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Asymptotic behavior of the fermion and gluon exchange amplitudes in massive quantum electrodynamics in the Regge limit

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We study the $e^+e^-\to\gamma\gamma$ amplitude in massive quantum electrodynamics in the large-s and fixed-t limit. We compute the amplitude in the leading-logarithm and the next-to-the-leading-logarithm approximations, to all orders in perturbation theory, and also find the general form of the full amplitude up to any non-leading-logarithm approximation. We do not use any transverse-momentum cutoff for our calculation. We find that, up to the next-to-the-leading-logarithm approximation, the contribution to the positive-signature amplitude is given by a single Regge pole. We find the contribution to the Regge trajectory up to two-loop order. The contribution to the negative-signature channel is consistent with the exchange of a gluon and a Reggeized fermion, interacting with each other through a four-point Reggeon vertex. The technique we have used to calculate the fermion exchange amplitude may also be used to calculate the vector-particle exchange amplitude in massive quantum electrodynamics. We have calculated the gluon exchange amplitude in massive QED in the positive- and the negative-signature channels in the leading-logarithm approximation.

I. INTRODUCTION

In this paper we shall study the amplitude for fermion-antifermion annihilation into two gluons in massive quantum electrodynamics in the limit of very large center-of-mass energy $\sqrt{s}$ and finite momentum transfer q [Fig. 1(a)]. We also study the nearly backward Compton scattering amplitude in the same limit [Fig. 1(b)]. We define the positive- and the negative-signature amplitudes as the sum and the difference of these two amplitudes (a more precise definition is given in Sec. II). This problem was first tackled by Gell-Mann *et al.*,[1] who predicted, on the basis of a one-loop calculation, that the positive-signature amplitude behaves as

$$-2ig^2\bar{\psi}_1\not{\epsilon}_b(s/m^2)^{\alpha(q)}(\not{q}-m)^{-1}\not{\epsilon}_a\psi_2\ . \qquad (1.1)$$

Here ψ_1 and ψ_2 are the external spinors, ϵ_a and ϵ_b are the polarizations of the external gluons, m is the fermion mass, g is the coupling constant, and $\alpha(q)$ is a γ-matrix function of q. They also predicted (erroneously) a similar s dependence of the negative-signature amplitude, with the exponent $\alpha(q)$ replaced by $-\alpha(q)$.

Polkinghorne[2] and Cheng and Wu[3] verified Eq. (1.1) up to two-loop orders in the leading-logarithm approximation. They, however, could not get any result for the negative-signature channel, since the negative-signature amplitude does not receive any contribution in the leading-logarithm approximation. The major difficulty in extending even the leading-logarithm results to higher orders in perturbation theory is that in r-loop order $[O((g^2)^{r+1})]$, individual diagrams in the Feynman gauge have terms of order $\ln^{2r-1}s$. On the other hand, if we expand (1.1) in a power series in g^2, then, since $\alpha(q)\sim g^2$, the term of order $(g^2)^{r+1}$ in the expansion will have at most r powers of $\ln s$. Thus, in order to verify (1.1) in higher orders in g^2, one has to first show the cancellation of all the terms having more than r powers of $\ln s$ in r-loop order. This was done by the authors of Refs. 2 and 3 at the two-loop order, but this becomes an extremely difficult task as one goes to higher orders in perturbation theory.

McCoy and Wu[4] avoided this problem by assuming a hypothesis of transverse cutoff. In each Feynman integral, if we cut off the transverse-momenta integrals at some value Λ, then, in r-loop order, we get terms of the form $\ln^p s\,\ln^q\Lambda$ $(p<r+1, q<r)$. Of course, when we set $\Lambda\gg\sqrt{s}$, the logarithms of Λ are converted to logarithms of s. According to the hypothesis of transverse-momentum cutoff, in evaluating the leading-logarithm contribution in the r-loop order, we keep only those terms that are of the form $\ln^r s\,\ln^q\Lambda$ $(q<r)$, but ignore all terms of the form $\ln^p s\,\ln^q\Lambda$ for $p<r$, $q<r$. If we now sum the left-over terms, all the logarithms of Λ cancel mi-

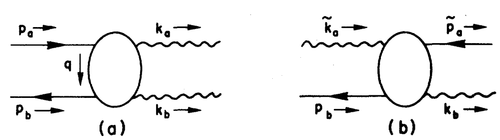


FIG. 1. Process considered.

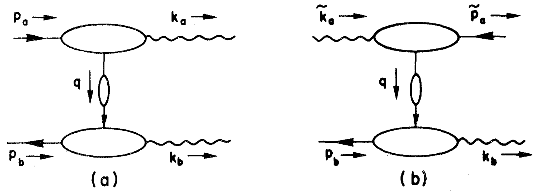


FIG. 2. The factorized diagrams.

raculously, and we get a finite answer. The sum of such terms then gives the leading Regge behavior (1.1). There is, however, no justification for throwing away terms of the form $\ln^p s \ln^q \Lambda$ $(p,q<r)$, since they may contain as many as $2(r-1)$ logarithms of s when the cutoff Λ is removed. If the transverse logarithms do not cancel, these apparently nonleading terms may completely upset the leading-logarithm result. McCoy and Wu[4] also calculated the imaginary part of the first nonleading logarithms using the same hypothesis.

This problem was tackled by Mason[5] in another clever way. He showed that, if, instead of working in the Feynman gauge, we work in the Coulomb gauge, then, in low orders in perturbation theory, the leading-logarithm contribution comes from the factorized diagrams of the form shown in Fig. 2. In the Coulomb gauge, the individual Feynman diagrams contain $2r$ powers of $\ln s$ in r-loop order; hence, apparently, the behavior of the individual terms is worse than that in the Feynman gauge. However, if we assume that only the factorized diagrams contribute in the leading-logarithm approximation, then the constraint that the final result must be Lorentz covariant immediately leads to the result (1.1). Mason, however, could not show the factorization of the amplitude in the leading-logarithm approximation beyond the two-loop order. As a result, he could prove (1.1) in the leading-logarithm approximation, only up to the three-loop order.

Attempts have also been made to show that the Regge behavior holds for the gluon and the fermion exchange reactions in non-Abelian gauge theories.[5–10]

In this paper, we shall prove the cancellation of the double logarithms in perturbation theory, without assuming any transverse cutoff, and show how to systematically calculate the amplitudes for the processes shown in Fig. 1 in the leading-logarithm, next-to-the-leading-logarithm, second-non-leading-logarithm, . . . approximations. We shall illustrate the method by calculating the contribution to the positive- and the negative-signature amplitude in the leading-logarithm and the first-non-leading-logarithm approximation (here by first nonleading logarithm we mean that in r-loop order we keep terms carrying $r-1$ powers of $\ln s$). We find that the sole effect of the first nonleading logarithms in the positive-signature channel is to give an $O(g^4)$ contribution to the trajectory function $\alpha(q)$ and some overall multiplicative constant, independent of s, without changing the form of (1.1). The contribution to the negative-signature amplitude may be interpreted as due to the exchange of a Reggeized fermion and a gluon in the t channel, interacting with each other through a four-point Reggeon vertex.[9] We have expanded our results in powers of g^2 and compared them with the results of explicit calculations by McCoy and Wu[4] up to the four-loop order. Our results for the leading-logarithm terms and the imaginary part of the first-non-leading-logarithm terms agree with the results of McCoy and Wu. The real part of the first nonleading logarithms, which gives rise to the $O(g^4)$ term in the trajectory function, was, however, not calculated by McCoy and Wu. This is a new result. We have also calculated the asymptotic behavior of the gluon exchange amplitude in the positive- and negative-signature channels in the leading-logarithm approximation using the same method. Also, as we have mentioned earlier, our approach provides a systematic way of calculating the amplitudes up to any nonleading logarithms. The techniques developed in this paper are in no way limited to Abelian gauge theories, and may be generalized to non-Abelian gauge theories for gluon and fermion exchange amplitudes.

The various stages involved in our analysis are as follows. In Sec. II we specify the kinematics, the gauge, and the renormalization procedure that we shall be using throughout this paper. We work in the Coulomb gauge. Since the annihilation and the Compton scattering amplitudes may be analyzed in the same way, we concentrate on the annihilation amplitude. We choose a frame in which the external fermion and one of the outgoing gluons move with very large momenta p_a and k_a, respectively, in the $+Z$ direction, and the external antifermion and the other outgoing gluon are moving with very large momenta p_b and k_b, respectively, in the $-Z$ direction. In Sec. III, we analyze the contribution to the Feynman diagrams, contributing to the amplitude, from different regions of integrations in the loop momentum space. For this we use a power-counting method developed by Sterman.[11] We find that if we neglect all terms that are suppressed by some power of $m/\sqrt{s}$, then, the momentum-space region, which contributes to the amplitude, consists of a connected set of lines moving along the Z axis with large momenta, a connected set of lines moving along the $-Z$ axis with large momenta, and a set of soft lines with all components $\lesssim q$, exchanged between the two oppositely moving jets. The factorized diagrams of Fig. 2 are special cases of this, where only one soft-fermion line is exchanged between the two jets. Using this picture, we express the full amplitude as a sum of convolution of two Green's functions $G^{(n)}$ and $F^{(n')}$. $G^{(n)}$ is a Green's function with an on-shell fermion line carrying momentum p_a, an on-shell gluon line carrying momentum k_a, and a set of n soft gluons and a soft-fermion line as its external lines. Similarly, $F^{(n')}$ contains an on-shell antifermion line carrying momentum p_b, an on-shell gluon line carrying momentum k_b, and a set of soft gluons and a soft-fermion line as its external lines.

In Sec. IV, we study the properties of the Green's functions $G^{(n)}$. $F^{(n')}$ may be analyzed in an exactly similar way. We derive various relations between the $G^{(n)}$'s. In particular, we study the variation of $G^{(n)}$ under an infinitesimal boost along the Z axis; this gives us some nontrivial relations among the $G^{(n)}$'s. We show in Sec. V that using the relations derived in Sec. IV the full amplitude may be expressed as a sum of the convolutions of the functions $\Gamma^{(n)}$ and $\Phi^{(n')}$ in the transverse-momentum space, where the functions $\Gamma^{(n)}$ and $\Phi^{(n')}$ are defined in terms of $G^{(n)}$ and $F^{(n')}$, respectively. In Sec. VI we show that in r –loop order, $\Gamma^{(n)}$ can have at most r logarithms of p_a^+/m and $\Phi^{(n')}$ can have at most r logarithms of p_b^-/m. This essentially shows the cancellation of double logarithms, so that only the factorized diagrams contribute to the amplitude in the leading-logarithm approximation, and only the factorized diagrams and the diagrams which have a one-gluon–one-fermion intermediate state in the t channel contribute in the next-to-the-leading-logarithm approximation, and so on. In Sec. VII, we show, how with the help of the equations derived in Sec. IV, we can systematically compute the asymptotic behavior of the functions $\Gamma^{(n)}$ and $\Phi^{(n')}$. In Sec. VIII we explicitly evaluate the contribution to the amplitude in the leading-logarithm and the first-non-leading-logarithm approximation, using the general method developed in Sec. VII.

We summarize our results in Sec. IX.

In Appendices A and B, we prove some of the technical results not proved in the text. In Appendix C we show how our formalism may be applied to study the gluon exchange processes in the Regge limit. We find the expression for the fermion-fermion scattering amplitude in the odd-C-parity and the even-C-parity channels in the leading-logarithm approximation.

