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Dynamical breaking of gauge symmetry in Abelian gauge theories in two dimensions

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Two-dimensional Abelian gauge theories with nonstandard fermion content are analyzed. One model contains N fermions with different electric charges and the other has a left-right-asymmetric fermion content. The operator solutions are explicitly constructed in a covariant gauge and a noncovariant gauge. In both models the gauge and the chiral symmetries are broken spontaneously. The direction of this breakdown is dictated by the axial anomaly. The spectrum of the minimal theory consists of one massive boson and one massless boson for both types of theories. It is argued that the left-right-asymmetric model is inappropriate to test the most-attractive-channel hypothesis.

I. INTRODUCTION

The conventional method of generating spontaneous symmetry breaking in quantum field theories is to introduce the requisite number of elementary scalar fields which develop nonvanishing vacuum expectation values. This is, however, possible only at the price of having many additional parameters in the resulting theory. Furthermore it was stressed recently that this mechanism, though it works well, is unnatural[1,2]; it must involve a very accurate fine tuning of the parameters in the context of grand unified theories.

One way to overcome this unnaturalness is to utilize the dynamical symmetry breaking.[1,3] In quantum chromodynamics (QCD) embedded into grand unified theories we naturally expect that the mass scale characterizing the chiral-symmetry breaking can be vastly smaller ($\sim 10^{-15}$) than the grand-unifying mass scale. It is therefore of great interest to study the dynamical breaking of gauge symmetry in the context of grand unified theories. A very interesting scenario called tumbling was proposed by Raby, Dimopoulos, and Susskind[4] in which a hierarchy of mass scales can arise because of the sequential breaking of gauge symmetry.

Another aspect of the dynamical gauge-symmetry breaking concerns its possible relevance to the light-composite-fermion problem.[5] There exists rather strong feeling that in unbroken gauge theory like QCD the chiral symmetry is realized in the Nambu-Goldstone mode. If this is true and if we want massless fermions it might be necessary to break gauge symmetry to achieve the symmetric realization of chiral symmetry.[6]

In this paper we examine some two-dimensional Abelian gauge theories as models of gauge-symmetry breaking. They are the variants of the Schwinger model[7] with more complex fermion content; the multifermion model with fermions belonging to different U(1) representations, and the left-right-asymmetric model with nonreal fermion content.

The two-dimensional Abelian gauge theory is a natural place to investigate dynamical gauge-symmetry breaking. Lowenstein and Swieca[8] observed in their thorough analysis of the Schwinger model that the gauge symmetry is broken in the "physical" gauge although it may be masked by spurious gauge excitations in other gauges. It is one of the purposes of our work to examine the stability of this phenomenon under the variation of the fermion content of the theory.

Another purpose of our work is to find regularity (if any) in the dynamical gauge-symmetry breaking. It will be best if our study in two dimensions provides any hint to understand the breaking pattern of the gauge-symmetry breaking in four dimensions. Of course our theories, being superrenormalizable, cannot be directly used to test the tumbling scenario. Nevertheless we believe that any solid information can be important for our understanding of gauge-symmetry breaking at the present stage of the subject. Moreover, in view of the difficulty in formulating chiral fermions on the lattice,[9] the two-dimensional gauge theory seems to be the unique place to study the nonperturbative aspects of gauge theory with nonreal fermion content.

We emply a method similar to that of Ref. 8 combined with that of Belvedere, Swieca, Rothe, and Schroer[10] to investigate our model. We first construct the operator solution in a covariant gauge. Transforming into a noncovariant gauge called a "physical" gauge we can obtain a physical interpretation of the theory. In this gauge all the Heisenberg operators can be expressed solely in terms of the asymptotic fields. Thanks to this feature of the physical gauge, we will be able to have a clear understanding of the structure of the physical vacuum.

In Sec. II we discuss the multifermion model following the procedure described above. In Sec. III we introduce the left-right-asymmetric model. After reviewing the general property of the theory resulting from the axial anomaly we construct the operator solution. We discuss in Sec. IV the properties of the physical vacuum of both models. Our main concern is the broken-symmetry aspect of the models. The last section is devoted to conclusions and discussions.

Throughout this paper the γ-matrix convention

$$\gamma^0=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\ \gamma^1=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix},$$

$$\gamma_5=\gamma^0\gamma^1=\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}$$

is adopted. An extensive use is made of the identity $\gamma^\mu\gamma_5=\epsilon^{\mu\nu}\gamma_\nu$, where $\epsilon^{10}=\epsilon_{01}=1$.

II. MULTIFERMION MODEL

The multifermion Schwinger model is massless quantum electrodynamics in two dimensions with an arbitrary number of fermions belonging to different U(1) representations, i.e., with different electric charges.[11] The Lagrangian density of the N-fermion model is given by

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\sum_{i=1}^{N}\bar{\psi}_i i\gamma_\mu(\partial^\mu+ig_iA^\mu)\psi_i\ . \tag{1}$$

The equations of motion are

$$i\gamma_\mu\partial^\mu\psi_i(x)=g_i\gamma_\mu A^\mu(x)\psi_i(x)\quad(i=1,2,\ldots,N)\ , \tag{2}$$

$$\partial_\nu F^{\nu\mu}(x)=\sum_{i=1}^{N}g_i\bar{\psi}_i(x)\gamma^\mu\psi_i(x)\ . \tag{3}$$

In this paper we investigate the two-fermion model in greater detail because all the qualitative features of the multifermion model already exist in it. The generalization to the N-fermion model will be briefly discussed in Sec. I C.

