In the above expressions $\alpha=1/137.036$, $s^2\equiv\sin^2\hat{\theta}_W(m_W)$, $c^2=1-s^2$, $\xi=m_\phi{}^2/m_Z{}^2$ (m_ϕ = Higgs-scalar mass), $m=m_u$ or m_d, M is a hadronic mass scale associated with the onset of the asymptotic behavior in the γZ box diagrams (in a free quark calculation $M=m$), Q_i = fermion electric charge, C_{3i} = twice the weak isospin (e.g., $C_{3e}=-1$) and the subscripts PV in $\rho_{\rm PV}$ and $\kappa_{\rm PV}$ remind us that these parameters arise naturally in the parity-violating amplitudes. The summation in Eq. (3b) is over all fermions (a color factor of 3 must be included for quarks). We have renormalized these parity-violating amplitudes by expressing them in terms of G_μ and introducing the renormalized weak mixing angle $\sin^2\hat{\theta}_W(m_W)$ which is defined by modified minimal subtraction ($\overline{\rm MS}$) using dimensional regularization with the 't Hooft unit of mass $=m_W$, the $W^\pm$ intermediate-vector-boson mass ≈ 83 GeV. Employing this prescription, all remaining radiative corrections are finite and given by Eqs. (2) and (3). (Our renormalization procedure is explained in detail in Refs. 14—16; those papers also contain formulas which allow one to express our results in terms of $\sin^2\theta_W\equiv 1-m_W{}^2/m_Z{}^2$.) The universal corrections in $\rho_{\rm PV}$ and $\kappa_{\rm PV}(0)$ can be extracted from Ref. 15. The specific $O(\alpha)$ corrections in these two parameters are determined by our choice of G_μ and $\sin^2\hat{\theta}_W(m_W)$ as renormalized quantities. The residual $O(\alpha)$ corrections in Eq. (2) which are not absorbed into $\rho_{\rm PV}$ or $\kappa_{\rm PV}(0)$ come from the following sources: The first $O(\alpha)$ correction is due to an electromagnetic renormalization of the axial-vector current at $q^2=0$.[17] To obtain these terms required a new calculation which yielded an overall factor of $(1-(\alpha/2\pi)Q_i{}^2)$ at each axial-vector-current vertex. The corrections proportional to

$$\ln\left[\frac{m_W{}^2 \text{ or } m_Z{}^2}{m^2 \text{ or } m_e{}^2}\right]+\tfrac{1}{6}$$

in Eq. (2) come from charge-radii contributions in which the photon couples to the intermediate fermion. In the quark sector these graphs involve either a virtual Z or W. In the latter case we have made the approximation of neglecting quark mixing, i.e., we have set $\theta_C=0$ where θ_C is the Cabibbo angle. Note that charge-radii contributions in which the photon is coupled to a W have been absorbed into $\kappa_{\rm PV}(0)$. Finally, the bracketed corrections come from box diagrams (WW, γZ, and ZZ in that order). The box-diagram corrections were previously discussed in Ref. 10. Here we have added $\frac{3}{2}$ to the $\ln(m_Z{}^2/M^2)$ term in the γZ contribution, so as to include the entire $O(\alpha)$ effect, corresponding to a free-quark-model calculation.[18] [We have neglected $O(\alpha m_e/M)$ corrections.]

As explained in Ref. 15, a number of the contributions to Eqs. (2) and (3) can be obtained using current-algebra and short-distance-expansion techniques; still others involve only loops in the leptonic or weak-boson sectors. These two classes of contributions remain valid in the presence of strong interactions. However, as evidenced by the logarithmic dependence on quark masses and M, some terms are affected by the strong interactions. The question naturally arises as to how to take into account the associated uncertainties in our analysis. We now address these questions.

