# Parity violation in $\vec{\gamma}+d\to n+p$

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We calculate the asymmetry $A_{\rm cir}=[\sigma(+)-\sigma(-)]/[\sigma(+)+\sigma(-)]$ for the reaction $\vec{\gamma}d\to np$, where $\sigma(+)$ $[\sigma(-)]$ is the total cross section for right [left] circularly polarized photons. The calculation is carried out for photon energies up to 30 MeV above threshold in the laboratory system. The result for $A_{\rm cir}$ is expressed in terms of the effective parity-violating $NN\pi$, $NN\rho$, and $NN\omega$ coupling constants.

## I. INTRODUCTION

Studies of parity-violating effects in nonleptonic nuclear processes are of considerable importance since they give information on the strangeness conserving nonleptonic weak interactions. One of the outstanding puzzles in this field is the experimental value[1] of the photon circular polarization ($P_\gamma$) in the $n+p\to d+\gamma$ reaction, which is about 50 times larger than the predicted one.[2]

In this paper we report on a calculation of the asymmetry $A_{\rm cir}$ in the total cross section for the reaction $\vec{\gamma}+d\to n+p$ for circularly polarized incident photons, defined as

$$A_{\rm cir}=[\sigma(+)-\sigma(-)]/[\sigma(+)+\sigma(-)]\ ,$$

where $\sigma(+)$ and $\sigma(-)$ are the total cross sections for right-handed and left-handed photons, respectively. As pointed out by Lee,[3] for photon energies near threshold, $A_{\rm cir}$ is equal to the photon circular polarization in $n+p\to d+\gamma$. The latter depends only on the $\Delta I=0$ and $\Delta I=2$ parts of the parity-violating interaction.

The calculation of Lee was carried out for photon energies up to 1 MeV above threshold, using the Hulthén-Sugawara[4] wave function. The results were given for specific values of the effective parity-violating nucleon-nucleon-meson coupling constants. Here we study $A_{\rm cir}$ for photon energies up to 30 MeV above threshold. For photon energies larger than 1 MeV the contribution of the $\Delta I=1$ part of the interaction can no longer be neglected. For the strong nucleon-nucleon force we use the realistic potential proposed by Hamada and Johnston[5] and express our results in terms of the seven parity-violating nucleon-nucleon-meson coupling constants involved in the parity-violating potential arising from the exchange of $\pi^\pm$, $\rho$, and $\omega$ mesons.

In Sec. II the parity-violating nucleon-nucleon potential is described. The formula for the asymmetry $A_{\rm cir}$ is given in Sec. III. In the final section we present our results and conclusions.

## II. THE PARITY-VIOLATING NUCLEON-NUCLEON POTENTIAL

We shall assume, as usual, that the parity-violating nucleon-nucleon interaction can be represented by a parity-violating potential $V_{\rm PV}$, arising from the exchange of single pseudoscalar and vector mesons: the $\pi^\pm$, $\rho^{\pm,0}$, and $\omega$. In particular, we shall ignore the effects of $2\pi$ exchange. This potential is determined by seven effective parity-violating nucleon-nucleon-meson coupling constants, defined by[6]

$$H_\pi^{\rm PV}=\frac{f_\pi}{\sqrt{2}}\bar{\psi}(\vec{\tau}\times\vec{\phi})_z\psi$$

and

$$H_V^{\rm PV}=-\bar{\psi}\left[h_\rho^0\vec{\tau}\cdot\vec{\rho}_\mu+h_\rho^1\rho_\mu^z+\frac{h_\rho^2}{2\sqrt{6}}(3\tau_z\rho_\mu^z-\vec{\tau}\cdot\vec{\rho}_\mu)\right]\gamma^\mu\gamma_5\psi-\bar{\psi}(h_\omega^0\omega_\mu+h_\omega^1\tau_z\omega_\mu)\gamma^\mu\gamma_5\psi$$
$$+\frac{h_\rho^{1\prime}}{2M}\bar{\psi}\sigma^{\mu\nu}(p'-p)_\nu\gamma_5(\vec{\tau}\times\vec{\rho}_\mu)_z\psi\ , \qquad (1)$$

where $\psi$, $\phi$, $\rho$, and $\omega$ are the fields of the nucleon, pion, $\rho$ meson, and $\omega$ meson, respectively. The $\tau$'s are Pauli matrices. For the $\gamma$ matrices and the metric we follow the conventions of Ref. 7.