A. Operator solution of the two-fermion model

We propose the following form of the operator solution[12]:

$$\psi_1(x)=\exp\{-i\sqrt{\pi}\gamma_5 a[\phi(x)-\tilde{\phi}(x)]\}\chi_1(x)\ , \tag{4a}$$

$$\psi_2(x)=\exp\{-i\sqrt{\pi}(g_2/g_1)\gamma_5 a[\phi(x)-\tilde{\phi}(x)]\}\chi_2(x)\ , \tag{4b}$$

where ϕ ($\tilde{\phi}$) denotes a free boson field quantized with definite (indefinite) metric and χ_i $(i=1,2)$ denote free fermion fields. The negative-metric field $\tilde{\phi}$ is taken to be massless since it will eventually be used to cancel out the free-fermion singularity of χ's. The combination ϕ-$\tilde{\phi}$ guarantees that in short distances ψ_i develops a free-fermion singularity, which is required by the superrenormalizability of the theory. The parameter a will be determined later.

Strictly speaking we define our free "fermion" operator χ_i in terms of free boson operators as[13]

$$\chi_i(x)=\left[\frac{\mu}{2\pi}\right]^{1/2}e^{-i\pi\gamma_5/4}\xi N_\mu\exp\left[i\sqrt{\pi}\left[\gamma_5\varphi_i(x)+\int_x^\infty d\xi\dot{\varphi}_i(\xi)\right]\right]u\quad(i=1,2)\ , \tag{5}$$

where N_μ implies to take normal ordering with respect to mass μ and $\xi=e^{i\pi N_1}$ with the number operator N_1 of χ_1 denoting the Klein transformation operator which ensures the anticommutativity of χ_1 and χ_2. $\varphi_i (i=1,2)$ stand for free massless boson fields and u is a constant spinor $\binom{1}{1}$. The operator χ_i defined by (5) is essentially the free fermion operator. By taking proper care of renormal ordering,[14] one can show that the right-hand side of Eq. (5) does carry the charge and the chirality of the original "fermions."

It should be noticed, however, that our fermions themselves do not have the constant operators found in Ref. 8 which lead to the θ vacuums irrespective of the presence of a gauge field. In our case their existence would lead to the θ vacuum involving four θ parameters. Without these constant operators we can successfully construct the operator solution as will be seen later.[15] Notice, however, that this does not mean that our theory has no θ vacuum. Its emergence is due to the presence of the indefinite metric as will be shown later.

From (2) the electromagnetic field A_μ takes the form

$$A^{\mu}(x)=\frac{\sqrt{\pi}}{g_1}\epsilon^{\mu\nu}\partial_\nu a[\phi(x)-\tilde{\phi}(x)] \; . \tag{6}$$

Using Schwinger's definition of the gauge-invariant current

$$j^{\mu}_{L,R}(x)=\lim_{\epsilon\to 0}\frac{1}{2}\left[\bar{\psi}_{L,R}(x+\epsilon)\gamma^{\mu}\psi_{L,R}(x)\exp\left[-ig\int_x^{x+\epsilon}d\xi_\mu A^{\mu}(\xi)\right] + \bar{\psi}_{L,R}(x-\epsilon)\gamma^{\mu}\psi_{L,R}(x)\exp\left[ig\int_{x-\epsilon}^{x}d\xi_\mu A^{\mu}(\xi)\right]\right] , \tag{7}$$

we obtain the expression of the electromagnetic current in a similar way as in Ref. 8:

$$J^{\mu}(x)=\frac{1}{\sqrt{\pi}}\left[\frac{g_1{}^2+g_2{}^2}{g_1}\right]\epsilon^{\mu\nu}\partial_\nu a[\phi(x)-\tilde{\phi}(x)] - \frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_\nu\left[\sum_j g_j\varphi_j(x)\right] . \tag{8}$$

Thus we encounter the same situations as the original Schwinger model in which the Maxwell equations are not satisfied as operator equations.

We set up the physical-state condition

$$\left[\epsilon^{\mu\nu}\partial_\nu\left[\sum_j g_j\varphi_j(x) + (g_1{}^2+g_2{}^2)^{1/2}\tilde{\phi}(x)\right]\right]_+ |\,\text{phys}\rangle=0 , \tag{9}$$

where $[\;]_+$ indicates to take the positive-frequency part of the operator in the square brackets. The parameter a is determined as

$$a=\frac{g_1}{(g_1{}^2+g_2{}^2)^{1/2}}\equiv\bar{g}_1 \tag{10}$$

so that the Maxwell equations are valid in the physical subspace. Then the equation turns out to be

$$\left[\Box+\frac{g_1{}^2+g_2{}^2}{\pi}\right]\phi(x)=0 \tag{11}$$

which indicates that $\phi(x)$ describes the massive free boson as in the original Schwinger model.

We thus obtain the operator solution in the Lorentz gauge [with $\bar{g}_i=g_i/(g_1{}^2+g_2{}^2)^{1/2}$]

$$\psi_i(x)=\exp\{-i\sqrt{\pi}\gamma_5\bar{g}_i[\phi(x)-\tilde{\phi}(x)]\chi_i(x) \; . \tag{12}$$

All the n-point functions can be calculated straightforwardly by using this operator solution. For instance we reproduce the form of scalar density correlation function given by Segrè and Weisberger.[11]

B. Physical gauge

The operator solution (12) does not mean the goal of our investigation. We still have important unsolved questions like the complete spectrum of the theory and the precise properties of the physical vacuum including the possibility of broken symmetries. The clearest way to answer these questions is to express the operator solution solely in terms of the asymptotic fields. In our theory one has to choose a suitable gauge to accomplish this.