Consider first the amplitudes in Eq. (1) proportional to C_{1u} and C_{1d}. Because the lowest order hadronic vertex is pure vector (and conserved), it is not renormalized by strong interactions. Strong interactions do, however, affect the $O(\alpha)$ corrections associated with γZ box diagrams and the hadronic contributions to γ-Z mixing in $\kappa_{\rm PV}(0)$. The terms proportional to $\ln(m_Z{}^2/M^2)$ in Eqs. (2) and (3) represent the leading short-distance contributions arising from the γZ box diagrams. Although the coefficient of this term can be determined in the presence of strong interactions,[19] there is some uncertainty regarding the value of M. Fortunately, these terms are suppressed by a $(1-4s^2)$ factor (which is ≈ 0.14) and the very small-loop momenta contributions (where strong interactions are most important) cancel to some extent when crossed and uncrossed diagrams are added. In the case of $\kappa_{\rm PV}(0)$, strong interactions, are more important. We can attempt to incorporate their effect by employing the dispersive analysis of e^+e^- annihilation data outlined in Sec. III C of Ref. 15. On the basis of our previous calculations,[15] we expect that employing an effective mass $m \simeq 0.1$ GeV for m_u, m_d, and m_s in Eq. (3b) should be consistent with such an analysis. Even this small uncertainty arising from the hadronic contribution to $\kappa_{\rm PV}(0)$ can be overcome by comparison of several precise neutral-current experiments at low q^2. For example, one can extract the value of

$$\sin^2\theta_W(0) = \kappa_{\rm PV}(0)\sin^2\hat{\theta}_W(m_W)$$

from one experiment and predict, at the one-loop level, the cross section for a different process. In summary, uncertainties in C_{1u} and C_{1d} due to strong-interaction effects are expected to be quite small; so, those parameters are good candidates for precise atomic-physics measurements.

It is useful to reparametrize the radiative corrections as follows:

$$C_{1u} = \tfrac{1}{2}\rho'_{\rm PV}[1 - \tfrac{8}{3}\kappa'_{\rm PV}(0)\sin^2\hat{\theta}_W(m_W)]\ , \tag{4a}$$

$$C_{1d} = -\tfrac{1}{2}\rho'_{\rm PV}[1 - \tfrac{4}{3}\kappa'_{\rm PV}(0)\sin^2\hat{\theta}_W(m_W)]\ , \tag{4b}$$

where

$$\rho'_{\rm PV} = \rho_{\rm PV} - \frac{\alpha}{2\pi}\left[1 + \frac{1}{s^2} + 4(1-4s^2)\left[\ln\frac{m_Z{}^2}{M^2} + \frac{3}{2}\right] + \frac{9}{16s^2c^2}(1 - \tfrac{16}{9}s^2)[1 + (1-4s^2)^2]\right]\ , \tag{5a}$$

$$\kappa'_{\rm PV}(0) = \kappa_{\rm PV}(0) - \frac{\alpha}{2\pi s^2}\left[\frac{9-8s^2}{8s^2} - \frac{(1-4s^2)}{6}\left[\ln\frac{m_Z{}^2}{m_e{}^2} + \frac{1}{6}\right] + (\tfrac{9}{4} - 4s^2)(1-4s^2)\left[\ln\frac{m_Z{}^2}{M^2} + \frac{3}{2}\right]\right.$$
$$\left. + \frac{9}{16s^2c^2}(\tfrac{1}{2} - 2s^2 + \tfrac{16}{9}s^4)[1 + (1-4s^2)^2]\right]\ . \tag{5b}$$

This parametrization is convenient for experimental examinations of C_{1u} and C_{1d}. For example, using ordinary hydrogen one may determine the combination $C_{1p} \equiv 2C_{1u} + C_{1d}$ while deuterium experiments will try to measure $C_{1\rm D} \equiv 3(C_{1u} + C_{1d})$. In terms of $\rho'_{\rm PV}$ and $\kappa'_{\rm PV}(0)$ we find

$$C_{1p} = \tfrac{1}{2}\rho'_{\rm PV}[1 - 4\kappa'_{\rm PV}(0)\sin^2\hat{\theta}_W(m_W)]\ , \tag{6a}$$

$$C_{1\rm D} = -2\rho'_{\rm PV}\kappa'_{\rm PV}(0)\sin^2\hat{\theta}_W(m_W)\ . \tag{6b}$$