The effective Hamiltonian for the strong nucleon-nucleon-meson couplings is given by

 

$$H_\pi^s = -ig_\pi \bar{\psi}\gamma_5 \vec{\tau}\cdot\vec{\phi}\psi$$

and

$$H_V^s = g_\rho \bar{\psi}\left[\gamma^\mu + \frac{\chi_V}{2M}\sigma^{\mu\nu}i(p'-p)_\nu\right]\vec{\tau}\cdot\vec{\rho}_\mu\psi + g_\omega\bar{\psi}\left[\gamma^\mu + \frac{\chi_s}{2M}\sigma^{\mu\nu}i(p'-p)_\nu\right]\omega_\mu\psi \ . \tag{2}$$

The parity-violating potential resulting from (1) and (2) is[6]

$$V_{\rm PV} = V_\pi^{\Delta I=1} + V_\rho^{\Delta I=0,1,2} + V_\omega^{\Delta I=0,1} \ , \tag{3}$$

where

$$V_\pi^{\Delta I=1} = \frac{g_\pi f_\pi}{2\sqrt{2}M} i\left[\frac{\vec{\tau}^1\times\vec{\tau}^2}{2}\right]_z (\vec{\sigma}^1+\vec{\sigma}^2)\cdot[\vec{p}^1-\vec{p}^2, f(m_\pi r)]_- \ , \tag{3a}$$

$$\begin{aligned} V_\rho^{\Delta I=0,1,2} = -\frac{g_\rho}{2M}\Bigg[ & \left[h_\rho^0 \vec{\tau}^1\cdot\vec{\tau}^2 + h_\rho^1\frac{\tau_z^1+\tau_z^2}{2} + h_\rho^2\frac{3\tau_z^1\tau_z^2 - \vec{\tau}^1\cdot\vec{\tau}^2}{2\sqrt{6}}\right] \\ & \times((\vec{\sigma}^1-\vec{\sigma}^2)\cdot\{\vec{p}^1-\vec{p}^2, f(m_\rho r)\}_+ + (1+\chi_V)i\vec{\sigma}^1\times\vec{\sigma}^2\cdot[\vec{p}^1-\vec{p}^2, f(m_\rho r)]_-) \\ & - h_\rho^1\frac{\tau_z^1-\tau_z^2}{2}(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\{\vec{p}^1-\vec{p}^2, f(m_\rho r)\}_+ \\ & + h_\rho^{1'} i\frac{(\vec{\tau}^1\times\vec{\tau}^2)_z}{2}(\vec{\sigma}^1+\vec{\sigma}^2)\cdot[\vec{p}^1-\vec{p}^2, f(m_\rho r)]_-\Bigg] \ , \end{aligned} \tag{3b}$$

and

$$\begin{aligned} V_\omega^{\Delta I=0,1} = -\frac{g_\omega}{2M}\Bigg[ & \left[h_\omega^0 + h_\omega^1\frac{\tau_z^1+\tau_z^2}{2}\right]((\vec{\sigma}^1-\vec{\sigma}^2)\cdot\{\vec{p}^1-\vec{p}^2, f(m_\omega r)\}_+ \\ & + (1+\chi_s)i\vec{\sigma}^1\times\vec{\sigma}^2\cdot[\vec{p}^1-\vec{p}^2, f(m_\omega r)]_-) + h_\omega^1\frac{\tau_z^1-\tau_z^2}{2}(\vec{\sigma}^1+\vec{\sigma}^2)\cdot\{\vec{p}^1-\vec{p}^2, f(m_\omega r)\}_+\Bigg] \ , \end{aligned} \tag{3c}$$

where $f(m_i r) = e^{-m_i r}(4\pi r)$.

## III. THE ASYMMETRY $A_{\rm cir}$ IN THE CROSS SECTION FOR THE PHOTODISINTEGRATION OF DEUTERON

In calculating the cross section for deuteron photodisintegration we shall include only the dipole transitions. The contributions from higher multipole transitions are negligible for photon energies we consider.[4,8]

In the absence of parity violation there are four possible dipole transitions: an $M1$ transition to the $^1S_0(I=1)$ final state and $E1$ transitions to the $^3P_0(I=1)$, $^3P_1(I=1)$, and $(^3P_2+{}^3F_2)(I=1)$ states. These are shown in Figs. 1(a) and 1(b) by double solid lines. Owing to the presence of the parity-violating force (3), states of opposite parity [in Figs. 1(a), and 1(b) connected by dotted lines with the original states] are admixed into the $^1S_0$, $^3S_1$ and $^3P_0$, $^3P_1$, $^3P_2+{}^3F_2$ states, allowing for further electromagnetic transitions [shown on Figs. 1(a) and 1(b) by single solid lines]. Only those admixed states are indicated which can be connected with the initial state by a dipole transition.