One of the most transparent ways to carry out this program is to follow the method of Belvedere, Swieca, Rothe, and Schroer.[10] We note the Bose form of the free fermion operators (5). To obtain the physical interpretation of the theory we make a gauge transformation so that the fermion fields take the following form:

$$\psi_i(x)=\left[\frac{\mu}{2\pi}\right]^{1/2} e^{-i\pi\gamma_5/4}\xi e^{-i\sqrt{\pi}\gamma_5\bar{g}_i\phi(x)}N_\mu\exp\left\{i\sqrt{\pi}\left[\gamma_5\Phi_i(x)+\int_x^{\infty}d\xi\dot{\Phi}_i(\xi)\right]\right\}u \; . \tag{13}$$

It is easy to realize that such a gauge transformation is unique and Φ's are given by

$$\Phi_i(x)=\varphi_i(x)+\bar{g}_i\tilde{\phi}(x) \;\; (i=1,2) \; . \tag{14}$$

The remaining task is to find the operators Ψ and $\tilde{\Psi}$ which have the properties

$$[\dot{\Psi}(x),\Psi(y)]_{x_0=y_0}=-i\delta(x-y) , \tag{15a}$$

$$[\dot{\tilde{\Psi}}(x),\tilde{\Psi}(y)]_{x_0=y_0}=0 , \tag{15b}$$

$$[\dot{\Psi}(x),\tilde{\Psi}(y)]_{x_0=y_0}=[\dot{\tilde{\Psi}}(x),\Psi(y)]_{x_0=y_0} =0 , \tag{15c}$$

and to express Φ's in terms of them. Obviously Ψ describes the physical excitation and (15b) and (15c) enable us to set up the subsidiary condition $[\epsilon^{\mu\nu}\partial_\nu\tilde{\Psi}]_+\,|\,\text{phys}\rangle=0$. It is not difficult to recognize that the only possible representation consists of one Ψ and one $\tilde{\Psi}$. The Φ's in (14) can be expressed in terms of them as

$$\Phi_1(x)=-\bar{g}_2\Psi(x)+C_1\tilde{\Psi}(x)\ , \tag{16a}$$

$$\Phi_2(x)=\bar{g}_1\Psi(x)+C_2\tilde{\Psi}(x)\ , \tag{16b}$$

where C's are undetermined constants. The physical-state conditions (9) now take the form $[\epsilon^{\mu\nu}\partial_\nu\tilde{\Psi}]_+\,|\,\text{phys}\rangle=0$.

We have obtained the operator solution in the physical gauge,

$$\psi_1(x)=\left[\frac{\mu}{2\pi}\right]^{1/2} e^{-i\pi\gamma_5/4}\xi e^{-i\sqrt{\pi}\gamma_5\bar{g}_1\phi(x)}N_\mu\exp\left\{-i\sqrt{\pi}\bar{g}_2\left[\gamma_5\Psi(x)+\int_x^\infty d\xi\,\dot{\Psi}(\xi)\right]\right\}$$
$$\times\exp\left\{i\sqrt{\pi}C_1\left[\gamma_5\tilde{\Psi}(x)+\int_x^\infty d\xi\,\dot{\tilde{\Psi}}(\xi)\right]\right\}u\ , \tag{17a}$$

$$\psi_2(x)=\left[\frac{\mu}{2\pi}\right]^{1/2} e^{-i\sqrt{\pi}\gamma_5/4}\xi e^{-i\sqrt{\pi}\gamma_5\bar{g}_2\phi(x)}N_\mu\exp\left\{i\sqrt{\pi}\bar{g}_1\left[\gamma_5\Psi(x)+\int_x^\infty d\xi\,\dot{\Psi}(\xi)\right]\right\}$$
$$\times\exp\left\{i\sqrt{\pi}C_2\left[\gamma_5\tilde{\Psi}(x)+\int_x^\infty d\xi\,\dot{\tilde{\Psi}}(\xi)\right]\right\}u\ . \tag{17b}$$

The spectrum of this theory contains a free massive boson ϕ (with mass $[(g_1{}^2+g_2{}^2)/\pi]^{1/2}$) and a free massless boson Ψ. The last exponentials in (17a) and (17b) are constant operators which allow immediate interpretation to form the θ vacuum as in Ref. 10.

The last factor in (17) can be expressed as $(\sigma)^{\text{const}}$ in terms of σ defined by

$$\sigma=\exp\left[i\sqrt{\pi}\left\{\gamma_j\left[\sum_j\bar{g}_j\varphi_j(x)+\tilde{\phi}(x)\right]+\int_x^\infty d\xi\left[\sum_j\bar{g}_j\dot{\varphi}_j(\xi)+\dot{\tilde{\phi}}(\xi)\right]\right\}\right]=\begin{bmatrix}\sigma_1 & 0\\ 0 & \sigma_2\end{bmatrix}. \tag{18}$$

The operator σ_α $(\alpha=1,2)$ commutes with all the observables of the theory. Therefore it is a constant operator in the physical subspace which merely carries the bare charge and the chiral selection rule:

$$[q,\sigma_\alpha]=\sigma_\alpha\ , \tag{19a}$$

$$[q_5,\sigma_\alpha]=-(\gamma_5)_{\alpha\alpha}\sigma_\alpha\ , \tag{19b}$$

where

$$q=\frac{-1}{\sqrt{\pi}}\int dx\,\partial^1\sum_j\bar{g}_j\varphi_j$$

and q_5 is the normalized axial charge $q_5=(g_1{}^2+g_2{}^2)^{-1/2}Q_5$. We are taking the axial charge Q_5 associated with the conserved and gauge-invariant axial-vector current. It is well known that the existence of such operators implies the infinitely degenerate vacuums

$$|\,n_1n_2\rangle=(\sigma_1)^{n_1}(\sigma_2)^{n_2}\,|\,0\rangle \tag{20}$$

and the violation of clustering. We have to form the θ vacuum

$$|\,\theta_1,\theta_2\rangle=\frac{1}{2\pi}\sum_{n_1,n_2}e^{i\theta_1n_1}e^{i\theta_2n_2}\,|\,n_1,n_2\rangle \tag{21}$$

in order to recover the cluster property. Then

$$\sigma_\alpha\,|\,\theta_1,\theta_2\rangle=e^{-i\theta\alpha}\,|\,\theta_1,\theta_2\rangle\ . \tag{22}$$