II. KINEMATICS AND GAUGE

We shall consider the process of fermion-antifermion annihilation into two gluons [Fig. 1(a)] and the backward Compton scattering amplitude [Fig. 1(b)] in the limit of very large center-of-mass energy and fixed momentum transfer. Let m and μ denote the physical masses of the fermion and the gluon, respectively. We choose a frame in which different momenta, shown in Fig. 1, are as follows:

$$\begin{aligned} p_a &= ((p^2+m^2+\alpha^2\vec{q}^{\,2})^{1/2}, \alpha q_1, \alpha q_2, p) , \\ p_b &= ((p'^2+m^2+\alpha^2\vec{q}^{\,2})^{1/2}, -\alpha q_1, -\alpha q_2, -p') , \\ k_a &= p_a - q, \quad k_b = p_b + q , \\ q &= (0, q_1, q_2, 0) , \\ \tilde{p}_a &= ((p^2+m^2+\alpha^2\vec{q}^{\,2})^{1/2}, -\alpha q_1, -\alpha q_2, p) , \\ \tilde{k}_a &= \tilde{p}_a + q . \end{aligned} \tag{2.1}$$

α is determined from

$$m^2+\alpha^2\vec{q}^{\,2} = \mu^2+(1-\alpha)^2\vec{q}^{\,2} . \tag{2.2}$$

We take the limit $p,p' \to \infty$ at fixed q. Thus the particles labeled a move with large momenta along the positive Z axis, while the particles labeled b move with large momenta along the negative Z axis. In this limit, we have

$$s = (p_a+p_b)^2 \simeq 4pp' , \tag{2.3}$$

$$t = q^2 = -\vec{q}^{\,2} . \tag{2.4}$$

In this paper we shall consider the case of tranversely polarized gluons only. We shall compute the amplitudes for the processes shown in Fig. 1 in an Abelian gauge theory with massive vector bosons. Such a theory is renormalizable[12] and also gauge invariant, in the sense that we may add any term of the form $(u^\mu k^\nu + u^\nu k^\mu)/(k^2-\mu^2+i\epsilon)$ to the gluon propagator, without changing any physical scattering amplitude, provided we include an extra wave-function renormalization factor in the external fermion lines (here u is any vector).

The amplitudes for the processes shown in Fig. 1 may be expressed as γ-matrix functions of s, q, and the polarization vector of the external gluons, sandwiched between the external Dirac spinors. If A_1 and A_2 are these γ-matrix functions for Figs. 1(a) and 1(b), respectively, we define the positive- and the negative-signature amplitudes as

$$A^{\pm} = A_1 \pm A_2 . \tag{2.5}$$

The amplitudes are calculated by calculating the sum of all the Feynman diagrams, including self-energy insertions on external lines, and then dividing it by the wave-function renormalization constant for each of the external lines. The renormalization mass is chosen to be of order m,μ,q.

The convention about the γ-matrices and the external spinors are the same as those used by Bjorken and Drell.[13] We define

$$\gamma^{\pm}=\gamma^0\pm\gamma^3 . \quad (2.6)$$

Then,

$$(\gamma^+)^2=(\gamma^-)^2=0 , \ \{\gamma^+,\gamma^-\}=4 ,$$
$$\{\gamma^{\pm},\gamma^i\}=0, \ i=1,2 . \quad (2.7)$$

For any four-vector k we define

$$k_{\perp}=(0,k^1,k^2,0), \ k^{\pm}=k^0\pm k^3 . \quad (2.8)$$

We choose to work in the Coulomb gauge, in which the unphysical longitudinal degrees of freedom are absent from the gluon propagator. The propagator of a gluon, carrying momentum k, is given in this gauge by

$$-iN^{\mu\nu}(k)/(k^2-\mu^2+i\epsilon)=-i\left[g^{\mu\nu}-\frac{k^{\mu}\bar{k}^{\nu}+\bar{k}^{\mu}k^{\nu}-k^{\mu}k^{\nu}}{k\cdot\bar{k}}\right]\Big/(k^2-\mu^2+i\epsilon) , \quad (2.9)$$

where

$$\bar{k}=(0,\vec{k}) . \quad (2.10)$$

For future reference, we shall list the various components of N below:

$$N^{++}=N^{--}=-k^+k^-/\vec{k}^2 ,$$
$$N^{+-}=N^{-+}=(-2\vec{k}_{\perp}^2+k^+k^-)/(-\vec{k}^2) ,$$
$$N^{ij}=g^{ij}+k^ik^j/\vec{k}^2, \ i,j=1,2 ,$$
$$N^{\pm i}=N^{i\pm}=\pm(k^+-k^-)k^i/2\vec{k}^2 , \ i=1,2 . \quad (2.11)$$

III. ANALYSIS OF FEYNMAN GRAPHS: POWER COUNTING

In this section we shall study the contribution to the Feynman integrals, contributing to the amplitudes under consideration, from different regions of integration in the loop momentum space, and identify the important regions of integration that contribute to the amplitude in the $s\to\infty$, t-fixed limit, in the leading power in s. We follow a method developed by Sterman.[11] We scale all the masses and momenta involved in this problem by $\sqrt{s}$, so that in the $s\to\infty$ limit, the problem reduces to the scattering of finite-energy massless particles at zero momentum transfer but finite c.m. energy. The powers of $\ln s$ will now appear as powers of $\ln(1/M)$, M being some mass of the order of the masses of the particles involved, scaled by $\sqrt{s}$. The analysis then reduces to the investigation of the singular structure of the integral in the $m,\mu, |\vec{q}|\to 0$ limit.

The singularities of a Feynman integral come from those regions of integration in the loop momentum space where the integrand becomes singular due to the vanishing of some of the denominators. However, in order that the integral becomes singular, it is not enough to get a singular point of the integrand. The variables of integration must also be trapped at the singular point. Otherwise by deforming the contours of integration in the complex plane, we may move away from the singular point. Singular points, where the integration variables are trapped, will be called the pinch singular points of the integral. The pinch singular points of a Feynman integral may be found out by standard analysis.[11,14] Following Ref. 11 we define the following regions of integration in the momentum space.

(1) A momentum k is called collinear to p_a if $k^+\sim p_a^+$, $k_{\perp}\sim\lambda^{1/2}p_a^+$, and $k^-\sim\lambda p_a^+$, λ being a number small compared to unity.

(2) A momentum k is called collinear to p_b, if $k^-\sim p_b^-$, $k_{\perp}\sim\lambda^{1/2}p_b^-$, and $k^+\sim\lambda p_b^-$.

(3) A momentum k is called soft if $k^{\mu}\sim\lambda\sqrt{s}$ for every μ.

(4) A momentum k is called hard if $k^{\mu}\sim\sqrt{s}$ for every μ.

With every singular point, we may associate a reduced diagram, which is obtained by contracting all the hard lines at the particular singular point. As was shown in Ref. 14, and generalized for massless particles in Ref. 11, the reduced diagram for a pinch singular point must describe a real physical process, each vertex of the reduced diagram describing a real space-time point. According to this result, the two types of reduced diagrams that may contribute to

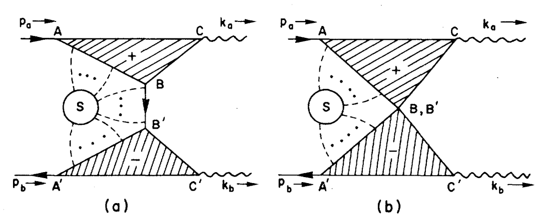


FIG. 3. Two types of reduced diagrams which may contribute to the amplitude. Here the blob marked plus contains lines moving parallel to p_a, the blob marked minus contains lines moving parallel to p_b, and the lines inside blob S, as well the gluons coming out of it, carry soft momenta. Also in (a), the exchange fermion line BB' carries soft momenta.

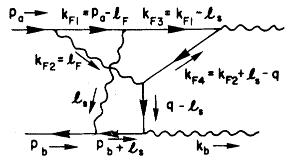


FIG. 4. Illustration of the pinching of soft-loop momenta by the jet lines. k_{Fi}'s are finite momentum lines, l_s and $(q-l_s)$ are soft lines. l_F is a finite loop momentum, l_s is a soft loop momentum.

the process we are considering are as shown in Fig. 3. The blobs marked plus and minus contain lines that are collinear to p_a and p_b, respectively. We shall call these blobs positive and negative jets, respectively. The blob marked S, as well as the lines coming out of S, are soft lines. For a real physical process, two oppositely moving jets may interact only through the exchange of soft lines [Fig. 3(a)], or they may actually meet at a single point in space-time, besides exchanging soft quanta [Fig. 3(b)].

The soft-loop momenta are usually pinched by the double poles in the soft denominators $(k^2-\mu^2+i\epsilon)^{-1}$ $[(k^2-m^2+i\epsilon)^{-1}$ for a soft-fermion line]. However, the soft-loop momenta may also be pinched by the singularities of the jet lines (jet lines have the same meaning as collinear lines). For example, in Fig. 4, consider the region where k_{F1}, k_{F2}, k_{F3}, and k_{F4} belong to the positive jet, p_b+l_s belongs to the negative jet, and l_s and $q-l_s$ are soft. l_F is a jet loop momentum and l_S is a soft-loop momentum. The denominators of the lines k_{F4} and k_{F3} give rise to singularities in the l_S^- plane at

$$l_S^- = \frac{(\vec{k}_{F2}+\vec{l}_S-\vec{q})_\perp^2+m^2-i\epsilon}{(k_{F2}+l_S)^+} - k_{F2}^- ,$$
$$l_S^- = -\frac{(\vec{k}_{F1}-\vec{l}_S)_\perp^2+m^2-i\epsilon}{(k_{F1}-l_S)^+} + k_{F1}^- , \qquad (3.1)$$

respectively. In the region considered, $(k_{F1})_\perp$, $(k_{F2})_\perp$, l_S^μ, k_{F1}^-, k_{F2}^-, m, and q are small (when scaled by $\sqrt{s}$), and k_{F1}^+ and k_{F2}^+ are finite and positive. Thus l_S^- is pinched at the origin between these two poles. In an exactly similar way, the plus component of the soft lines, attached to the jet collinear to p_b, may be pinched between the two jet lines. The importance of these extra pinches will become clear later. It will allow different components of soft momenta to scale differently, e.g., instead of having $|k^\mu|\sim\lambda\sqrt{s}$, we may have $|k^+|$, $|k_\perp|\sim\sigma\sqrt{s}$ and $|k^-|\sim\lambda\sqrt{s}$, where $\lambda \ll \sigma \ll 1$.

Now that we have found the pinch singular points of the Feynman graphs under consideration, we shall use a power-counting method, developed in Ref. 11, to estimate the degree of divergence of the integral near a pinch singular point. We shall briefly point out the main ideas that go behind this development; for more details the reader is referred to the original paper. Let us consider a line carrying momentum k, which is a part of the positive jet. In the massless limit, its Feynman denominator is given by $k^+k^- - \vec{k}_\perp^2 + i\epsilon$. Since the jet line is massless, and moving close to the $+Z$ direction, we have $k_\perp \simeq 0$, and $k^- \simeq 0$. Since the denominator depends linearly on k^- and $\vec{k}_\perp^2$, we expect to obtain maximum divergence when k^- and $\vec{k}_\perp^2$ scale to zero at the same rate. Let us call λ the common scale of all the minus components and the square of the transverse components of the positive jet momenta. Then for each jet line we get a factor of λ^{-1}. The integration volume for each jet loop scales as λ^2 (one factor of λ due to the minus-momentum integration and one due to the transverse-momenta integrations). The fermion numerators as well as the $N^{\mu\nu}(k)$ factors from the gluon propagators may give rise to some extra powers of λ in the numerator. As was shown in Ref. 11, for each three-point jet-gluon-jet fermion vertex, we get at least a factor of $\lambda^{1/2}$.