The asymmetry

$$A_{\rm cir} \equiv \frac{\sigma(+)-\sigma(-)}{\sigma(+)+\sigma(-)} \ , \tag{4}$$

is given by

$$A_{\rm cir} = \frac{2\,{\rm Re}\sum_{f_z i_z}[(M1)^*_{f_z i_z}(\tilde{E}1)_{f_z i_z} + (E1)^*_{f_z i_z}(\tilde{M}1)_{f_z i_z}]}{\sum_{f_z i_z}[\,|(M1)_{f_z i_z}|^2 + |(E1)_{f_z i_z}|^2]} = \frac{2\,{\rm Re}(\hat{M}_1+\hat{E}_1+\hat{E}_2+\hat{E}_3)}{M_1+E_1+E_2+E_3} \ , \tag{5}$$

where

$$M_1=\left[\frac{1+\mu_V}{M}\right]^2\left|\int dr\, U^*({}^1S_0)U_d({}^3S_1)\right|^2, \tag{5a}$$

$$E_1=\tfrac{1}{27}\left|\int r\,dr\, U^*({}^3p_0)[U_d({}^3S_1)-\sqrt{2}U_d({}^3D_1)]\right|^2, \tag{5b}$$

$$E_2=\tfrac{1}{9}\left|\int r\,dr\, U^*({}^3p_1)\left[U_d({}^3S_1)+\frac{1}{\sqrt{2}}U_d({}^3D_1)\right]\right|^2, \tag{5c}$$

$$E_3=\tfrac{5}{27}\left|\int r\,dr\left\{U^*({}^3p_2)\left[U_d({}^3S_1)-\frac{1}{5\sqrt{2}}U_d({}^3D_1)\right]+\frac{3\sqrt{3}}{5}U^*({}^3F_2)U_d({}^3D_1)\right\}\right|^2, \tag{5d}$$

$$\hat{M}_1=\left[\frac{1+\mu_V}{M}\int dr\, U^*({}^1S_0)U_d({}^3S_1)\right]^*\left[\frac{i}{3}\int r\,dr\tilde{U}^*({}^3p_0)[U_d({}^3S_1)-\sqrt{2}U_d({}^3D_1)] - \frac{i}{\sqrt{3}}\int r\,dr\, U^*({}^1S_0)\tilde{U}_d({}^1p_1)\right], \tag{5e}$$

$$\hat{E}_1=\left[\frac{i}{9}\int r\,dr\, U^*({}^3p_0)[U_d({}^3S_1)-\sqrt{2}U_d({}^3D_1)]\right]^* \times\left[\frac{1+\mu_V}{M}\int dr\left[\tilde{U}^*({}^1S_0)U_d({}^3S_1)-\frac{1}{\sqrt{3}}U^*({}^3p_0)\tilde{U}_d({}^1p_1)\right]-\frac{\sqrt{2}\mu_s}{\sqrt{3}M}\int dr U^*({}^3p_0)\tilde{U}_d({}^3p_1)\right], \tag{5f}$$

$$\hat{E}_2=\left[\frac{i}{3\sqrt{6}}\int r\,dr\, U^*({}^3p_1)\left[U_d({}^3S_1)+\frac{1}{\sqrt{2}}U_d({}^3D_1)\right]\right]^* \times\left[\frac{\sqrt{2}(1+\mu_V)}{M}\int dr\, U^*({}^3p_1)\tilde{U}_d({}^1p_1)+\frac{2+\mu_s}{M}\int dr\, U^*({}^3p_1)\tilde{U}_d({}^3p_1)\right], \tag{5g}$$

and

$$\hat{E}_3=\left\{-\frac{5i}{9\sqrt{3}}\int r\,dr\left[U^*({}^3p_2)\left[U_d({}^3S_1)-\frac{1}{5\sqrt{2}}U_d({}^3D_1)\right]+\frac{3\sqrt{3}}{5}U^*({}^3F_2)U_d({}^3D_1)\right]\right\}^* \times\left[\frac{1+\mu_V}{M}\int dr\left[U^*({}^3p_2)\tilde{U}_d({}^1p_1)-\frac{\sqrt{3}}{\sqrt{5}}\tilde{U}^*({}^1D_2)U_d({}^3D_1)\right] - \frac{\mu_s}{\sqrt{2}M}\int dr\left[U^*({}^3p_2)\tilde{U}_d({}^3p_1)+\frac{3}{\sqrt{5}}\tilde{U}^*({}^3D_2)U_d({}^3D_1)\right]\right]. \tag{5h}$$

Here $i_z$ and $f_z$ are the third components of the spin of the initial and final nucleon systems, respectively. In Eq. (5) the wave functions $U$ and $U_d$ represent the $np$ scattering state and the deuteron state, respectively. They are eigenstates of the Hamada-Johnston potential.[5] The states admixed by the parity-violating potential are denoted by $\tilde{U}$ and $\tilde{U}_d$. They satisfy the Schrödinger equations given in the Appendix.