In heavy atoms,[1–5] coherence effects make the dominant source of parity violation proportional to the weak charge $Q_W(Z,A)$

$$Q_W(Z,A) = 2[(A+Z)C_{1u} + (2A-Z)C_{1d}]\ , \tag{7a}$$

which in terms of Eq. (4) becomes

$$Q_W(Z,A) = \rho'_{\rm PV}[2Z - A - 4Z\kappa'_{\rm PV}(0)\sin^2\hat{\theta}_W(m_W)]\ , \tag{7b}$$

e.g., for bismuth

$$Q_W({}^{209}_{83}\mathrm{Bi}) = 584C_{1u} + 670C_{1d}\ , \tag{8a}$$

$$Q_W(\mathrm{Bi}) = \rho'_{\rm PV}[-43 - 332\kappa'_{\rm PV}(0)\sin^2\hat{\theta}_W(m_W)]\ . \tag{8b}$$

Unfortunately, parity-violating effects in heavy atoms have large uncertainties in their atomic-physics calculations. This difficulty seems to make heavy-atom experiments poor probes of radiative corrections.

For the amplitudes in Eq. (1) proportional to C_{2u} and C_{2d}, strong-interaction modifications should be significant, since they involve the axial-vector current. Cahn and Kane have partially estimate such effects by relating their isoscalar and isovector components to measured charged-current weak decays on the basis of SU(3) symmetry. They find[12]

$$C_{2p} \simeq 2FC_{2u} + (F-D)C_{2d} \ , \quad (9a)$$

$$C_{2D} \simeq (3F-D)(C_{2u}+C_{2d}) \ ,$$

$$F \simeq 0.425, \ \ D \simeq 0.825 \ , \quad (9b)$$

in going from free electron-quark to electron-proton and electron-deuterium hadronic axial-vector-current amplitudes. Here we recall that in the $SU(2)_L \times U(1)$ theory the neutral axial-vector current is purely isovector; therefore, the tree-level amplitude can be related to the neutron-proton axial-vector matrix element by invoking only isospin symmetry. When loop effects are considered, isoscalar axial currents are induced. Thus, the SU(3) symmetry is actually used only in the study of matrix elements arising from loop effects, which makes its application much more acceptable. There are additional effects not accounted for by Eq. (9). For example, Collins, Wilczek, and Zee[20] have shown within the framework of quantum chromodynamics (QCD) that the axial-vector-current anomaly induces an axial-vector isoscalar neutral current which effectively increases C_{2u} and C_{2d} by[20,21]

$$\Delta C_{2u} = \Delta C_{2d} \simeq 0.10 C_{2u} \simeq -0.10 C_{2d} \ . \quad (10)$$

Taken together with Eq. (9), this implies

$$C_{2p} \simeq 0.935 C_{2u} - 0.360 C_{2d} \ , \quad (11a)$$

$$C_{2D} \simeq 0.45(1.1 C_{2u} + 0.9 C_{2d}) \ , \quad (11b)$$

with C_{2u} and C_{2d} given by Eqs. (2c) and (2d). There are also strong-interaction corrections to γZ box diagrams (here not suppressed by a $1-4s^2$ factor) and $\kappa_{PV}(0)$ that we previously mentioned. In addition, for C_{2u} and C_{2d} the charge-radii corrections in Eqs. (2c) and (2d) are modified by QCD corrections. (That effect appears to increase their magnitude[22] and is somewhat accounted for by using $m \simeq 0.1$ GeV in the logarithmic contribution.) Given these various sources of uncertainty, it seems unlikely that one could predict C_{2p} at the one-loop level with great reliability. On the other hand, C_{2D} vanishes in lowest order; so it does provide a useful laboratory to test and compare a number of loop-level calculations, such as the QCD-induced effect in Eq. (10).

