Systematics of large-p_T pion and kaon production in $\pi^- p$ and pp collisions

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Recent data on the production of large-transverse-momentum (p_T) pions and kaons in $\pi^- p$ and pp collisions at 200 GeV/c are examined and compared with the leading-order predictions of QCD. The theory gives an excellent qualitative description of all the features of the data including the large-p_T particle ratios, beam ratios, and angular dependence. In some cases, however, the predictions do not agree precisely with the data and the significance of this is discussed.

When a high-energy π^- or proton collides with a target proton the overall electric charge is conserved resulting in a net charge of zero for the former and $+2$ for the latter. However, overall charge conservation has little to do with what happens at large transverse momentum. Any charge can be produced at large p_T with the resulting low-momentum hadrons balancing the total. For example, there is no *a priori* reason for more positives than negatives to be produced at large p_T in pp collisions. In fact, the positive-to-negative ratio of large-transverse-momentum hadrons could, in principle, be the same for $\pi^- p$ and pp collisions. On the other hand, if quarks are involved in the production process then there are striking predictions for the types of large-p_T hadrons produced in these reactions. It is the purpose of this paper to compare the leading-order quantum-chromodynamics (QCD) predictions with recent data on the production of pions and kaons in $\pi^- p$ and pp collisions at 200 GeV/c.[1] These predictions were made before the data became available and thus give an honest indication of how well one can do with the leading-order QCD formula.

In leading-order QCD, mesons are produced at large transverse momentum as the result of a hard parton-parton collision, one parton from the incoming beam hadron and one from the target hadron (parton=quark, antiquark, or gluon). The resulting elastic scattering produces two outgoing partons which subsequently "fragment" onto "jets" of hadrons, one containing the large-p_T trigger particle. The invariant cross section for the reaction $A+B\to C+X$ can be written as

$$E\frac{d\sigma}{d^3p}(AB\to C+X;W,p_T,\theta_{\text{c.m.}})=\int\frac{dx_a}{\pi}\int\frac{dx_b}{z_c}\,G_{A\to a}(x_a,Q)G_{B\to b}(x_b,Q)D_{c\to C}(z_c,Q)\frac{d\hat\sigma}{d\hat t}(ab\to cd;\hat s,\hat t)\,, \tag{1}$$

where $d\hat\sigma/d\hat t(ab\to cd,\hat s,\hat t)$ is the hard-scattering parton differential cross section, $a+b\to c+d$, calculated to order $\alpha_s^2(Q)$ from perturbation theory with the strong-interaction coupling given by

$$\alpha_s(Q)=12\pi/25\ln(Q^2/\Lambda^2)\,.$$

One must sum Eq. (1) over all eight two-to-two parton subprocesses $qq\to qq$, $\bar qq\to\bar qq$, $\bar q\bar q\to\bar q\bar q$, $gq\to gq$, $g\bar q\to g\bar q$, $gg\to\bar qq$, $\bar qq\to gg$, and $gg\to gg$. The invariants $\hat s$, $\hat t$, and $\hat u$ are the usual Mandelstam invariants but for the constituent subprocess and are given by

$$\hat s=x_ax_bs,\quad \hat t=-\tfrac{1}{2}x_asx_T\tan(\tfrac{1}{2}\theta_{\text{c.m.}})\,,$$

$$\hat u=-\tfrac{1}{2}x_bsx_T\cot(\tfrac{1}{2}\theta_{\text{c.m.}})\,,$$

where W is the incoming hadron-hadron c.m. energy ($s=W^2$), $\theta_{\text{c.m.}}$ is the c.m. angle of the large-p_T trigger hadron, and x_T is its fractional transverse momentum, $x_T=2p_T/W$. One must integrate over the incoming fractional longitudinal momenta carried by the partons a and b, x_a and x_b, respectively, with the trigger hadron carrying a fraction z_c of the longitudinal momentum of the jet to which it belongs. The two-body scattering constraint for massless partons, $\hat s+\hat t+\hat u=0$, yields

$$z_c=\tfrac{1}{2}x_T[\tan(\tfrac{1}{2}\theta_{\text{c.m.}})/x_b+\cot(\tfrac{1}{2}\theta_{\text{c.m.}})/x_a]\,.$$

The effects of collinear and soft gluon emission off the incoming partons are treated to leading-logarithmic order by assigning the proper Q dependence to the probability that an incoming parton carries longitudinal momentum fraction x,

$G_{A\to a}(x,Q)$. Similarly, gluon bremsstrahlung off the outgoing partons results in a Q dependence of the "fragmentation functions," $D_{i\to h}(z,Q)$ (the number of hadrons of type h within a jet initiated by a parton of type i carrying a fraction of the momentum between z and $z+dz$). The Q dependence of structure and fragmentation functions are prescribed by perturbation theory (e.g., the Altarelli-Parisi equations[2]). However, both the parton distribution functions and the fragmentation functions must be determined experimentally at some reference value $Q=Q_0$.