## IV. RESULTS AND CONCLUSIONS

Assuming for the tensor-type strong couplings the values $\chi_V=\mu_p-\mu_n=3.71$ and $\chi_S=\mu_p+\mu_n=-0.12$ and taking $m_\rho=m_\omega=775$ MeV, we obtain the following result for the asymmetry $A_{\rm cir}$ as a function of the incident photon energy $\omega_L$:

$$A_{\rm cir}(\omega_L)=\alpha(\omega_L)g_\rho h_\rho^0+\alpha'(\omega_L)g_\omega h_\omega^0+\beta(\omega_L)g_\rho h_\rho^2 +\gamma(\omega_L)(g_\rho h_\rho^1-g_\omega h_\omega^1)+\gamma'(\omega_L)g_\rho h_\rho^{1\prime} +\delta(\omega_L)g_\pi f_\pi\,. \tag{6}$$

Thus $A_{\rm cir}$ is sensitive to six independent combinations of the seven constants. This is due to the fact that there is no contribution from the spin-changing parts of the $\Delta I=1$ parity-violating potential, and

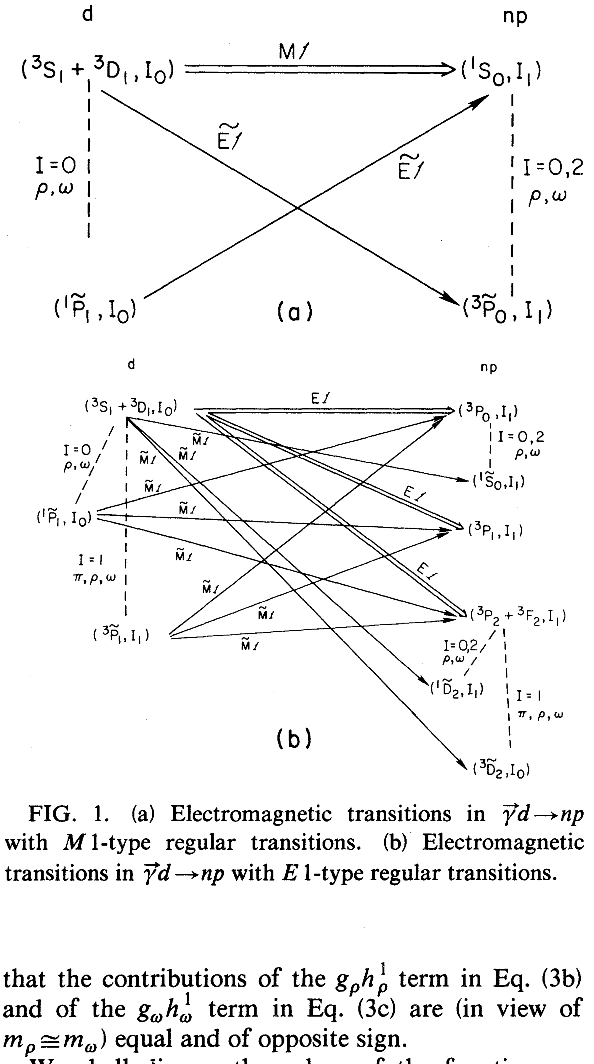


FIG. 1. (a) Electromagnetic transitions in $\vec{\gamma} d \rightarrow np$ with $M1$-type regular transitions. (b) Electromagnetic transitions in $\vec{\gamma} d \rightarrow np$ with $E1$-type regular transitions.

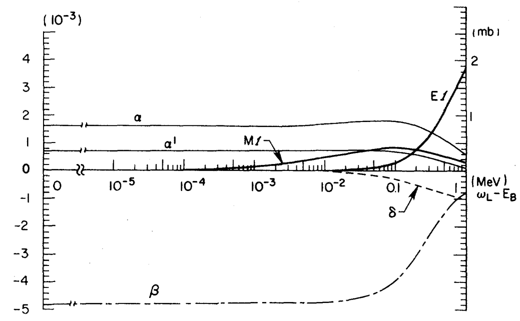


FIG. 2. The quantities $\alpha$, $\alpha'$, $\beta$, and $\delta$ as functions of $\omega_L - E_B$ for $0 \leq \omega_L - E_B \leq 1$ MeV (plotted on the left-side vertical axis in units of $10^{-3}$). Also shown are the integrated $\gamma d \rightarrow np$ cross sections for $M1$ and $E1$ transitions as functions of $\omega_L - E_B$ (plotted on the right-side vertical axis in units of mb).

that the contributions of the $g_\rho h_\rho^1$ term in Eq. (3b) and of the $g_\omega h_\omega^1$ term in Eq. (3c) are (in view of $m_\rho \cong m_\omega$) equal and of opposite sign.

We shall discuss the values of the functions $\alpha$, $\alpha', \dots, \delta$ separately for the regions $\omega_L - E_B \leq 1$ MeV and $\omega_L - E_B \geq 1$ MeV. For photon energies $\omega_L - E_B \geq 1$ MeV the dominant regular transition is $E1$, while near threshold the regular $M1$ transition dominates.

