PHYSICAL REVIEW D VOLUME 27, NUMBER 12 15 JUNE 1983

Comments

Comments are short papers which comment on papers of other authors previously published in the **Physical Review**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Bounds on the entropy

P. F. González-Díaz
Instituto de Optica "Daza de Valdés," Consejo Superior de Investigaciones Cientificas
Serrano, 121, Madrid-6, Spain
(Received 4 October 1982; revised manuscript received 11 February 1983)

It is shown that the bound on entropy-to-energy ratio recently proposed by Bekenstein actually implies the existence of a limitation on the entropy-to-surface ratio $S/A \leq \frac{1}{4}$ for any bounded spherical system. The implications of this on the full number of particle species that may exist are briefly discussed.

Bekenstein recently suggested[1] that for any bounded system with entropy S and rest energy E there exists a universal upper limit on the entropy-to-energy ratio which reads

$$S/E \leq 2\pi R \ , \tag{1}$$

where R is the radius of the sphere circumscribing the system (if the system is topologically compact, R is to be defined in terms of the system's volume) and we have used units $\hbar = c = k = G = 1$. Such a bound can give considerable new insights into ordinary thermodynamics, particle physics, and communication theory. By judiciously applying (1), Bekenstein showed that black holes set a limitation on the information rate for given message energy by any communication system,[2] and on the full number of particle species.[3] Examples of fields with negative vacuum energy have now been adduced[4–6] against Bekenstein's bound on specific entropy. Nevertheless, Bekenstein himself has showed[7] that bound (1) still holds if applied to a complete system.

I note first that the upper bound in (1) corresponds to black-hole systems for which $R = 2E$ and $S = A/4$. It is easy to show that bound (1) implies necessarily an upper bound on the entropy-to-surface-area ratio of the form

$$S/A \leq \tfrac{1}{4} \ . \tag{2}$$

In fact, the physical interpretation of the general theory of relativity requires that[8] $1 + 2\Phi \geq 0$. At a distance R from the center of the spherical system the potential Φ becomes $\Phi = -E/R$. Hence, $R \geq 2E$ which can be combined with (1) to yield (2).

The upper limit in (2) coincides with the familiar expression $S = A/4$ for black holes. Thus, (2) suggests some sort of unification between the thermodynamical concepts for black holes and external systems. For example, one could think about a bound $T \geq \kappa_g/2\pi$, where T is the surface temperature of any spherical bounded system with "surface gravity" $\kappa_g = 8\pi \partial E/\partial A \leq 1/4E$.

In the case of an elementary particle, it is known[9,10] that it cannot be bounded in a spherical region whose radius is less in order of magnitude than the Planck length, i.e., $R \gtrsim 2^{1/2}$. The direct application of this bound to (2) leads to

$$S \leq \pi R^2 \gtrsim 2\pi \tag{3}$$

which does not set, in principle, any 2π bound on S. We conjecture, however, that there exists a bound

$$S \leq 2\pi \tag{4}$$

for individual, elementary systems—quarks and leptons—and add reasonable arguments in favor of (4). The main of these arguments is that the rest energy of any individual, elementary system cannot be greater than the Planck energy:

$$E \leq (\tfrac{1}{2})^{1/2} \equiv \tfrac{1}{2}(\text{Planck length}) \tag{5}$$

which is, of course allowed by the present status of experimental and theoretical particle physics. The upper limit in (5) should correspond to a black hole with minimum energy, as suggested by Hawking's theory of radiation from black holes, according to which every black hole loses mass until it reaches the Planck mass, at which point it disappears in a burst of radiation containing all species of elementary particles. Thus, such particles should all obey bound (5).

A black hole with the Planck mass is quantum mechanically made up of only one particle with minimum energy ("elementary black hole"). There is also a cosmological argument in favor of (5). It is that cosmological times earlier than the Planck time ($\sim 10^{-43}$ sec) do not make physical meaning, so that we must take the Planck temperature ($\sim 10^{32}$ K) as the initial (maximum) temperature of the Universe.[11] Thermal equilibrium at that temperature then predicts also the Planck energy as the upper limit for the rest energy of elementary particles.

