Effects of ρ, ω, ϕ, and ψ exchanges on the $K_1 \to \gamma\gamma$ decay

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The effect of incorporating the new data on the rates $\rho^- \to \pi^-\gamma$ and $\psi \to \pi^0\gamma$ in determining the vector-meson-exchange contributions to the decay rate of $K_1 \to \gamma\gamma$ is to alter the rate with the pion exchange alone by a considerable percentage, the alteration increasing with the cutoff mass that is introduced.

The object of this Brief Report is to present the result of calculating the exchange contributions of the vector mesons ρ(776 MeV), ω(782 MeV), ϕ(1019 MeV), and ψ(3097 MeV) to the rate of decay of the kaon K_1 to two photons. Previous works, which made use of the then-existing experimental data on this topic have been published[1,2]; however, the recent refinement in the experimental measurements[3,4] of the rates $\rho^- \to \pi^-\gamma$ and $\psi \to \pi^0\gamma$ warrants the updating of the calculation.

No experimental determination of the rate of decay of the short-lived neutral kaon into two photons, $K_S \to \gamma\gamma$, has been done since 1973 when Barmin *et al.*[5] established an upper bound on its branching ratio:

$$\Gamma(K_S \to \gamma\gamma)/\Gamma(K_S \to \text{all}) < 4.0 \times 10^{-4} \ . \tag{1}$$

The study of this decay mode is important because of the possibility of observing CP violations[6] which are well known to be exhibited by the other decay modes of the kaon system.[7]

Theoretically, the present status of the decay mode $K_S \to \gamma\gamma$ is as follows:

(a) A unitarity-relation calculation,[8] with the assumption of dominant two-pion state, yields $\Gamma(K_S \to \gamma\gamma) = 2.6 \times 10^4 \text{ sec}^{-1}$. Together with the value[9] of $(0.8923 \pm 0.002 \text{ sec})$ for the mean life of the K_S, the unitarity-relation calculation predicts the branching ratio

$$\Gamma(K_S \to \gamma\gamma)/\Gamma(K_S \to \text{all}) = 2.53 \times 10^{-6} \ , \tag{2}$$

which is well below the experimental upper bound.

(b) Gaillard and Lee[10] found that in a free-quark model of gauge theories, the branching ratio for $K_S \to \gamma\gamma$ is about 1.4×10^{-6}.

(c) In the exact-SU(3) limit, $K_S \to \gamma\gamma$ is forbidden by U-spin conservation.[11]

(d) The effect of incorporating the new data[3,4] on the rates $\rho^- \to \pi^-\gamma$ and $\psi \to \pi^0\gamma$ in determining the vector-meson-exchange contributions to the $K_S \to \gamma\gamma$ decay rate is to alter the rate with the pion exchange by a considerable percentage, the alteration increasing with the cutoff mass that is introduced. This is essentially the result conveyed by this paper. How this result is obtained is explained in the rest of the paper.

For lucidity, a brief outline of the method used in deriving the result is presented.[12] CP conservation is assumed in this paper, hence the short-lived kaon will be indicated by K_1.

In the decay mode $K_1 \to \gamma\gamma$, which is graphically represented in Fig. 1, the Lorentz-invariant matrix element M which satisfies gauge invariance and Bose statistics can be written as

$$M = H(s)[(\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_2) - (\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1)] \ , \tag{3}$$

where ϵ_i and k_i refer to the polarization and momentum, respectively, of the ith photon. $H(s)$, in the rest frame of the K_1, is a function of the square of the kaon mass $s = M_K{}^2$. In terms of H, the decay rate of $K_1 \to \gamma\gamma$ is

$$\Gamma(K_1 \to \gamma\gamma) = \frac{M_K{}^3}{64\pi} |H|^2 \ . \tag{4}$$

Assuming the dominance of the 2π intermediate state, one-pion exchange, and the interaction Lagrangians

$$L_{K\pi\pi} = \lambda \psi_K \phi_\pi^\dagger \phi_\pi \ , \tag{5}$$

$$L_{\pi\pi\text{-}\gamma\gamma} = -ieA_\mu(\phi_\pi^\dagger \overleftrightarrow{\partial}^\mu \phi_\pi) + e^2 A_\mu A^\mu \phi_\pi^\dagger \phi_\pi \tag{6}$$

for the vertices in Fig. 2, it is straightforward to write down the corresponding decay matrix element M_π.[13] Using the Cutkosky rule and a dispersion relation,

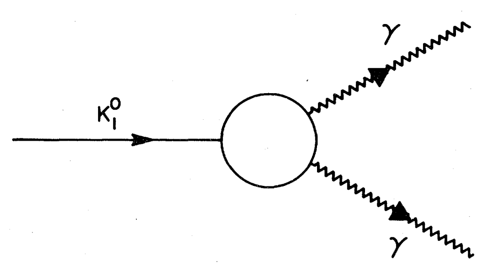

FIG. 1. Graphical representation of the decay process $K_1 \to \gamma\gamma$.