Let us ignore the presence of the soft lines for the time being and estimate the degree of divergence from the jet lines and jet loops only. We can estimate the degree of divergence from the positive jet and the negative jet separately. We shall concentrate on the positive jet only, power counting for the negative jet may be carried out in an exactly similar way. We first identify three special points on the jet in Fig. 3, the point A, where the external fermion breaks up into two or more jet lines, the point C, where two or more jet lines meet to produce the external gluon, and the point B, where either the soft exchange fermion line leaves the jet [for reduced diagrams (RD's) of type Fig. 3(a)], or the positive jet meets the negative jet [for RD's of type Fig. 3(b)]. Let γ, δ, and ϵ be the number of positive jet lines attached to the contracted vertices A, C, and B, respectively; j and l be the number of jet lines and jet loops in the positive jet, respectively; and x_α be the number of internal vertices of the positive jet with α jet lines attached to it (vertices A, B, and C are excluded in counting x_α).

As we have seen before, j jet denominators contribute a factor of $(\lambda^{-1})^j$, while l jet loops contribute a factor of $(\lambda)^{2l}$. Let us define the number $n_A(n_C)$ such that $n_A(n_C)$ is $\frac{1}{2}$ if $A\,(C)$ is a three-point vertex and is zero otherwise. Total suppression from the numerator factors then goes at least as $(\lambda^{1/2})^{n_A+n_C+x_3}$. The total power d of λ^{-1} in the integral then satisfies the inequality

$$d \le j-2l-(n_A+n_C+x_3)/2 . \qquad (3.2)$$

Now,

$$j=\frac{1}{2}\sum_{\alpha\geq 3}\alpha x_\alpha+\frac{1}{2}(\epsilon+\gamma+\delta)\ . \qquad (3.3)$$

The total number of vertices v in the positive jet (including A, B, and C) is

$$v=\sum_{\alpha\geq 3}x_\alpha+3\ . \qquad (3.4)$$

Euler's identity tells us that

$$l=j-v+1\ . \qquad (3.5)$$

Using (3.3) to (3.5) we may write (3.2) as

$$d\leq-\left[\frac{1}{2}\sum_{\alpha\geq 5}(\alpha-4)x_\alpha+\frac{1}{2}\epsilon+\frac{1}{2}\gamma+n_A+\frac{1}{2}\delta+n_C-4\right]. \qquad (3.6)$$

Now, if $\gamma=2$, $n_A=\frac{1}{2}$. If $\gamma>2$, $n_A=0$. Thus,

$$(\tfrac{1}{2}\gamma+n_A)\geq\tfrac{3}{2}\ . \qquad (3.7)$$

Similarly,

$$(\tfrac{1}{2}\delta+n_C)\geq\tfrac{3}{2}\ . \qquad (3.8)$$

Also,

$$\sum_{\alpha\geq 5}(\alpha-4)x_\alpha\geq 0\ . \qquad (3.9)$$

Hence,

$$d\leq 0 \qquad (3.10)$$

which shows that the contribution from the positive jet is at most logarithmically divergent. The equality sign in (3.10) is satisfied only if (1) $x_\alpha=0$ for $\alpha>4$, (2) $\gamma,\delta=2$ or 3, and (3) $\epsilon=2$. Also we must choose the numerator factors in such a way that they do not have more than $n_A+n_C+x_3$ powers of $\lambda^{1/2}$. This means that we have to choose as many $p^+\gamma^-$ terms from the fermion numerators and as many $\gamma_\perp$ terms from the gluon-fermion vertices as possible. This is because if we look at the expressions (2.11) for $N^{\mu\nu}(k)$, we see that N^{+-}, N^{++}, and $N^{--}\sim\lambda$, $N^{\pm i}\sim\lambda^{1/2}$, and $N^{ij}\sim 1$ $(i,j=1,2)$ if k is a collinear momentum. Hence for each γ^+ or γ^- at the gluon-fermion vertex, we get a suppression factor of $\lambda^{1/2}$ from the gluon propagator. Amongst many things, this implies that in the chain of γ matrices on the fermion line inside the positive jet, the leftmost longitudinal γ matrix (longitudinal plus or minus) must be a γ^-. There may of course be transverse γ matrices to the left of this γ^{-1} matrix, but there is no γ^+ to the left of this γ^- matrix.

Let us now compare the contribution from the RD's of the type shown in Figs. 3(a) and 3(b), ignoring the presence of soft lines, except for the single fermion exchange line in Fig. 3(a). In both the figures, the two jets give logarithmic divergence ($d=0$). However, in Fig. 3(a), we have an extra soft-fermion line which gives a contribution of $i/(\not q-m)$. Such terms are not present in RD's of the type shown in Fig. 3(b), i.e., the $i/(\not q-m)$ factor is replaced by a factor of order $1/\sqrt{s}$. Thus, since we are interested only in terms contributing in the leading power of s, RD's of the type shown in Fig. 3(a) are the only ones we shall be interested in. This conclusion does not change even after we attach soft-gluon lines to the jets, since, as we shall see, the presence of soft gluons does not increase the degree of divergence of the integral.

Let us now consider the effect of attaching soft lines to the reduced diagram. First, let us ignore the pinching of soft momenta by jet lines (as was illustrated in Fig. 4). Then all components of soft momenta scale together for maximum divergence. Let us call this scale σ. Let us now consider the effect of attaching one end of a soft-gluon line to a jet and the other end to a soft-fermion line (say). The gluon propagator gives a factor of σ^{-2}, the extra soft-fermion denominator gives a factor of σ^{-1}. If $\sigma<\lambda$, then the extra jet denominator gives a factor of λ^{-1}. The soft-loop integration volume gives a factor of σ^4. Thus the net extra factor goes as $\sigma^{-3}\lambda^{-1}\sigma^4\sim\sigma/\lambda$, which gives a suppression unless $\sigma\sim\lambda$. If, on the other hand, $\sigma>\lambda$, then all the jet denominators, through which this soft momentum flows, scale as σ^{-1} instead of λ^{-1}, and there is again an extra suppression. The only way we can avoid the extra suppression is to keep $\sigma\sim\lambda$. A similar result also holds for soft-gluon lines exchanged between the two jets. In order to avoid any extra suppression factor from the jet numerator due to the attachment of the soft lines, the soft-gluon lines must attach to the positive jet through a three-point γ^+ vertex and the negative jet through a three-point γ^- vertex. Also, not more than one soft gluon should be connected to the jet at a point (in a reduced diagram, we can, in principle, have many lines attached to the same vertex), otherwise we lose some jet denominators that we could have gotten by attaching them to different points of the jet. Finally, one can see by simple dimensional analysis that soft fermions attached to jet lines produce extra suppression factors[11]; hence we do not have any other soft-fermion line attached to the positive or the negative jet, except the soft exchange fermion line.

At this point, one may ask the following question: How small should the momenta of the soft lines be? To answer this, let us note that when soft exchange lines are present, the factor $1/(\not q-m)$ from the exchange fermion line is replaced by some term of order $1/\sigma\sqrt{s}$. Thus, in order to get a contribution in the leading power in s, we must keep the momenta $(\sigma\sqrt{s}\,)$ of the exchange lines to be of order q. Other soft lines, which are attached solely to the positive jet or solely to the negative jet, may have their momenta lying anywhere between m and $\sqrt{s}$.

Let us now consider the effect of pinching the soft momenta from the jet lines. As we have seen before, if a soft line of momentum k is attached to the upper jet, then k^- may be pinched between the two jet denominators and scale as λ, while the other components of k may be larger ($\sim\sigma$ say). If we now again consider the effect of attaching one end of such a soft line to a jet line, and the other end to a soft fermion line, the extra contribution due to the extra soft denominators goes as σ^{-3}, while the extra jet denominator gives a factor of λ^{-1}. The integration volume due to the extra soft-loop integration goes as $\sigma^3\lambda$. Thus there is no extra suppression factor. A similar result holds when the two ends of the soft line are attached to the two jets. This result holds as long as the transverse components of the soft momenta are less than the transverse components of the jet momenta, i.e., $\sigma \lesssim \lambda^{1/2}$. As before, for the exchange lines σ must be of order $q/\sqrt{s}$. This shows that the minus components of the momenta of the exchange lines, attached to the positive

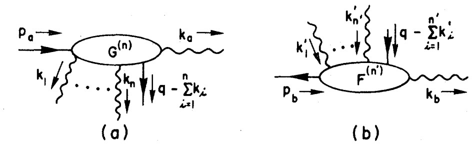


FIG. 5. The Green's functions $G^{(n)}$ and $F^{(n')}$.

jet, and the plus components of the momenta of the exchange lines attached to the negative jet, may go down to $\sqrt{s}\times q^2/s \sim m^2/\sqrt{s}$, without changing the degree of divergence of the integral, while the transverse components of the momenta must be of order q.

The above analysis gives us the regions of integration which contribute to the amplitude in the leading power in s. Let us now define by $G^{(n)}(p_a,q,\epsilon_a,k_1,\ldots,k_n)$ and $F^{(n')}(p_b,q,\epsilon_b,k_1',\ldots,k_{n'}')$ the Green's functions shown in Figs. 5(a) and 5(b), respectively. Here, q, $k_1,\ldots,k_n$, and $k_1',\ldots,k_{n'}'$ are all soft momenta. $G^{(n)}$ and $F^{(n')}$ are one-particle irreducible in the external soft-gluon and the soft-fermion lines. In $G^{(n)}$, the external soft gluons carry polarization $\frac{1}{2}(1,0,0,-1)$; in $F^{(n')}$, the external soft gluons carry polarization $\frac{1}{2}(1,0,0,1)$. Then the contribution from an RD of the type shown in Fig. 3(a) may be written in the form

$$\int d^4l_1 \cdots d^4l_{n_s} F^{(n')}(p_b,q,\epsilon_b,k_1',\ldots,k_{(n')}')S(q,l_1,\ldots,l_{n_s})G^{(n)}(p_a,q,\epsilon_a,k_1,\ldots,k_n) \tag{3.11}$$

if the momenta inside the positive and the negative jets in Fig. 3(a) are fully integrated over. In (3.11), $l_1,\ldots,l_{n_s}$ are the n_s independent soft-loop momenta, $k_1',\ldots,k_{n'}'$ are the momenta of the n' soft lines attached to the negative jet, $k_1,\ldots,k_n$ are the momenta of the n soft lines attached to the positive jet (k_i's and k_i''s are linear combinations of the l_i's), and S is the total contribution from the soft numerator and the denominator factors. Now, by fully integrating over the internal loop momenta of F and G, we include some extra contribution in (3.11). For example, if we take the RD of Fig. 6(a) and fully integrate over the internal loop momenta of the posi-

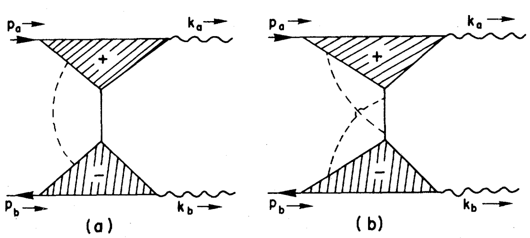


FIG. 6. Two typical RD's contributing to the amplitude. The broken lines are soft lines.

tive and the negative jets, we also include the contribution from the RD of Fig. 6(b). However, these extra contributions may also be expressed as an integral of the form (3.11). As a result, the sum of all the Feynman diagrams in the leading power in s may be expressed as a sum of expressions like (3.11). A prescription for systematically expressing the full amplitude in terms of integrals of the form (3.11) is given in Appendix A using the "tulip-garden" formalism of Collins and Soper.[15] In each of these integrals, the integrations over the l_i's lie only in the soft region.