### A. $0 \leq \omega_L - E_B \leq 1$ MeV

In Fig. 2 we show the calculated values of the integrated cross sections for $M1$ and $E1$ transitions (plotted on the right-side vertical axis). The total cross section is obtained by adding the $M1$ and $E1$ parts. The values of $\alpha$, $\alpha'$, $\beta$, and $\delta$ are also given in this figure (plotted on the left-side vertical axis). The values of $\gamma$ and $\gamma'$ are too small to be seen in this figure. The following features are to be noted. For comparable coupling strength the $\Delta I = 1$ contribution is suppressed in this energy region, and consequently $A_{\text{cir}}$ is dominated by the $\Delta I = 0$ (involving $\alpha$ and $\alpha'$) and $\Delta I = 2$ (proportional to $\beta$) parts.

For photon energies $0 < \omega_L - E_B < 0.01$ MeV the quantities $\alpha$, $\alpha'$, and $\beta$ are almost constants, and the asymmetry is given to a good approximation by

$$A_{\text{cir}} \cong (1.60 g_\rho h_\rho^0 + 0.72 g_\omega h_\omega^0 - 4.74 g_\rho h_\rho^2) \times 10^{-3} \,. \tag{7}$$

The same expression holds for the photon circular polarization in the capture of thermal neutrons by protons, $n + p \rightarrow d + \gamma$. Our numerical result (7) is consistent with previous calculations[2,9] of $P_\gamma$. Thus, as emphasized by Lee,[3] a measurement with low-energy photons provides a test of Lobashov's experimental result.[1]

$A_{\text{cir}}$ calculated with our $\alpha$, $\alpha'$, and $\beta$, with the coupling constants used in Ref. 3, shows the same behavior as Lee's calculation,[3] which was carried out up to 1 MeV above threshold, employing the Hulthén-Sugawara[4] wave function.

### B. $1 \leq \omega_L - E_B \leq 30$ MeV

In Fig. 3 we give the integrated cross section for $E1$ and $M1$ transitions and the values for $\alpha, \alpha', \dots, \delta$. An important feature of our result is that in the energy region $1 \leq \omega_L - E_B \leq 30$ MeV the $\Delta I = 1$ component in $A_{\text{cir}}$ can no longer be neglected. In fact, the weak pion-exchange term (proportional to $\delta$) becomes more and more dominant (for comparable couplings) as the photon energy increases. To see this more precisely, in Table I we give $\alpha, \alpha', \dots, \delta$ for several values of $\omega_L - E_B$.

To estimate $A_{\text{cir}}$, we need information on the strength of the parity-violating couplings

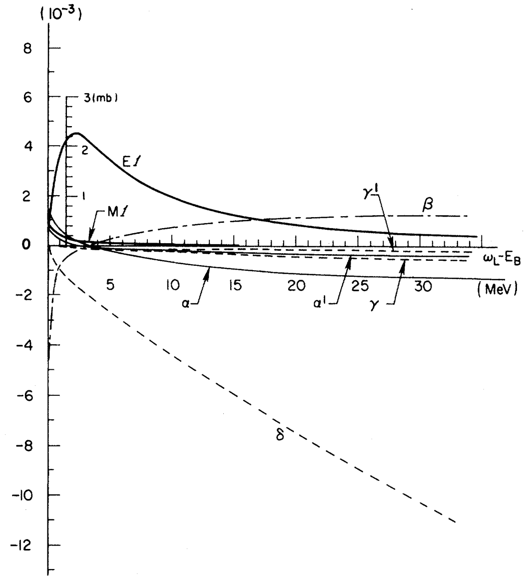


FIG. 3. The quantities $\alpha$, $\alpha'$, $\beta$, $\gamma$, $\gamma'$, and $\delta$ for $1 \le \omega_L - E_B \le 30$ MeV. See also figure caption for Fig. 2.

$h_\rho^0, \ldots, h_\rho^{1'}$ and $f_\pi$, as well as on the strong coupling constants $g_\rho$, $g_\omega$, and $g_\pi$. For $g_\rho$ we take $g_\rho{}^2/4\pi=0.50$. This value follows, assuming vector-meson dominance for the $NN\rho$ vertex, from the $\rho\rightarrow e^+e^-$ decay with $\Gamma_{\rho\rightarrow e^+e^-}=6.8\pm1.0$ keV.[10] For $g_\omega$ we shall take the value $g_\omega{}^2/4\pi=4.5$, obtained using SU(3) symmetry with ideal $\omega$-$\phi$ mixing. The pion-nucleon coupling constant is $g_\pi{}^2/4\pi=14.4$.[11] As mentioned earlier, the magnetic coupling $\chi_V$ is taken to be $\chi_V=\mu_p-\mu_n=3.71$, in accordance with the vector-dominance assumption. The value $\chi_S=\mu_p+\mu_n=-0.12$ used follows again from SU(3) symmetry with ideal $\omega$-$\phi$ mixing.

