PHYSICAL REVIEW D VOLUME 27, NUMBER 2 15 JANUARY 1983

Self-consistent spin structures

C. E. Jones and J. Uschersohn
Behlen Laboratory of Physics, University of Nebraska, Lincoln, Nebraska 68588-0111
(Received 28 June 1982)

The general self-consistent spin structures employed at the "zero-entropy" level of the topological expansion in the dual topological unitarization approach to particle theory are examined in detail. Special attention is given to the problem of the phase factors to be associated with the zero-entropy terms. We consider the discontinuity equation satisfied by the zero-entropy scalar function and find that the discontinuity has a sign that varies as $(-1)^L$ where L is the number of quark loops. This result is shown to be a consequence of the requirement that the sum of zero-entropy terms be Hermitian analytic. The effects of charge conjugation, parity, and time reversal on the zero-entropy S-matrix elements are derived. The two- and four-component spin formalisms are also compared. Normalization conventions for amplitudes and phase space are given explicitly.

I. INTRODUCTION

Recently, a major advance has occurred in the dual topological unitarization[1] (DTU) approach to particle theory through Stapp's[2] discovery of self-consistent spin structures for the simplest "zero-entropy" terms in topological expansion. Stapp's two-component formalism has been transcribed into a four-component formalism by Chew *et al.*[3] which is particularly useful for application to electromagnetic problems. A remarkably accurate set of relations[4] on strong-interaction coupling constants has been derived using a set of self-consistent spin structures.

Our purpose is to study in detail the spin structures that have been proposed. Although for simplicity we consider explicitly here only meson amplitudes, our results can be generalized to amplitudes involving more complicated hadrons such as baryons and baryonium. We are particularly interested in the question of what phases should multiply each self-consistent spin term. These phases determine the sign of the discontinuity for the zero-entropy bootstrap equations. These bootstrap equations determine the spin-independent scalar function which multiplies the spin structure factor. If Hermitian analyticity is required to hold at the zero-entropy level, we find the discontinuity acquires a negative sign for each quark loop.[2]

We also determine the effects of charge conjugation $\mathscr{C}$, parity $\mathscr{P}$, and time reversal $\mathscr{T}$ on the general self-consistent spin structures. Our approach is to first describe the spin structure of particle states in terms of two-component spinors. We then determine the effect of $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$ on meson states composed of two-component spinors. Once the transformation properties of the states is known, one can deduce the behavior of the zero-entropy amplitudes under $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$.

As mentioned, a central focus of our discussion is the determination of the multiplicative phases for the zero-entropy terms. Although our final conclusions are in agreement with Stapp[2] in every case where the results overlap, our treatment and emphasis differ considerably from his. Stapp[2] gives a specific proposal for the zero-entropy M functions with phases determined at the outset by a prescription involving the permutation of quark lines based on fermion statistics. His proposal leads to a self-consistent zero-entropy spin structure. The phases in his model imply a negative sign for discontinuities involving individual quark loops. This, perhaps, might be expected since his model explicitly incorporates the fermion character of the individual quarks. The question arises whether the phases given by Stapp's[2] model are unique or whether some other choice of phases could produce self-consistent spin structures and perhaps lead to positive discontinuities for an individual quark loop.

In our approach we begin with spin structures which reproduce themselves in connected sums. The phases of the individual terms at the zero-entropy level of the topological expansion are at first unspecified. We then begin progressively restricting the phases based on the general requirement that at the zero-entropy level the S matrix incorporate as many general analytic and symmetry properties of the full physical S matrix as possible without sacri-

ficing the basic self-consistency of the individual terms. We then find by appropriately selecting the phases of the zero-entropy terms that pole factorization, crossing symmetry, and $\mathscr{P}$, $\mathscr{C}$, and $\mathscr{T}$ invariance can all be built in at the zero-entropy level. (Flavor symmetry holds automatically regardless of the choice of phases.)

At this point we find that the phases still possess enough arbitrariness to yield either a positive or negative quark loop. However, by enforcing the additional requirement[5] that the zero-entropy amplitude be Hermitian analytic (a well-known property of the full physical amplitude), the phases become restricted in such a way that the discontinuity involving a single quark loop is negative. Remarkably, the negative sign is found to follow from general requirements placed on the S matrix rather than from the need to involve a detailed description of the Fermi statistics for individual quarks.

Our motivation in picking the phases so as to make the zero-entropy amplitude have as many properties as possible enjoyed by the full physical amplitude is to produce a more rapid convergence of the topological expansion. In fact, to do otherwise might lead to inconsistencies.

In the development that follows, we shall define the S matrix in a basis of two-component spinors. In our manipulations we shall work almost entirely with the S-matrix elements in this spinor space and with the spinors themselves. This contrasts with Stapp's work where the focus is on the M functions, which are scarcely even mentioned in the present work. It may seem curious to consider analytic continuations to the crossed channel of S-matrix elements, when it is well known that the M functions have a simple analytic structure amenable to crossing whereas S-matrix elements have kinematic singularities introduced by the spinors. Nonetheless the S matrix can be analytically continued to the crossed channel where certain particles must be interpreted as antiparticles and certain spin states modified. For our purposes here no detailed knowledge of sheet structure is required, just the fact that a particle can be crossed by analytically continuing its four-momentum $p\rightarrow -p$ along some path while holding it on the mass shell. Different paths for the continuation $p\rightarrow -p$ will in general lead to the evaluation of the amplitude on different sheets in the crossed channel, but again, as we shall see, this fact does not interfere with the simple general conclusions we wish to draw. One advantage of using the S-matrix elements is that a determination of the quantum phases for states consisting of quark-antiquark pairs resulting from $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$ transformations can be made in a particularly succinct manner. The argument determining these phases is quite general and does not depend upon the form of the zero-entropy S matrix. We explicitly relate our formalism to Stapp's two-component trace formalism[2] to the four-component formalism.[3]

In Sec. II we give a brief review of four two-dimensional representations of the Lorentz group associated with dotted, undotted, upper, and lower indices. The use of such spaces in defining the S matrix is discussed in Sec. III, along with the crossing properties of such spaces. In Sec. IV we discuss how the particle states defined in terms of spinor states transform under $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$ transformations. The zero-entropy scattering amplitudes are introduced in Sec. V and in Sec. VI the role of Hermitian analyticity in determining the sign of the quark loop is studied. Properties of the zero-entropy amplitudes under $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$ transformations are discussed in Sec. VII. Appendix A treats some subtle points connected with charge conjugation and Appendix B relates our two-component formalism to Stapp's trace formalism.

II. REVIEW OF RELATIVISTIC TWO-COMPONENT SPINOR FORMALISM

To establish our notation and to bring together a collection of useful formulas and conventions needed in later sections, we begin with a brief review of relativistic spinor formalism. A general finite-dimensional irreducible representation of the Lorentz group is designated by $D(F,G)$ where F and G can assume the values $0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$. We shall focus mainly on the two simplest nontrivial inequivalent representations $D(0,\frac{1}{2})$ and $D(\frac{1}{2},0)$. A particular transformation in $D(0,\frac{1}{2})$ we shall designate by $A(\theta\hat{u},\lambda\hat{w})$, where

$$A=e^{-i\theta\hat{u}\cdot\vec{\sigma}/2}e^{\lambda\hat{w}\cdot\vec{\sigma}/2}. \tag{2.1}$$

In (2.1), A is a 2×2 unimodular matrix and $\vec{\sigma}$ correspond to the standard Pauli matrices. Our point of view is that of active transformations so (2.1) means a boost along the direction $\hat{w}$ followed by a rotation around the direction $\hat{u}$; by convention we always take the boost and rotation parameters λ and θ to be positive.

Expressed in the $D(\frac{1}{2},0)$ representation, the transformation (2.1) will be designated $B(\theta\hat{u},\lambda\hat{w})$ and is given by the expression

$$B=e^{-i\theta\hat{u}\cdot\vec{\sigma}/2}e^{-\lambda\hat{w}\cdot\vec{\sigma}/2}. \tag{2.2}$$

Since no similarity transformation exists which will take A into B, the representations $D(0,\frac{1}{2})$ and $D(\frac{1}{2},0)$ are inequivalent. Even though in both cases the representations are by 2×2 unimodular matrices

$SL(2,C)$, the particular matrix associated with a given physical transformation is, in general, different for the two cases, (We note, however, that for the case of pure rotations $\lambda=0$ the representations are identical.) The general relation between A and B is seen to be

$$B=(A^{\dagger})^{-1} . \tag{2.3}$$

The matrix representations we are discussing act on two-dimensional vector spaces and our convention is to associate with the transformations A in $D(0,\frac{1}{2})$ lower undotted indices and with the transformations B in $D(\frac{1}{2},0)$ upper dotted indices. Thus for a two-component spinor η we have the transformation properties

$$\eta'_{\alpha}=A_{\alpha}{}^{\beta}\eta_{\beta}, \quad \eta'^{\dot{\alpha}}=B^{\dot{\alpha}}{}_{\dot{\beta}}\eta^{\dot{\beta}} , \tag{2.4}$$

where we have written the components of η with a lower undotted index if it transforms according to $D(0,\frac{1}{2})$ and with an upper dotted index if it transforms according to $D(\frac{1}{2},0)$.

It turns out to be convenient to introduce two other irreducible representations by unimodular 2×2 matrices associated with lower dotted and upper undotted indices. The former is the representation by the matrices A^* and the latter by the matrices B^*, where the asterisk denotes complex conjugation. Thus, we can write

$$\eta'_{\dot{\alpha}}=A^{*\dot{\beta}}_{\dot{\alpha}}\eta_{\dot{\beta}}, \quad \eta'^{\alpha}=B^{*\alpha}{}_{\beta}\eta^{\beta} . \tag{2.5}$$

The irreducible representations embodied in (2.5) will be denoted $D^*(0,\frac{1}{2})$ and $D^*(\frac{1}{2},0)$ but they are equivalent to the representations already introduced. In particular a similarity transformation of the form $C\times C^{-1}$ will convert $D^*(0,\frac{1}{2})$ into $D(\frac{1}{2},0)$ and $D^*(\frac{1}{2},0)$ into $D(0,\frac{1}{2})$, where $C=-i\sigma_2$. Correspondingly the transformation C on a two-component spinor with lower indices will convert it into a spinor with upper indices and *vice versa*.

A principal advantage of working with the four irreducible representations just discussed—even though only two are inequivalent—is that scalar products of the form $\eta^{\dot{\alpha}}\eta_{\dot{\alpha}}$ are Lorentz invariant as can be easily seen using (2.3)—(2.5).