In general, we gain two powers of $\ln s$ for each loop integration in a Feynman diagram, one due to the collinear divergence, and the other due to the soft divergence. Thus, from an r-loop graph, we can get a maximum of $2r$ powers of $\ln s$. The nonfactorized diagrams, however, lose some of the logarithms, since, according to Fig. 3(a), there must be one soft-loop momentum, which is constrained to be of order q, and hence, cannot produce a $\ln s$ term after integration. Only the factorized diagrams of the form shown in Fig. 2 do not need to have any soft-gluon exchange and can have $2r$ powers of $\ln s$ in r-loop order. But as already mentioned in the Introduction, explicit calculations[2–5] in low orders in perturbation theory show that, when we sum the contribution from all the Feynman graphs, all the double logarithms cancel, and the largest term in the r-loop order is proportional to $\ln^r s$. Hence we should define the leading-logarithm approximation as the sum of terms with not less than r logarithms of s in r-loop order. Then we must consider the contribution from the factorized as well as the nonfactorized diagrams, even in the leading-logarithm approximation.

In (3.11), if we first integrate over the internal loop momenta of $G^{(n)}$ and $F^{(n')}$, and look at the integrand as a function of l_i's and the external momenta, the potential sources of the $\ln(s/m^2)$ in the final integral are the following. There will be explicit logarithms of (p_a^+/m) from $G^{(n)}$ and (p_b^-/m) from $F^{(n')}$. Besides these logarithms, since the integral receives contribution from small-k_i^- and small-$k_i'^+$ regions, there may be logarithms of m/k_i^- and $m/k_i'^+$ present in $G^{(n)}$ and $F^{(n')}$ respectively, which are converted to logarithms of p_a^+/m and p_b^-/m, respectively, after the k_i^- and $k_i'^+$ integrals are done. Thus the analysis of the logarithms seems to be a complicated multivariable problem. However, we shall show in Sec. V that expression (3.11) may be expressed as a sum of integrals of the form

$$\sum_{n,n'}\int d^2k_{1\perp}\cdots d^2k_{n\perp}d^2k'_{1\perp}\cdots d^2k'_{n'\perp}\Phi^{(n')}(p_b,q,\epsilon_b,k'_{1\perp},\ldots,k'_{n'\perp})\times\tilde{S}^{(n,n')}(q,k_{1\perp},\ldots,k_{n\perp},k'_{1\perp},\ldots,k'_{n'\perp})\Gamma^{(n)}(p_a,q,\epsilon_a,k_{1\perp},\ldots,k_{n\perp})\,, \quad (3.12)$$

where

$$\Gamma^{(n)}(p_a,q,\epsilon_a,k_{1\perp},\ldots,k_{n\perp}) =\int_{-M}^{M}dk_1^-\cdots\int_{-M}^{M}dk_n^-G^{(n)}(p_a,q,\epsilon_a,k_1^-,\ldots,k_n^-,k_{1\perp},\ldots,k_{n\perp},k_1^+=0,\ldots,k_n^+=0) \quad (3.13)$$

and

$$\Phi^{(n')}(p_b,q,\epsilon_b,k'_{1\perp},\ldots,k'_{n'\perp}) =\int_{-M}^{M}dk_1'^+\cdots\int_{-M}^{M}dk_{n'}'^+F^{(n')}(p_b,q,\epsilon_b,k_1'^+,\ldots,k_{n'}'^+,k'_{1\perp},\ldots,k'_{n'\perp},k_1'^-=0,\ldots,k_{n'}'^-=0)\,, \quad (3.14)$$

where M is any arbitrary parameter of order m. The final result is independent of the choice of M, although, in (3.12), $\Phi^{(n')}$, $\tilde{S}$, and $\Gamma^{(n)}$ may individually depend on M. In (3.13) and (3.14), instead of cutting off the k_i^- and $k_i'^+$ integrals sharply at M, we could also have defined $\Gamma^{(n)}$'s and $\Phi^{(n')}$'s as integrals of $G^{(n)}$'s and $F^{(n')}$'s with a smooth cutoff on the k_i^- and $k_i'^+$ integrals. $\tilde{S}$ is a calculable function of its arguments. After we prove (3.12), the analysis of the s dependence of the full amplitude will essentially reduce to the analysis of the p_a^+ and p_b^- dependence of $\Gamma^{(n)}$ and $\Phi^{(n')}$, respectively.

IV. STUDY OF $G^{(n)}$

We have seen in Sec. III that the study of the fermion-antifermion annihilation amplitude in the Regge limit reduces to the analysis of the Green's functions $G^{(n)}$ and $F^{(n')}$. For definiteness, we shall limit ourselves to the study of $G^{(n)}$, since $F^{(n')}$ may be studied exactly in the same way. In Sec. IV A we shall study the behavior of $G^{(n)}$ when the minus component of the momentum of one of the external soft gluons is of order m. In Sec. IV B we shall find the change in $G^{(n)}$ under an infinitesimal boost along the Z axis. This will later be useful to us to derive a differential equation involving the Green's functions. In Sec. IV C, we shall analyze the dependence of $G^{(n)}$ on the plus components of the external soft momenta.

A. Behavior of $G^{(n)}$ when the minus component of one of its external soft momenta is of order m

Let us consider the Green's function $G^{(n)}(p_a,q,\epsilon_a,k_1,\ldots,k_n)$, and let us assume for definiteness, that k_n^- is of order m. Following a technique due to Grammer and Yennie,[16] we decompose the γ^+ vertex, at which the nth gluon is attached to the jet line, as

$$-ig\gamma^+/2=-ig[G^{+\sigma}(k_n)+K^{+\sigma}(k_n)]\gamma_\sigma/2\,, \quad (4.1)$$

where

$$K^{\lambda\sigma}(k)=p_a^\lambda k^\sigma/(p_a\cdot k-i\epsilon)\,, \quad (4.2)$$

$$G^{\lambda\sigma}(k)=g^{\lambda\sigma}-K^{\lambda\sigma}(k)\,. \quad (4.3)$$

The K gluon carries a polarization proportional to k^μ. Hence, when we sum over all insertions of the K gluon to $G^{(n)}$, the K gluon decouples from the rest

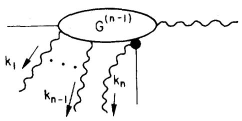


FIG. 7. Sum over all insertions of the K part of the nth gluon.

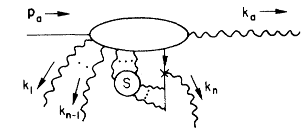


FIG. 8. Regions of integration contributing to the G part of the nth gluon. All the lines inside the blob S, as well as the gluon lines coming out of it, are soft.

of the diagram due to the Ward identity, and the contribution has the form

$$-\frac{g}{k_n^- - i\epsilon} G^{(n-1)}(p_a, q, \epsilon_a, k_1, \ldots, k_{n-1}) \quad (4.4)$$

which is graphically represented as in Fig. 7. The circled vertex in Fig. 7 carries a factor of $-g/(k_n^- - i\epsilon)$.

For the G part, we see that G^{+-} is identically zero . Hence we cannot get a γ^+ term at the vertex where the G gluon is attached inside $G^{(n)}$. From the analysis of Sec. III, we know that the soft gluon must attach to the jet gluons through a γ^+ vertex. Thus the fermion line to which the G gluon is attached cannot be a jet line; hence it must be soft. Thus, if k_n^- is of order m, and we consider the contribution from the G part of the nth gluon, the region of integration, which will contribute to the amplitude in the leading power in s, looks like Fig. 8. The crossed vertex represents a G vertex. This picture, however, is not valid if $|k_n^-| \ll m$, since then the large factor $k_\perp / k^-$ in $G^{+\perp}$ may compensate for the suppression due to the attachment of the G part to a jet line through a $\gamma_\perp$ vertex.

B. Variation of $G^{(n)}$ under an infinitesimal boost along the Z axis

Let us consider an infinitesimal boost along the Z axis, which changes a momentum k to k' as

$$k'^+ = \gamma k^+(1+\beta) ,$$
$$k'^- = \gamma k^-(1-\beta) , \quad (4.5)$$
$$k'_\perp = k_\perp ,$$

where

$$\gamma = (1-\beta^2)^{-1/2} = 1 + O(\beta^2) . \quad (4.6)$$

Let us write the transformation (4.5) as

$$k'^\mu = \Lambda^\mu{}_{\mu'} k^{\mu'} . \quad (4.5')$$

Any tensor $T^{\mu_1 \cdots \mu_n}$ then transforms to

$$T'^{\mu_1 \cdots \mu_n} = \Lambda^{\mu_1}{}_{\mu_1'} \cdots \Lambda^{\mu_n}{}_{\mu_n'} T^{\mu_1' \cdots \mu_n'} . \quad (4.7)$$

Let S be the representation of this Lorentz transformation in Dirac space such that

$$S k\cdot\gamma S^{-1} = k'\cdot\gamma . \quad (4.8)$$

Then,

$$S u(p_a) = u(p_a') . \quad (4.9)$$

The total contribution to $G^{(n)}$ in s-loop order from all the Feynman diagrams may be expressed abstractly as

$$G^{(n)}(p_a, q, \epsilon_a, k_1, \ldots, k_n)$$
$$= \sum_F \int \left[\prod_{i=1}^{s} \frac{d^4 l_i}{(2\pi)^4} \right] \mathcal{F}_{\mu_1 \cdots \mu_r \nu_1 \cdots \nu_r}(k_1, \ldots, k_n, l_1, \ldots, l_s, p_a, q, \epsilon_a) [Z_2(p_a)]^{-1/2} (\not{p}_a - m) u(p_a)$$
$$\times \left[\prod_{j=1}^{r} N^{\mu_j \nu_j}(q_j) \Big/ (q_j{}^2 - \mu^2 + i\epsilon) \right] , \quad (4.10)$$

where $\sum_F$ denotes the sum over all Feynman diagrams, including self-energy insertions on the external fermion p_a. $l_1, \ldots, l_s$ are the independent loop momenta, $[Z_2(p_a)]^{-1/2}$ is the fermion wave-function renormalization constant, $N^{\mu_j \nu_j}(q_j)/(q_j{}^2 - \mu^2 + i\epsilon)$ are the gluon propagators of the r internal gluons present in the diagram (q_j's are linear combinations of the l_i's), and $\mathcal{F}_{\mu_1 \cdots \mu_r \nu_1 \cdots \nu_r}$ is the contribution to the Feynman integrand from all the fermion loops and lines. Hence $\mathcal{F}$ is a matrix in the Dirac space. The wave-function renormalization constant of the external gluon k_a is included in $\mathcal{F}$. $\mathcal{F}$ has a Lorentz-covariant expression in terms of its arguments, except for the presence of the n γ^+ matrices at the vertices where the n external soft gluons attach to the fermion line. Noting that

$$S\gamma^+S^{-1}=(1-\beta)\gamma^+ +O(\beta^2)\ , \tag{4.11}$$

we may write

$$S\mathcal{F}_{\mu_1\cdots\mu_r\nu_1\cdots\nu_r}(k_i,l_j,p_a,q,\epsilon_a)S^{-1}\left[\prod_{j=1}^{r}N^{\mu_j\nu_j}(q_j)\right]$$
$$=\mathcal{F}_{\mu_1\cdots\mu_r\nu_1\cdots\nu_r}(k_i',l_j',p_a',q,\epsilon_a)(1-\beta)^n\left[\prod_{j=1}^{r}N'^{\mu_j\nu_j}(q_j)\right]\ , \tag{4.12}$$

where N' is related to N by the tensor transformation law (4.7). Thus we may write

$$SG^{(n)}(p_a,q,\epsilon_a,k_1,\ldots,k_n)(1-\beta)^{-n}$$
$$=\sum_F\int\left[\prod_{i=1}^{s}\frac{d^4l_i'}{(2\pi)^4}\right]\mathcal{F}_{\mu_1\cdots\mu_r\nu_1\cdots\nu_r}(k_1',\ldots,k_n',l_1',\ldots,l_n',p_a',q,\epsilon_a)S[Z_2(p_a)]^{-1/2}S^{-1}(\not p_a'-m)u(p_a')$$
$$\times\prod_{j=1}^{r}[N'^{\mu_j\nu_j}(q_j)/(q_j'^2-\mu^2+i\epsilon)] \tag{4.13}$$

using the facts $d^4l_i=d^4l_i'$ and $q_j^2=q_j'^2$.