For the parity-violating constant $h_\rho^0, \ldots, f_\pi$ no firm prediction can be made at present, in view of the uncertainties involved in treating the effects of the strong interactions. To estimate $A_{\rm cir}$, we shall take the recommended values of Desplanques, Donoghue, and Holstein[6,12]:

$$h_\rho^0=-1.14\times10^{-6},\ h_\omega^0=-0.19\times10^{-6}\ ,$$
$$h_\rho^2=-0.95\times10^{-6},\ h_\rho^1=-0.02\times10^{-6}\ ,$$
$$h_\omega^1=-0.11\times10^{-6},\ h_\rho^{1'}=-0.07\times10^{-6}\ , \qquad (8)$$

and

$$f_\pi=0.46\times10^{-6}\ .$$

With the strong coupling constants we use, these give

$$g_\rho h_\rho^0=-2.86\times10^{-6}\ ,$$
$$g_\omega h_\omega^0=-1.43\times10^{-6}\ ,$$
$$g_\rho h_\rho^2=-2.38\times10^{-6}\ , \qquad (9)$$
$$g_\rho h_\rho^1-g_\omega h_\omega^1=0.78\times10^{-6}\ ,$$
$$g_\rho h_\rho^{1'}=-0.18\times10^{-6}\ ,$$

and

$$g_\pi f_\pi=6.19\times10^{-6}\ .$$

The parity-violating vector-meson and pion-exchange contributions to $A_{\rm cir}$ are given in Table II. As seen, the weak pion-exchange contribution dominates even at photon energies as low as $\omega_L-E_B\cong1$ MeV. This is in part due to the fact that for $1\le\omega_L-E_B\le30$ MeV there is a cancellation among the contributions of the vector-meson terms. In addition, for $10\le\omega_L-E_B\le30$ MeV the magnitude of the function $\delta$ is larger than the magnitudes of the other functions $\alpha,\alpha',\ldots\gamma'$.

Recently calculations of the parity-violating asymmetry in the differential cross section in the electrodisintegration of the deuteron by longitudinally polarized electrons were reported.[13,14] At the $q^2\rightarrow0$ limit this asymmetry is related to the asymmetry $A_{\rm cir}$ in $\vec{\gamma}+d\rightarrow n+p$ and when the energy is near the threshold of disintegration, both asymmetries are related to the $P_\gamma$ in $n+p\rightarrow d+\gamma$. Like us, Hwang, Henley, and Miller[13] find that as the energy of the incident electron increases the weak

TABLE I. Values of $\alpha,\alpha',\ldots,\delta$ (in units of $10^{-3}$) as functions of $\omega_L-E_B$.

| | 1 MeV | 10 MeV | 20 MeV | 30 MeV |
|---|---|---|---|---|
| $\alpha$ | 0.600 | −0.701 | −1.098 | −1.273 |
| $\alpha'$ | 0.197 | −0.220 | −0.343 | −0.395 |
| $\beta$ | −0.704 | 0.674 | 1.051 | 1.195 |
| $\gamma$ | −0.022 | −0.100 | −0.186 | −0.277 |
| $\gamma'$ | 0.046 | 0.215 | 0.407 | 0.615 |
| $\delta$ | −1.074 | −4.392 | −7.484 | −10.294 |

TABLE II. $A_{\rm cir}$ as a function of $\omega_L-E_B$, calculated with $g_\rho h_\rho^{1'}=-0.18\times10^{-6}$ (Ref. 12) and the recommended values of Desplanques, Donoghue, and Holstein (Ref. 6) for the remaining coupling constants. $A_{\rm cir}^V$ and $A_{\rm cir}^\pi$ denote the vector-meson and pion contributions, respectively. $A_{\rm cir}=A_{\rm cir}^V+A_{\rm cir}^\pi$. All values are quoted in units of $10^{-8}$.

| | 1 MeV | 10 MeV | 20 MeV | 30 MeV |
|---|---|---|---|---|
| $A_{\rm cir}^V$ | −0.04 | 0.06 | 0.09 | 0.10 |
| $A_{\rm cir}^\pi$ | −0.66 | −2.72 | −4.63 | −6.37 |
| $A_{\rm cir}$ | −0.70 | −2.66 | −4.54 | −6.27 |

pion-exchange term becomes more and more dominant. However as the kinematics of electrodisintegration and photodisintegration are different, a precise comparison of our results with theirs is difficult.

It is also interesting to give an upper limit for $A_{\rm cir}$ implied by the bounds on the parity-violating coupling constants deduced from the existing experimental data. The bound

$$-4.1\times10^{-6}<f_\pi<2.9\times10^{-6},\quad 90\%\ \text{C.L.} \tag{10}$$

obtained by Bowman, Gibson, Herczeg, and Henley[15] (from their fit $A$ in which data on parity violation in heavy nuclei and the experimental result on the photon circular polarization in $np\to d\gamma$ are excluded) implies

$$\begin{aligned} |A_{\rm cir}| &\le 0.6\times10^{-7} \ \text{for}\ \omega_L-E_B=1\ \text{MeV}\ ,\\ |A_{\rm cir}| &\le 2.4\times10^{-7} \ \text{for}\ \omega_L-E_B=10\ \text{MeV}\ ,\\ |A_{\rm cir}| &\le 5.7\times10^{-7} \ \text{for}\ \omega_L-E_B=30\ \text{MeV}\ , \end{aligned} \tag{11}$$

provided that the weak vector-meson couplings are not much larger than the limits (10) on $f_\pi$. To have $A^V_{\rm cir}$ comparable to $A^\pi_{\rm cir}$ would require $h_{\rho,\omega}/f_\pi\cong3$ for $\omega_L-E_B\cong1$ MeV, $h_{\rho,\omega}/f_\pi\cong12$ for $\omega_L-E_B\cong10$ MeV, and $h_{\rho,\omega}/f_\pi\cong15$ for $\omega_L-E_B\cong30$ MeV.