I have taken the fragmentation functions from our paper on large-p_T production in pp collisions.[3] At $Q=2$ GeV they correspond to the Field-Feynman quark-jet parametrization,[4] which has been shown to agree with intermediate-energy jets produced in e^+e^- annihilation. At higher Q they evolve according to the leading-order perturbative (Altarelli-Parisi) equations. The quark and gluon distributions within a proton $G_{p\to i}(x,Q)$ are taken from Ref. 5 and are shown at $Q=4$ and 20 GeV in Fig. 1.

The quark and gluon distributions within a pion $G_{\pi^-\to i}(x,Q)$ are more difficult to determine.

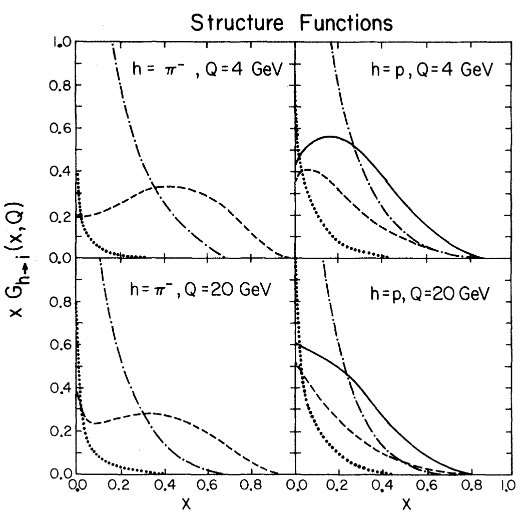

FIG. 1. Parton distributions $xG_{h\to i}(x,Q)$ within a π^- and a proton. The distributions were set at a reference point $Q_0=2$ GeV by comparison with deep-inelastic scattering data for the proton and $\pi^-p\to\mu^+\mu^-+X$ data for the π^-. For $Q>Q_0$ the distributions are arrived at by use of the leading-order QCD evolution equations (i.e., Altarelli-Parisi equations). The solid and dashed curves correspond to $i=u$ and $i=d$, respectively, whereas the dot-dashed curve corresponds to $i=g$. The dotted curves are sea contributions and correspond to $\bar{u}+\bar{d}$ for the proton and $u+\bar{d}$ for the π^-. In addition, $G_{\pi^-\to\bar{u}}(x,Q)=G_{\pi^-\to d}(x,Q)$.

Feynman's original guess of a flat quark distribution within a pion which was presented in our quark-quark scattering paper[6] is not consistent with QCD and does not agree with recent data on the Feynman x_F dependence of muon pairs produced in π^-p collisions.[7] The muon-pair data indicates a distribution within the π^- of roughly

$$xG_{\pi^-\to d}(x,Q_0)\approx 0.52\,(1-x)^{1.01}$$

for the region $0.2<x<0.6$. QCD, on the other hand, suggests a distribution at large x and low Q of the form $A(1-x)^2+B/Q^2$.[8] What I have done here is to ignore the $1/Q^2$ term and refit the muon-pair data with a distribution of the form

$$xG_{\pi^-\to d}(x,Q_0)=(0.1+1.0x+8x^3)(1-x)^2$$

at a reference energy (dimuon mass) of $Q_0=M=2$ GeV. This form gives an equally good fit to the $\pi^-p\to\mu^+\mu^-+X$ data and has the desired large-x behavior. The gluon distribution within a pion was guessed and set at $Q_0=2$ GeV/c to be

$$xG_{\pi\to g}(x,Q_0)=2(1-x)^3\ .$$

As for the proton, the higher Q values are arrived at by the use of the QCD Q^2 evolution formulas. The resulting pion distributions at $Q=4$ and 20 GeV are shown, together with the proton distributions, in Fig. 1. The valence quarks $d+\bar{u}$ carry 44% of the momentum of the π^- at $Q=4$ GeV, whereas $u+d$ quarks carry 43% of the proton's momentum. Gluons carry 53% of the pion's momentum and 51% of the proton momentum at this value of Q.