$G^{(n)}$ at p_a', $k_1',\ldots,k_n'$ may be evaluated by replacing the external variables by the corresponding primed variables on the right-hand side of (4.10). For the internal variables, we can change the variable name from l_i and q_j to l_i' and q_j'; q_j''s are then the same linear combinations of l_i''s as q_j's are of l_i's. Thus we have

$$G^{(n)}(p_a',q,\epsilon_a,k_1',\ldots,k_n')$$
$$=\sum_F\int\left[\prod_{i=1}^{s}\frac{d^4l_i'}{(2\pi)^4}\right]\mathcal{F}_{\mu_1\cdots\mu_r\nu_1\cdots\nu_r}(k_1',\ldots,k_n',l_1',\ldots,l_n',p_a',q,\epsilon_a)[Z_2(p_a')]^{-1/2}u(p_a')$$
$$\times\prod_{j=1}^{r}[N^{\mu_j\nu_j}(q_j')/(q_j'^2-\mu^2+i\epsilon)]\ . \tag{4.14}$$

Now, $[Z_2(p_a')]^{-1/2}$ and $S[Z_2(p_a)]^{-1/2}S^{-1}$ differ from each other by a term of order β. Also $N^{\mu\nu}(q')$ and $N'^{\mu\nu}(q)$ differ from each other by a term of order β. Let us define

$$\beta S^{\mu\nu}(q')=N^{\mu\nu}(q')-N'^{\mu\nu}(q)\ . \tag{4.15}$$

Then, up to terms of order β, the difference between (4.13) and (4.14) may be written as

$$G^{(n)}(p_a',q,\epsilon_a,k_1',\ldots,k_n')-(1-\beta)^{-n}SG^{(n)}(p_a,q,\epsilon_a,k_1,\ldots,k_n)$$
$$=\sum_F\int\left[\prod_{i=1}^{s}\frac{d^4l_i'}{(2\pi)^4}\right]\mathcal{F}_{\mu_1\cdots\mu_r\nu_1\cdots\nu_r}(k_i',l_j',p_a',q,\epsilon_a)\{[Z_2(p_a')]^{-1/2}-S[Z_2(p_a)]^{-1/2}S^{-1}\}$$
$$\times(\not p_a'-m)u(p_a')\prod_{j=1}^{r}\left[\frac{N^{\mu_j\nu_j}(q_j')}{q_j'^2-\mu^2+i\epsilon}\right]$$
$$+\sum_F\int\left[\prod_{i=1}^{s}\frac{d^4l_i'}{(2\pi)^4}\right]\mathcal{F}_{\mu_1\cdots\mu_r\nu_1\cdots\nu_r}(k_i',l_j',p_a',q,\epsilon_a)[Z_2(p_a')]^{-1/2}(\not p_a'-m)u(p_a')$$
$$\times\left[\sum_{i=1}^{r}\frac{\beta S^{\mu_i\nu_i}(q_i')}{q_i'^2-\mu^2+i\epsilon}\prod_{\substack{j=1\\ j\neq i}}^{r}\left[\frac{N^{\mu_j\nu_j}(q_j')}{q_j'^2-\mu^2+i\epsilon}\right]\right]+O(\beta^2)\ . \tag{4.16}$$

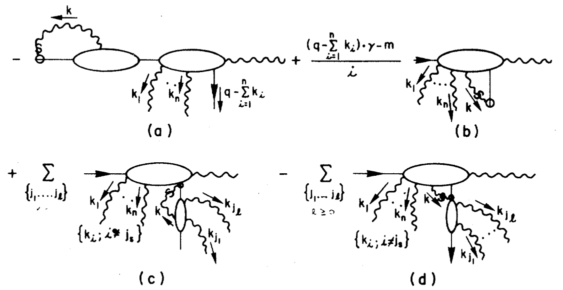


FIG. 9. Sum over all insertions of the S gluon. $\sum_{\{j_1,\ldots,j_l\}}$ implies sum over all choices of the set $\{j_1,\ldots,j_l\}$ out of $1,\ldots,n$. The propagators marked S are proportional to $S^\mu(k)/(k^2-\mu^2+i\epsilon)$, μ being the Lorentz index carried by the uncircled end. The circled vertex just gives a factor of g.

The second term on the right-hand side of the above equation may be interpreted in the following way. Let us define by S gluon a gluon with propagator $S^{\mu\nu}(q)/(q^2-\mu^2+i\epsilon)$. Then this term is the sum of all Feynman diagrams where one of the gluons is an S gluon, the rest being ordinary Coulomb gauge gluons. It is straightforward to calculate $S^{\mu\nu}$ using Eqs. (4.15). It has the form

$$S^{\mu\nu}(k)=S^\mu(k)k^\nu+S^\nu(k)k^\mu\ , \tag{4.17}$$

where

$$\begin{aligned} S^+(q)&=(q^-\vec{q}_\perp^{\,2}+q^3q^+q^-)/(\vec{q}^{\,2})^2\ ,\\ S^-(q)&=(-q^+\vec{q}_\perp^{\,2}+q^3q^+q^-)/(\vec{q}^{\,2})^2\ ,\\ S^j(q)&=-q^0q^3q^j/(\vec{q}^{\,2})^2,\ \ j=1,2\ . \end{aligned} \tag{4.18}$$

Since the S gluon carries polarization proportional to its momentum, in order to sum over all possible S-gluon insertions in a given Feynman diagram, we may use the Ward identity. If we take into account the fact that $G^{(n)}$ includes only those diagrams which are one-particle irreducible in the soft-fermion line, the sum over all insertions of the S gluon will be given by the diagrams of Fig. 9 (ignoring terms which do not have poles at $\not{p}_a=m$). Of these, Fig. 9(a) may be shown to give a contribution, exactly opposite to the first term on the right-hand side of Eq. (4.16),[17] and these two contributions cancel. Thus we are left with the contributions from Figs. 9(b)–9(d). Note that the circled vertices marked S carry a factor of g, as opposed to the circled vertices coming from the K gluon which carry a factor of $-g/(k^--i\epsilon)$.

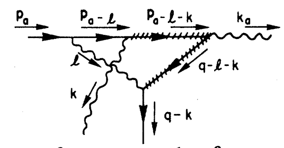


FIG. 10. Flow of an external soft momentum inside a diagram.

To analyze the contribution from these diagrams, we need to know some properties of the S gluon. Suppose the S gluon is a part of the jet, so that its negative momentum scales as λ, and transverse momenta scale as $\lambda^{1/2}$. Then, we can see from Eq. (4.18) that S^-, $S^+\sim\lambda$ and $S_\perp\sim\lambda^{1/2}$. Thus the S gluon carries extra suppression factor compared to the ordinary Coulomb gauge gluons in the jetlike region. This shows that the S gluon cannot be the part of a jet. In Figs. 9(b) and (9d), it cannot be hard either, since $S^\mu(k)$ has three powers of k in the denominator, as opposed to $N^{\mu\nu}(k)$, which has only two powers of k in the denominator. Thus it must be soft.

We shall use these results in our later analysis.

C. Analysis of $\partial G^{(n)}/\partial k_i^+$

In this subsection we shall analyze the quantity $k_i^+\partial G^{(n)}/\partial k_i^+$ (the factor of k_i^+ makes the quantity have the same dimension as $G^{(n)}$). For a given graph in $G^{(n)}$, we may label the internal loop momenta in such a way that k_i flows along a particular path inside the graph (e.g., in Fig. 10, a possible choice of the flow of the momentum k_i is indicated by the shaded lines). Then, if we take the $\partial/\partial k_i^+$ operator inside the integral, it will act on the shaded lines only. Let us denote the operation of $k_i^+\partial/\partial k_i^+$ on any part of the path of k_i^+ by a cross on that part. If $P+k_i$ is the momentum flowing through this part (P is a linear combination of the other external momenta and the loop momenta), then

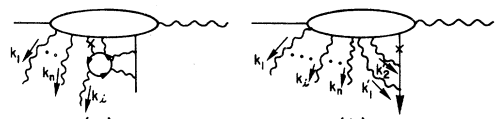
$$k_i^+\frac{\partial}{\partial k_i^+}\frac{(P+k_i)\cdot\gamma+m}{(P+k_i)^2-m^2+i\epsilon}=-\frac{k_i^+(P^-+k_i^-)}{[(P+k_i)^2-m^2+i\epsilon]^2}[(P+k_i)\cdot\gamma+m]+\frac{k_i^+}{P^+}\frac{P^+\gamma^-}{(P+k_i)^2-m^2+i\epsilon}\ . \tag{4.19}$$

Since the denominator is $\gtrsim(P^++k_i^+)(P^-+k_i^-)$, we see that on the right-hand side of (4.19), each term is suppressed relative to the original undifferentiated term by a factor of k_i^+/P^+. This is expected, since k_i^+ always appears in the combination $k_i^++P^+$ in the propagator of the line $P+k_i$, and the dependence of the propagator on k_i^+ will be negligible unless k_i^+ is of order P^+. This analysis shows that in order to get a nonsuppressed contribution to $k_i^+\partial G^{(n)}/\partial k_i^+$, the crossed line must carry positive momentum of order k_i^+, i.e., it must be soft.

A more careful analysis shows that it is not enough to have only the crossed line to be soft. For example, if both ends of the crossed line are attached to jet lines, then, we can first replace the $\partial/\partial k_i^+$ operation on the line carrying momentum $P+k_i$ by $\partial/\partial P^+$ operation, and then integrate the P^+ integral by parts, so that the $\partial/\partial P^+$ operator now acts on the rest of the integrand. Such a contribution is suppressed, since the contribution from the jet lines will be insensitive to P^+. Similar suppression occurs from any region of integration where the crossed line cannot be continuously connected to the point where the momentum k_i enters the graph, or the point where the soft-fermion line, carrying momentum $q-\sum k_j$, leaves the graph, by a set of soft lines. Figures 11(a) and 11(b) show us typical regions of integration which give nonsuppressed contribution to $\partial G^{(n)}/\partial k_i^+$. All the lines, shown explicitly in these graphs, are constrained to carry soft momenta. These look like soft-loop integrals of some $G^{(n')}$'s which have less number of loops than the original $G^{(n)}$. We must, however, be careful about counting the graphs. In Appendix B we have shown how to systematically express the contribution to $k_i^+\partial G^{(n)}/\partial k_i^+$ in the form

$$\sum_{n'}\int f_+^{(n')}(k_1,\ldots,k_n,k'_1,\ldots,k'_{n'})\times G^{(n')}(p_a,q,\epsilon_a,k'_1,\ldots,k'_{n'})d^4k'_1\cdots d^4k'_{n'}\,, \tag{4.20}$$

where $f_+^{(n')}$ is a function which goes down sufficiently rapidly as $k'_i\to\infty$ so as to restrict the k'_i integrals in (4.20) in the soft region. The Green's functions $G^{(n')}$ have less number of loops than the original function $G^{(n)}$. The function $f_+^{(n')}$ may contain factors of $\delta(k'_l-k_j)$, which set some of the k'_l in $G^{(n')}$ to be equal to some of the k_j's.



(a) (b)

FIG. 11. Typical contributions to $k_i^+\partial G^{(n)}/\partial k_i^+$. All the lines outside the blob, shown explicitly in these figures, are constrained to be soft.

