Analysis of multiplicities and correlations of charged particles produced in neutrino-nucleon interactions

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We study general features of hadron multiplicities observed in high-energy neutrino-induced reactions. We find that when multiplicity of pairs of oppositely charged particles is considered, the energy dependence of mean multiplicity $\langle n_{\rm pair} \rangle$ and the relation between the mean value and dispersion are very similar in νp, νn, $\bar{\nu} p$, and $\bar{\nu} n$ interactions. We analyze available data on forward-backward correlations and find good agreement with a simple hypothesis of binomial distribution of hadrons between forward and backward hemispheres. We compare data from νN interactions with those from e^+e^- annihilations and nondiffractive pp collisions.

I. INTRODUCTION

In terms of the quark-parton model[1] of deep-inelastic lN scattering the interaction between the incident lepton and the nucleon proceeds via the exchange of a virtual boson, γ for $e^{\pm},\mu^{\pm}$ and W^+ (W^-) for ν ($\bar{\nu}$) charged-current reactions. A quark in the target nucleon absorbs the intermediate boson and is separated from the spectator system of partons. Similarly, e^+e^- annihilation leads to two colored objects, a quark and an antiquark, moving apart. Since free quarks are not observed in the final state, it is important to understand how color-carrying partons convert into hadrons.

In the most widely used approach,[2] the highly energetic partons produced in the primary interaction are allowed to "fragment" independently into jets of hadrons. An alternative model,[3] considering production of showers of many partons, has recently been developed for e^+e^- annihilation. According to this model, the initial quarks, having in general high off-shell mass, radiate gluons, which may in turn also radiate, leading to a multiparton state. In the final stage, color-singlet subsystems are formed and subsequently "decay" uniformly in their center-of-mass frame, according to two-body phase space, or turn into single hadrons. In this approach, the distinction between the fragmentation products of the initial quarks is less clear.

In this paper we point out some features of charged-particle multiplicity distributions in high-energy neutrino-induced reactions which may throw light on the mechanism by which partons convert into hadrons.

In Sec. II we show that the data on the mean value and dispersion of charged-hadron multiplicity for νN and $\bar{\nu} N$ interactions are very similar, once the difference in the total charge of the hadronic state is properly accounted for.

Correlations between multiplicity distributions in the forward and backward hemispheres in the hadronic center-of-mass frame are discussed in Sec. III. We confront available νN data with the hypothesis that the division of hadrons between the forward and backward hemispheres is governed by the binomial distribution. We find that this assumption gives a qualitative fit to the data.

Throughout the paper we compare νN data with those from e^+e^- annihilations and nondiffractive pp collisions.

II. AVERAGE MULTIPLICITIES AND DISPERION

In order to compare data for processes with different values of the total charge of the hadronic system Q, we introduce the number of pairs of charged particles created in addition to the minimum number of charged hadrons in the final state, $n_{\rm min}$, required by charge conservation:

$$n_{\rm pair} = (n_{\rm ch} - n_{\rm min})/2 \ . \tag{1}$$

For reactions νp, νn, and $\bar{\nu} n$, $n_{\rm min} = |Q|$, that is, $n_{\rm min} = 2$ for νp and $n_{\rm min} = 1$ for $\nu(\bar{\nu})n$ interactions. For reasons which we shall explain later, in the case of $\bar{\nu} p$ interaction we set $n_{\rm min} = 1$, rather than $n_{\rm min} = |Q| = 0$.

We study the dependence of $\langle n_{\rm pair} \rangle$ on the available energy defined as

$$E_a = W - \sum M_{\rm in} \ , \tag{2}$$

where W is the center-of-mass energy of the hadron-

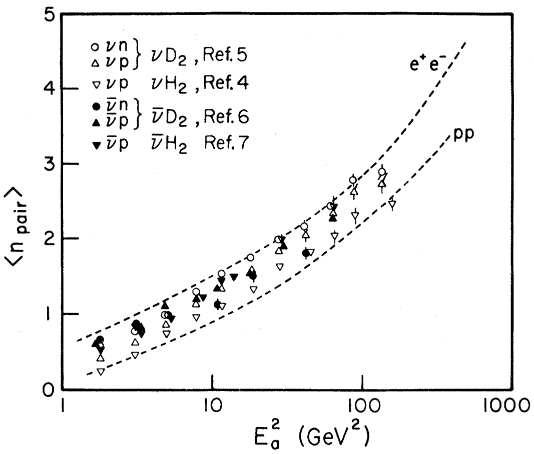

FIG. 1. Average multiplicity of pairs of oppositely charged particles in νN and $\bar{\nu}N$ scattering vs available energy squared $E_a{}^2$. (The $\bar{\nu}p$ data are lowered by 0.5 as explained in the text.)