Although in principle Eq. (1) is the correct leading-order formula, in practice (at finite energies) there are many problems and uncertainties associated with its use. First, there is the choice for the energy variable Q, that sets the strength of the strong-interaction coupling, $\alpha_s(Q)$. To leading order all choices that increase linearly with the parton-parton c.m. energy are equivalent, but predictions at existing energies are sensitive to the precise form of this choice. Here I stick which the choice made in Ref. 3,

$$Q^2=2\hat{s}\hat{t}\hat{u}/(\hat{s}^2+\hat{t}^2+\hat{u}^2)\ .$$

This choice is not independent from the selection of the QCD perturbative parameter Λ, which sets the energy scale and which has been asigned the value 0.4 GeV.

Second, there is the question of higher-order corrections to the leading-order formula. There are corrections of order $\alpha_s(Q)$, which in principle could be calculated and might be large at these energies. At present only the corrections to the $qq\to qq$ scattering term have been calculated and, indeed, are found to be sizable.[9] One cannot yet make any definite statements since this subprocess makes up only a piece of the large-p_T rate. However, it is quite likely that order $\alpha_s(Q)$ corrections are important when estimating the magnitude of the large-p_T cross section. On the other hand, these corrections are probably not as important when one is considering the ratios of various large-p_T reactions. Therefore, in this paper we will be concerned primarily with large-p_T trigger-particle and beam-particle ratios.

At low and intermediate values of p_T there may also be corrections that fall off by powers of p_T (or Q) faster than leading order. There is no systematic way of correctly including all corrections that are down by powers of p_T. One type of power-damped correction comes from the fact that the initial partons are not precisely parallel to the incoming hadrons. Similarly particles in the outgoing jets are not exactly parallel to the quark that initiated the jet. Corrections for this can be made (called "smearing" corrections), but they effect primarily the rate for large-p_T production (and the event structure) and not the cross-section ratios. Precisely how much these smearing corrections effect the single-particle rates is a debated issue but does not concern us here since we will be dealing with ratios.

The leading-order formula assumes that the hard scattering is effectively a two-to-two process. To leading-logarithmic order all radiation of gluons from the initial or outgoing partons can be considered as collinear or soft emissions. If one is interested in the overall event shape of a large-p_T process then the assumption of parallel kinematics may give misleading results. For example, the large transverse momentum of the parton that produced the trigger hadron often is balanced on the away side by two medium-p_T partons. Events of this type are not approximated accurately by the leading-order formula but can be handled properly by Monte Carlo techniques.[10] These more sophisticated techniques, however, give similar results for the particle and beam ratios presented here.

Subprocesses involving the large-angle scattering of hadrons (e.g., $\pi q \rightarrow \pi q$) rather than partons constitute another form of corrections that are down by powers of p_T from the leading-order parton-parton terms. It has been postulated that corrections of this form might be important (at intermediate-p_T values) since, for example, one produces directly the large-p_T pion rather than demanding the higher parton-parton c.m. energy necessary to produce a jet of hadrons containing the trigger pion.[11] Some of these nonleading subprocesses can be estimated, but it is not at present possible to include all such terms in a systematic manner. These terms can drastically affect the predictions for the large-p_T particle and beam ratios and the angular dependences. By examining such ratios one can, in some cases, ascertain the importance of some of these nonleading subprocesses.

In spite of all the above caveats, if one were asked to make a prediction concerning large p_T, the first step would be to try Eq. (1). As I have indicated this is particularly true for particle ratios. Particle ratios are not very sensitive to ones choice for Q (or of the choice for lambda) and it is safe to assume collinear kinematics. In addition, higher-order QCD corrections are more likely to affect the magnitude of the cross section then the ratio of cross sections. At present energies large-p_T particle and beam ratios provide the best test for the systematics predicted by Eq. (1). In fact, by studying particle ratios one might be able to ascertain whether quark-quark, quark-gluon, and gluon-gluon scatterings are the dominant subprocesses at a given p_T or