To unequivocally probe radiative corrections, one must compare two distinct experiments. At present, deep-inelastic ν_μ-hadron scattering (including radiative corrections) gives[16,23]

$$\sin^2\hat{\theta}_W(m_W) = 0.215 \pm 0.014 \quad (12)$$

as the best determination of the weak mixing angle. (Eventually, ν_μ-e scattering will provide comparable precision.) Inserting $\sin^2\hat{\theta}_W(m_W)=0.215$ into our expressions for ρ'_{PV} and $\kappa'_{PV}(0)$, we find (for $\xi=1$, $M=m=0.1$ GeV, and $m_t=20$ GeV)

$$\rho'_{PV} = 0.974 \ , \quad (13a)$$

$$\kappa'_{PV}(0) = 1.0033 \ , \quad (13b)$$

$$C_{1p} = 0.487[1-4.013\sin^2\hat{\theta}_W(m_W)] \ , \quad (13c)$$

$$C_{1D} = -1.954\sin^2\hat{\theta}_W(m_W) \ , \quad (13d)$$

$$Q_W(\mathrm{Bi}) = -41.9 - 324.4\sin^2\hat{\theta}_W(m_W) \ , \quad (13e)$$

$$C_{2p} = 0.6567[1-4.080\sin^2\hat{\theta}_W(m_W)] \ , \quad (13f)$$

$$C_{2D} = 0.052[1-3.580\sin^2\hat{\theta}_W(m_W)] \ . \quad (13g)$$

[For $M=1$ GeV, we find $\rho'_{PV}=0.977$ and $\kappa'_{PV}(0)=1.0081$.] Using $\sin^2\hat{\theta}_W(m_W)=0.215$, Eq. (13c) gives $C_{1p} \simeq 0.067$ (or 0.065 for $M=1$ GeV). This one-loop-corrected prediction is significantly larger than the lowest-order prediction[24] of 0.046. As previously mentioned, C_{2D} is interesting since it vanishes in lowest order. Equation (13g) is the induced axial-vector isoscalar effect due to higher-order QCD and electroweak corrections. Again using $\sin^2\hat{\theta}_W(m_W)=0.215$, we find $C_{2D} \simeq 0.012$. About half of this effect is QCD induced while the other half is due to electroweak corrections (fortunately they have the same sign).

In conclusion, precise measurements of C_{1u} and C_{1d} via parity-violating effects in ordinary hydrogen and deuterium may provide a probe of higher-order electroweak radiative corrections. For those quantities, strong-interaction effects are expected to be small; so our calculations should be very reliable. What one needs is other precise determinations of $\sin^2\hat{\theta}_W(m_W)$ for comparison. Ongoing ν_μ-e scattering will measure $\sin^2\theta_W(0)$ to within 5%. Comparison of that low-q^2 experiment with parity violation in hydrogen or deuterium is almost devoid of strong-interaction uncertainties. Further away is the anticipated determination of $\sin^2\hat{\theta}_W(m_W)$ at the Z^0 resonance via measurement of m_Z used in conjunction with the relationship[15,16]

$$\sin 2\hat{\theta}_W(m_W) = 77.1 \text{ GeV}/m_Z \ . \quad (14)$$

This procedure should determine $\sin^2\hat{\theta}_W(m_W)$ to within 1%.

Predictions for C_{2u} and C_{2d} are plagued by uncertainties induced through the strong interactions. Nevertheless, it is of great importance to measure C_{2D}, since this parameter vanishes in lowest order [for the $SU(2)_L \times U(1)$ model]. So, one can directly detect a higher-order effect.

Parity violation in hydrogen and deuterium may provide a viable test of the standard model's higher-order corrections. Hopefully, ongoing experiments will attain the high precision necessary to make comparison with our calculations possible.

Note added. After submitting this work for publication, we were sent copies of a thesis by Bryan Lynn (Columbia University Dissertation, Jan. 1982) which also examines electroweak radiative corrections to atomic parity violation in the $SU(2)_L \times U(1)$ model.

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