V. EXPRESSING THE FULL AMPLITUDE IN TERMS OF $\Gamma^{(n)}$'s AND $\Phi^{(n)}$'s

In Appendix A we show that the full amplitude may be expressed as a sum of terms of the form given in Eq. (3.11). In this section, we shall show that a term of the form (3.11) may be expressed as a sum of the integrals given in Eq. (3.12). To do this, let us first look at the k_i^- integrals involving the $G^{(n)}$'s in (3.11). The integral may be written as

$$\int\prod_{i=1}^{n}dk_i^-f(k_1^-,\ldots,k_n^-,\ldots)\times G^{(n)}(p_a,q,\epsilon_a,k_1,\ldots,k_n)\,, \tag{5.1}$$

where the ellipsis in the argument of f denotes various other momenta and mass parameters, on which the function f may depend. f has a smooth limit as $k_i^-\to 0$, i.e., f is independent of k_i^- if $|k_i^-|\ll m$. We shall show that (5.1) may be expressed completely in terms of the functions $\Gamma^{(n)}$. We first express f as

$$\begin{aligned}f(k_1^-,\ldots,k_n^-)=&f(k_1^-=0,\ldots,k_n^-=0)\prod_{i=1}^{n}\theta(M-|k_i^-|)\\&+\sum_{i=1}^{n}[f(k_1^-,\ldots,k_i^-,k_{i+1}^-=0,\ldots,k_n^-=0)\\&-f(k_1^-,\ldots,k_{i-1}^-,k_i^-=0,\ldots,k_n^-=0)\theta(M-|k_i^-|)]\prod_{j=i+1}^{n}\theta(M-|k_j^-|)\,.\end{aligned} \tag{5.2}$$

The first term on the right-hand side of (5.2), when substituted in (5.1), gives

$$f(k_1^-=0,\ldots,k_n^-=0)\int_{-M}^{M}dk_1^-\cdots\int_{-M}^{M}dk_n^-G^{(n)}(p_a,q,\epsilon_a,k_1,\ldots,k_n)\,, \tag{5.3}$$

which is almost in the form of the right-hand side of (3.13), except for the fact that in (3.13), the $G^{(n)}$'s are evaluated at $k_i^+=0$, whereas in (5.3) they are still evaluated at finite k_i^+. We shall show later how (5.3) may be reduced to the form (3.13). The $i=1$ term on the right-hand side of (5.2) vanishes in the limit $|k_1^-|\ll M$. Thus, if we substitute this term in (5.1), the k_1^- integral will receive contribution only from the $|k_1^-|\sim m$ region. In this region we may apply the Grammer-Yennie decomposition to the k_1 gluon. The K part factorizes and the integral looks like

$$\left[\int dk_1^-[f(k_1^-,k_2^-=0,\ldots,k_n^-=0)-\theta(M-|k_1^-|)f(k_1^-=0,\ldots,k_n^-=0)]\frac{(-g)}{k_1^- -i\epsilon}\right] \times\int_{-M}^{M}dk_2^-\cdots\int_{-M}^{M}dk_n^- G^{(n-1)}(p_a,q,\epsilon_a,k_2,\ldots,k_n)\ . \quad (5.4)$$

For the G part of the k_1 gluon, the contribution comes from the regions of integration shown in Fig. 8. This can again be expressed in terms of integrals of the form (5.1), involving some $G^{(n')}$ with less number of loops. We can then repeat the whole procedure mentioned so far in this section with this new integral. The successive terms in the summation in (5.2) may be analyzed in the same way, and we may finally express (5.1) in terms of the integrals

$$\int_{-M}^{M}dl_1^-\cdots\int_{-M}^{M}dl_{n'}^- G^{(n')}(p_a,q,\epsilon_a,l_1,\ldots,l_{n'})\ . \quad (5.5)$$

Next we must show how (5.5) may be expressed in terms of integrals of $G^{(n')}$'s at $l_i^+=0$. To do this, we express $G^{(n')}$ in (5.5) as

$$G^{(n')}(p_a,q,\epsilon_a,l_1^-,\ldots,l_{n'}^-,l_{1\perp},\ldots,l_{n'\perp},l_1^+=0,\ldots,l_{n'}^+=0)$$
$$+\sum_{i=1}^{n'}\int_0^{l_i^+}dl_i'^+\frac{\partial}{\partial l_i'^+}G^{(n)}(p_a,q,\epsilon_a,l_{1\perp},\ldots,l_{n'\perp},l_1^-,\ldots,l_{n'}^-,l_1^+,\ldots,l_{i-1}^+,l_i'^+,l_{i+1}^+=0,\ldots,l_n^+=0)\ . \quad (5.6)$$

The first term in (5.6), when substituted into (5.5), has the desired form (3.13). As shown in Appendix B, contribution to the other terms in (5.5) may be expressed as soft-loop integrals of $G^{(n)}$'s with less number of loops. If we substitute this in (5.5), we may express the contribution in terms of integrals of the form (5.1). We now repeat all the steps, mentioned so far in this section, with these new integrals. Proceeding in this manner, we may finally express (5.1) in terms of integrals of $\Gamma^{(n)}$'s defined in (3.13).

We can similarly look at the $k_i'^+$ integrals in (3.11), and express it in terms of $\Phi^{(n')}$ defined in (3.14). The full amplitude may then be expressed as a sum of terms of the form given in (3.12). The function $\widetilde{S}$ is obtained by integrating over all the soft-loop momenta in (3.11) and also those which appear during the reduction of (3.11) into the form (3.12), except the momenta $k_{1\perp},\ldots,k_{n\perp}$, $k_{1\perp}',\ldots,k_{n'\perp}'$.

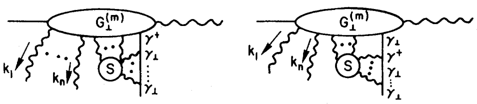
VI. COUNTING THE NUMBER OF LOGARITHMS IN $\Gamma^{(n)}$ AND $\Phi^{(n')}$

In this section we shall count the number of logarithms of p_a^+/m in $\Gamma^{(n)}$, in a given loop order. The number of logarithms of p_b^-/m in $\Phi^{(n')}$ may be counted in an exactly similar way; hence we shall not carry out the counting for $\Phi^{(n')}$ explicitly. We shall show that in r-loop order, $\Gamma^{(n)}$ has at most r logarithms of p_a^+/m; similarly, in r'-loop order, $\Phi^{(n')}$ has at most r' logarithms of p_b^-/m. Then if we consider the expression (3.12), the maximum number of logarithms of s in this from an l-loop graph will be given by $l-n_s$, n_s being the minimum number of soft exchange loops that the graph must have. This shows that in the leading-logarithm approximation, only the factorized diagrams contribute to the amplitude, since they are the only diagrams for which $n_s=0$. In the next-to-the-leading-logarithm approximation, the contribution comes from the factorized diagrams, as well as the diagrams which have a one-gluon–one-fermion intermediate state in the t channel, and so on.

The result mentioned in the previous paragraph is a nontrivial result, since, as we have mentioned before, each individual diagram contributing to $G^{(n)}$ will contribute $2r$ logarithms of p_a^+/m to $\Gamma^{(n)}$ in r-loop order. To prove the above result, we shall use the method of induction. First, following Ref. 5, let us break up $\Gamma^{(n)}$ as

$$\Gamma^{(n)}=\Gamma_\perp^{(n)}u(p_a)+(\gamma^+/2)\Gamma_+^{(n)}u(p_a)\ , \quad (6.1)$$

where $\Gamma_\perp^{(n)}$ and $\Gamma_+^{(n)}$ are products of transverse γ matrices only. To see how such a decomposition is possible, let us note that, given a string of $\gamma^\pm$ and $\gamma_\perp$ matrices from the internal fermion numerators and the gluon-fermion vertices in a given graph contributing to $G^{(n)}$, we can always bring it into a sum of terms with the γ^- matrices at the extreme right and the γ^+ matrices at the extreme left, with the transverse γ matrices in the middle, using Eqs. (2.7). γ^- acting on $u(p_a)$ may always be expressed in terms of $p_{a\perp}\cdot\gamma$ and $p_a^-\gamma^+$ acting on $u(p_a)$, by using the Dirac equation. This new γ^+ may again be commuted to the extreme left through the transverse γ matrices, using (2.7). We are then left with terms which are



(a) (b)

FIG. 12. Contribution to $G_+^{(n)}$. The lines inside the blob S, as well as all the gluon lines coming out of the blob S, carry soft momenta.

either products of transverse γ matrices only, or products of transverse γ matrices, multiplied by a γ^+ at the left. There cannot be more than one γ^+ at the left, since $(\gamma^+)^2=0$. This way $\Gamma^{(n)}$ can be brought into the form (6.1).

We shall show that, in r-loop order, (1) $\Gamma_\perp^{(n)}$ does not have more than r logarithms of p_a^+/m for any n, and (2) $\Gamma_+^{(n)}$ does not have more than $(r-1)$ logarithms of p_a^+/m for any n.

We shall assume that the above result is valid up to $(r-1)$-loop order, and then show that it is valid up to r-loop order. We shall start with $\Gamma_+^{(n)}$. To analyze this, we draw the reader's attention to the discussion after Eq. (3.10), where we showed that if all lines in $G^{(n)}$ carry jetlike momenta, the left-most longitudinal γ matrix in the corresponding Feynman integrand must be a γ^-. Hence only $G_\perp^{(n)}$ receives contribution from such a configuration. So, the left-most γ^+ matrix in $(\gamma^+/2)G_+^{(n)}$ must come from the numerator of a soft-fermion line, or a soft-fermion—soft-gluon vertex. Thus $G_+^{(n)}$ must receive contribution from a configuration of the form shown in Fig. 12. These contributions, however, are soft-loop integrals of $G_\perp^{(n)}$'s with less than r loops, and hence, using the method of Sec. V, may be expressed as transverse-momentum integrals of $\Gamma_\perp^{(n)}$'s with less than r-loops. This, by assumption, has less than r logarithms. Hence, in r-loop order, $\Gamma_+^{(n)}$ has less than r logarithms.

There is an important consequence of the results obtained in the previous paragraph. Since $\Gamma_+^{(n)}$ may be expressed as transverse-momentum integrals of $\Gamma_\perp^{(m)}$'s (and similarly $\Phi_-^{(n')}$ as transverse-momentum integrals of $\Phi_\perp^{(m')}$'s), we may reexpress (3.12) as

$$\int d^2k_{1\perp}\cdots d^2k_{n\perp}d^2k'_{1\perp}\cdots d^2k'_{n'\perp}\Phi_\perp^{(n')}(p_b,q,\epsilon_b,k'_{1\perp},\ldots,k'_{n'\perp})\times\hat{S}(q,k_{1\perp},\ldots,k_{n\perp},k'_{1\perp},\ldots,k'_{n\perp})\Gamma_\perp^{(n)}(p_a,q,\epsilon_a,k_{1\perp},\ldots,k_{n\perp})\,, \tag{6.2}$$

where $\hat{S}$ is some new function of $k_{i\perp}$ and $k'_{i\perp}$.