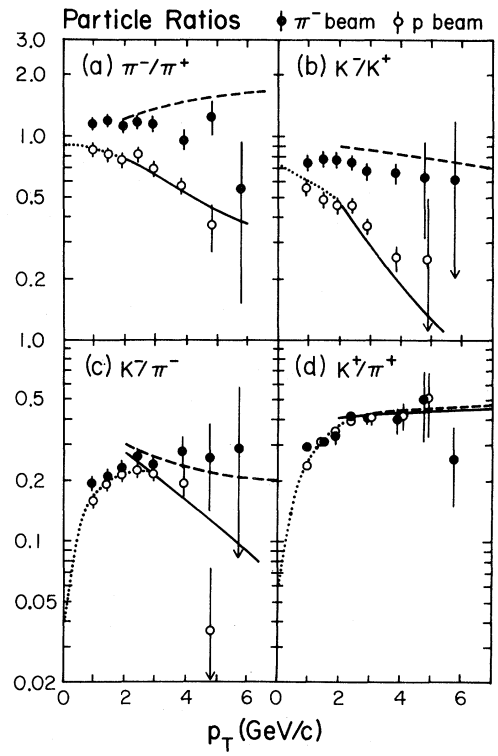

FIG. 2. Comparison of the large-p_T trigger-particle ratios for $\pi^- p$ and pp collisions at 200 GeV/c and $\theta_{c.m.} = 90°$ from Ref. 1 with the leading-order QCD predictions of Eq. (1). The solid and dashed curves are the predictions for pp and $\pi^- p$ collisions, respectively, and should only be compared with data above p_T of about 3 GeV/c. The dotted curves are extrapolations of the low-p_T pp data ($p_T < 1$ GeV/c) drawn (by hand) to meet the QCD predictions at about $p_T = 3$ GeV/c.

whether nonleading processes like $\pi q \to \pi q$ are important.

The leading-order QCD predictions for the large-p_T particle ratios in $\pi^- p$ and pp are compared with the data at 200 GeV/c and $\theta_{\text{c.m.}} = 90°$ (Ref. 1) in Fig. 2. Although in some cases the agreement is not exact, the systematics of the data are described correctly. The proton contains more u than d quarks and since u-quark jets contain (at high z) more π^+ than π^-, one expects and sees a π^-/π^+ ratio considerably less than one in pp collisions. For $\pi^- p$ collisions, on the other hand, one predicts a π^-/π^+ ratio slightly greater than one. This is because the d (and $\bar{u}$) quark in the π^- has on the average a higher momentum than the u quark within the proton which at 90° results in a slight excess of π^- over π^+ at large p_T. The data show a π^-/π^+ ratio that is slightly greater than one but not quite as large as the prediction.

To appreciate the significance of the agreement seen in Fig. 2 one must contrast it with predictions from other subprocesses. At one time it was suggested that the process in which the incoming pion scatters off a quark in the proton thereby gaining a large transverse momentum, $\pi^- q \to \pi^- q$, might make a major contribution to the cross section for $\pi^- p$ collisions at these p_T values. Since the double charge exchange subprocess $\pi^- q \to \pi^+ q$ does not exist, one expects a large π^-/π^+ ratio in $\pi^- p$ collisions. Jones and Gunion[12] predicted a π^-/π^+ ratio from the $\pi q \to \pi q$ term of about 35 at 200 GeV/c and $p_T = 6$ GeV/c, which is clearly ruled out by the data.[13]

In pp collisions one can only produce large-p_T K^-'s by scattering partons out of the proton sea or by producing them as nonleading particles in the quark jets. A very small K^-/K^+ ratio for pp collisions is therefore predicted. On the other hand, K^-'s can be produced directly in $\pi^- p$ collisions through the scattering of the valence $\bar{u}$ quark within the π^- so that the K^-/K^+ ratio here should not be as small. This is indeed what is observed in Fig. 2(b).

While the π^-/π^+, K^-/K^+, and K^-/π^- ratios are predicted to be quite different from π^- and proton beams, the K^+/π^+ ratio is predicted to be essentially identical for the two beams. This is because neither the π^- nor the proton contains any valence strange quarks. In both reactions large-p_T K^+'s arise from the fragmentation of u-quark jets. The K^+/π^+ ratio for both reactions simply measures the extra difficulty of producing an $s\bar{s}$ pair over a $d\bar{d}$ pair during the fragmentation process and should be the same for both reactions. The data in Fig. 2(d) confirm this.