ic system and $\sum M_{\text{in}}$ is the sum of masses of hadrons in the initial state. The results for reactions νp (Refs. 4 and 5), νn (Ref. 5), $\bar{\nu}p$ (Refs. 6 and 7), and $\bar{\nu}n$ (Ref. 7) are shown in Fig. 1. A clear similarity of mean multiplicities for all four reactions is observed, although the νp data[4] lie lower than others by approximately 0.3 units.

These results are well described by the quark-parton model,[2] as depicted in Fig. 2(a): in the $\nu(\bar{\nu})N$ collision the primary reaction leads to the separation of a quark from a remaining diquark system. The colored field between the quark and the diquark is discharged by creating $q\bar{q}$ pairs which combine to produce mesons. The minimum number of particles in the final state corresponds to the situation when only one $q\bar{q}$ pair is created. This is shown in Fig. 3 for all four νN and $\bar{\nu}N$ reactions. We see that in the cases νp, νn, and $\bar{\nu}n$ the minimum number of charged particles observed is independent of the flavor of the quark generated from the sea and equal to $n_{\min}=2$ for νp and $n_{\min}=1$ for $\nu(\bar{\nu})n$. In the case

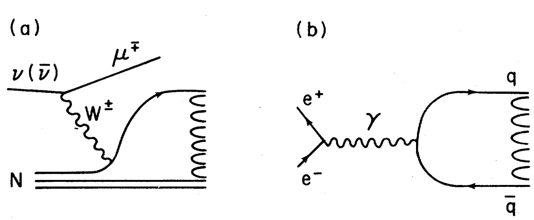

FIG. 2. Quark-parton-model diagrams for (a) $\nu(\bar{\nu})N \rightarrow \mu^{\mp}+$hadrons and (b) $e^+e^- \rightarrow$hadrons.

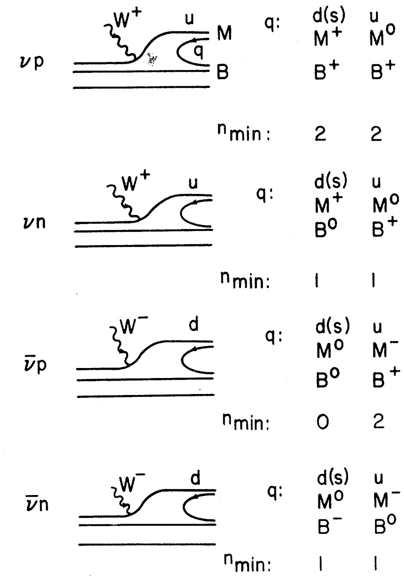

FIG. 3. Diagram of hadron production in νN and $\bar{\nu}N$ interactions with one $q\bar{q}$ loop. M and B denote a meson and a baryon containing the primary quarks. $n_{\min}$ denotes the minimum number of charged hadrons in the final state corresponding to different flavors of the sea quark.

of $\bar{\nu}p$ interaction we have $n_{\min}=0$ if $q=d$ (or s) and $n_{\min}=2$ if $q=u$. Assuming that the two possibilities are approximately equally probable we set $n_{\min}=1$. Figure 3 explains also why the observed average multiplicity $\langle n_{\text{pair}} \rangle$ is slightly lower in the νp case as compared with other reactions. The difference may be attributed to the possible decay of the neutral isobars B^0 into two charged particles in all cases except νp.

Also shown in Fig. 1 are results of the best fit to the data for nondiffractive pp collision[8] ($n_{\min}=2$), and for e^+e^- annihilation,[9] for which, as in the case of $\bar{\nu}p$, we assumed $n_{\min}=1$. We find the energy dependence of the average number of pairs of charged particles produced in all three types of interactions to be similar, suggesting that similar mechanisms lead to the formation of hadronic final states in those processes. Small differences, however, are observed. At the same available energy, the pp data lie lower than the νN data, which, in turn, are below the e^+e^- data. These differences may be tentatively attributed to the necessity to form baryons in the final states.