Next, let us turn towards counting the number of logarithms in $\Gamma_\perp^{(n)}$ in r-loop order. We start with the definition (3.13) and make a change of variables from k_i^- to $k_i'^-$ according to (4.5). Thus, we may write

$$\Gamma_\perp^{(n)}(p_a,q,\epsilon_a,k_{1\perp},\ldots,k_{n\perp})=\int_{k_i'^-=-M(1-\beta)}^{M(1-\beta)}\left[\prod_{i=1}^{n}dk_i'^-\right](1-\beta)^{-n}G_\perp^{(n)}(p_a,q,\epsilon_a,k_i^-,k_{i\perp},k_i^+=0)\,. \tag{6.3}$$

On the other hand, we have

$$\Gamma_\perp^{(n)}(p'_a,q,\epsilon_a,k_{1\perp},\ldots,k_{n\perp})=\int_{k_i'^-=-M}^{M}\left[\prod_{i=1}^{n}dk_i'^-\right]G_\perp^{(n)}(p'_a,q,\epsilon_a,k_{i\perp},k_i'^-,k_i'^+=0)\,, \tag{6.4}$$

which is obtained from (6.2) by changing p_a to p'_a on both sides, and changing the name of the integration variable from k_i^- to $k_i'^-$. Taking the difference between (6.4) and (6.3), and keeping only up to first-order terms in β, we get

$$\begin{aligned}\beta p_a^+\partial\Gamma_\perp^{(n)}/\partial p_a^+ &=\sum_{j=1}^{n}\int_{k_i'^-=-M}^{M}\left[\prod_{\substack{i=1,\\ i\neq j}}^{n}dk_i'^-\right][\beta M G_\perp^{(n)}(p'_a,q,\epsilon_a,k_{i\perp},k_i'^+=0,k_1'^-,\ldots,k_{j-1}'^-,k_j'^-=M,k_{j+1}'^-,\ldots,k_n'^-)\\ &\qquad-(-\beta M)G_\perp^{(n)}(p'_a,q,\epsilon_a,k_{i\perp},k_i'^+=0,k_1'^-,\ldots,k_{j-1}'^-k_{j-1}'^-k_j'^-=-M,k_{j+1}'^-,\ldots,k_n'^-)]\\ &\quad+\int_{k_i'^-=-M}^{M}\left[\prod_{i=1}^{n}dk_i'^-\right][G_\perp^{(n)}(p'_a,q,\epsilon_a,k_{i\perp},k_i'^-,k_i'^+=0)-(1-\beta)^{-n}G_\perp^{(n)}(p_a,q,\epsilon_a,k_{i\perp},k_i^-,k_i^+=0)]\,.\end{aligned} \tag{6.5}$$

Let us first consider the first term on the right-hand side of the above equation. In the jth term in the summation, $G_\perp^{(n)}$ has to be evaluated at $k_j'^{-}=M$ and $-M$. Hence we may apply the Grammer-Yennie technique to analyze the contribution. The contribution from the K terms cancels between the two terms. The contributions from the G terms come from the regions of integration shown in Fig. 8. They are soft-loop integrals of $G^{(n)}$'s with less than r loops, and, following the procedure mentioned in Sec. V, may be expressed as transverse integrals of $\Gamma^{(n)}$'s with less than r loops. Hence these terms have at most $(r-1)$ logarithms of p_a^+/m.

We now turn to the second term in (6.5). If from (4.16), we project out the term proportional to the product of the transverse γ matrices on both sides, the left-hand side reduces to

$$[G_\perp^{(n)}(p_a',q,\epsilon_a,k_1',\ldots,k_n') -(1-\beta)^{-n}G_\perp^{(n)}(p_a,q,\epsilon_a,k_1,\ldots,k_n)]u(p_a') \tag{6.6}$$

since S commutes with the transverse γ matrices. This, according to (4.16), is given by the sum of the diagrams shown in Figs. 9(b)–9(d). When substituted into the second term on the right-hand side of (6.5), each of these terms becomes a soft-loop integral of some $G^{(n')}$ with less than r loops, and hence may be expressed as transverse-momentum integral of $\Gamma^{(n')}$'s with less than r loops. Hence these terms have less than r logarithms of p_a^+/m.

Thus $p_a^+\partial\Gamma_\perp^{(n)}/\partial p_a^+$ has less than r logarithms of p_a^+/m in r-loop order. This shows that $\Gamma_\perp^{(n)}$ has at most r logarithms of p_a^+/m in the r-loop order. This completes our proof by induction. Since at the tree level, $\Gamma_\perp^{(n)}$ does not have any logarithm of p_a^+/m [this is easy to verify by considering the sum of tree-graph contributions to $G_\perp^{(n)}$, which is proportional to $\prod_{i=1}^n(k_i^- -i\epsilon)^{-1}$, and integrating over the minus components of its external momenta], we can conclude that $\Gamma_\perp^{(n)}$ does not have more than r logarithms in r-loop order, for any r.

VII. EVOLUTION OF THE $\Gamma^{(n)}$'S

In Sec. VI we saw that in the r-loop order, $\Gamma^{(n)}$'s do not have more than r logarithms. In this section, we shall show that the equations derived in Sec. VI are also sufficient to find out the asymptotic behavior of the $\Gamma^{(n)}$'s as functions of p_a^+, including the leading, as well as the nonleading logarithms. We shall first define some new quantities. Let $\Gamma_\perp^{(n,r)}$ be the total contribution to $\Gamma_\perp^{(n)}$ in r-loop order. [We shall not analyze $\Gamma_+^{(n)}$ separately, since, according to (6.2), the total contribution to the amplitude may be expressed in terms of $\Gamma_\perp^{(n)}$ and $\Phi_\perp^{(n')}$.] The result of Sec. VI shows that $\Gamma_\perp^{(n,r)}$ may be expressed as

$$\Gamma_\perp^{(n,r)}(p_a^+/m,q,\epsilon_a,k_{i\perp})=\sum_{j=0}^{r} a_j^{(n,r)}(q,\epsilon_a,k_{i\perp})\ln^j\frac{p_a^+}{m} . \tag{7.1}$$

In order to find the amplitude in the kth nonleading-logarithm approximation, we need to know $\Gamma_\perp^{(n)}$ in the $(k-n)$th non-leading-logarithm approximation, i.e., we need to know $a_j^{(n,r)}$'s for

$$r+n-k\le j\le r . \tag{7.2}$$

First we shall prove the following result. Suppose we are trying to evaluate the contribution to the amplitude in the kth-non-leading-logarithm approximation, and we know the relevant $a_j^{(n',r')}$'s for $r'<r$. Then, we shall show that the equations derived in Sec. VI are sufficient to determine the required $a_j^{(n,r)}$'s for all n and j, satisfying (7.2), except for $j=0$.

Equation (7.1) gives

$$\partial\Gamma_\perp^{(n,r)}/\partial\ln p_a^+=\sum_{j=0}^{r} ja_j^{(n,r)}(q,\epsilon_a,k_{i\perp})\times\ln^{j-1}\left[\frac{p_a^+}{m}\right] . \tag{7.3}$$

This is given by the right-hand side of (6.5). As was shown in Sec. VI, the first term on the right-hand side of (6.5) may be analyzed by using Grammer-Yennie decomposition. The K-gluon contribution vanishes, while the G-gluon contribution is given by contributions of the form shown in Fig. 8. These contributions are soft-loop integrals of $G^{(n',r')}$'s with $n'+r'<n+r$ and $r'<r$, and by using the procedure given in Sec. V, may be expressed as a sum of transverse-momentum integrals of $\Gamma_\perp^{(n',r')}$'s with $n'+r'<n+r$ and $r'<r$. Thus we know the coefficient of $\ln^{j'}(p_a^+/m)$ in the contribution to $\partial\Gamma^{(n)}/\partial\ln p_a^+$ from these terms for

$$j'\ge n'+r'-k . \tag{7.4}$$

Then, comparing (7.3) with this expression, we can find the contribution to $a_j^{(n,r)}$'s from these terms for

$$j-1\ge n'+r'-k . \tag{7.5}$$

Since $n'+r'<n+r$, we know the contribution to $a_j^{(n,r)}$'s for

$$j\ge n+r-k \tag{7.6}$$

except for $j=0$.

Let us now try to analyze the contribution from

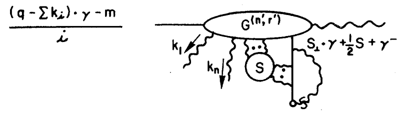


FIG. 13. Regions of integration contributing to Fig. 12(b) when we choose the $S_\perp \cdot \gamma + \frac{1}{2} S^+ \gamma^-$ term from the S gluon. All the internal lines of the blob marked S, as well the lines coming out of it, carry soft momenta.

the second term in the right-hand side of (6.5). As shown in Sec. VI, these contributions are given by the sum of the diagrams shown in Figs. 9(b)—9(d). Of these, the contribution from the diagrams in Figs. 9(c) and 9(d) may be expressed as soft-loop integrals of $G^{(n',r')}$'s with $n'+r'<n+r$ and $r'<r$, and hence their contribution to $a_j^{(n,r)}$ is known for $j \geq n+r-k$. Analysis of the contribution from Fig. 9(b), however, is slightly more tricky. Apparently, this involves a soft-loop integral of $G^{(n',r')}$ with $n'=n+1$ and $r'=r-1$. Hence $n'+r'=n+r$, and our previous argument to show that the contribution to $a_j^{(n,r)}$ is known for $j \geq n+r-k$ breaks down.

A more careful analysis of the contribution from Fig. 9(b) shows that there is actually no problem. First, note that, if we choose the $S_\perp \cdot \gamma$ or the $S^+\gamma^-$ term from the S gluon propagator, it constrains the fermion line, to which it is attached, to be soft. The contribution then comes from the regions of integration shown in Fig. 13, which are all soft-loop integrals of $G^{(n',r')}$'s with $n'+r'<n+r$. Hence their contribution to $a_j^{(n,r)}$'s are known for $j \geq n+r-k$. The contribution to (6.5) from the $S^-\gamma^+$ term in Fig. 9(b) may be written as

$$g \int_{-M}^{M} dk_1^- \cdots \int_{-M}^{M} dk_n^- \frac{\left[q-\sum_{i=1}^{n} k_i\right]\cdot\gamma}{i} S^-(k) \frac{(-i)}{k^2-\mu^2+i\epsilon} \frac{i\left[\left[q-k-\sum_{i=1}^{n} k_i\right]\cdot\gamma+m\right]}{\left[q-k-\sum_{i=1}^{n} k_i\right]^2-m^2+i\epsilon}$$
$$\times G^{(n+1,r-1)}(p_a,q,\epsilon_a,k_1,\ldots,k_n,k)\Big|_{k_1^+=0,\ \ldots,\ k_n^+=0} . \tag{7.7}$$

First, let us consider the contribution to the above integral from the region where either k^- or at least one of the k_i^-'s is of order m. Then we can decompose the gluon, carrying the order-m minus momentum, into G and K parts. The K-part contribution will involve integrals of $G^{(n,r-1)}$, and hence the contribution from this term to $a_j^{(n,r)}$ can be found for $j \geq n+r-k$. The contribution from the G part will involve soft-loop integrals of $G^{(n',r')}$ with $n'+r'<n+r$ and $r'<r$, and hence this contribution is also known for $j \geq n+r-k$. Thus the only troublesome contribution comes from the region where all the $|k_i^-|$'s and $|k^-|$ are small compared to m. In this region, the fermion denominator carrying momentum $q-k-\sum k_i$ and the gluon denominator $(k^2-\mu^2+i\epsilon)$ are independent of k^+. If we choose the perpendicular component of $G^{(n+1,r-1)}$, then we are forced to choose the $(q-k-\sum k_i)_\perp \cdot \gamma + m$ term from the fermion numerator. Hence this is also independent of k^+. $S^-(k)$, on the other hand, is an odd function of k^+, in the limit $|k^-| \ll |k^+|$. Thus, if we ignore the dependence of $G^{(n+1,r-1)}$ on k^+, the integrand of (7.7) is an odd function of k^+, and hence the k^+ integral vanishes by symmetry.