Figure 3 compares the beam ratios for the production of large-p_T pions and kaons. The beam ratios are sensitive to the absolute normalization of the π^- parton distributions relative to the proton distributions. (The large-p_T particle ratios in Fig. 2 are sensitive to the shapes of the distributions and to the relative amount of glue but not to the overall normalizations.) Again the trends of the data are predicted correctly. It is considerably easier for a π^- beam to produce a large-p_T π^- or K^- than a proton beam. At around $p_T = 4$ GeV/c both beams are equally proficient at producing large-p_T π^+'s and K^+'s.

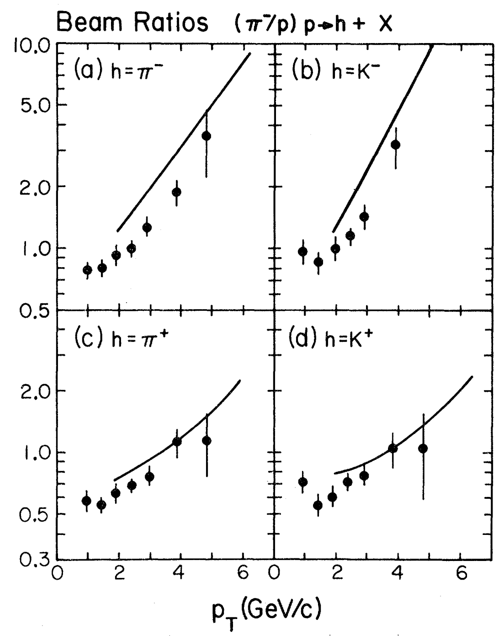

FIG. 3. Comparison of the large-p_T beam ratios $(\pi^-/p)p \to h + X$, at 200 GeV/c and $\theta_{\text{c.m.}} = 90°$ from Ref. 1 with the leading-order QCD predictions of Eq. (1).

Both the QCD subprocesses $qq \to qq$ and $qg \to qg$ are peripheral in nature (i.e., the cross sections are larger in the forward rather than the backward direction). (The quarks do not like to change direction by large angles.) This is not true for all subprocesses, for example, the constituent interchange diagram $\pi q \to \pi q$ does not have this property. It is true, however, for the dominant leading QCD subprocesses. This feature together with the kinematic fact that demanding a large-p_T hadron at small (large) $\theta_{\text{c.m.}}$ requires a constituent collision involving a high-x beam (target) parton results in the dramatic predictions for the angular dependence of the particle ratios shown in Fig. 4.

The large-p_T particle ratios in pp collisions are symmetric about 90°, but this is not the case for $\pi^- p$ interactions. As one changes to a more forward angle (at fixed p_T) it becomes more and more

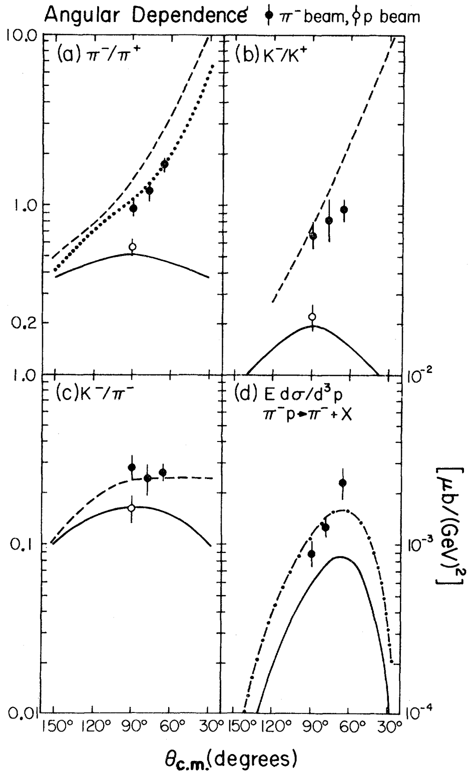

FIG. 4. (a)—(c) Comparison of the angular dependence of the large-p_T trigger-particle ratios for $\pi^- p$ and pp collisions at 200 GeV/c and $p_T = 4$ GeV/c from Ref. 1 with the leading-order QCD predictions of Eq. (1). For the pp case data only exist at $\theta_{\text{c.m.}} = 90°$. The solid and dashed curves are the predictions for pp and $\pi^- p$, respectively, made before the data existed. The dotted curve for $\pi^- p \to (\pi^-/\pi^+) + X$ in (a) is the result of changing the gluon fragmentation function and the gluon distributions within the pion and proton (see text). (d) Comparison of the angular dependence of the $\pi^- p$ invariant cross section with the leading-order QCD predictions of Eq. (1) (solid curve). The dot-dash curve is the same as the solid curve except that it has been multiplied by an arbitrary factor of 2.