The dispersion of the distribution of multiplicity n is defined as

$$D \equiv (\langle n^2 \rangle - \langle n \rangle^2)^{1/2} . \quad (3)$$

Along with the dispersion, one can use the correlation parameter f_2:

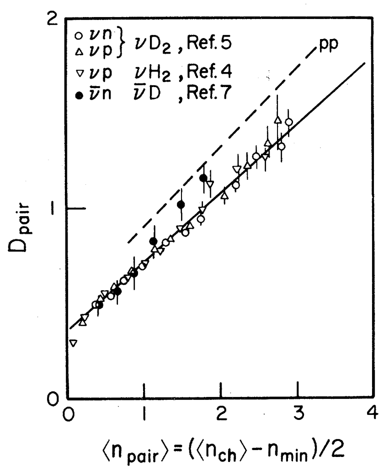

FIG. 4. Dispersion D_{pair} of pair multiplicity vs average pair multiplicity $\langle n_{\text{pair}} \rangle$ for νp (Refs. 4 and 5), νn (Ref. 5), and $\bar{\nu} n$ (Ref. 7).

$$f_2 \equiv \langle n(n-1) \rangle - \langle n \rangle^2 = D^2 - \langle n \rangle \ . \tag{4}$$

The parameter f_2 will be discussed in Sec. III.

The dispersion of n_{pair} distribution as a function of $\langle n_{\text{pair}} \rangle$ for νp (Refs. 4 and 5), νn (Ref. 5), and $\bar{\nu} n$ (Ref. 7) interactions is shown in Fig. 4. It can be seen that all the data agree with the relation

$$D_{\text{pair}} = 0.36 \langle n_{\text{pair}} \rangle + 0.36 \ . \tag{5}$$

(We do not include $\bar{\nu} p$ data since $n_{\min}$ is not unique, and hence D_{pair} is not well defined in this case. For the same reason, we do not make a comparison with e^+e^- data.)

For comparison, the result of the best fit to the nondiffractive pp data is also shown in Fig. 3. In the multiplicity range available, $\langle n_{\text{pair}} \rangle < 3$, the pp data are displaced from νN data by about 0.25 units. The additional fluctuation of the number of pairs of charged particles may be due to the presence of two "diquarks" in the pp case as opposed to only one in lN interactions.

III. FORWARD-BACKWARD CORRELATIONS

To discuss correlations between the number of particles emitted forward, n_F, and backward, n_B, in the hadronic center-of-mass system we shall use the quantity called covariance of n_B and n_F defined[10] as

$$\text{cov}(n_F, n_B) \equiv \langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle \tag{6}$$

and the coefficient of correlation, given by

$$\rho = \text{cov}(n_F, n_B)/(D_F D_B) \ , \tag{7}$$

where D_F (D_B) is the dispersion of the multiplicity distribution in the forward (backward) hemisphere.

As mentioned before, in the quark-parton model of νN interaction a d quark of the target nucleon absorbs the intermediate boson W^+ and is separated from the remaining diquark system. One usually associates the hadrons produced in the forward and backward hemispheres in the hadronic center-of-mass system with the "fragmentation" of the current quark and the spectator diquark, respectively.

If the quark and the diquark indeed fragment independently, the observed forward and backward multiplicities should be (asymptotically) independent, i.e., the probability $P(n, n_F)$, where $n = n_F + n_B$, should be given by

$$P(n, n_F) = P(n_F) P(n_B) \ . \tag{8}$$

Since at finite energies there is some overlap between the quark and the target fragmentation regions one can expect a linear dependence of the mean multiplicity in one hemisphere on the multiplicity in the opposite hemisphere:

$$\langle n \rangle_{F,B} = \alpha + \beta n_{B,F} \tag{9}$$

with the slope β positive and decreasing as energy W increases. In that case the covariance of n_B and n_F, Eq. (6), and the coefficient of correlation ρ, Eq. (7), should be positive, and ρ should decrease with $\langle n \rangle$.