For use in the development of subsequent sections, we shall now introduce a set of two-component spinors and discuss some of their properties. We start by defining a general rest-frame two-component spinor designated ϕ:

$$\phi=\begin{bmatrix}\alpha\\ \beta\end{bmatrix} . \tag{2.6}$$

We shall also have occasion to use two basis spinors ϕ_i defined by

$$\phi_{+1}=\begin{bmatrix}1\\ 0\end{bmatrix}, \quad \phi_{-1}=\begin{bmatrix}0\\ 1\end{bmatrix} . \tag{2.7}$$

The rest-frame spinors in (2.6) and (2.7) will be used with all four spinor spaces described up to now, i.e., those associated with both upper and lower, dotted and undotted indices.

The spinors we shall work with can be simply defined in terms of the rest-frame spinors of (2.6) or (2.7). We define general spinors by applying a pure boost (velocity transformation) to the rest-frame spinors. The resulting spinor will have its components labeled by upper, lower, dotted, or undotted indices depending on which of the four representations is used for the boost. The four different types of two-component spinors are defined as follows:

$$\eta_{\alpha}(v\phi)=(e^{\lambda\hat{v}\cdot\vec{\sigma}/2}\phi)_{\alpha} , \tag{2.8a}$$

$$\eta^{\dot{\alpha}}(v\phi)=(e^{-\lambda\hat{v}\cdot\vec{\sigma}/2}\phi)^{\dot{\alpha}} , \tag{2.8b}$$

$$\eta_{\dot{\alpha}}(v\phi)=(e^{\lambda\vec{v}\cdot\vec{\sigma}^*/2}\phi)_{\dot{\alpha}} , \tag{2.8c}$$

$$\eta^{\alpha}(v\phi)=(e^{-\lambda\hat{v}\cdot\vec{\sigma}^*})^{\alpha} , \tag{2.8d}$$

where λ is the boost parameter and v is the four-vector defined by

$$\vec{v}=\hat{v}\sinh\lambda, \quad v_0=\cosh\lambda . \tag{2.9}$$

When we associate the spinors in (2.8) with particle states, v will be the four-momentum of the particle divided by its mass. This explains the use of the term "rest frame" for the states ϕ. (Actually, we will never associate particle states with a single spinor but always with a direct product of several spinors.)

For the rest-frame spin state ϕ_i in (2.7) we have the normalization

$$\sum_{\dot{\alpha}}\eta^{\dot{\alpha}}(v\phi_i)\eta_{\dot{\alpha}}(v\phi_j)=\sum_{\alpha}\eta^{\alpha}(v\phi_i)\eta_{\alpha}(v\phi_j) =\delta_{ij} . \tag{2.10}$$

Other useful relations are

$$\sum_i\eta^{\dot{\alpha}}(v\phi_i)\eta_{\dot{\beta}}(v\phi_i)=\delta^{\dot{\alpha}}{}_{\dot{\beta}} , \qquad \sum_i\eta^{\alpha}(v\phi_i)\eta_{\beta}(v\phi_i)=\delta^{\alpha}{}_{\beta} . \tag{2.11}$$

We close this section by indicating the relationship between the two-component spinors and the four-component spinors in the Weyl representation. We define

$$U(v\phi)=\frac{1}{\sqrt{2}}\begin{bmatrix}\eta_\alpha(v\phi)\\ \eta^{\dot\alpha}(v\phi)\end{bmatrix},$$
$$V(v\phi)=\frac{1}{\sqrt{2}}\begin{bmatrix}\eta_\alpha(v\phi)\\ -\eta^{\dot\alpha}(v\phi)\end{bmatrix}, \tag{2.12}$$

where the notation in (2.12) means the four-component object has as its first two components those of the lower undotted spinor and the second two coming from the upper dotted spinor. The normalization is

$$\bar U(v\phi)U(v\phi)=\phi^\dagger\phi=1,$$
$$\bar V(v\phi)V(v\phi)=-\phi^\dagger\phi=-1, \tag{2.13}$$

where

$$\bar U=U^\dagger\gamma_0,$$

and in this representation,

$$\gamma_\mu=\begin{bmatrix}0 & \sigma_\mu\\ \tilde\sigma_\mu & 0\end{bmatrix},\quad \gamma^5=\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix},$$
$$\sigma_\mu=(\sigma_0,\vec\sigma),\quad \tilde\sigma_\mu=(\sigma_0,-\vec\sigma),$$
$$(\gamma_\mu p^\mu-m)U=(\gamma_\mu p^\mu+m)V=0, \tag{2.14}$$
$$p^\mu=m(v_0,\vec v).$$

The normalizations in (2.13) while evaluated in the Weyl representation are, of course, valid in any representation.

III. THE S MATRIX IN SPINOR SPACE AND CROSSING

We wish to discuss here the use of two-component spinors in describing the spin properties of particle states. We shall imagine scattering amplitudes to be written as matrix elements in the two-component spinor space. As previously mentioned, we shall never actually use individual spinors to describe the spin of particle states but only to describe spin degrees of freedom of particle constituents which we call quarks or antiquarks. Thus the scattering matrix in spinor space will always have several spinor indices corresponding to each incoming and outgoing physical particle.

Nevertheless, as convenience, we shall now temporarily think of a spinor as representing particle spin so we can derive its transformation and crossing properties.

It is natural and convenient to employ spinors of the form $\eta^{\dot\alpha}(v\phi)$ and $\eta_\alpha(v\phi)$ to describe incoming spin-$\frac{1}{2}$ quarks having the rest-frame spin state ϕ. The choice of spinors $\eta^{\dot\alpha}$ and η_α both to represent incoming quarks is a consistent one because they transform in the same way under rotations. Under a general Lorentz transformation of the spinor $\eta^{\dot\alpha}(v\phi)$ [represented by the matrices $B(\theta\hat u,\lambda\hat w)$ in Sec. II], the four-vector v will be Lorentz transformed and the state ϕ will undergo an appropriate rotation (often called a Wigner or Stapp rotation). The spinor η_α will transform similarly with the matrices $A(\theta\hat u,\lambda\hat w)$. Having taken the space of spinors $\eta^{\dot\alpha}(v\phi)$ or $\eta_\alpha(v\phi)$ to represent the spin state of the incoming quark states, we must take the dual spinors $\eta_{\dot\alpha}(v\phi)$ and $\eta^\alpha(v\phi)$ to describe the outgoing quarks.

The relation between the two-component spinors (which give the basis for a finite-dimensional nonunitary representation of the Lorentz group) and the Hilbert-space states $|p,\phi\rangle$ of definite momentum which can be used to describe the particle (which give a unitary representation of the Lorentz group) is given by

$$|p\phi\rangle=|p\rangle\otimes\eta^{\dot\alpha}(v\phi) \tag{3.1}$$

with

$$\langle p\phi|=\eta_{\dot\alpha}(v\phi^*)\otimes\langle p|,\quad p=mv,$$

where ϕ is the rest-frame spin state.

We choose our normalization as follows:

$$\langle p'\phi'|p\phi\rangle=(2\pi)^3 2E\delta^3(\vec p\,'-\vec p)\eta_{\dot\alpha}(v'\phi'^*)\eta^{\dot\alpha}(v\phi)$$
$$=(2\pi)^3 2E\delta^3(\vec p\,'-\vec p)\phi'^\dagger\phi. \tag{3.2}$$

The generalization of (3.1) and (3.2) to the case of undotted spinors is obvious. We shall see that a ket state describing an incoming antiquark will involve a two-component spinor with a lower dotted (or upper undotted) index.

We shall now write down a matrix element for the S operator and give its form in terms of the two-component spinors. Displaying explicitly only the variables and indices associated with a single incoming and outgoing quark (using as an example dotted indices) we have

$$\langle p_B\phi_B|S|p_A\phi_A\rangle=\eta_{\dot B}(v_B\phi_B^*)M^{\dot\beta}{}_{\dot\alpha}(p_Bp_A)\eta^{\dot\alpha}(v_A\phi_A). \tag{3.3}$$

The scattering process is thus fully described by the 2×2 momentum-dependent matrix $M^{\dot\beta}{}_{\dot\alpha}$ whose elements are invariant functions of the four-momenta. The matrix $M^{\dot\beta}{}_{\dot\alpha}$ is just a Stapp[2] M function (see also Ref. 6) although the indices are of a different type than those frequently employed.[2]

If in (2.3) we analytically continue $v_B\to-v_B$, we must then interpret the spinor labeled with B as characterizing the spin state of an incoming antiquark. The transformation properties of the anti-

quark spinor will then follow from those of the quark spinor. Thus to find the properties of antiquark spin states we must determine what happens to $\eta_{\dot{\alpha}}(v_B\phi_B)$ when we analytically continue the four-vector v_B into $-v_B$. First, we must decide upon a proper path of continuation. In the physical processes we shall be interested in, $p_B=m_Bv_B$ will represent the four-momentum of particle B with mass m_B. For definiteness we think of particle B as outgoing and imagine the multiparticle process of Fig. 1 as taking place. In Fig. 1(a) the process is given in the center-of-mass frame and B is the upper-left particle. The other two lines in Fig. 1(a) correspond to clusters of ingoing and outgoing particles. For convenience we take the direction of $\vec{p}_B$ to be along z. Figure 1(b) gives a path of continuation in p_z which takes $p_B\rightarrow -p_B$ and gives the variables in a physical region where particle B is an incoming antiparticle, without crossing any of the other particles.

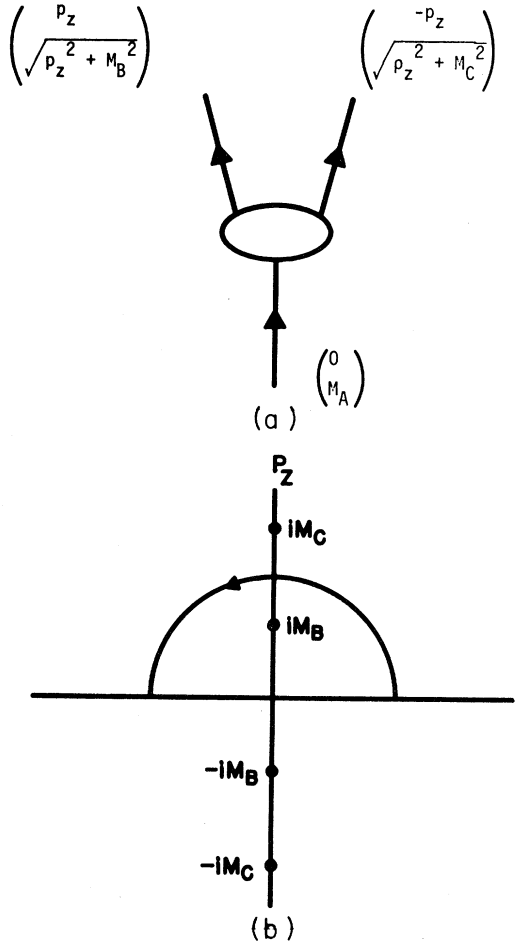


FIG. 1. (a) Center-of-mass scattering process for cluster A→ particle B + cluster C. (b) Path of continuation for crossing particle B.