One may feel the emergence of the θ vacuum strange since we did not introduce the constant operators of Lowenstein-Swieca type. Our θ vacuum arises for different reasons; it arises because of the presence of an indefinite metric associated with the gauge field. Our θ vacuum, having only two θ parameters independent of the complexity of fermion content, may allow the physical interpretation as the background electric field.[16]

C. N-fermion model

It is straightforward to extend our analysis to the case with an arbitrary number of fermions. We only describe our main results. The spectrum of the N-fermion model includes one free massive boson with mass $(\Sigma_i g_i{}^2/\pi)^{1/2}$ and $N-1$ free massless bosons. This fact can also easily be seen by using the boson representation in the Coulomb gauge as done in Ref. 17. We have the θ vacuum with only two θ parameters in complete parallelism with the two-fermion model.

III. LEFT-RIGHT-ASYMMETRIC SCHWINGER MODEL

For simplicity and clarity, we consider the Schwinger model with one left- and two right-handed fermions each of which belongs to a different U(1) representation. The Lagrangian of the system has the form

$$\mathscr{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_L i\gamma_\mu(\partial^\mu + igA^\mu)\psi_L + \sum_{i=1,2} \bar{\psi}_{Ri} i\gamma_\mu(\partial^\mu + ig_i A^\mu)\psi_{Ri} \ . \tag{23}$$

For brevity we shall refer to A_μ as the "electromagnetic" field and use the term "photon" instead of "weak boson." The equations of motion are

$$i\gamma_\mu\partial^\mu\psi_L = g\gamma_\mu A^\mu\psi_L \ , \tag{24}$$

$$i\gamma_\mu\partial^\mu\psi_{Ri} = g_i\gamma_\mu A^\mu\psi_{Ri} \quad (i=1,2) \ , \tag{25}$$

$$\partial_\nu F^{\nu\mu} = g\bar{\psi}_L\gamma^\mu\psi_L + \sum_{i=1,2} g_i\bar{\psi}_{Ri}\gamma^\mu\psi_{Ri} \ . \tag{26}$$

As we will soon see, this model is the minimal one among a variety of the possible left-right-asymmetric models. In this paper we concentrate our attention on this minimal model although the extension to more complicated cases can be done straightforwardly.

This model was first discussed by Banks, Frishman, and Yankielowicz[18] using a Feynman diagram approach. Since we are interested in the broken-symmetry aspect of the model it may be too dangerous to rely on perturbation theory. Therefore we shall employ the operator method. Our method not only reproduces the results in Ref. 18, but also enables us to discuss clearly the dynamical breaking of gauge and chiral symmetries.

A. Axial anomaly constraint

First one has to realize that a nontrivial constraint exists among gauge couplings of the left- and right-handed fermions. To see this we notice that the theory possesses the two-dimensional Adler-Bell-Jackiw anomaly

$$\partial_\mu j_L^\mu = +\frac{g}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} \ , \tag{27a}$$

$$\partial_\mu j_{Ri}^\mu = -\frac{g_i}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} \quad (i=1,2) \ , \tag{27b}$$

which will be seen later. [Take the divergence of Eq. (33).] The anomaly equations (27) are consistent with the Maxwell equations (26) if and only if the condition

$$g^2 = g_1{}^2 + g_2{}^2 \tag{28}$$

is met.[19] Because of this condition our model (and its left-right inverted one) turns out to be the minimal version of the possible left-right-asymmetric Schwinger models.

Next we take a derivative $\epsilon_{\rho\mu}\partial^\rho$ in the Maxwell equation (26). Noticing that $\epsilon_{\mu\nu}j_{L,R}^\nu = \mp j_{L,R,\mu}$ and the anomaly free condition (28), we obtain

$$\left[\Box + \frac{g_2}{\pi}\right]F = 0 \ , \tag{29}$$

where F is defined by $F_{\mu\nu} = \epsilon_{\mu\nu}F$. Therefore the photon becomes massive because of the axial anomaly in the same way as in the original Schwinger model.

B. Operator solution in covariant gauge

Now we proceed to the construction of the operator solution, going through the same steps as in the analysis of the multifermion model. We propose the operator solution[12]

$$\psi_L(x) = \exp\{i\sqrt{\pi}[a\phi(x) - c\tilde{\phi}_R(x)]\}\chi_L(x) \ , \tag{30a}$$

$$\psi_{Ri}(x) = \exp\{-i\sqrt{\pi}(g_i/g)[a\phi(x) - b\tilde{\phi}_L(x)]\}\chi_{Ri}(x) \quad (i=1,2) \ , \tag{30b}$$

$$A^\mu(x) = \frac{\sqrt{\pi}}{g}\left[-a\epsilon^{\mu\nu}\partial_\nu\phi(x) + \frac{b}{2}(\epsilon^{\mu\nu}\partial_\nu - \partial^\mu)\tilde{\phi}_L(x) + \frac{c}{2}(\epsilon^{\mu\nu}\partial_\nu + \partial^\mu)\tilde{\phi}_R(x)\right] \ , \tag{30c}$$

where $\chi_L(\chi_R)$ stands for the left- (right-) handed free massless spinor and the negative-metric fields $\tilde{\phi}_L$ and $\tilde{\phi}_R$ are introduced to cancel out separately the fermionic singularities of χ_L and χ_{Ri}, respectively. This cancellation is quite plausible also in our model since it is a mathematical expression of the total screening of the electric charge, originally due to Schwinger.[7] Notice that the screening in fact occurs in our model in view of the generation of the photon mass as demonstrated in (29).