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## APPENDIX

In this appendix we give the equations for the parity-violating scattering and deuteron states. The scattering states $\tilde U(^1S_0)$, $\tilde U(^3P_0)$, $\tilde U(^1D_2)$, and $\tilde U(^3D_2)$ satisfy

$$\begin{aligned}(D_0+V_s)\tilde U(^1S_0)=8i\Bigg\{&g_\rho\left[h_\rho^0-\frac{2}{\sqrt6}h_\rho^2\right]\left[\chi_V\tilde V_{B\rho}+2\tilde V_{E\rho}\left[\frac{1}{\chi}+\frac{\vec\partial}{\partial\chi}\right]\right]\\ &+g_\omega h_\omega^0\left[\chi_S\tilde V_{B\omega}+2\tilde V_{E\omega}\left[\frac{1}{\chi}+\frac{\vec\partial}{\partial\chi}\right]\right]\Bigg\}U(^3P_0)\ ,\end{aligned} \tag{A1}$$

$$\begin{aligned}(D_1+V_s)\tilde U(^3P_0)=-8i\Bigg\{&g_\rho\left[h_\rho^0-\frac{2}{\sqrt6}h_\rho^2\right]\left[(2+\chi_V)\tilde V_{B\rho}+2\tilde V_{E\rho}\left[\frac{1}{\chi}-\frac{\vec\partial}{\partial\chi}\right]\right]\\ &+g_\omega h_\omega^0\left[(2+\chi_S)\tilde V_{B\omega}+2\tilde V_{E\omega}\left[\frac{1}{\chi}-\frac{\vec\partial}{\partial\chi}\right]\right]\Bigg\}U(^1S_0)\ ,\end{aligned} \tag{A2}$$

$$\begin{aligned}(D_2+V_s)\tilde U(^1D_2)=-8\frac{\sqrt2}{\sqrt5}i\Bigg\{&g_\rho\left[h_\rho^0-\frac{2}{\sqrt6}h_\rho^2\right]\left[\chi_V\tilde V_{B\rho}+2\tilde V_{E\rho}\left[-\frac{2}{\chi}+\frac{\vec\partial}{\partial\chi}\right]\right]\\ &+g_\omega h_\omega^0\left[\chi_S\tilde V_{B\omega}+2\tilde V_{E\omega}\left[-\frac{2}{\chi}+\frac{\vec\partial}{\partial\chi}\right]\right]\Bigg\}U(^3P_2)\\ +8\frac{\sqrt3}{\sqrt5}i\Bigg\{&g_\rho\left[h_\rho^0-\frac{2}{\sqrt6}h_\rho^2\right]\left[\chi_V\tilde V_{B\rho}+2\tilde V_{E\rho}\left[\frac{3}{\chi}+\frac{\vec\partial}{\partial\chi}\right]\right]\\ &+g_\omega h_\omega^0\left[\chi_S\tilde V_{B\omega}+2\tilde V_{E\omega}\left[\frac{3}{\chi}+\frac{\vec\partial}{\partial\chi}\right]\right]\Bigg\}U(^3F_2)\ ,\end{aligned} \tag{A3}$$

and

$$
\begin{aligned}
(D_2+V_s)\tilde{U}(^3D_2)=&-4\frac{\sqrt{6}}{\sqrt{5}}ig_\pi f_\pi \tilde{V}_\pi\left[U(^3P_2)+\frac{\sqrt{2}}{\sqrt{3}}U(^3F_2)\right]+8\frac{\sqrt{3}}{\sqrt{5}}ig_\rho h_\rho^{1'}\tilde{V}_{B\rho}\left[U(^3P_2)+\frac{\sqrt{2}}{\sqrt{3}}U(^3F_2)\right]\\
&+8\frac{\sqrt{3}}{\sqrt{5}}ig_\rho h_\rho^1\left\{\left[\tilde{V}_{B\rho}-2\tilde{V}_{E\rho}\left[-\frac{2}{\chi}+\frac{\vec{\partial}}{\partial\chi}\right]\right]U(^3P_2)+\frac{\sqrt{2}}{\sqrt{3}}\left[\tilde{V}_{B\rho}-2\tilde{V}_{E\rho}\left[\frac{3}{\chi}+\frac{\vec{\partial}}{\partial\chi}\right]\right]U(^3F_2)\right\}\\
&-8\frac{\sqrt{3}}{\sqrt{5}}ig_\omega h_\omega^1\left\{\left[\tilde{V}_{B\omega}-2\tilde{V}_{E\omega}\left[-\frac{2}{\chi}+\frac{\vec{\partial}}{\partial\chi}\right]\right]U(^3P_2)\right.\\
&\left.+\frac{\sqrt{2}}{\sqrt{3}}\left[\tilde{V}_{B\omega}-2\tilde{V}_{E\omega}\left[\frac{3}{\chi}+\frac{\vec{\partial}}{\partial\chi}\right]\right]U(^3F_2)\right\}. \qquad \text{(A4)}
\end{aligned}
$$