A few remarks are perhaps in order concerning our crossing procedure for S-matrix elements. One question that might arise is whether the two branch points in Fig. 1(b) might coalesce, thereby pinching the path of continuation. However, since our continuation is on the energy shell, this means that before crossing we have

$$M_A=(p_z^2+M_B^2)^{1/2}+(p_z^2+M_c^2)^{1/2}$$

and afterwards we must have

$$M_A'+(p_z^2+M_B^2)^{1/2}=(p_z^2+M_c'^2)\ .$$

We see that if the former equation is satisfied, the latter cannot be if both $M_A'=M_A$, $M_C'=M_C$. Thus either M_C or M_A or both must vary during the continuation process which crosses particle B. The latter equation shows that during the continuation M_C must become larger than M_B if it is not already so. Thus the path of continuation which threads the two singularities in Fig. 1(b) must indeed be possible if a physical channel exists in which only particle B is crossed relative to the original channel.

It is, of course, the M functions such as $M^{\dot{\beta}}{}_{\dot{\alpha}}$ in (3.3) which possess a simple physical singularity structure and are normally used in discussions of crossing. However, nothing prevents us from carrying along the two-component spinors in the continuation process. If we wish to cross particle B above and be certain to arrive at some well-specified sheet (say, the physical sheet in the crossed channel), this may entail a particular path in Fig. 1(b); e.g., it may require that we use the path in the upper-half or lower-half plane or even a combination of the two but the end result must still be $p_B\rightarrow -p_B$ and the two-component spinor will have a definite continuation along whatever the appropriate path is. The physical conclusions we wish to draw using crossing (in Sec. IV, e.g.) turn out to be independent of the exact path of continuation chosen — they give the same result for any path along which $p_B\rightarrow -p_B$. For definiteness we shall choose the path in Fig. 1(b) but other paths (such as one in the lower-half plane) will produce the same physical results in Sec. IV. From (2.9) we see that the path in Fig. 1(b) results in a continuation of λ_B corresponding to $\lambda_B\rightarrow\lambda_B+i\pi$. We emphasize that under this continuation the initial-state antiparticle has the same value for its four-momentum as the original particle, namely p_B.

Thus to get the crossed two-component spinors we simply replace λ_B by $\lambda_B+i\pi$ in (2.8). For example, the crossing of the spinor $\eta_{\dot{\alpha}}(v_B\phi_B)$ is given by

$$\eta_{\dot{\alpha}}(v_B\phi_B)\rightarrow(e^{\lambda_B\hat{v}_B\cdot\vec{\sigma}^*/2}e^{i\pi\hat{v}_B\cdot\vec{\sigma}^*/2}\phi_B)_{\dot{\alpha}}\ . \qquad (3.4)$$

The meaning of (3.4) is that under crossing the rest-

frame spin state ϕ_B becomes rotated by an amount π around the direction $\hat{v}_B$ of the velocity. We can also write (3.4) in the form

$$\eta_{\dot{\alpha}}(v\phi)\rightarrow\eta_{\dot{\alpha}}(vR^*(\hat{v},\pi)\phi)\,, \tag{3.5}$$

where

$$R(\hat{v},\pi)=e^{-i\pi\hat{v}\cdot\vec{\sigma}/2}\,. \tag{3.6}$$

The crossing properties of the remainder of the two-component spinors for basis spinors are given by

$$\eta^{\dot{\alpha}}(v\phi)\rightarrow\eta^{\dot{\alpha}}(vR(\hat{v},\pi)\phi)\,, \tag{3.7a}$$

$$\eta_{\alpha}(v\phi)\rightarrow\eta_{\alpha}(vR(\hat{v},-\pi)\phi)\,, \tag{3.7b}$$

$$\eta^{\alpha}(v\phi)\rightarrow\eta^{\alpha}(vR^*(\hat{v},-\pi)\phi)\,. \tag{3.7c}$$

We see above that under crossing the spinors with undotted indices have their rest-frame spin states rotated by $(-\pi)$ about the direction $\hat{v}$ while the spinors with dotted indices are rotated by $(+\pi)$ around the same direction.

From (3.4)—(3.7) we see that the effect of crossing an outgoing quark spinor to make it an incoming antiquark spinor is to rotate the spinor but not to change its basic transformation properties. Thus we have the general conclusion that an incoming antiquark spinor is of the same type as an outgoing particle spinor. Similar arguments show that outgoing antiquark spinors are of the same type as incoming quark spinors.

To summarize the specification of incoming and outgoing spin states by two-component spinors is the following:

$$\eta^{\dot{\alpha}},\eta_{\alpha}\leftrightarrow\begin{cases}\text{incoming quarks}\\ \text{or outgoing antiquarks}\,,\end{cases}$$

$$\eta_{\dot{\alpha}},\eta^{\alpha}\leftrightarrow\begin{cases}\text{outgoing quarks}\\ \text{or incoming antiquark}\,.\end{cases}$$

As we have seen, once $\eta^{\dot{\alpha}}$ and η_{α} are chosen to represent incoming quark spinors, the rest of the above specification is a consequence of crossing.

IV. TRANSFORMATION OF SPINOR STATES UNDER $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$

Before discussing the S matrix which defines the zero-entropy amplitudes in the topological expansion we shall first discuss the transformation properties under $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$ of the particle states represented by composite states of spinors introduced in Secs. II and III. Having established the transformation properties of the states, we shall be able to determine the symmetry properties of the topological expansion of zero-entropy S-matrix elements under $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$.

The transformation properties of the composite spinor states under $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$ are determined by the previously discussed requirement that when an outgoing particle state is crossed, it becomes an incoming antiparticle state and when an incoming particle state is crossed it becomes an outgoing antiparticle state. First we consider the form of a possible particle state in terms of its constituent two-component spinor. In this paper we only consider meson states which are quark-antiquark pairs. Ignoring the momentum ket vector discussed in Sec. III, possible particle states can be represented by the following spinor combinations:

$$\mathscr{M}^{\dot{\alpha}}{}_{\dot{\gamma}}=\eta^{\dot{\alpha}}(v\phi)\eta_{\dot{\gamma}}(v\psi)\,, \tag{4.1a}$$

$$\mathscr{M}^{\dot{\alpha}\gamma}=\eta^{\dot{\alpha}}(v\phi)\eta^{\gamma}(v\psi)\,, \tag{4.1b}$$

$$\mathscr{M}_{\alpha}{}^{\gamma}=\eta_{\alpha}(v\phi)\eta^{\gamma}(v\psi)\,, \tag{4.1c}$$

$$\mathscr{M}_{\alpha\dot{\gamma}}=\eta_{\alpha}(v\phi)\eta_{\dot{\gamma}}(v\psi)\,. \tag{4.1d}$$

In (4.1) mv is the momentum of the meson which has mass m and if the particle is incoming, the first spinor designates the quark and the second spinor the antiquark. A flavor index has been suppressed in specifying the state. If the particle is outgoing the first spinor designates the antiquark and the second spinor the quark in accordance with the discussion in Sec. III. The states in (4.1) can be thought of as 2×2 matrices and it will be useful to express them by matrix multiplication as follows:

$$\mathscr{M}^{\dot{\alpha}}{}_{\dot{\gamma}}=(\sqrt{v\cdot\tilde{\sigma}}\phi\psi^T\sqrt{v\cdot\sigma})^{\dot{\alpha}}\,, \tag{4.2a}$$

$$\mathscr{M}^{\dot{\alpha}\gamma}=(\sqrt{v\cdot\tilde{\sigma}}\phi\psi^T\sqrt{v\cdot\tilde{\sigma}})^{\dot{\alpha}\gamma}\,, \tag{4.2b}$$

$$\mathscr{M}_{\alpha}{}^{\gamma}=(\sqrt{v\cdot\sigma}\phi\psi^T\sqrt{v\cdot\tilde{\sigma}})_{\alpha}{}^{\gamma}\,, \tag{4.2c}$$

$$\mathscr{M}_{\alpha\dot{\gamma}}=(\sqrt{v\cdot\sigma}\phi\psi^T\sqrt{v\cdot\sigma})_{\alpha\dot{\gamma}}\,, \tag{4.2d}$$

where $\sqrt{v\cdot\sigma}$ is short-hand for the boost of type (2.8a) and $\sqrt{v\cdot\tilde{\sigma}}$ is shorthand for the boost of the type (2.8b). In (4.2) ϕ is written as a 2×2 matrix with the second column consisting of zeros.

Let us begin by considering the effect of the charge conjugation transformation $\mathscr{C}$ on individual quark states. Using the example of dotted indices we have

$$\mathscr{C}\eta^{\dot{\alpha}}(v\phi)=\epsilon_c\eta_{\dot{\alpha}}(v\phi')\,, \tag{4.3}$$

where ϵ_c is a phase factor. If $\eta^{\dot{\alpha}}$ represents an incoming quark state, then $\eta_{\dot{\alpha}}$ represents an incoming antiquark state. The question is what is the spin state ϕ'? The physical meaning of charge conjugation requires that under this transformation the spin not change. Thus the antiquark rest-frame spin

state ϕ' represents the same spin state that ϕ does for the quark. However, this does not imply $\phi'=\phi$.