As in the previous section we employ the Bose form of the free "fermion" operators,

$$\chi_L(x)=\left[\frac{\mu}{2\pi}\right]^{1/2} e^{-i\pi\gamma_5/4}\xi N_\mu \exp\left\{i\sqrt{\pi}\left[\gamma_5\varphi(x)+\int_x^\infty d\xi\,\dot{\varphi}(\xi)\right]\right\}u_L\ , \tag{31a}$$

$$\chi_{Ri}(x)=\left[\frac{\mu}{2\pi}\right]^{1/2} e^{-i\pi\gamma_5/4}\xi_i N_\mu \exp\left\{i\sqrt{\pi}\left[\gamma_5\varphi_i(x)+\int_x^\infty d\xi\,\dot{\varphi}_i(\xi)\right]\right\}u_R\ , \tag{31b}$$

where $u_L=\binom{1}{0}$ and $u_R=\binom{0}{1}$. The Klein transformation operators ξ's, whose explicit forms will be given in the Appendix, are responsible for the anticommutativity of χ's. These boson representations play a key role in the physical interpretation of our theory as in the multifermion model.

By noticing the identities

$$i\gamma_\mu\partial^\mu\phi(x)\cdot\chi_{L,R}(x)=\frac{i}{2}\gamma_\mu(\partial^\mu\pm\epsilon^{\mu\nu}\partial_\nu)\phi(x)\cdot\chi_{L,R}(x)\ ,$$
$$\gamma_\mu(\epsilon^{\mu\nu}\partial_\nu\mp\partial^\mu)\phi\cdot\chi_{L,R}(x)=0\ , \tag{32}$$

one can easily verify that the operator solution candidate (30) satisfies the equations of motion (24) and (25). From (30c) it is clear that we are working with the Lorentz gauge.

The expressions of the gauge-invariant currents are as follows:

$$j_L^\mu=:\bar{\chi}_L\gamma^\mu\chi_L:-\frac{g}{2\pi}(A^\mu-\epsilon^{\mu\nu}A_\nu)$$
$$=\frac{1}{2\sqrt{\pi}}(\epsilon^{\mu\nu}\partial_\nu-\partial^\mu)(a\phi-\varphi-b\tilde{\phi}_L)\ , \tag{33a}$$

$$j_{Ri}^\mu=:\bar{\chi}_{Ri}\gamma^\mu\chi_{Ri}:-\frac{g_i}{2\pi}(A^\mu+\epsilon^{\mu\nu}A_\nu)$$
$$=\frac{\bar{g}_i}{2\sqrt{\pi}}(\epsilon^{\mu\nu}\partial_\nu+\partial^\mu)\left[a\phi-\frac{1}{\bar{g}_i}\varphi_i-c\tilde{\phi}_R\right]\ , \tag{33b}$$

where $\bar{g}_i=g_i/g$. In this procedure the fermion fields should have the property that they develop free-fermion singularities at short distances. This gives the important constraints to possible values of the parameters,

$$a^2=b^2=c^2\ . \tag{34}$$

This requirement is again possible because of the superrenormalizability of the theory.

From (30c) and (33) the Maxwell equations take the form

$$-\frac{\sqrt{\pi}}{g}a\epsilon^{\mu\nu}\partial_\nu\Box\phi(x)=\frac{g}{\sqrt{\pi}}a\epsilon^{\mu\nu}\partial_\nu\phi(x)-\frac{g}{2\sqrt{\pi}}(\epsilon^{\mu\nu}\partial_\nu-\partial^\mu)(\varphi+b\tilde{\phi}_L)$$
$$-\frac{g}{2\sqrt{\pi}}(\epsilon^{\mu\nu}\partial_\nu+\partial^\mu)\left[\sum_j\bar{g}_j\varphi_j+c\tilde{\phi}_R\right]. \tag{35}$$

Notice that $\partial^\mu\phi$ terms in (33) cancel out in the right-hand side of (35) because of the anomaly-free condition (28).

We set up the subsidiary conditions

$$\{(\epsilon^{\mu\nu}\partial_\nu-\partial^\mu)[\varphi(x)+\tilde{\phi}_L(x)]\}_+\,|\,\mathrm{phys}\rangle=0\ , \tag{36a}$$

$$\left\{(\epsilon^{\mu\nu}\partial_\nu+\partial^\mu)\left[\sum_j\bar{g}_j\varphi_j(x)+\tilde{\phi}_R(x)\right]\right\}_+\,|\,\mathrm{phys}\rangle=0\ , \tag{36b}$$

where $\{\ \}_+$ implies to take the positive-frequency part of the operator in the curly brackets. Under the choice of the parameters

$$b=c=1 \tag{37}$$

which is consistent with the second equality in (34), the Maxwell equations are satisfied in the physical subspace. The parameter a is determined as

$$a=1 \tag{38}$$

from (34).

Now the Maxwell equations are transcribed into the form

$$\left[\Box+\frac{g^2}{\pi}\right]\phi=0 \tag{39}$$

in the physical subspace. This is in complete agreement with (29) obtained from the axial anomaly equations.

Thus we have verified that the operator solution (30) with the parameters (37) and (38) satisfies the equations of motion of fermions as operator equations and the Maxwell equations in the physical subspace. Everything seems to be all right. However, we are still missing one important point; our operator solution does not transform correctly under the gauge transformation. In fact, using the charge operator

$$Q=\frac{g}{\sqrt{\pi}}\int dx\left\{\partial^1\phi(x)-\tfrac{1}{2}(\partial^1-\partial^0)[\varphi(x)+\tilde{\phi}_L(x)]-\tfrac{1}{2}(\partial^1+\partial^0)\left[\sum_j \bar{g}_j\varphi_j(x)+\tilde{\phi}_R(x)\right]\right\}, \tag{40}$$

it is easy to see that $\psi_L(x)\to e^{+i\alpha Q}\psi_L(x)e^{-i\alpha Q}=e^{i\alpha g/2}\psi_L(x)$ (charge $g/2$).