Another extreme assumption one can make is that of a random distribution of particles between the forward and backward hemispheres. In this case, the probability $P(n, n_F)$ is given by the binomial distribution

$$P(n, n_F) = P(n) \binom{n}{n_F} f^{n_F} b^{n-n_F} \ , \tag{10}$$

where f (b) is the probability that in a given event a given particle is emitted forward (backward). Assuming the hypothesis (10) and using empirical information on the distribution $P(n)$ we may calculate distributions $P(n_F)$ and $P(n_B)$ and their moments, as well as the covariance and the coefficient of correlation defined in Eqs. (6) and (7). Some simple relations following from Eq. (10) are given in the Appendix. In particular, if the probability f in Eq. (10) remains constant with increasing $\langle n \rangle$ the covariance (6) is expected to vary with $\langle n \rangle$ in direct proportion to f_2 [see Eq. (A4)].

We have studied the implications of the assumption (10) for the forward-backward correlations for negative particles produced in νp (Refs. 4 and 5) and νn (Ref. 5) interactions. By considering negative particles only we avoid spurious correlations due to the conservation of the total charge, which may be important at low energies. We used the relation $D_-(\langle n_- \rangle)$ given in Eq. (4) (here, $n_- = n_{\text{pair}}$), and we took an empirical value of $f = \langle n_- \rangle_F / \langle n_- \rangle$.

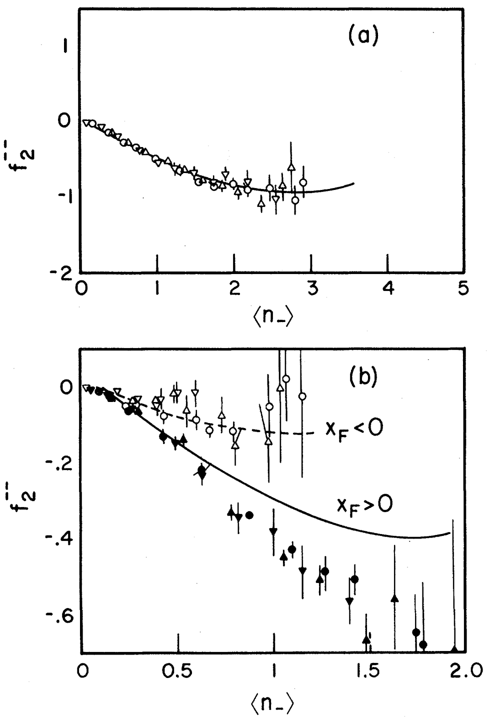

FIG. 5. Correlation parameter f_2^{--} vs average multiplicities of negative particles: (a) in νp (Refs. 4 and 5) and νn (Ref. 5) interactions and (b) in νp (Refs. 4 and 5) and νn (Ref. 5) interactions, for forward and backward c.m. hemispheres. The curve in (a) follows from the form $D_-=0.36\langle n_-\rangle+0.36$. The curves in (b) follow the above fit of $D_-(\langle n_-\rangle)$ and the assumption of the binomial distribution.

In the νp and νn interactions, the value of f is found to be approximately constant with $\langle n_-\rangle$ and equal to $f=0.6$.[4,5]

The dependence of the correlation parameter (7) for negative particles, f_2^{--}, on $\langle n_-\rangle$ is given in Fig. 5(a). The solid curve follows from the relation $f_2^{--}=D_-^{\,2}-\langle n_-\rangle$ and from Eq. (4). This curve is then used as an input to calculate $(f_2^{--})_F$ and $(f_2^{--})_B$ under the assumption of the binomial distribution, from Eqs. (A2). The results are presented in Fig. 5(b) as a solid and a broken curve, respectively. The calculated curves and the experimental data show a similar trend, although the observed difference between $(f_2^{--})_F$ and $(f_2^{--})_B$ is more pronounced. The covariance $\mathrm{cov}((n_-)_F,(n_-)_B)$ which is related to the correlation parameters by Eq. (A3), is shown as a function of $\langle n_-\rangle$ in Fig. 6. The calculated curve closely follows the data.

We conclude that the available data for νN interaction at $\langle n_-\rangle \lesssim 3$ are in a qualitative agreement with the assumption of the binomial distribution. It will be interesting to confront this assumption with data from νN scattering and other deep-inelastic processes, such as μp scattering, at higher average multiplicities.