The equations for the deuteron state $\tilde{U}_d(^1P_1)$ and $\tilde{U}_d(^3P_1)$ are

$$
\begin{aligned}
(D_1+V_s)\tilde{U}_d(^1P_1)=&8\sqrt{3}i\left\{g_\rho h_\rho^0\left[\chi_V\tilde{V}_{B\rho}+2\tilde{V}_{E\rho}\left[-\frac{1}{\chi}+\frac{\vec{\partial}}{\partial\chi}\right]\right]\right.\\
&\left.-\frac{g_\omega h_\omega^0}{3}\left[\chi_S\tilde{V}_{B\omega}+2\tilde{V}_{E\omega}\left[-\frac{1}{\chi}+\frac{\vec{\partial}}{\chi}\right]\right]\right\}U_d(^3S_1)\\
&-8\sqrt{6}i\left\{g_\rho h_\rho^0\left[\chi_V\tilde{V}_{B\rho}+2\tilde{V}_{E\rho}\left[\frac{2}{\chi}+\frac{\vec{\partial}}{\partial\chi}\right]\right]-\frac{g_\omega h_\omega^0}{3}\left[\chi_S\tilde{V}_{B\omega}+2\tilde{V}_{E\omega}\left[\frac{2}{\chi}+\frac{\vec{\partial}}{\partial\chi}\right]\right]\right\}U_d(^3D_1) \qquad \text{(A5)}
\end{aligned}
$$

and

$$
\begin{aligned}
(D_1+V_s)\tilde{U}_d(^3P_1)=&\frac{8i}{\sqrt{3}}g_\pi f_\pi\tilde{V}_\pi[U_d(^3S_1)+\frac{1}{\sqrt{2}}U_d(^3D_1)]-8\frac{\sqrt{2}}{\sqrt{3}}ig_\rho h_\rho^{1'}\tilde{V}_{B\rho}\left[U_d(^3S_1)+\frac{1}{\sqrt{2}}U_d(^3D_1)\right]\\
&+8\frac{\sqrt{2}}{\sqrt{3}}ig_\rho h_\rho^1\left\{\left[\tilde{V}_{B\rho}-2\tilde{V}_{E\rho}\left[-\frac{1}{\chi}+\frac{\vec{\partial}}{\partial\chi}\right]\right]U_d(^3S_1)+\frac{1}{\sqrt{2}}\left[\tilde{V}_{B\rho}-2\tilde{V}_{E\rho}\left[\frac{2}{\chi}+\frac{\vec{\partial}}{\partial\chi}\right]\right]U_d(^3D_1)\right\}\\
&-8\frac{\sqrt{2}}{\sqrt{3}}ig_\omega h_\omega^1\left\{\left[\tilde{V}_{B\omega}-2\tilde{V}_{E\omega}\left[-\frac{1}{\chi}+\frac{\vec{\partial}}{\partial\chi}\right]\right]U_d(^3S_1)\right.\\
&\left.+\frac{1}{\sqrt{2}}\left[V_{B\omega}-2\tilde{V}_{E\omega}\left[\frac{2}{\chi}+\frac{\vec{\partial}}{\partial\chi}\right]\right]U_d(^3D_1)\right\}. \qquad \text{(A6)}
\end{aligned}
$$

In Eqs. (A1)–(A6)

$$\tilde{V}_{Bi}=\frac{m_\pi{}^2}{16\pi\chi^2}(1+\alpha_i\chi)e^{-\alpha_i\chi}\quad(i=\rho,\omega)\ ,$$

$$\tilde{V}_{Ei}=\frac{m_\pi{}^2}{16\pi\chi}e^{-\alpha_i\chi}\quad(i=\rho,\omega),\quad \tilde{V}_\pi=\frac{m_\pi{}^2}{16\pi\chi^2}(1+\chi)e^{-\chi}\ ,$$

where $\alpha_i=m_i/m_\pi$ and $\chi=m_\pi r$. $V_s$ denotes the strong-interaction potential and $D_l=d^2/dr^2+k^2-l(l+1)/r^2$ ($l\equiv$ orbital angular momentum.)

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