The origin of this trouble is clear. We have assumed that all the free boson operators are independent degrees of freedom. That is, however, wrong because there is the relation (28) among the charges of the fermions. The required relation takes a very simple form,

$$\varphi(x)=\sum_j \bar{g}_j\varphi_j(x)\ , \tag{41a}$$

and correspondingly,

$$\tilde{\phi}_L(x)=\tilde{\phi}_R(x)\equiv\tilde{\phi}(x)\ . \tag{41b}$$

Now it is a simple exercise to check that the gauge transformation property is repaired. The relations (41) imply a very nontrivial feature of our theory; the electromagnetic current becomes pure vector. We will return to this point in Sec. V. We thus obtained the operator solution of the left-right-asymmetric model.

C. Physical gauge

In order to have the physical interpretation of the theory we take the gauge

$$\psi_L(x)=\left[\frac{\mu}{2\pi}\right]^{1/2}e^{-i\pi\gamma_5/4}\xi e^{i\sqrt{\pi}\phi(x)}N_\mu\exp\left\{i\sqrt{\pi}\left[\gamma_5\Phi_L(x)+\int_x^\infty d\xi\,\dot{\Phi}_L(\xi)\right]\right\}u_L\ , \tag{42a}$$

$$\psi_{Ri}(x)=\left[\frac{\mu}{2\pi}\right]^{1/2}e^{-i\pi\gamma_5/4}\xi_i e^{-i\sqrt{\pi}\bar{g}_i\phi(x)}N_\mu\exp\left\{i\sqrt{\pi}\left[\gamma_5\Phi_i(x)+\int_x^\infty d\xi\,\dot{\Phi}_i(\xi)\right]\right\}u_R\ , \tag{42b}$$

where

$$\Phi_L(x)=\varphi(x)+\tilde{\phi}(x)\ , \tag{43a}$$

$$\Phi_i(x)=\varphi_i(x)+\bar{g}_i\tilde{\phi}(x)\quad (i=1,2)\ . \tag{43b}$$

As in Sec. II it is easy to show that Φ's can be uniquely expressed in terms of one Ψ and one $\tilde{\Psi}$ obeying (15) as

$$\Phi_L=C\tilde{\Psi}\ , \tag{44a}$$

$$\Phi_1=-\bar{g}_2\Psi+C_1\tilde{\Psi}\ , \tag{44b}$$

$$\Phi_2=\bar{g}_1\Psi+C_2\tilde{\Psi}\ , \tag{44c}$$

where C's are undetermined constants. The physical state conditions (36) now take the form $[\epsilon^{\mu\nu}\partial_\nu\tilde{\Psi}]_+\,|\,\text{phys}\rangle=0$.

Thus we have obtained the operator solution in the physical gauge,

$$\psi_L(x)=\left[\frac{\mu}{2\pi}\right]^{1/2}e^{-i\pi\gamma_5/4}\xi e^{i\sqrt{\pi}\phi(x)}N_\mu\exp\left\{i\sqrt{\pi}C\left[\gamma_5\tilde{\Psi}(x)+\int_x^\infty d\xi\,\dot{\tilde{\Psi}}(\xi)\right]\right\}u_L\ , \tag{45a}$$

$$\psi_{Ri}(x)=\left[\frac{\mu}{2\pi}\right]^{1/2}e^{-i\pi\gamma_5/4}\xi_i e^{-i\sqrt{\pi}\bar{g}_i\phi(x)}N_\mu\exp\left\{-i\sqrt{\pi}\bar{\epsilon}^{ij}\bar{g}_j\left[\gamma_5\Psi(x)+\int_x^\infty d\xi\,\dot{\Psi}(\xi)\right]\right\}$$
$$\times\exp\left\{i\sqrt{\pi}C_i\left[\gamma_5\tilde{\Psi}(x)+\int_x^\infty d\xi\,\dot{\tilde{\Psi}}(\xi)\right]\right\}u_R\quad (i=1,2)\ , \tag{45b}$$

where we use the notation $\bar{\epsilon}^{12}=-\bar{\epsilon}^{21}=1$.

The spectrum of the theory consists of the massive free boson ϕ with mass $g/\sqrt{\pi}$ and the massless free boson Ψ. $\tilde{\Psi}$ is the constant operator by which we are to form the θ vacuum as in the multifermion model.

The left- (right-) handed fermion operator in (45) exactly imitates the one of the original (two-fermion) model. This implies that the left- and the right-handed sectors essentially do not communicate with each other,[18] due to the two-dimensionality of our model.

IV. PROPERTIES OF THE PHYSICAL VACUUM

We first investigate the question of which combinations of the fermion operators are Lorentz scalar and have the chance to condense. It is a highly nontrivial problem in two dimensions because the Lorentz transformation properties of fermion operators are determined quite dynamically rather than kinematically.