In Fig. 7 we present the covariance of forward and backward multiplicities for all charged particles. A similar pattern of the covariance changing sign from negative to positive is expected under the assumption of binomial distribution, but the detailed form of the relation of $\mathrm{cov}((n_{ch})_F,(n_{ch})_B)$ vs $\langle n_{ch}\rangle$ depends on the total charge of the hadronic state. The two curves in Fig. 7 are given for $Q=2$, as in νp scattering and for $|Q|=1$, as in νp, $\bar{\nu} n$, and $l^{\pm}p$ interactions. In the case of νp scattering, the relation $D=0.36\langle n_{ch}\rangle$, which is equivalent to Eq. (5), and Eqs. (A2) and (A5) imply that the change of sign should occur near $\langle n_{ch}\rangle=7$. The dependence of the mean multiplicity of charged particles in one hemisphere on the multiplicity in the opposite hemi-

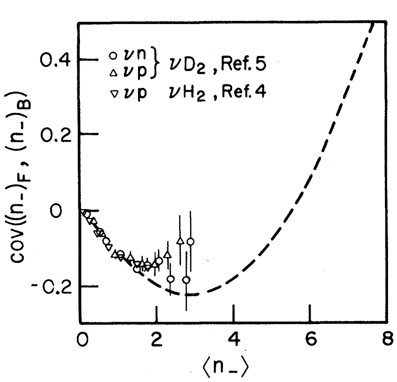

FIG. 6. Covariance of the multiplicity of negative particles in the forward and backward hemispheres vs $\langle n_-\rangle$ in νp (Refs. 4 and 5) and νn (Ref. 5) scattering. The dashed curve follows from the fit $D_-=0.36\langle n_-\rangle+0.36$ and the assumption of the binomial distribution.

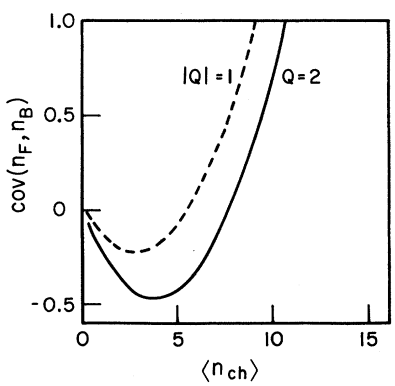

FIG. 7. Covariance of the multiplicity of charged particles in the forward and backward hemispheres as a function of $\langle n_{ch}\rangle$, calculated for lN interactions under the assumption of the binomial distribution, and the relation (5). The solid line applies to the case $Q=2$, as in νp scattering the dashed line to the $|Q|=1$ case, as in νn and $l^{\pm}p$.

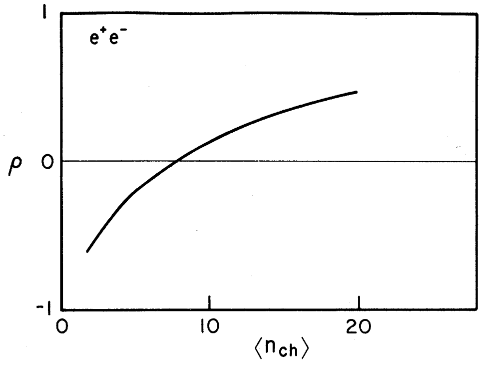

FIG. 8. Coefficient of forward-backward correlation ρ, for e^+e^- annihilation, calculated under the assumption of the binomial distribution and the relation $D=0.36\langle n_{ch}\rangle$.

sphere has been studied by Allen *et al.*,[11] for three bins of energy W. Indeed, it has been found that $\langle n\rangle_F$ is decreasing with n_B (the same for $F\leftrightarrow B$) for small W, and it starts rising with n_B at $W=8$ GeV, where $\langle n_{ch}\rangle\approx 6$.

The hypothesis of binomial distribution may also be tested in other types of interactions. In the case of e^+e^- and pp interactions $f=b=\frac{1}{2}$ and we obtain a simple relation between the coefficient of correlation, Eq. (7), and dispersion and mean value of n_{ch}:

$$\rho=\frac{D^2-\langle n_{ch}\rangle}{D^2+\langle n_{ch}\rangle}\ . \tag{11}$$

For e^+e^- annihilation we assume the relation $D=0.36\langle n_{ch}\rangle$ which gives a good fit to the data up to $\langle n_{ch}\rangle=11$. The resulting dependence $\rho(\langle n_{ch}\rangle)$ is given in Fig. 8. No data on the forward-backward multiplicity correlations are available at present. (Preliminary results obtained by PLUTO collaboration[12] show a trend similar to that presented in Fig. 8.)