likely that the observed particle came from the fragmentation of a parton that originated in the beam hadron. Thus, for example, in $\pi^- p$ collisions the π^-/π^+ ratio should become larger as $\theta_{\text{c.m.}}$ decreases and should deviate more and more from the pp case. On the other hand, in the backward direction this ratio should become similar for the two beams since the quarks responsible are predominately target quarks. There are no data on the angular dependence of the π^-/π^+ ratio in pp collisions. The data for $\pi^- p$ interactions is shown in Fig. 4(a). The ratio does increase as $\theta_{\text{c.m.}}$ decreases, but the effect is not as strong as predicted.

Because the quarks within the pion contain on the average larger longitudinal momentum (higher-x values) than do the quarks within the proton, one expects the $\pi^- p \to \pi^- + X$ invariant cross section $E\, d\sigma/d^3p$ to have an interesting angular dependence (at fixed p_T). The cross section is predicted to be shifted toward the forward direction with a peak at about 65°. A comparison with the data is shown in Fig. 4(d). One cannot expect to predict the magnitude precisely, but a comparison with the shape of the angular distribution is meaningful. The data do show an increase toward forward angles. Unfortunately the data do not extend to small enough angles to judge if there is a peak at 65°.

As mentioned earlier all of the predictions appearing in Figs. 2—4 (solid and dashed curves) were made before the data were available. One might ask at this stage if it is worth changing some of the inputs into Eq. (1) in order to get a more precise fit to the data. Personally I feel that this is asking too much of a leading-order calculation. One should be content (and, in fact, very pleased) at the overall qualitative agreement between the theory and experiment at this energy and these p_T values. The systematics are explained quite nicely and one really does not know, at present, the best way to improve the accuracy of the calculations. The agreement, of course, does not prove that the leading-order QCD diagrams are the dominant subprocesses at these energies. If, however, they are not dominant, then whatever is responsible for large-p_T hadron production at these energies and p_T values must exhibit systematics that resembles closely the leading-order QCD predictions presented here.

Although I do not believe that a more precise fit to the data at this stage would be any more significant than the qualitative agreement already obtained, it is instructive to see how one would have to modify the input in order to fit the data more precisely. One could easily improve the beam-ratio predictions in Fig. 3 by changing slightly the normalization of the parton distributions (at high x) within the pion (Fig. 1). This could be done without vitiating agreement with the muon-pair data on $\pi^- p \to \mu^+\mu^- + X$. It is a bit more difficult to improve agreement with the angular dependence of the π^-/π^+ ratio shown in Fig. 4(a). The dotted curve shows a "postdiction" that was arrived at in the following manner. First, one allows gluon jets to resemble more closely quark jets. Initially, I had assumed that gluon jets were "softer" (i.e., less high-momentum hadrons) which resulted in large-p_T hadrons that arise predominantly from quark jets (although quite often the high-p_T quark scatters off glue through the process $qg \to qg$). With more high-p_T hadrons now being produced by glue jets all

π^-/π^+ and K^-/K^+ ratios become closer to one (gluons produce equal amounts of positive and negative hadrons). This improves the agreement with the π^- beam data but worsens the agreement with the π^-/π^+ and K^-/K^+ ratios for pp collisions [Fig. 2(a) and 2(b)]. In order to restore the agreement for the pp case one changes the gluon distribution within the proton so that there is less high-momentum glue and thus less gluons produced at large p_T. For the π^-, on the other hand, one leaves a sizable amount of high-momentum glue so that numerous large-momentum gluons are produced thus keeping the large $p_T \pi^-/\pi^+$ ratio closer to one.

It is possible that the data are providing us with information about the gluon distributions within pions and protons and about gluon jets. However, before one could take this too seriously one would have to have a better handle on the corrections to the leading-order predictions. It is possible that higher-order QCD corrections or nonleading subprocesses (or even uncalculable $1/Q^2$ effects) might change the predictions in such a way as to agree perfectly with experiment without changing the gluon distributions. Unfortunately, at present we must be satisfied with qualitive QCD predictions for hadron-hadron collisions. However, let us not forget that it was not too many years ago when we had no theoretical predictions at all for the production of large-p_T mesons in hadron-hadron collisions.

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