According to Klaiber's prescription[20] the fermion operator

$$\psi(x)=\prod_i \exp\left\{i\sqrt{\pi}\left[\gamma_5 a_i\varphi_i(x)+b_i\int_x^\infty d\xi\,\dot{\varphi}_i(\xi)\right]\right\}F(x)u \tag{46}$$

transforms as

$$U(\Lambda)^{-1}\psi(x)U(\Lambda)=\exp\left[\tfrac{1}{2}\sum_i a_i b_i\chi\gamma_5\right]\psi(\Lambda^{-1}x)\,, \tag{47}$$

under the Lorentz transformation $x\to\Lambda x$ and χ stands for the Lorentz angle. Here φ_i denotes the free massless scalar field, F includes all the massive degrees of freedom, and $u=\binom{1}{1}$. Therefore ψ_R's in our theory do transform under the Lorentz transformation in contrast to the case of the original Schwinger model.[8]

Next we investigate whether the selected Lorentz scalar operators condense. This is in general a highly dynamical question but in two dimensions we have a simple criterion to decide it. It is now well known that Coleman's theorem[21] can be transcribed into the following statement: The vacuum expectation value of the exponential of the massless boson field vanishes, for short, $\langle e^{i\Psi}\rangle=0$, meaning that there is no order parameter in two dimensions. Irrespective of the validity of Coleman's proof in the indefinite-metric theory, it is entirely reasonable to expect that $\langle e^{i\Psi}\rangle=0$ in the physical gauge. Here $\langle\ \rangle$ should be understood to take the expectation value in terms of the physical vacuum. In fact the renormal-ordering procedure with respect to the physical mass of Ψ guarantees that $\langle e^{i\Psi}\rangle=0$, as mentioned in Sec. II A. We emphasize that taking the physical gauge is quite essential in deciding whether the operators condense. In other gauges the broken-symmetry aspect of the theory may be masked because of the spurious gauge excitations.[8]

A. Multifermion model

Following the above prescription it is easy to show that the Lorentz scalar operator in the two-fermion model is of the following type[22]:

$$(\psi_{1L,R})^{\bar{g}_i}(\psi_{2L,R})^{\bar{g}_2},\ \ (\psi_{1L,R})^{\bar{g}_1}(\psi_{2R,L})^{\bar{g}_2}\ ,$$

and

$$(\psi^*_{1L,R})^{\bar{g}_1}(\psi_{2R,L})^{\bar{g}_2}\ , \tag{48}$$

where ψ_L (ψ_R) implies the upper (lower) component of the spinor not the left- (right-) handed spinor itself.

Now we turn to the question whether these operators condense. The answer is obviously yes (no) for the first (second and third) operators in (48) according to the aforementioned criterion.

The global symmetry present in the Lagrangian ($N=2$ case) is chiral $[U(1)\times U(1)]^2$. The nonvanishing vacuum expectation value of $(\psi_{1L,R})^{\bar{g}_1}(\psi_{2L,R})^{\bar{g}_2}$ implies the breakdown of the global gauge invariance and the chiral U(1) invariance under the transformation $\psi_i\to e^{i\bar{g}_i\alpha\gamma_5}\psi_i$. The symmetries left unbroken are the invariances under the U(1)′ transformation $\psi_1\to e^{i\bar{g}_2\alpha}\psi_1$, $\psi_2\to e^{-i\bar{g}_1\alpha}\psi_2$ and the chiral U(1)′ transformation $\psi_1\to e^{i\bar{g}_2\alpha\gamma_5}\psi_1$, $\psi_2\to e^{-i\bar{g}_1\alpha\gamma_5}\psi_2$. The breakdown of the former two invariances does not accompany the Nambu-Goldstone bosons. Their absence results from the occurrence of the Higgs phenomenon in the case of the electric charge conservation[23] and from the axial anomaly for the chiral U(1) invariance.[8] The latter two invariances are kept exact because of the infrared behavior of the massless Ψ field.[24]

In the N-fermion model one can show that the only operator which develops the vacuum expectation value is

$$\prod_{i=1}^N(\psi_{iL,R})^{\bar{g}_i}\ . \tag{49}$$

We thus end up with the broken gauge and the chiral invariances and the unbroken chiral $[U(1)'\times U(1)']^{N-1}$.

B. Left-right-asymmetric model

In a similar way we can count all the Lorentz scalar operators in the left-right-asymmetric model with the result

$$\psi_L \text{ and } (\psi_{R1})^{\bar{g}_1}(\psi_{R2})^{\bar{g}_2}\ . \tag{50}$$

It should be noticed that all the operators in (50) develop nonvanishing vacuum expectation values. It is quite exceptional that all the Lorentz scalar operators have nonvanishing vacuum expectation values. This happens because of the nonreal nature of the fermion content.

The global symmetry of the Lagrangian (23) is $[U(1)]^3$. We choose the first and the second U(1) to be the gauge and the chiral U(1) generated by Q and Q_5, respectively. We call the third U(1) the "orthogonal" U(1) invariance. This is the invariance under the transformation $\psi_{R1} \to e^{i\alpha \bar{g}_2} \psi_{R1}$ and $\psi_{R2} \to e^{-i\alpha \bar{g}_1} \psi_{R2}$. The nonvanishing vacuum expectation value of the operators (50) implies the breakdown of the gauge and the chiral symmetries whereas the orthogonal U(1) invariance remains unbroken. The Goldstone bosons are absent for the same reasons as in the multifermion model.

V. CONCLUSION AND DISCUSSION

In this paper we have analyzed the variant of the Schwinger model, the models with different charge fermions, and with left-right-asymmetric fermion content. We have succeeded in solving these models by explicitly constructing the operator solutions.

We have focused our attention on the broken-symmetry aspect of the models. The Lorentz scalar products of the fermion operators in the physical gauge have been used to decide what kind of charges condense in the physical vacuum. We have found that in both models the global gauge and the chiral symmetries are spontaneously broken whereas the remaining U(1) invariances are kept exact. We have also observed that the broken gauge invariance results in the Higgs phenomenon. We emphasize the stability of these results over the Abelian gauge theories with various fermion content.