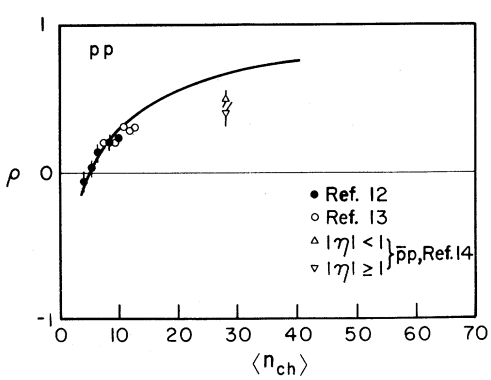

FIG. 9. Coefficient of forward-backward correlation ρ for nondiffractive pp interactions. The solid curve was calculated under the assumption of the binomial distribution and the relation (Ref. 8) $D=0.416\langle n_{ch}\rangle+0.063$.

In the case of nondiffractive pp interaction we used the relation[8] $D=0.416\langle n_{ch}\rangle+0.063$ which gives the best linear fit to the data for $\langle n_{ch}\rangle$ between 4 and 10. Assuming that the same relation holds for higher multiplicities and inserting it in Eq. (11), we obtain the dependence $\rho(\langle n_{ch}\rangle)$ which is presented as a solid line in Fig. 9, along with experimental data from Fermilab,[13] from CERN ISR,[14] and from a recent study of $\bar{p}p$ collisions at the CERN SPS collider.[15] It can be seen that in the case of pp interactions the hypothesis of binomial distribution correctly reproduces the observed data on forward-backward multiplicity correlations over a wide energy region.

IV. CONCLUSIONS

Data on the average values and dispersion of the multiplicity of charged particle pairs in the hadronic final state show a clear similarity for νn and $\bar{\nu}N$ charged-current interactions. Comparison of the behavior of $\langle n_{\text{pair}}\rangle$ as a function of the available energy E_a for νN, e^+e^- annihilation, and nondiffractive pp interactions indicates that hadron-production processes in all those types of reactions are similar.

Correlations between the number of particles produced in the forward and backward hemispheres in νN interactions at c.m. energies below 15 GeV have been found in qualitative agreement with a simple hypothesis of binomial distribution. Study of forward-backward correlations in deep-inelastic lN scattering at higher energies, as well as in e^+e^- annihilation, should determine the nature of this correlation. In particular, it is interesting to see whether the covariance continues to rise in direct proportion to the correlation parameter f_2, or whether the correlation observed at modest values of energy is a consequence of energy-momentum conservation and disappears when this restriction becomes less important.

In the case of pp and $\bar{p}p$ collisions, where the forward-backward correlation data up to $\sqrt{s}=540$ GeV are available, the hypothesis of binomial distribution gives a qualitative fit over a wide range of energy.

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APPENDIX

Below we give some relations between moments of forward-backward and overall multiplicity distributions which follow from the assumption of the binomial distribution (10).

If the overall dependence of dispersion on the mean multiplicity is given by a function $D(\langle n \rangle)$ then for the forward hemisphere we have

$$D_F{}^2 = f^2 D^2 + fb\langle n \rangle \tag{A1}$$

(for $D_B{}^2$ replace f by b). f (b) is the probability for a given particle to be emitted forward (backward), $f+b=1$. A particularly simple relation holds between the correlation parameter f_2, Eq. (4), and the corresponding parameters for the forward- and backward-going particles:

$$F_{2,F} = f^2 f_2, \quad f_{2,B} = b^2 f_2 \; . \tag{A2}$$

For the covariance $\mathrm{cov}(n_B, n_F)$, defined in Eq. (6), which can be expressed in terms of the correlation parameters as

$$\mathrm{cov}(n_B, n_F) = \tfrac{1}{2}(f_2 - f_{2,F} - f_{2,B}) \; , \tag{A3}$$

we obtain

$$\mathrm{cov}(n_B, n_F) = \tfrac{1}{2}(1 - f^2 - b^2) f_2 = fbf_2 \; . \tag{A4}$$

Hence, the sign of covariance is the same as the sign of the function f_2. In particular, for distributions which are narrower (broader) than the Poisson distribution (for which $f_2 \equiv 0$) $\mathrm{cov}(n_B, n_F)$ is negative (positive).

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