Reference to Sec. II shows that whereas $\vec{\sigma}/2$ are the generators of rotations for the upper dotted spinors, $-\vec{\sigma}^*/2$ are the generators of rotations for the upper undotted spinors. Thus in (4.3) if the quark state ϕ satisfies

$$\frac{\vec{\sigma}\cdot\hat{s}_r}{2}\phi=\tfrac{1}{2}\phi\ , \tag{4.4}$$

corresponding to a polarization in the direction $\hat{s}_r$, then the antiquark spin state ϕ' must satisfy

$$\frac{-\vec{\sigma}^*\cdot\hat{s}_r}{2}\phi'=\tfrac{1}{2}\phi'\ . \tag{4.5}$$

The most general ϕ' satisfying this condition (up to a phase) is

$$\phi'=c\phi\ , \qquad c=-i\sigma_2=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}. \tag{4.6a}$$

The possibility of a phase has to be taken into account by the ϵ_c in (4.3). The result (4.6a) follows from

$$c\vec{\sigma}c^{-1}=-\vec{\sigma}^*\ . \tag{4.6b}$$

If we apply $\mathscr{C}$ to an incoming antiquark state we obtain in a similar fashion

$$\mathscr{C}\eta_{\dot{\alpha}}(v\phi)=\bar{\epsilon}_c\eta^{\dot{\alpha}}(vc\phi)\ . \tag{4.7}$$

The phases ϵ_c and $\bar{\epsilon}_c$ may in principle depend upon the quark type (or flavor) but cannot depend upon v. We now show that consistency with crossing demands

$$\epsilon_c\bar{\epsilon}_c=+1\ .$$

Taking the dual of the right side of (4.3) defines the effect of c on an outgoing quark state:

$$\mathscr{C}\eta_{\dot{\alpha}}(v\psi)=\epsilon_c^*\eta^{\dot{\alpha}}(vc\psi)\ , \qquad \psi=\phi^*\ . \tag{4.8}$$

Here we come the the critical part of our argument which involves crossing. When (4.8) is used as the outgoing state of a scattering process, the corresponding S-matrix element involves an outgoing state which is the charge conjugate of $\eta_{\dot{\alpha}}(v,\psi)$. If this outgoing particle state is crossed then we must arrive at the S-matrix element with an incoming state which results from first crossing the state as follows:

$$\eta_{\dot{\alpha}}(v\psi)\rightarrow\eta_{\dot{\alpha}}(vR^*(\hat{v},\pi)\psi)\ ,$$

and then charge conjugating it. In simple terms we are just saying that the operations of charge conjugation and crossing commute. It is, in a sense, a matter of consistent definition: to specify the meaning of an amplitude with $\mathscr{C}$ acting on an incoming antiquark, we start with the amplitude involving $\mathscr{C}$ acting on an outgoing quark and then cross. From the above we have the cross of the right side of (4.8) must be $\mathscr{C}$ on the incoming antiquark state gotten from crossing $\eta_{\dot{\alpha}}(v,\psi)$. Thus we have

$$\mathscr{C}\eta_{\dot{\alpha}}(vR^*(\hat{v},\pi)\psi)=\epsilon_c^*\eta^{\dot{\alpha}}(vR(\hat{v},\pi)c\psi)\ . \tag{4.9a}$$

But from (4.7) we have

$$\mathscr{C}\eta_{\dot{\alpha}}(vR^*(\hat{v},\pi)\psi)=\bar{\epsilon}_c\eta^{\dot{\alpha}}(vcR^*(\hat{v},\pi)\psi)\ . \tag{4.9b}$$

According to our general argument the right sides of the equations in (4.9) are equal. From (4.6b) the two spinors are identical so we obtain the result $\bar{\epsilon}_c=\epsilon_c^*$ or $\epsilon_c\bar{\epsilon}_c=1$. Results (4.3) and (4.7) also hold with undotted indices and again $\epsilon_c\bar{\epsilon}_c=1$. This means that states of neutral quark-antiquark pairs [such as those given in (4.1)] can be given a physical conjugation quantum number independent of the phase ϵ_c. If the quark and antiquark in states of (4.1) are combined to produce a state of well-defined total spin either $S=1$ or $S=0$, then application of (4.3) and (4.7) leads immediately to the result that these states are eigenstates of charge conjugation with eigenvalue $C=(-1)^S$ agreeing with the usual rule (here, of course, $l=0$). More explicitly if $\sum_{jK}a_{jK}(S)\phi_j\psi_K$ corresponds to a rest-frame quark-antiquark spin state with $S=1,0$ then it follows [taking (4.1a) as an example] that

$$\sum_{jK}a_{jK}(S)\eta^{\dot{\alpha}}(v\phi_j)\eta_{\dot{\gamma}}(v\psi_K)=(-1)^S\sum_{jK}a_{jK}(S)\eta^{\dot{\alpha}}(vc\psi_K)\eta_j(vc\phi_j)\ . \tag{4.9c}$$

The result (4.9c) applies equally well to any of the spinor combinations in (4.1).

Now we consider the effect of $\mathscr{P}$ the parity transformation on the state (4.1b). Physically we must have

$$\mathscr{P}\eta^{\dot{\alpha}}(v\phi)=\epsilon_p\eta^{\dot{\alpha}}(\tilde{v}\phi)=\epsilon_p\eta_{\alpha}(v\phi)\ , \tag{4.10}$$

where $\tilde{v}$ is just v with the three-vector reversed in sign, ϵ_p is a phase factor, and the last equality follows from (2.8). The spin states in (4.10) are of

course unaffected by the parity transformation. Applying $\mathscr{P}$ to an incoming antiquark state gives

$$\mathscr{P}\eta_{\dot{\alpha}}(v\phi)=\bar{\epsilon}_p\eta^{\alpha}(v\phi)\ . \tag{4.11}$$

As before the ϵ_p and $\bar{\epsilon}_p$ may depend upon the quark type but not on v if we are to have the correct commutation relations between $\mathscr{P}$ and the boost generators.

As before the use of crossing will enable us to determine the value of $\epsilon_p\bar{\epsilon}_p$. To proceed with this determination we let the state $\eta_{\dot{\alpha}}$ represent an incoming quark. Taking the dual of the right side of (4.10) gives $\mathscr{P}$ on outgoing quark state:

$$\begin{aligned}&\mathscr{P}\eta_{\dot{\alpha}}(v\psi)=\epsilon_p^*\eta^{\alpha}(v\psi)\ ,\\ &\psi=\phi^*\ ,\end{aligned} \tag{4.12}$$

where the complex conjugate occurs because it is an outgoing state and $\mathscr{P}$ is assumed to be a unitary transformation. As in the case of $\mathscr{C}$, we cross the right side of (4.12) and that will define $\mathscr{P}$ on the incoming antiquark state which results from crossing $\eta_{\dot{\alpha}}(v,\psi)$. Thus we obtain

$$\mathscr{P}\eta_{\dot{\alpha}}(vR^*(\hat{v},\pi)\psi)=\epsilon_p^*\eta^{\alpha}(vR^*(\hat{v},-\pi)\psi)\ . \tag{4.13}$$

Using (4.11) to evaluate the left side of (4.13) gives

$$\bar{\epsilon}_p\eta^{\alpha}(vR^*(\hat{v},\pi)\psi)=\epsilon_p^*\eta^{\alpha}(vR^*(\hat{v},-\pi)\psi)\ . \tag{4.14}$$

Now the two rotations in (4.14) differ by a minus sign since they are rotations on spinors so we obtain

$$\bar{\epsilon}_p=-\epsilon_p^*,\quad \epsilon_p\bar{\epsilon}_p=-1\ . \tag{4.15}$$

Our results also hold if we exchange dotted with undotted indices.

The effect of time reversal $\mathscr{T}$ on the state (4.1b) can be determined in an analogous manner. First, time reversal $\mathscr{T}$ on an incoming quark spinor gives

$$\begin{aligned}\mathscr{T}\eta^{\dot{\alpha}}(v\phi)&=\epsilon_T\eta^{\dot{\alpha}}(\tilde{v}\phi')\\ &=\epsilon_T\eta_{\alpha}(v\phi')\ ,\end{aligned} \tag{4.16}$$

where ϕ' must have the opposite spin polarization to ϕ. Thus if

$$\frac{\vec{\sigma}\cdot\hat{s}_r}{2}\phi=\tfrac{1}{2}\phi$$

we must have

$$\frac{\vec{\sigma}\cdot\hat{s}_r}{2}\phi'=-\tfrac{1}{2}\phi'\ .$$

It is easy to verify from (4.6b) that

$$\phi'=c\phi^*\ , \tag{4.17}$$

up to all-over phase which is accounted for by the phase ϵ_T. Thus we can write for incoming quarks

$$\begin{aligned}\mathscr{T}\eta^{\dot{\alpha}}(v\phi)&=\epsilon_T\eta^{\dot{\alpha}}(\tilde{v}c\phi^*)=\epsilon_Tc\eta^{\dot{\alpha}*}(v\phi)\\ &=\epsilon_T\eta_{\alpha}(vc\phi^*)\end{aligned} \tag{4.18a}$$

and for incoming antiquarks

$$\mathscr{T}\eta_{\dot{\alpha}}(v\phi)=\bar{\epsilon}_T\eta^{\alpha}(vc\phi^*)\ . \tag{4.18b}$$

Once again the phases ϵ_T and $\bar{\epsilon}_T$ can only depend on quark type and not on v. Following our previous procedure, $\mathscr{T}$ on an outgoing quark state [from the dual of (4.18a)] is

$$\begin{aligned}&\mathscr{T}\eta_{\dot{\alpha}}(v\psi)=\epsilon_T^*\eta^{\alpha}(vc\psi^*)\ ,\\ &\psi=\phi^*\ .\end{aligned} \tag{4.19}$$

Crossing then yields the following expression for $\mathscr{T}$ on an incoming antiquark state:

$$\mathscr{T}\eta_{\dot{\alpha}}(vR^*(\hat{v},\pi)\psi)=\epsilon_T^*\eta^{\alpha}(vR^*(\hat{v},-\pi)c\psi^*)\ . \tag{4.20}$$

Using (4.18b) on the left side of (4.20) gives

$$\bar{\epsilon}_T\eta^{\alpha}(vcR(\hat{v},\pi)\psi^*)=\epsilon_T^*\eta^{\alpha}(vR^*(\hat{v},-\pi)c\psi^*)\ . \tag{4.21}$$

From the identity (4.6b) we can conclude

$$cR(\hat{v},\pi)c^{-1}=R^*(\hat{v},\pi)\ , \tag{4.22}$$

and thus

$$\bar{\epsilon}_T=-\epsilon_T^*,\quad \epsilon_T\bar{\epsilon}_T=-1\ .$$

Again, the results derived are also valid if we exchange dotted and undotted indices.