What can we learn from the study of these two-dimensional models besides the stability? In the author's opinion the most interesting feature of these models is that the gauge-symmetry breaking is governed by the axial anomaly. It is not only "triggered" by the anomaly, but also the direction of the breakdown is dictated by the anomaly. Namely, the anomaly-free U(1) subgroups which are orthogonal to the anomalous one remain unbroken.[25]

We believe that the idea that the gauge-symmetry breaking is triggered by the anomaly can generalize to four dimensions. In fact, Veneziano[26] observed that this really occurs in his effective-Lagrangian approach to gauge-symmetry breaking.

What about the second point? Is the direction of the breakdown dictated by the axial anomaly in four dimensions? We can only say that if it is the case it provides us with some interesting consequences. In quantum chromodynamics it requires the absence of diquark condensation. According to Srednicki and Susskind[27] this implies the Nambu-Goldstone realization of chiral symmetry, the desired answer in an as yet unsettled problem.[28]

Furthermore this principle, when applied to a wider class of theories, implies the formation of a multifermion condensate rather than a bilinear condensate. For instance, in SU(5) theory with $\underline{5}$ and $\overline{\underline{10}}$ fermions, the condensate (if it forms) involves at least four fermion operators. The multibody condensate may be the key ingredient in constructing the composite model of leptons and quarks with realistic mass spectrum.[29]

One would expect that the left-right-asymmetric model can be used to test the most-attractive-channel (MAC) hypothesis of Raby, Dimopoulos, and Susskind.[4] Unfortunately this is not the case as we will see below. The MAC hypothesis assumes that if the coupling is large enough the most attractive left- and right-handed fermion pair condense. In our case the MAC is between ψ_L and ψ_{R1} provided that $g_1 > g_2$. Therefore $\langle \bar{\psi}_L \psi_{R1} \rangle \neq 0$ according to the MAC criterion, which is of course wrong as we have seen in Sec. III. Thus one may conclude[18] that the MAC hypothesis fails in our model.

On the contrary, one can argue that $\langle \bar{\psi}_L \psi_{R1} \rangle = 0$ only because of the untamable infrared behavior of a massless boson which is peculiar to two-dimensional space-time. Then we have a chance to get nonzero $\langle \bar{\psi}_L \psi_{R1} \rangle$ in four dimensions as if $\bar{\psi}\psi$ really condenses in quantum chromodynamics.

Both arguments, however, are subject to criticism. The operator $\bar{\psi}_L \psi_{R1}$ does not condense in our model because it is not a Lorentz scalar. Therefore it has no four-dimensional analog unlike the case of $\bar{\psi}\psi$. Unfortunately, our model is not the appropriate place to test the MAC hypothesis.

Finally we make a brief comment on the peculiar property of the electromagnetic current in the left-right-asymmetric model. One naturally expects that the left-right asymmetry in the fermion content results in parity-violating effects. This, however, turns out to be illusory. Even if we introduce a weak external field which couples to our electromagnetic current, the effect of parity violation is unobservable. This stems from the pure vector nature of the electromagnetic current and from the absence of the charged-fermion spectrum. The former is ultimately due to the charge relation (28) arising from the axial-anomaly constraint and the latter due to the total screening of the electric charge.

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APPENDIX

The free "fermion" operators defined by (31) without ξ factors satisfy the canonical equal-time anticommutation relations

$$\{\chi_i(x),\chi_i^+(y)\}_{x_0=y_0}=\delta(x-y) \quad (i=L,\ R1,\ \text{and}\ R2)\ , \tag{A1}$$
$$\{\chi_i(x),\chi_i(y)\}_{x_0=y_0}=0\ .$$

They, however, do not satisfy the anticommutation relations between different fields. Instead they obey, at $x_0=y_0$,

$$\chi_L(x)\chi_{Ri}(y)=e^{i\pi\bar{g}_i}\chi_{Ri}(y)\chi_L(x) \quad (i=1,2)\ ,$$
$$\chi_L(x)\chi_{Ri}^+(y)=e^{-i\pi\bar{g}_i}\chi_{Ri}^+(y)\chi_L(x)\ , \tag{A2}$$
$$[\chi_{R1}(x),\chi_{R2}(y)]=[\chi_{R1}(x),\chi_{R2}^+(y)]=0\ .$$

We show here that the appropriate Klein transformation exists guaranteeing the canonical anticommutation relations among χ's. Its existence is not so obvious in our case because all the boson fields φ's are not independent as shown in Sec. III.

We introduce the number operators N_i of the right-handed fermions obeying the commutation relations

$$[N_i,\chi_{Rj}(x)]=-\delta_{ij}\chi_{Rj}(x) \quad (i,j=1,2)\ . \tag{A3}$$

Natural candidates for such operators are

$$N_i=\frac{1}{\sqrt{\pi}}\int dx\,\partial^1\varphi_i(x)\ . \tag{A4}$$

Using (A4) one can see that the left-handed fermion has fractional fermion numbers, i.e.,

$$[N_i,\chi_L(x)]=-\bar{g}_i\chi_L(x)\ . \tag{A5}$$

Now it is easy to verify that the following Klein transformation operators

$$\xi=1,\quad \xi_1=\exp\left[i\pi\frac{1-\bar{g}_1}{\bar{g}_1}N_1\right],$$
$$\xi_2=\exp\left[i\pi\left[N_1+\frac{1-\bar{g}_1-\bar{g}_2}{\bar{g}_2}N_2\right]\right], \tag{A6}$$

do guarantee the canonical anticommutation relations among different fields as well as (A1). Notice, however, that the form of the Klein transformation (A6) is by no means unique.

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[24]To be honest, we have to admit that our treatment is ineffective for the symmetry breaking which is not signaled by a nonvanishing order parameter. Therefore we are not addressing the realization of the symmetry in the strict sense but only dealing with the question of what kind of charges condense in the physical subspace.

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