On the other hand, a nonrotating, uncharged black hole with rest energy E requires (even if it is considered quantum mechanically[12,13]) a smaller deal of information to describe it than any spherical uncharged non-black-hole system with the same rest energy. The no-hair theorems[14] ensure that the specification of E suffices for knowledge of all relevant physical quantities—radius, entropy, temperature—of a nonrotating, uncharged black hole. This is not the case for a non-black-hole system for which at least the radius (to say that $T > 2E$ does not suffice, of course, to set up the radius of the system) needs to be specified in addition to E for a complete description of the system. Relating entropy with information,[15] we can say then that the entropy of a black hole must be always larger than that of any closed non-black-hole system with the same rest energy, i.e., $S \leqslant 4\pi E^2$ [note that (1) arises immediately from this bound when it is combined with the condition $R \geqslant 2E$]. Equation (4) then results from combining this bound with (5). Or, in other terms, by taking into account the quantum nature of a closed spherical system, we cannot choose particles of arbitrarily small energy to constitute the closed system: The minimum particle energy allowed by quantum theory will be $\epsilon = R^{-1}$. Hence the maximum number of such particles that go to make up a closed system with radius R and rest energy E is ER. An estimate of the entropy of the full closed system should be then $S = \sigma ER$, where σ is a number of order unity or so[16] which is to be interpreted as the specific entropy (entropy/Boltzmann's constant) due to each particle constituting the system. The parameter σ should be calculated from a proper, full quantum theory for the closed system; however, the value of this parameter may be bounded by comparing the expression $S = \sigma ER$ with (1). This yields bound (4) again.

Following the idea attributable to Bekenstein himself,[3] (4) can be used to set a fundamental limitation on the number of the elementary-particle species—quarks, leptons, neutrinos—which may exist. Thus, considering a baryon as a bag containing three quarks,[3] $S = \ln W(q,s_z)$ where W is the number of permitted three-quark (antiquark) combinations corresponding to a baryon or antibaryon with charge q and spin projection s_z, we obtain $W = 3(g^3 + 2g)$, g being the number analogous generations. From (4) it follows then that there may be a compelling maximum of five generations of quarks, leptons, and neutrinos, all exactly analogous in structure. This prediction is allowed by the present status of particle physics where three such generations are already known. It is worth noting that $g = 5$ is precisely the constraint estimated from standard big-bang cosmology for helium production,[17] though recent astronomical evidence and theoretical particle-physics considerations have raised[18] some doubts about it.

1. J. D. Bekenstein, Phys. Rev. D 23, 287 (1981).
2. J. D. Bekenstein, Phys. Rev. Lett. 46, 623 (1981).
3. J. D. Bekenstein, Gen. Relativ. Gravit. 14, 355 (1982).
4. W. G. Unruh and R. M. Wald, Phys. Rev. D 25, 942 (1982).
5. S. D. Unwin, Phys. Rev. D 26, 944 (1982).
6. D. N. Page, Phys. Rev. D 26, 947 (1982).
7. J. D. Bekenstein, Phys. Rev. D 26, 950 (1982).
8. L. D. Landau and E. Lifshitz, *Teoría Clásica de Campos,* translated by R. Ortiz Fornaguera (Reverté, Barcelona, 1966), Chap. 11.
9. C. D. Mead, Phys. Rev. 135, B849 (1964).
10. C. D. Mead, Phys. Rev. 143, 990 (1966).
11. D. N. Schramm, Ann. N.Y. Acad. Sci. 375, 54 (1981).
12. J. D. Bekenstein, Phys. Rev. D 7, 2333 (1973).
13. S. W. Hawking, Nature (London) 248, 30 (1974); Commun. Math. Phys. 43, 199 (1975).
14. B. Carter, in *General Relativity: An Einstein Centenary Survey,* edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, 1979), p. 359.
15. P. C. W. Davies, *The Physics of Time Asymmetry* (University of California Press, Berkeley, 1974).
16. S. Frautschi, Science 217, 593 (1982).
17. G. Steigman, D. N. Schramm, and J. E. Gunn, Phys. Lett. 66B, 202 (1977).
18. F. W. Stecker, Phys. Rev. Lett. 44, 1237 (1980).