We can also conclude from this section how $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$ act on four-component spinors. From (2.12) and the results of this section we can deduce.

$$\begin{aligned}&\begin{bmatrix}0 & c\\ c^{-1} & 0\end{bmatrix}U^*(v\phi)=\epsilon_c^*V(vc\phi^*)\ ,\\ &\mathscr{P}U(v\phi)=\epsilon_p\gamma_0U(v\phi)\ ,\\ &\mathscr{P}V(v\phi)=-\epsilon_p\gamma_0V(v\phi)\ ,\\ &\mathscr{T}U(v\phi)=\epsilon_T\gamma_0U(vc\phi^*)\ ,\\ &\mathscr{T}V(v\phi)=-\epsilon_T\gamma_0V(vc\phi^*)\ .\end{aligned} \tag{4.23}$$

V. THE ZERO-ENTROPY SCATTERING AMPLITUDE

We have defined in the previous sections our quark-antiquark states and the transformation prop-

erties of the two-component spinors under $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$. In this section we shall discuss the zero-entropy[1,2] S-matrix elements for meson amplitudes written in terms of these two-component spinors. We want to work with scattering amplitudes with clearly defined normalizations so that pole factorization properties of these amplitudes will be evident. We begin with a scattering operator S defined in terms of a T operator as follows:

$$S=I+(2\pi)^4 iT \ . \tag{5.1}$$

Let us now consider S-matrix elements between states of the type (3.1). In general the incoming and outgoing states will consist of direct products of many such quark (and antiquark) states. For illustrative purposes, however, we shall display only one such state explicitly, the extension to the general case being obvious. Thus we have

$$\begin{aligned}\langle p'\psi \mid S \mid p\phi\rangle &= (2\pi)^3 2E\delta^3(\vec{\mathrm{p}}-\vec{\mathrm{p}}')\psi^*\phi+(2\pi)^4 i\delta^4(p'-p)\eta_{\dot\alpha}(p'\psi^*)A^{\dot\alpha}{}_{\dot\beta}(p'p)\eta^{\dot\beta}(p\phi)\ ,\\ \langle p'\psi \mid T \mid p\phi\rangle &= \delta^4(p'-p)\eta_{\dot\alpha}(p'\psi^*)A^{\dot\alpha}{}_{\dot\beta}(p'p)\eta^{\dot\beta}(p\phi)\ ,\end{aligned} \tag{5.2}$$

where $A^{\dot\alpha}{}_{\dot\beta}$ may be thought of as an M function for the connected part and variables specifying the other incoming and outgoing quark or antiquark states have been suppressed. It is $A^{\dot\alpha}{}_{\dot\beta}$ which possesses the simple physical threshold cuts for the scattering problem. Thinking of a general initial and final state i and f but again displaying explicitly only the spinors associated with one quark we introduce the notation

$$A_{fi}(\pm)=\eta_{\dot\alpha}(f)A^{\dot\alpha}{}_{\dot\beta}(\pm)\eta^{\dot\beta}(i)\ , \tag{5.3}$$

where $+$ refers to above and $-$ below a physical cut in amplitude. Under very general circumstances it has been found[2] that A_{fi} satisfies a discontinuity equation of the form

$$\begin{aligned}&A_{fi}(+)-A_{fi}(-)\\ &\quad=(2\pi)^4 i\sum_n A_{fn}(+)A_{ni}(-)\delta^4(p_n-p_i)\ ,\end{aligned} \tag{5.4}$$

where the sum $\sum_n$ includes for each intermediate particle both a sum over a complete set of rest-frame spin states as well as a momentum integration of the form

$$\int \frac{d^3\vec{\mathrm{p}}_n}{(2\pi)^3 2E}\ .$$

We shall assume that all the amplitudes discussed in this paper satisfy the discontinuity formula (5.4). It may be verified by applying the discontinuity formula in (5.4) to poles (single-particle intermediate states) occurring in the amplitude A_{fi}, that the residues of such poles are products of A-type amplitudes involving the intermediate and external particles but with no other constant or kinematical factors.

It is important to realize that to assume the discontinuity formula (5.4) is to assume less than unitarity of the S operator, Hermitian analyticity of the amplitude, or time-reversal invariance. Although unitarity and Hermitian analyticity will both be true for the physical S-matrix elements, we shall see that they do not hold for individual self-consistent zero-entropy amplitudes as Stapp has emphasized.[2] The individual zero-entropy amplitudes will, however, obey self-consistently the discontinuity equation (5.4).

We note here that Hermitian analyticity expressed in terms of the amplitudes A_{fi} implies

$$A^*_{if}(+)=A_{fi}(-)\ . \tag{5.5}$$

It may be easily checked that if both (5.4) and (5.5) are true, then unitarity holds, although time-reversal invariance need not hold.

Now we shall define the zero-entropy S-matrix by giving the zero-entropy form of the amplitude A_{fi}, which we denote A^0_{fi}. The amplitude A^0_{fi} will consist of a series of individual terms determined by an M function of the general type (5.3) which consists of Kronecker δ functions in the dotted and undotted spinor indices and which is multiplied by a scalar function f. Thus the individual zero-entropy terms appear in a form in which the dependence on the spinors is a distinct factor which multiplies f. The individual terms also introduce an order to the particle states and are characterized by a planar topology. Each zero-entropy term must have the correct Lorentz properties and in addition we require (1) that it satisfy (5.4) when planar connected sums are used on the right side (this means that each term is self-reproducing), (2) that the rules for determining the terms be consistent with crossing, (3) that residues of poles in the individual terms be factorized into other zero-entropy terms as required by (5.4) as discussed earlier, and (4) that phases be attached to individual zero-entropy terms in such a way as to make the sum of the terms obey as many properties of the full physical S matrix as possible. This last requirement is rather vaguely worded but its meaning should become clearer as we proceed.

Figure 2 illustrates diagrammatically an example

of one zero-entropy-term contribution to A_{fi}^0 for a four-meson amplitude $AB \to CD$. (There will be a phase for the amplitude but this is not indicated in Fig. 2.) The dashed lines within Fig. 2 represent the Landau or momentum graph for the process. The center orientation arrow gives a global orientation which agrees with that of the so-called Harari-Rosner (HR) quark arcs lying at the perimeter of the two-dimensional graph. According to a recent proposal[7] we have also oriented the patches bounded by the HR arcs and the lines of the momentum graph. The convention is that spinors with dotted indices are used if the HR arc bounds a patch whose orientation agrees with the global orientation and spinors with undotted indices are used if the orientations disagree. The indices u, d, s, c in Fig. 2 denote flavor.

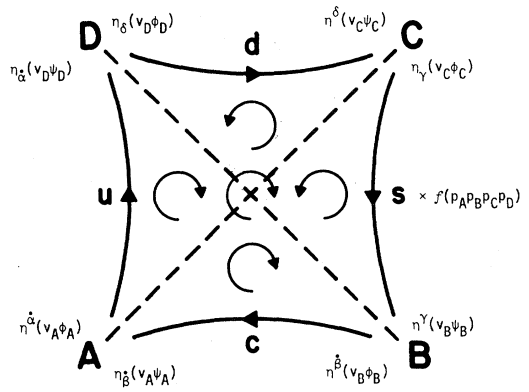


FIG. 2. Example of a zero-entropy term in meson-meson scattering.

Corresponding to the process of Fig. 2, there are actually 2^4 individual zero-entropy terms corresponding to the two possibilities for orienting each patch. For an N-meson amplitude there will in general be 2^N zero-entropy terms if all the quark flavors are different. However, if two or more quark flavors are the same, there will be even more than 2^N zero-entropy terms contributing to the same process.

We designate by $S_i(A,B,C,D)$ the spin structure factors of Fig. 2 where it denotes one of the 2^4 different patch orientation combinations. For the term shown we have

$$S_i(A,B,C,D)=\eta_{\dot\alpha}(D)\eta^{\dot\alpha}(A)\eta_{\dot\beta}(A)\eta^{\dot\beta}(B)\eta^{\gamma}(B)\eta_{\gamma}(C)\eta^{\delta}(C)\eta_{\delta}(D)\ .$$

This can, of course, be immediately generalized to an arbitrary number of particles. The complete amplitude for Fig. 2 can then be written as

$$\Gamma_i S_i(A,B,C,D) f(p_A,p_B,p_C,p_D)\ , \tag{5.6}$$

where f is an invariant scalar function and Γ_i is an all-important phase factor. The scalar function f, in the case of mesons, does not change its value under a cyclic permutation of its variables nor if the order of its variables is completely reversed. The index i in (5.6) is not summed over unless specifically indicated. In much of what follows we shall not refer to the scalar function f but shall concentrate on the factors Γ_i and $S_i(A,B,C,D)$.

Ignoring for the moment the question of the phases Γ_i, it may be verified using (2.10) and (2.11), that if we go to a pole in f that S_i does indeed factorize into two spin factors of the same form as is required by (5.4). We shall see later that the requirement of factorization of pole residues restricts the choice of the phases Γ_i.

The rule for calculating $S_i(A,B,C,D)$ in Fig. 2 is valid in any channel. However, the phase factor Γ_i, as we shall see, is generally channel dependent. The interpretation of the spin states also depends on the channel. If we assume A and B are incoming and C and D are outgoing, $S_i(A,B,C,D)$ can also be written in terms of the four-component spinors defined in Sec. II as follows:

$$\Gamma_i S_i(A,B,C,D)=\tilde\Gamma_i \tilde S_i(A,B,C,D)\ , \tag{5.7a}$$

$$\begin{aligned}\tilde S_i(A,B,C,D)=&\bar U(v_D\psi_D^*)(1+\gamma_5)U(v_A\phi_A)\bar V(v_A\psi_A^*)(1+\gamma_5)U(v_B\phi_B)\\ &\times\bar V(v_B\psi_B^*)(1-\gamma_5)V(v_c\phi_c)\bar U(v_c\psi_c^*)(1-\gamma_5)V(v_D\phi_D)\ ,\end{aligned} \tag{5.7b}$$

where $\tilde\Gamma_i$ is the phase associated with the four-component spinor form $\tilde S_i$. We see in the four-component form that the factors $1\pm\gamma_5$ are associated with HR arcs whose orientations, respectively, agree or disagree with the global orientation. We note that $V\bar U$ represents an outgoing particle and $U\bar V$ an incoming particle. We can also write S_i in a form which is closely related to that given in Stapp's original paper,[2] namely,

$$S_i=\mathrm{Tr}(\mathscr{M}(A)\mathscr{M}(B)\mathscr{M}(C)\mathscr{M}(D))\ , \tag{5.8}$$

using the notation of (4.2) with appropriate indices. Thus, for example, we have

$$\mathcal{M}(B)^{\dot{\beta}\gamma}=\left[\sqrt{v_B\cdot\tilde{\sigma}}\phi_B\psi_B^T\sqrt{v_B\cdot\tilde{\sigma}}\right]^{\dot{\beta}\gamma}. \tag{5.9}$$

The zero-entropy S matrix for the process in Fig. 2 as previously mentioned will consist of a sum of 2^4 terms. [That is, summing over i in (5.6).] All such terms correspond to the same physical process (same momentum and spin variables). The spin-structure factors S_i will be different for the different terms as will the phase factors Γ_i, i being the label which distinguishes these different terms.