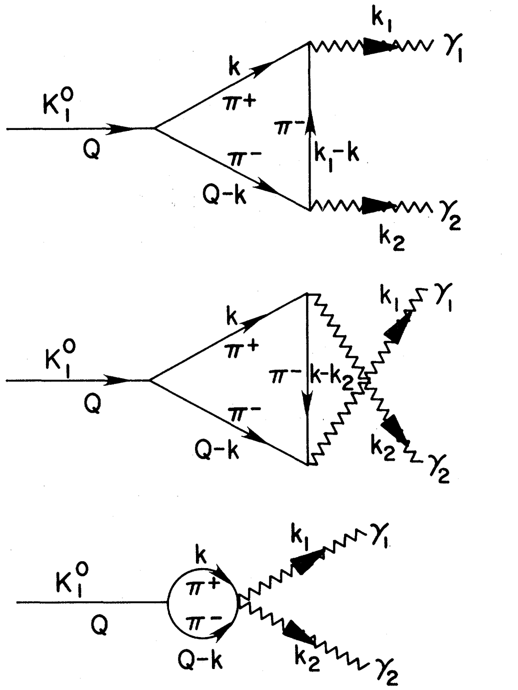

FIG. 2. The three Feynman diagrams contributing to the perturbation calculations of $K_1 \rightarrow \gamma\gamma$ with the pion as the exchange particle. The seagull term is needed for gauge invariance.

one can obtain the real and imaginary parts of H_π:

$$\mathrm{Im}H_\pi = 2\lambda\alpha\frac{\mu^2}{s^2}\ln\left[\frac{1+\beta}{1-\beta}\right] , \tag{7}$$

$$\mathrm{Re}H_\pi = 2\lambda\alpha\frac{\mu^2}{2\pi s}\left\{-\frac{1}{\mu^2}+\frac{1}{s}\left[\pi^2-\ln^2\left[\frac{1+\beta}{1-\beta}\right]\right]\right\} , \tag{8}$$

where

$$\begin{gathered}\mu = \text{pion mass},\quad M_K = K_S \text{ mass},\quad s = M_K{}^2 ,\\ \alpha=\tfrac{1}{137},\quad \beta=\left[1-\frac{4\mu^2}{s}\right]^{1/2} .\end{gathered} \tag{9}$$

The λ in Eqs. (7) and (8) is obtained from the known decay rate of $K_S \rightarrow \pi\pi$ and the relation

$$\Gamma(K\rightarrow\pi\pi)=\frac{\lambda^2}{M_K}\frac{1}{16\pi}\beta . \tag{10}$$

The result is that

$$\begin{aligned}\mathrm{Im}H_\pi &= 8.45\times10^{-23}\ \mathrm{cm} ,\\ \mathrm{Re}H_\pi &= -4.80\times10^{-23}\ \mathrm{cm} ,\end{aligned} \tag{11}$$

which gives the rate

$$\Gamma_\pi(K_1\rightarrow\gamma\gamma)=2.26\times10^4/\mathrm{sec} . \tag{12}$$

This agrees with the previous calculations.[8]

In considering the contributions of the vector mesons ρ, ω, ϕ, and ψ, we use the Lagrangian

$$L_{V\rightarrow\pi\gamma}=\frac{if_V}{M_V}\left[V^\mu\frac{\partial\phi^\dagger}{\partial x^\nu}F^{\alpha\beta}\right]\epsilon_{\mu\nu\alpha\beta}+\mathrm{H.c.} , \tag{13}$$

where

$$F^{\alpha\beta}=\frac{\partial A^\beta}{\partial x^\alpha}-\frac{\partial A^\alpha}{\partial x^\beta}$$

