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Comment on the nature of point sources in Yang-Mills theories

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Using transport it is argued that for a point source in classical Yang-Mills theories one must require that certain components of $\vec{A}_a$ be less singular than $1/r$ at the source. For time-independent solutions the requirement is that the Coulomb solution dominate near the source.

In a very nice paper Sikivie and Weiss[1] have shown that under the constraint of Gauss's law the lowest-energy classical Yang-Mills solution for any static source must be time independent. They then use this theorem to show that a static point source will always have time-independent solutions with energy lower than the Coulomb energy. Their only requirement for these point-source solutions is that the charge density be a δ function.

We will argue here for a more restrictive definition of a static point source. Deciding what is the best definition of a classical point source may be important in the second-quantized version of the theory. For example, in a second-quantized theory for charges and magnetic monopoles, Brandt, Neri, and Zwanziger[2] have shown that important point-particle properties carry over even though one represents charges and monopoles with current densities such as $\bar{\psi}\gamma^\mu\psi$. Using general path-integral techniques they find that classical point-particle trajectories play a dominant role.

Our definition for a static point source is physically quite simple: We require that all parts of the source point in the same internal direction in a gauge-independent way, e.g., in an SU(2) theory there is a single well-defined isovector associated with the point source. To clarify, suppose we consider a point source to be a small extended source in the limit of its size shrinking to zero. We pick some point in the source and by transport compare the charge at other points in the source with that at our comparison point. We have argued in favor of such gauge-independent comparisons earlier.[3] In the limit of the source size shrinking to zero we require all parts of the source to point in the same internal direction.

Let us consider the consequences of this requirement. From the equation of continuity we get the condition that the static source, ρ_a, and A_a^0 be "parallel," i.e.,

$$c_{abc}A_b^0\rho_c \equiv 0 \quad , \tag{1}$$

where c_{abc} are the structure constants of the theory. In addition, we will pick a gauge where all parts of the source "line up," i.e.,

$$\hat{\rho}_a(\vec{x}) = \hat{\rho}_a(\vec{x}') \quad , \tag{2a}$$

where

$$\hat{\rho}_a = \frac{\rho_a}{\sqrt{\rho_a\rho_a}} \quad . \tag{2b}$$

It is important to point out that Eqs. (2) do not satisfy our requirement for all parts of the source to point in the same internal direction. If we transport $\rho_a(\vec{x}')$ from $\vec{x}'$ to $\vec{x}$, (2a) will not necessarily still be true. Indeed we know from the equation of continuity that if we transport a test charge q_a then the change in q_a is given by

$$dq_a = gc_{abc}(-A_b^0 dt + \vec{A}_b\cdot d\vec{x})q_c \quad , \tag{3a}$$

with g the coupling constant. If q_a is a piece of the source then (1) tells us that

$$dq_a = gc_{abc}(\vec{A}_b\cdot d\vec{x})q_c \quad . \tag{3b}$$

Because of our choice of gauge, the requirement that all parts of a point source point in the same internal direction is satisfied by requiring the dq_a of (3b) go to zero as the size of our small extended source shrinks to zero, i.e.,

$$c_{abc}(\vec{A}_b\cdot d\vec{x})q_c \to 0 \quad . \tag{4}$$

Now $d\vec{x}$ is a displacement inside the source. As the source shrinks, $d\vec{x}$ must also. We can take $d\vec{x} \propto r$, with r the source size. Thus we conclude from (4) that the "nonparallel" part of A_a must be less singular than $1/r$ as the source shrinks.

This requirement is more restrictive than that made by Sikivie and Weiss. Indeed, the solution they present, which is time dependent, has a δ-function source but $\vec{A}_a$ becomes more and more singular at the source as time goes on, violating condition (4).

For time-independent solutions all components of $\vec{A}_a$ will be less singular than $1/r$. This is because there is no three-current in the source, and because of the above restriction of the nonparallel components of $\vec{A}_a$. The field equations then lead to all other components being less singular. It follows from Gauss's law that A_a^0 should be dominated by the Coulomb solution close to the source.

We claim then that exact time-independent solutions for static point sources should be dominated by the Coulomb solution near the source. Without this requirement, even though one has a δ-function source in a particular gauge, the source may actually be very complicated in internal structure with different parts pointing relatively in quite different internal directions. This will be reflected in very singular $\vec{A}_b$'s.

We have used this restricted definition elsewhere[4] to show that there can be no exact time-independent solutions with energy lower than the Coulomb energy for a static point source below the Mandula[5] threshold.

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