We turn to the problem of determining the phase factor Γ_i. We discuss first the *relative* phases between the terms S_i. For definiteness we describe phases defined relative to an N-point zero-entropy reference spin-structure factor S_R all of whose indices are *dotted* and whose phase Γ_R is $+1$. Thus S_R for Fig. 2 would look like (5.6) except that all indices would be dotted. As outlined earlier our rule for fixing the relative phases is such as to give the amplitude as many properties as possible possessed by the physical S matrix. In the case of strong-interaction processes—to which zero-entropy amplitudes apply—this means adding terms in such a way as to ensure $\mathscr{P}$, $\mathscr{C}$, and $\mathscr{T}$ invariance, still maintaining consistency with crossing and pole factorization. These requirements lead to a unique choice for the phase Γ_i relative to the term with all dotted indices:

$$\Gamma_i=(-1)^{N_u}, \tag{5.10}$$

where $N_u=$ number of quark lines with undotted indices except those joining incoming and outgoing particles. Here it is understood that the same rest-frame spin states are used in each term. The rule (5.10) is a direct consequence of the fact that $\epsilon_p\bar{\epsilon}_p=\epsilon_T\bar{\epsilon}_T=-1$ (from Sec. IV) for an incoming or outgoing quark-antiquark pair. It may be readily verified that the rule (5.10) guarantees that the total zero-entropy sum $\sum_i\Gamma_iS_i$ consists of sums over pairs of terms, each pair being invariant under $\mathscr{P}$ and $\mathscr{T}$. A given pair consists of the two terms whose patch orientations with respect to the global orientation are reversed; e.g., the second term to be paired with Fig. 2 is one in which all the patch orientations are reversed, giving an amplitude in which the spin states of the u and c quarks are represented by undotted spinors and the d and s quarks by dotted spinors. $\mathscr{C}$ invariance is also built into the sum of zero-entropy terms. We shall discuss more explicitly in Sec. VII how to construct the amplitudes related by $\mathscr{T}$, $\mathscr{C}$, and $\mathscr{P}$ transformations.

We re-emphasize that the rule for the phases Γ_i given by (5.10) for the individual zero-entropy terms in the topological expansion is consistent with pole factorization. Remarkably, if the sum $\sum_i\Gamma_iS_i$ is analytically continued to the crossed channel using the results (3.5)–(3.7), the end result is the same as if we had applied the rule (5.10) directly in the crossed channel. This is perhaps not so surprising since the arguments leading to the phase (5.10) were based on the relations $\epsilon_p\bar{\epsilon}_p=\epsilon_T\bar{\epsilon}_T=-1$ which were deduced using crossing.

The corresponding determination of the phase $\tilde{\Gamma}_i$ for the four-component spinor formulation is just

$$\tilde{\Gamma}_i=(-1)^{N_V}, \tag{5.11}$$

$N_V=$ number of final state mesons or V (not $\bar{V}$) spinors in S_i.

We now note that the phases (5.10) or (5.11) imply that a sum over intermediate spin states in a connected sum of two amplitudes such as that given in the left part of Fig. 3 produces an amplitude on the right with a correct spin factor S_K with the correct phase Γ_K. (Although illustrated here for a single-particle intermediate state, the self-reproducing property of the phases Γ and the spin factors S holds also for connected sums involving many-meson intermediate states.) The patch orientations in Fig. 3 are immaterial except that those which involve the quarks in particle A must match in the two amplitudes in order that the connected sum give a zero-entropy amplitude. The outgoing A particle in the amplitude in the lower half of Fig. 3 in a basis spin state is characterized by the state $\eta_{\dot{\alpha}}(v_A\psi_i)\eta_\gamma(v_A\phi_j)$ and the incoming A particle in the upper amplitude by the corresponding dual state $\eta^{\dot{\delta}}(v_A\psi_i)\eta^{\beta}(v_A\phi_j)$. (No complex conjugation of the states is needed because the basis states are real.) The connected sum over i and j according to (2.11) becomes just $\delta^{\dot{\delta}}{}_{\dot{\alpha}}\delta^{\beta}{}_{\gamma}$. The phase factor Γ_K for the new process is seen to be the product of the two phase factors for the amplitude in the left part of Fig. 3. Thus

$$\sum_A\Gamma_iS_i(K,H,G,F,A)\Gamma_jS_j(A,E,D,C,B)=\Gamma_KS_K(K,H,G,F,E,D,C,B),\quad \Gamma_i\Gamma_j=\Gamma_K, \tag{5.12}$$

where $\sum_A$ designates the complete sum over spin states for particle A. This illustrates the all-important pole-factorization property for zero-entropy amplitudes and shows it to be consistent with our choice of phases Γ_i which were selected in such a way to ensure $\mathscr{T}$, $\mathscr{C}$, and $\mathscr{P}$ invariance.

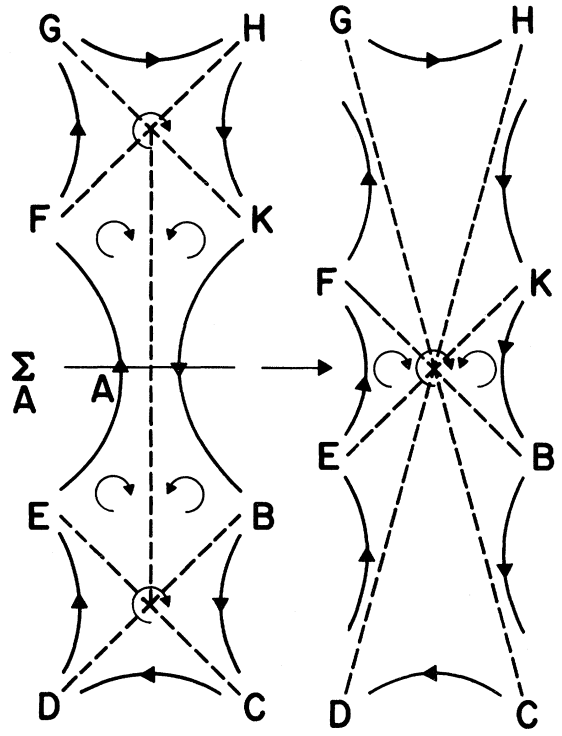


FIG. 3. Connected sum for single-particle intermediate state of a zero-entropy term.

VI. PHASES, HERMITIAN ANALYTICITY, AND THE SIGN OF THE QUARK LOOP

In the last section we derived a set of phases Γ_i relative to the reference amplitude of all dotted spinor indices whose phase is taken to be $\Gamma_R=1$. The phases Γ_i of individual zero-entropy terms were shown to be consistent with crossing and factorization of pole residues, and the sum of zero-entropy terms was invariant under $\mathscr{T}$, $\mathscr{C}$, and $\mathscr{P}$ transformations.

Here we wish to examine the residual freedom left in selecting the phases Γ_i. Since relative phases have been fixed by the arguments in the previous section, the only possible freedom remaining is an all-over phase multiplying each of the Γ_i. Such a change of phases can in no way affect either $\mathscr{T}$, $\mathscr{C}$, or $\mathscr{P}$ invariance or crossing which depend only on relative phases. Such a phase change, however, can affect consistent pole factorization since this is a bilinear relation in the phases [see (5.12)].

It can be shown that the most general multiplicative phase Γ_0 consistent with the requirements of pole factorization is[8]

$$\Gamma_0=e^{i\theta(N-2)}, \tag{6.1}$$

where θ is real and N is the total number of mesons in the amplitude. Thus the residue of a pole in an amplitude involving N mesons factorizes into amplitudes involving N_1 and N_2 mesons with $N_1+N_2=N+2$. The phase Γ_0 is thus consistent with factorization since

$$e^{i\theta(N_1-2)}e^{i\theta(N_2-2)}=e^{i\theta(N-2)}. \tag{6.2}$$

We now wish to address the question of whether there is any motivation for picking the phase Γ_0 to be other than unity or $\theta=0$. It will be useful to begin by examining the consequences of assuming that the individual zero-entropy terms satisfy the discontinuity equation (5.4) with planar connections in the intermediate state. As discussed in Sec. V this is a weaker assumption than one of unitarity. If we apply (5.4) to the situation of intermediate poles (single-particle intermediate states), the previously described factorization properties of the spin structure functions S_i (5.12) then implies that the residue of the pole in scalar function f factorizes into two other f-type scalar functions. Thus, near the pole corresponding to the A particle in Fig. 3 we have

$$f(K,H,G,F,E,D,C,B)\underset{s\to m_A^2}{=}\frac{f(K,H,G,F,A)f(A,E,C,B)}{m_A^2-s}, \tag{6.3}$$

where s is the usual invariant four-momentum squared and the letters stand for four-momenta. The absence of any numerical factors in (6.3) is a consequence of the normalization of the original A_{fi} amplitude discussed earlier. We recall that the scalar function f is invariant under a cyclic permutation of its variables.

Because of (6.2) the equation (6.3) is unchanged if each individual zero-entropy term is multiplied by the phase Γ_0 in (6.1). We now proceed by examining the requirement that the individual zero-entropy amplitudes satisfy (5.4) with planar connections for arbitrary intermediate states. First we consider the two-meson intermediate state characterized by a single quark loop and illustrated in Fig. 4. In Fig. 4 patch orientations have been omitted but the patch orientations of particles A and B must match in both amplitudes or we will not achieve self-consistency. The connected sum for the spin structure factors for Fig. 4 is of the form

$$\sum_{A,B,N_f}\Gamma_iS_i(K,H,G,F,A,B)\Gamma_jS_j(B,A,E,D,C)=4N_f\Gamma_KS_K(K,H,G,F,E,D,C),\quad \Gamma_i\Gamma_j=\Gamma_K, \tag{6.4}$$

where the summation in (6.4) is over a complete set of spin states, the two possible patch orientations and the total number of flavors N_f for the quark in the loop. [Thus the number four in (6.4) comes from two possible spin states and two possible patch orientations.] The phases Γ in (6.4) are those given in Sec. V and do not include multiplication by the all-over phase Γ_0.