in the $V\pi\gamma$ vertices in Fig. 3; f_V is the $V\pi\gamma$ coupling constant and M_V is the vector-meson mass. The invariant matrix elements corresponding to the Feynman graphs of Fig. 3 can easily be written down.[14] Applying the Cutkosky rule and a dispersion relation leads to the expressions

$$\mathrm{Im}H_V=\frac{\lambda}{16\pi}\left[\frac{f_V}{M_V}\right]^2\left[\frac{M_V{}^2}{s}\hat{V}-\beta\right] , \tag{14}$$

$$\mathrm{Re}H_V=\frac{\lambda}{16\pi}\left[\frac{f_V}{M_V}\right]^2\frac{1}{\pi}[M_V{}^2P(\Omega')-P(\Omega'')] , \tag{15}$$

where[15]

$$\hat{V}=\ln\left[\left[\frac{1+\beta}{1-\beta}\right]\left[\frac{\delta_V-\beta}{\delta_V+\beta}\right]\right] , \tag{16}$$

$$\begin{aligned}P(\Omega')=\frac{1}{s}\Bigg\{&\tfrac{1}{3}\pi^2-\tfrac{1}{2}\left[\ln\left[\frac{1+\beta}{1-\beta}\right]\right]^2-\tfrac{1}{2}[\ln(u_1)]^2-\tfrac{1}{2}[\ln(u_2)]^2+\tfrac{1}{2}[\ln(u_3)]^2\\ &-\mathrm{Li}\left[\frac{1}{u_1}\right]-\mathrm{Li}\left[\frac{1}{u_2}\right]-\mathrm{Li}\left[\frac{1}{u_3}\right]+\mathrm{Li}(u_4)+\tfrac{1}{2}\mathrm{Li}\left[\frac{1}{u_3{}^2}\right]-\tfrac{1}{2}\mathrm{Li}(u_4{}^2)\Bigg\} ,\end{aligned} \tag{17}$$

$$P(\Omega'')=\ln\left[\frac{1+\beta''}{1-\beta''}\right]-\beta\ln\left[\frac{\beta''+\beta}{\beta''-\beta}\right] , \tag{18}$$

$$\begin{gathered}\beta=\left[1-\frac{4\mu^2}{s}\right]^{1/2},\quad \beta''=\beta''(\Lambda^2)=\left[1-\frac{4\mu^2}{\Lambda^2}\right]^{1/2} ,\quad \delta_V=1+\frac{2\mu^2}{(M_V{}^2-\mu^2)} ,\\ u_1=\frac{\delta_V-\beta}{1-\beta},\quad u_2=\frac{\delta_V+\beta}{1+\beta} ,\quad u_3=\frac{\delta_V+\beta}{1-\beta},\quad u_4=\frac{\delta_V-\beta}{1+\beta} .\end{gathered} \tag{19}$$

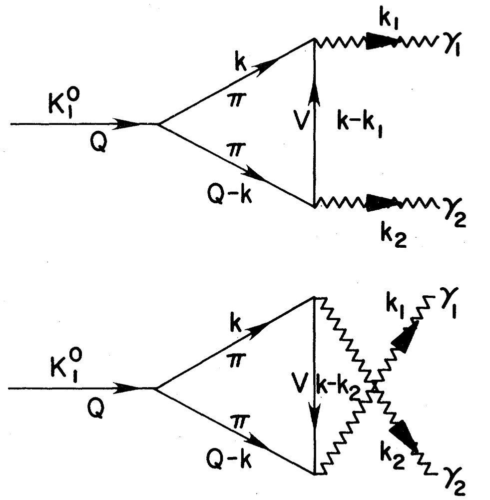

FIG. 3. The Feynman diagrams for $K_1 \to \gamma\gamma$ with vector-meson exchange in $\pi\pi \to \gamma\gamma$ scattering part. No seagull term is included because the two graphs are already gauge invariant in the model assumed.