The discontinuity equation for the quark loop in terms of the scalar functions becomes

$$f(K,H,G,F,E,D,C)_+ - f(K,H,G,F,E,D,C)_- = 4N_f(2\pi)^4 i \int \frac{d^3p_A}{(2\pi)^3 2E_A} \frac{d^3p_B}{(2\pi)^3 2E_B} f(K,H,G,F,A,B)_+ f(B,A,E,D,C)_- \delta^4(p_A+p_B-p_E-p_D-p_C) , \quad (6.5)$$

where $\pm$ refers to above or below the threshold cut. The equation (6.5) plus its generalization to more complicated many-body intermediate states would constitute the zero-entropy bootstrap,[1,2] in the case that we set $\Gamma_0=1$, or $\theta=0$ in (6.1).

If we now include the multiplicative phase Γ_0 in (6.1) and also generalize (6.5) to include intermediate states with N_I mesons, we obtain [using the notation of (5.4)]

$$f_{fi}(+)-f_{fi}(-)=(2\pi)^4 i \sum_n e^{2i\theta(N_I-1)}(4N_f)^{N_I-1} f_{fn}(+) f_{ni}(-) \delta^4(p_n-p_i) , \quad (6.6)$$

where f and i designate the initial and final momentum states for the scalar functions.

It is clear from (6.6) that the choice of θ can materially affect the form of the zero-entropy bootstrap problem. The question is: can any general principle relating to the S matrix be invoked to determine or restrict θ?

To answer this, we recall that the individual zero-entropy terms satisfy a discontinuity equation of the form (5.4) as a consequence of the fact that the scalar functions satisfy (6.6). However, the individual zero terms do not satisfy planar unitary because as Stapp has emphasized[2] these terms are not Hermitian analytic, i.e., they do not satisfy (5.5). As discussed earlier, our form for the zero-entropy S matrix is dictated by the requirement that it possess as many properties of the physical S matrix as possible. In the previous section we saw that the sum of zero-entropy terms with proper relative phases consists of sums of pairs of terms, each pair being invariant under $\mathscr{P}$ and $\mathscr{T}$. It can easily be checked that these same pairs will also be Hermitian analytic

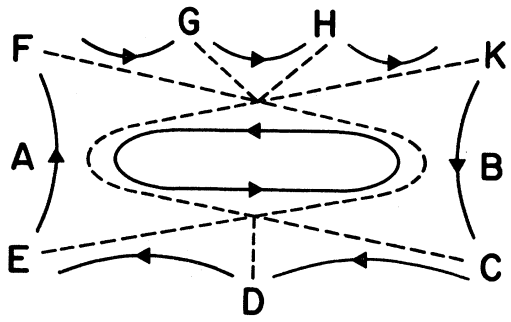


FIG. 4. Connected sum for a two-meson intermediate state.

if θ in Γ_0 is chosen as follows:

$$\theta=(n+\tfrac{1}{2})\pi, \quad n=\text{integer} \quad (6.7a)$$

with

$$f^*_{if}(+)=f_{fi}(-) . \quad (6.7b)$$

Thus in line with our general approach we take θ as given by (6.7a) in order to make the zero-entropy S matrix Hermitian analytic, thus sharing yet another property of the physical S matrix. It should be emphasized that requiring (6.7) does not make the zero-entropy S matrix unitary since the discontinuity formula is not linear. With (6.7a) we get the phase in (6.6) to be

$$e^{2i\theta(N_I-1)}=e^{i\pi(N_I-1)} . \quad (6.8)$$

For $N_I=$ even number, there are an odd number of quark loops and the phase (6.8) is negative.[2] Thus each quark loop contributes a negative sign to the discontinuity equation.

Chew[9] has emphasized the above result does not imply that the net two-particle discontinuity for the f amplitude is necessarily negative. At the zero-entropy level, the meson-meson and baryon-antibaryon intermediate states are degenerate and the latter contribute with a positive sign due to the presence of two quark loops and thus also carry a multiplicity of $(4N_f)^2$. Hence, the net two-particle discontinuity of f is positive.

We close by remarking that the all-over phase Γ_0 takes a different form than (6.1) when baryons are present. Even in this case, however, factorization and Hermitian analyticity are adequate to determine Γ_0. This point will be fully discussed elsewhere.

VII. PROPERTIES OF THE ZERO-ENTROPY S MATRIX UNDER $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$

Having established the transformation properties of the quark-antiquark states under $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$ in Sec. IV and having defined the zero-entropy S matrix in Secs. V and VI, we can now investigate the $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$ properties of the zero-entropy scattering amplitude. To give a concrete illustration we may think of these transformations being performed on the amplitude in Fig. 2 with AB the initial state and DC the final state, although we shall give the transformations in very general form. For Fig. 2 the zero-entropy amplitude consists of a sum

$$\sum_{i=1}^{2^4} \Gamma_0 \Gamma_i S_i(A,B,C,D) \tag{7.1}$$

over the permutations of patch orientations and Γ_0 is the all-over phase derived in the previous section.

The most economical way to discuss the effect of the $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$ on meson amplitudes of the type shown in Fig. 2 is to discuss what these transformations do to the individual quark lines in the diagram. We obtain a complete understanding of the effect of these transformations by considering their effect on two types of quark lines: (a) a quark line that connects an ingoing and an outgoing particle, and (b) a quark line that connects either two ingoing or two outgoing particles. Using the results of Sec. IV we then have the following results for charge conjugation on the quark lines of a zero-entropy amplitude:

$$\mathscr{C}\left[\overleftarrow{\eta_{\dot\alpha}(v'\psi)\quad \eta^{\dot\alpha}(v\phi)}\right]=\overrightarrow{\eta^{\dot\alpha}(v'c\psi)\quad \eta_{\dot\alpha}(vc\phi)}\,, \tag{7.2a}$$

$$\mathscr{C}\begin{bmatrix}\eta_{\dot\alpha}(v'\psi)\\ \eta^{\dot\alpha}(v\phi)\end{bmatrix}=\begin{matrix}\eta^{\dot\alpha}(v'c\psi)\\ \eta_{\dot\alpha}(vc\phi)\end{matrix}\,, \tag{7.2b}$$

where (7.2a) refers to a quark line connecting an ingoing and outgoing particle and (7.2b) refers to a quark line connecting two ingoing or two outgoing particles. The results above in (7.2) also apply when the indices are undotted except that the direction of the quark line must then be reversed. Applying (7.2) to each line in the diagram of an amplitude such as Fig. 2, we see that reverses the directions of all quark lines and also changes the rest-frame spin states. However, because of the identity

$$\eta^{\dot\alpha}(v'\psi)\eta_{\dot\alpha}(v\phi)=\eta_{\dot\alpha}(v'c\psi)\eta^{\dot\alpha}(vc\phi)\,, \tag{7.3}$$

the charge-conjugated amplitude is equal to the original amplitude and thus the individual zero-entropy terms are charge-conjugation invariant. We note that the operation of $\mathscr{C}$ does not change patch orientations. (Some of the subtleties of charge conjugation are discussed in Appendix A where the case of self-conjugate particles is explicitly treated.)

We now consider $\mathscr{P}$ and find for the transformation on individual quark lines (again using the results of Sec. IV):

$$\mathscr{P}\left[\overleftarrow{\eta_{\dot\alpha}(v'\psi)\quad \eta^{\dot\alpha}(v\phi)}\right]=\overleftarrow{\eta^{\alpha}(v'\psi)\quad \eta_{\alpha}(v\phi)}\,, \tag{7.4a}$$

$$\mathscr{P}=\left[\begin{matrix}\eta_{\dot\alpha}(v'\psi)\\ \eta^{\dot\alpha}(v\phi)\end{matrix}\right]=(-1)\times\begin{matrix}\eta^{\alpha}(v'\psi)\\ \eta_{\alpha}(v\phi)\end{matrix}\,. \tag{7.4b}$$

Again the equations (7.4) apply with dotted indices replaced by undotted indices and *vice versa* and quark-line directions reversed. It is clear that the effect of $\mathscr{P}$ is to produce an amplitude with patch orientations reversed and that the (-1) phase (7.4b) gives consistency with the relative phase Γ_i. That is, we have verified that pairs of zero-entropy terms corresponding to opposite patch orientations are invariant under $\mathscr{P}$. The phase Γ_i of (5.10) is a consequence of (7.4b). We now show that the same pairs of terms are invariant under $\mathscr{T}$. Here we must recall that the time-reversed amplitude is defined not only by applying $\mathscr{T}$ to the states but also exchanging ingoing and outgoing states. Thus we obtain for individual quark lines in the amplitude

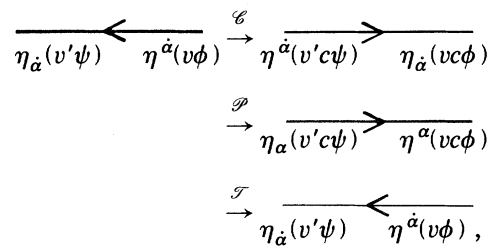
$$\mathscr{T}\left[\overleftarrow{\eta_{\dot\alpha}(v'\psi)\quad \eta^{\dot\alpha}(v\phi)}\right]=\overrightarrow{\eta_{\alpha}(v'c\psi)\quad \eta^{\alpha}(vc\phi)}\,, \tag{7.5a}$$

$$\mathscr{T}=\left\{\begin{matrix}\eta_{\dot\alpha}(v'\psi)\\ \eta^{\dot\alpha}(v\phi)\end{matrix}\right\}=(-1)\times\begin{matrix}\eta_{\alpha}(v'c\psi)\\ \eta^{\alpha}(vc\phi)\end{matrix}\,. \tag{7.5b}$$

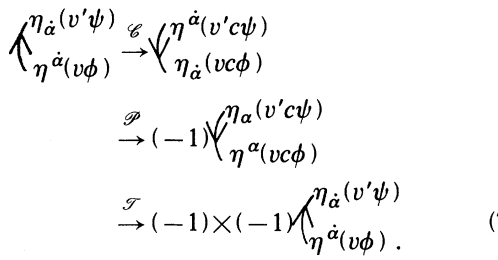
Again, as before, (7.5) applies when dotted and undotted indices are interchanged and the quark lines are reversed in direction. The absence of a complex conjugate on the rest-frame spin states [see (4.18a)] in (7.5b) is due to the complex conjugation which occurs when in states are converted into out states and *vice versa*. Comparing (7.5) and (7.4) and using identity (7.3) with undotted indices we see that the time-reversed amplitude is numerically equal to the parity-reversed amplitude (the amplitudes, of course, refer to different processes in the two cases since for $\mathscr{T}$ in and out states are switched). Thus the same pairs of terms which give $\mathscr{P}$ invariance also give $\mathscr{T}$

invariance.

Finally, using (7.2)–(7.5), we indicate the results of applying $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$ sequentially to the quark lines of a zero-entropy term:



(7.6a)



(7.6b)

Thus from (7.6a) and (7.6b) we can conclude immediately that individual zero-entropy terms are invariant under the combined $\mathscr{C}\mathscr{P}\mathscr{T}$ transformation. We have, of course, assumed throughout that the scalar function f is invariant under the separate $\mathscr{C}$, $\mathscr{P}$, and $\mathscr{T}$ transformations.

ACKNOWLEDGMENTS

This work was supported in part by National Science Foundation Grant No. PHY80-07643. Dr. Henry P. Stapp made a number of helpful suggestions while the work was in progress. One of us (C.E.J.) would like to thank Professor P. Finkler for helpful discussions.

APPENDIX A: MORE ON CHARGE CONJUGATION

In Sec. VII we have seen that individual zero-entropy amplitudes are invariant under charge conjugation. We can express this fact by means of the equation

$$\mathscr{C}^{-1}S^0\mathscr{C}=S^0 \tag{A1}$$

where S^0 designates the zero-entropy scattering operator.

The situation seems simple enough because the physical S operator also satisfies (A1). There is an important difference, however, between the two cases because the matrix elements of S^0 give rise to ordered amplitudes—that is, amplitudes which depend on the order of the particle variables (as discussed in Sec. V). The physical S-matrix elements, of course, do not depend upon the order of the variables. If we agree to always run the quark arrows one way (say clockwise), then charge conjugation changes the order of the variables for zero-entropy amplitudes. For example, for particles A, B, C, D, each of definite spin S_A, S_B, S_C, S_D (either zero or one), we have

$$\begin{aligned}\langle DC\,|\,S^0\,|\,AB\rangle&=\langle DC\,|\,C^{-1}S^0C\,|\,AB\rangle\\&=e^{i\phi}\langle\bar C\bar D\,|\,S^0\,|\,\bar B\bar A\rangle\ ,\end{aligned} \tag{A2}$$

where the antiparticles have exactly the same spin states as the particles. The phase $e^{i\phi}$ is determined—using both (4.7) and the result (4.9c) for states with well-defined spin—to be

$$e^{i\phi}=(-1)^{S_A+S_B+S_C+S_D}\ . \tag{A3}$$

Now an important distinction arises between the consequences of $\mathscr{C}$ invariance for the physical S matrix and the zero-entropy S matrix. This is best seen by considering the case of self-conjugate particles $A=\bar A$, $B=\bar B$, etc. In this situation, applying $\mathscr{C}$ invariance to the physical S matrix in the same form as (A2) would give immediately the result

$$e^{i\phi}=1\ , \tag{A4}$$

because the order of the variables in (A2) is immaterial for the physical S-matrix elements. In this case, of the physical S matrix, the phase $e^{i\phi}$ is just the product the charge-conjugation quantum numbers for each of the four particles. However, in the case of the zero-entropy S matrix one cannot require the phase in (A3) to be equal to unity in the self-conjugate case due to the different order of variables in (A2).

However, the topological expansion for the case of self-conjugate particles will include a sum over the pair terms

$$\langle DC\,|\,S^0\,|\,AB\rangle+\langle CD\,|\,S^0\,|\,BA\rangle\ , \tag{A5}$$

this sum will vanish unless

$$(-1)^{S_A+S_B+S_C+S_D}=1\ . \tag{A6}$$

This result is then consistent with the assignment of the $\mathscr{C}$ quantum of the $\mathscr{C}$ quantum number according to $\mathscr{C}=(-1)^S$, mentioned in Sec. IV.

The subtle feature illustrated here is that although individual zero-entropy terms are invariant under $\mathscr{C}$, the conservation of the $\mathscr{C}$ quantum number can only be understood by considering pairs of terms in the topological expansion. (See, in this regard, a similar earlier discussion of this phenomenon by

Chew and Rosenzweig before spin had been incorporated into the topological expansion.[1])

APPENDIX B: SPIN STRUCTURE FUNCTIONS IN THE TRACE FORMALISM

It is sometimes more convenient to write the spin structure factor S_i in a form in which no reference to the spins of the individual quarks and antiquarks is made as is done in Stapp's original paper.[2] To accomplish this with the formalism used in this paper we take the two-component rest-frame spinors $\phi_x(x=A,\ldots,N)$ and ψ_x to be ϕ_{i_x} and ϕ_{j_x} $(i_x,j_x=\pm 1)$ as defined in (2.7), then multiply with a factor $(s^r\cdot\tilde{\sigma})_{i_x j_x}/\sqrt{2}$ for each particle and then sum over i_x and j_x. s^r is the ordinary spin vector of a particle of spin 0 or spin 1, taken in the rest frame of the particle. For a spin-1 particle, s satisfies $s\cdot v=0$ and $s^2=-1$, while for spin-0 particle $s^\mu=v^\mu$.

We start from (5.3c) assuming all indices dotted and we arrive at the following explicit form:

$$\mathrm{Tr}(\sqrt{v_4\cdot\tilde{\sigma}}\phi_{i_A}\phi_{j_A}^T\sqrt{v_A\cdot\sigma}\sqrt{v_B\cdot\tilde{\sigma}}\phi_{i_B}\phi_{j_B}^T\sqrt{v_B\cdot\sigma}\cdots)\,. \tag{B1}$$

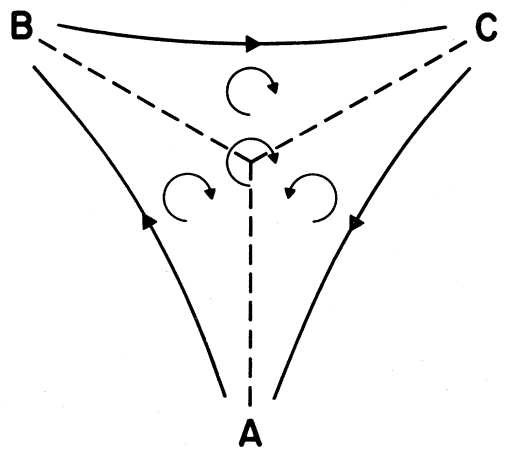


FIG. 5. Amplitude with mixed spin indices.

Using

$$(s^r\cdot\tilde{\sigma})_{ij}\phi_i\phi_j^T=s^r\cdot\tilde{\sigma}$$

the projection on definite hadron spin states gives

$$\left[\frac{1}{\sqrt{2}}\right]^N\mathrm{Tr}(\sqrt{v_A\cdot\tilde{\sigma}}s_A^r\cdot\tilde{\sigma}\sqrt{v_A\cdot\sigma}\sqrt{v_B\cdot\tilde{\sigma}}s_B^r\cdot\tilde{\sigma}\sqrt{v_B\cdot\sigma}\cdots)\,. \tag{B2}$$

Since $\sqrt{v\cdot\tilde{\sigma}}$ is the boost operator from the rest frame to the frame where $p/m=v$, using $\sqrt{v\cdot\tilde{\sigma}}s^r\cdot\tilde{\sigma}\sqrt{v\cdot\tilde{\sigma}}=s\cdot\tilde{\sigma}$ and $\sqrt{v\cdot\tilde{\sigma}}\sqrt{v\cdot\sigma}=\sqrt{v\cdot\sigma}\sqrt{v\cdot\tilde{\sigma}}=1$ we finally obtain[2]

$$\left[\frac{1}{\sqrt{2}}\right]^N\mathrm{Tr}(s_A\cdot\tilde{\sigma}v_A\cdot\sigma s_B\cdot\tilde{\sigma}v_B\cdot\sigma\cdots s_N\cdot\tilde{\sigma}v_N\cdot\sigma)\,. \tag{B3}$$

if all indices are undotted, the spin structure factor becomes[2]

$$\left[\frac{1}{\sqrt{2}}\right]^N\mathrm{Tr}(v_A\cdot\sigma s_A\cdot\tilde{\sigma}v_B\cdot\sigma s_B\cdot\tilde{\sigma}\cdots)\,. \tag{B4}$$

We treat the case of mixed indices with a simple example shown in Fig. 5. In this case we arrive at the spin structure factor

$$\left[\frac{1}{\sqrt{2}}\right]^3\mathrm{Tr}(s_A\cdot\tilde{\sigma}v_c\cdot\sigma s_C\cdot\tilde{\sigma}v_c\cdot\sigma s_B\cdot\tilde{\sigma}v_B\cdot\sigma)\,. \tag{B5}$$

The general rule is as follows: Going around the amplitude against the quark directions, write a factor $s\cdot\tilde{\sigma}$ for each vertex (hadron) and a factor $v\cdot\sigma$ for each "quark propagator" which v is the momentum of the leading (trailing) edge if the index is dotted (undotted). The appropriate phase, Γ_i in each of the examples discussed above is the same as that in (7.1).

[1]G. F. Chew and V. Poenaru, Z. Phys. C 11, 59 (1981); G. F. Chew and C. Rosenzweig, Phys. Rep. 41C, 263 (1978).

[2]H. P. Stapp, Report No. LBL-13310 (unpublished).

[3]G. F. Chew, J. Finkelstein, R. E. McMurray, and V. Poenaru, Phys. Rev. D 24, 2287 (1981).

[4]G. F. Chew, J. Finkelstein, and M. Levinson, Phys. Rev. Lett. 47, 767 (1981).

[5]We are indebted to H. P. Stapp for suggesting that the enforcement of Hermitian analyticity might further reduce the arbitrariness of the phases.

[6]This principle is employed extensively in the original paper giving an axiomatic foundation for S-matrix theory: H. P. Stapp, Phys. Rev. 125, 2139 (1962).

[7]G. F. Chew and J. Finkelstein, Z. Phys. C 13, 161 (1982). The graphs in this paper dispense with HR lines altogether and use the "belt" as a boundary. We find it notationally convenient here to retain the HR lines to carry the spin and flavor labels.

[8]We are grateful to Professor Paul Finkler for suggestions concerning the formulation of the general phase problem. In the case of baryons a more complicated phase structure than (6.1) is required, with Γ_0 depending not just on the total number of particles as here but on the particular channel as well. For the case of mesons (unlike baryons) no special signs are introduced in crossing from one channel to another and Γ_0 therefore is channel independent. The determination of Γ_0 for the case of baryons will be discussed elsewhere.

[9]G. F. Chew, Phys. Rev. Lett. 47, 764 (1981).