The Li functions are the dilogarithm functions[15] and Λ is the cutoff mass. The numerical value of the f_V's are obtained from the expression of the decay rate

$$\Gamma(V \to \pi\gamma) = \frac{f_V^2}{24\pi} M_V \left(1 - \frac{\mu^2}{M_V^2}\right)^3 , \tag{20}$$

and the known rates[3,4,9]

$$\begin{aligned}
&\Gamma(\rho^- \to \pi^-\gamma) = 67 \pm 7 \text{ keV} ,\\
&\Gamma(\omega \to \pi^0\gamma) = 8.8\%\Gamma(\omega \to \text{all}) = 0.89 \text{ MeV} ,\\
&\Gamma(\phi \to \pi^0\gamma) = 0.14\%\Gamma(\phi \to \text{all}) = 5.74 \text{ keV} ,\\
&\Gamma(\psi \to \pi^0\gamma) = 2.27 \text{ eV} .
\end{aligned} \tag{21}$$

The f_V's for $\rho^0 \to \pi^0\gamma$ and $\rho^+ \to \pi^+\gamma$ are assumed to be the same as that for $\rho^- \to \pi^-\gamma$, as predicted by SU(3) and charge-conjugation invariance.[16]

TABLE I. Values of ImH_V for different vector mesons.

Vector mesons	ImH_V (10^{-23} cm)
ρ^0	-1.47×10^{-2}
$\rho^\pm$	-2.15×10^{-2}
ω	-1.87×10^{-1}
ϕ	-3.35×10^{-4}
ψ	-5.56×10^{-10}
	$\sum_V \mathrm{Im}H_V = -2.24 \times 10^{-24}$ cm

The result of the numerical computation for ImH_V and ReH_V with different cutoff masses $\Lambda = 25\mu$, 50μ, and 100μ are shown in Tables I and II. Note from Eq. (15) that the ReH_V depends on the cutoff mass. Table III displays the percentage difference between the rate with both the pion and vector-meson exchanges included and the rate with only the pion exchange. From these tables, one can see that among the vector mesons, ω contributes the most and ψ the least. Although the alteration of the rate ranges from 11% to 36% as the cutoff mass increases from 25 to 100μ, it has to be kept in mind that choosing a cutoff mass that is commensurate with the mass of the most massive vector meson included, in this case ψ which has a mass $\approx 23\mu$, is sufficient. Furthermore, it is more likely that the more massive the vector meson, the less is its decay rate to $\pi\gamma$ because of the many other channels opened to it; hence its contribution is minimized.

TABLE II. Values of ReH_V for different vector mesons corresponding to different cutoff mass Λ.

		ReH_V (10^{-23} cm)	
	$\Lambda = 25\mu$	$\Lambda = 50\mu$	$\Lambda = 100\mu$
ρ^0	-8.44×10^{-2}	-1.39×10^{-1}	-1.93×10^{-1}
$\rho^\pm$	-1.26×10^{-1}	-2.08×10^{-1}	-2.89×10^{-1}
ω	-1.08	-1.79	-2.45
ϕ	-2.26×10^{-3}	-4.23×10^{-3}	-6.20×10^{-3}
ψ	$+1.14 \times 10^{-8}$	-1.21×10^{-8}	-3.85×10^{-8}
	$\sum_V \mathrm{Re}H_V = -1.30 \times 10^{-23}$ cm	$\sum_V \mathrm{Re}H_V = -2.14 \times 10^{-23}$ cm	$\sum_V \mathrm{Re}H_V = -2.98 \times 10^{-23}$ cm

TABLE III. The values of the real and imaginary parts of the total $H_T = H_\pi + H_V$, the corresponding rates of $K_1 \to \gamma\gamma$ with both pion and vector-meson exchanges included, and the percentage difference between the rate with pion exchange only and the rate with both pion and vector-meson exchange. Λ is the cutoff mass.

	$\mathrm{Im}H_V$ (10^{-23} cm)	$\mathrm{Re}H_V$ (10^{-23} cm)	$\Gamma_{\pi,V}(K_1 \to \gamma\gamma)$ (10^4 sec^{-1})	% difference
$\Lambda = 25\mu$	8.22	−6.09	2.51	11.0%
$\Lambda = 50\mu$	8.22	−6.94	2.77	22.6%
$\Lambda = 100\mu$	8.22	−7.77	3.06	35.7%

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[12]Some typographical errors in Eqs. (14)−(16) of Ref. 2 are corrected here.
[13]The complete expression is Eq. (4) of Ref. 1.
[14]The complete expression is Eq. (12) of Ref. 1.
[15]P indicates the principal value and the Ω's are the integrals:

$$\Omega' = \int_{4\mu^2}^{\infty} ds' \frac{\hat{V}(s')}{s'(s'-s)} ,$$

$$\Omega''(s,\Lambda^2) = \int_{4\mu^2}^{\Lambda^2} ds' \frac{\beta(s')}{(s'-s)} .$$

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