Thus, in order to get a nonsuppressed contribution to (7.7) from the region $|k_i^-| \ll m$ and $|k^-| \ll m$, we must choose the $G_+^{(n+1,r-1)}$ term from $G^{(n+1,r-1)}$, or we must consider the k^+-dependent part of $G^{(n+1,r-1)}$. Contributions to both the parts come from the region where some of the internal loop momenta of $G^{(n+1,r-1)}$ are soft (Figs. 11 and 12). These are soft-loop integrals of $G^{(n',r')}$ with $n'+r' \leq n+r$. Configurations shown in Fig. 14 are the only ones for which $n'+r'=n+r$. If, however, the minus component of all these internal soft-loop momenta l_i are small compared to m, then we do not get any contribution to $G_+^{(n+1,r-1)}$ since, in order to get a γ^+ somewhere on the fermion numerator or the gluon-fermion vertex inside $G^{(n+1,r-1)}$, we must have at least one power of the minus component of some soft-loop momentum in the numerator [note that $N^{--}(l) \propto l^-$]. We do not get any k^+-dependent part of $G^{(n+1,r-1)}$ either,

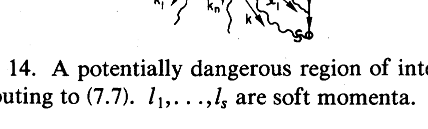


FIG. 14. A potentially dangerous region of integration contributing to (7.7). $l_1,\ldots,l_s$ are soft momenta.

since all the new soft-fermion denominators in Fig. 14 are independent of k^+. Thus the k^+ integral still vanishes by symmetry. If, on the other hand, one of the internal soft-loop momenta the minus component is of order m, it can be decomposed into G and K gluons, and the contribution from both the parts may be expressed as soft-loop integrals of $G^{(n',r')}$ with $n'+r'<n+r$ and $r'<r$. Thus the contribution from this term to $a_j^{(n,r)}$ may be found for $j\geq n+r-k$.

Hence we see that if we know the $a_{j'}^{(n',r')}$'s for $j'\geq n'+r'-k$ for $r'\leq r-1$, we may find out the $a_j^{(n,r)}$'s for $j\geq n+r-k$, except for $j=0$. We shall now interpret this result from a somewhat different angle. Equation (6.5) gives an expression for $\partial\Gamma^{(n)}/\partial\ln p_a^+$. The right-hand side is a sum of soft-loop integrals of $G^{(n')}$'s, and hence, using the method of Sec. V, may be expressed as a sum of terms of the form

$$\sum_{n'=0}^{\infty}\int A^{(n,n')}(q,k_{1\perp},\ldots,k_{n\perp},k'_{1\perp},\ldots,k'_{n'\perp})\Gamma_\perp^{(n')}(p_a,q,\epsilon_a,k'_{1\perp},\ldots,k'_{n'\perp})d^2k'_{1\perp}\cdots d^2k'_{n'\perp}\ . \tag{7.8}$$

The result of this section tells us that if we want to evaluate $\Gamma_\perp^{(n)}$ in the $(k-n)$th-non-leading-logarithm approximation, then (7.8) may be evaluated by knowing $\Gamma_\perp^{(n')}$ up to the $(k-n')$th-non-leading-logarithm approximation, for $n'=0,1,\ldots,k$. Thus it is enough to terminate the sum on the right-hand side of (7.8) at $n'=k$. The set of equations (7.8) will then read as

$$\frac{\partial\Gamma_\perp^{(n)}}{\partial\ln p_a^+}(p_a^+,q,\epsilon_a,k_{1\perp},\ldots,k_{n\perp})=\sum_{n'=0}^{k}\int A^{(n,n')}(q,k_{1\perp},\ldots,k_{n\perp},k'_{1\perp},\ldots,k'_{n'\perp})\Gamma_\perp^{(n')}(p_a^+,q,\epsilon_a,k'_{1\perp},\ldots,k'_{n'\perp})d^2k'_{1\perp}\cdots d^2k'_{n'\perp}. \tag{7.9}$$

If we regard A as a generalized matrix in the product space of the transverse-momentum space and the n space, and $\Gamma_\perp^{(n)}$ as a generalized vector in the same space, we may write (7.9) as

$$\partial\Gamma_\perp/\partial\ln p_a^+=A\Gamma_\perp \tag{7.10}$$

with the solution

$$\Gamma_\perp=\exp\left[A\ln\frac{p_a^+}{m}\right]B_\perp\ , \tag{7.11}$$

where, in evaluating the exponential, we must use the notion of generalized product for evaluating the powers of A. For example,

$$(A^2)^{(n,n')}(q,k_{1\perp},\ldots,k_{n\perp},k'_{1\perp},\ldots,k'_{n'\perp})=\sum_{n'=0}^{k}\int d^2k''_{1\perp}\cdots d^2k''_{n''\perp}A^{(n,n'')}(q,k_{1\perp},\ldots,k_{n\perp},k''_{1\perp},\ldots,k''_{n''\perp})A^{(n'',n')}(q,k''_{1\perp},\ldots,k''_{n''\perp},k'_{1\perp},\ldots,k'_{n'\perp})\ . \tag{7.12}$$

In (7.11), $B_\perp$ is an unknown p_a^+-independent vector in the product space of the transverse-momentum space and the n space. These are the constants of integration and correspond to the undetermined coefficients $a_0^{(n,r)}$'s, mentioned before in this section. These coefficients may be calculated by doing some low-order calculations. For example, in the leading-logarithm approximation, the only undetermined coefficient is $a_0^{(0,0)}$, which is trivially determined from the tree diagram. In the next-to-the-leading-logarithm approximation, the undetermined coefficients are $a_0^{(0,0)}$, $a_0^{(0,1)}$, and $a_0^{(1,0)}$. Of these, $a_0^{(0,0)}$ and $a_0^{(1,0))}$ are determined from the tree diagrams; $a_0^{(0,1)}$ may be calculated by a complete one-loop calculation of $\Gamma^{(0)}$.

Equations (7.9) become extremely simple in the leading-logarithm approximation. Here $k=0$, hence $\Gamma^{(0)}$ is the only relevant term. The equation is

$$\partial\Gamma_\perp^{(0)}/\partial\ln p_a^+=\alpha(q)\Gamma_\perp^{(0)}\ , \tag{7.13}$$

where $\alpha(q)=A^{(0,0)}(q)$. This equation has the solution

$$\Gamma_\perp^{(0)}=\exp\left[\alpha(q)\ln\frac{p_a^+}{m}\right]B_\perp^{(0)}(q,\epsilon_a)\ , \tag{7.14}$$

which shows the Regge behavior (1.1). $\Phi_\perp^{(n)}$ has a similar expression as $\Gamma_\perp^{(n)}$. In Sec. VIII we shall apply the techniques, we have developed so far, to calculate $\alpha(q)$. We shall also evaluate the complete

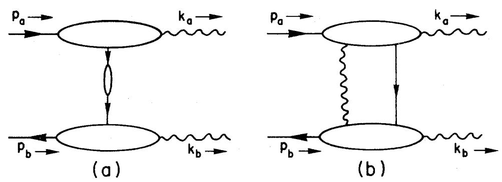


FIG. 15. Diagrams contributing to the amplitude in the leading- and the next-to-leading-logarithm approximation.

contribution in the next-to-the-leading-logarithm approximation, using these techniques.

VIII. CONTRIBUTION TO THE AMPLITUDE IN THE LEADING-LOGARITHM AND THE NEXT-TO-THE LEADING-LOGARITHM APPROXIMATION

In this section we shall find out the contribution to the amplitude in the leading-logarithm and the next-to-the-leading-logarithm approximation. From the discussion of the previous sections, it is clear that in the leading-logarithm approximation, the contribution comes from the factorized diagrams of the form shown in Fig. 15(a), while the first nonleading logarithms come from the factorized diagrams, as well as the diagrams which have a one-gluon—one-fermion intermediate state in the t channel [Fig. 15(b)]. Before evaluating the contribution from these diagrams, we shall isolate the contribution to the positive- and the negative-signature channels. To do this, let us define $\widetilde{G}^{(n)}(\widetilde{p}_a,q,\epsilon_a,k_1,\ldots k_n)$ to be the analog of $G^{(n)}$ for the backward Compton scattering amplitude (Fig. 16). There is a one-to-one correspondence between the diagrams contributing to $G^{(n)}$ and $\widetilde{G}^{(n)}$, which may be related by a simple transformation of the loop momenta ($l^{\pm}\to -l^{\pm}, l_\perp \to l_\perp$). The relationship is

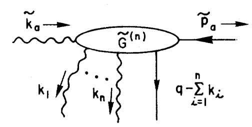


FIG. 16. Analog of $G^{(n)}$ for the backward Compton scattering amplitude.

$$G_\perp^{(n)}(p_a,q,\epsilon_a,k_1,\ldots,k_n) =(-1)^n\widetilde{G}_\perp^{(n)}(\widetilde{p}_a,q,\epsilon_a,\hat{k}_1,\ldots,\hat{k}_n)\ , \tag{8.1}$$

$$G_+^{(n)}(p_a,q,\epsilon_a,k_1,\ldots,k_n) =(-1)^{n+1}\widetilde{G}_+^{(n)}(\widetilde{p}_a,q,\epsilon_a,\hat{k}_1,\ldots,\hat{k}_n)\ , \tag{8.2}$$

where

$$\hat{k}_1^{\pm}=-k_1^{\pm},\ \ \hat{k}_{1\perp}=k_{1\perp}\ . \tag{8.3}$$

Up to the next-to-leading-logarithm level, only $G_\perp^{(0)}$ and $F_\perp^{(0)}$ contribute to Fig. 15(a). This is because, if we choose the $\gamma^+G_+^{(0)}$ term from the upper blob in Fig. 15(a), we must choose the $F_-^{(0)}\gamma^-$ term from the lower blob, in order to get a nonzero answer. Such a term contains at most $r-2$ logarithms in r-loop order, and hence may be ignored in the next-to-the-leading-logarithm level. Equation (8.1) then shows that the contribution from the factorized diagrams to the negative-signature channel vanishes in the next-to-the-leading-logarithm approximation.

In order to know the contribution from Fig. 15(b) up to the next-to-the-leading-logarithm approximation, we need to know the contribution from $G^{(1)}$ and $F^{(1)}$ in the leading-logarithm approximation. Hence we may ignore the dependence of $G^{(1)}$ and $F^{(1)}$ on k^+ and k^-, respectively. Also, we must choose the $G_\perp^{(1)}$ and $F_\perp^{(1)}$ terms. Thus the contribution to $A^{\pm}$ from the one-gluon exchange diagrams may be written as

$$g^2\int\frac{d^4k}{(2\pi)^4}F_\perp^{(1)}(p_b,q,\epsilon_b,k^+,k^-=0,k_\perp)\frac{i[(q-k_\perp)\cdot\gamma+m]}{(q-k)^2-m^2+i\epsilon}$$
$$\times[G_\perp^{(1)}(p_a,q,\epsilon_a,k^+=0,k^-,k_\perp)\pm\widetilde{G}_\perp^{(1)}(\widetilde{p}_a,q,\epsilon_a,k^+=0,k^-,k_\perp)](-i)N^{\mu\nu}(k)/(k^2-\mu^2+i\epsilon)\ . \tag{8.4}$$

In the region where either k^+, or k^-, or both are small compared to m, so that $|k^+k^-|\ll\vec{k}_\perp^{\,2}$, the contribution to A^+ from the above integral vanishes, since the integrand changes sign under $k^-\to -k^-$, according to (8.1). Thus, the contribution comes only from the $k^+,k^-\sim m$ region. In this region, we may use the Grammer-Yennie decomposition technique, and write the contribution from (8.4) to A^+ as

$$2F_\perp^{(0)}(p_b,q,\epsilon_b)\left[g^2\int\frac{d^4k}{(2\pi)^4}P\left[\frac{1}{k^-}\right]\frac{1}{k^++i\epsilon}(i)\frac{(q-k_\perp)\cdot\gamma+m}{(q-k)^2-m^2+i\epsilon}\frac{(-i)}{k^2-\mu^2+i\epsilon}N^{+-}(k)\right]G_\perp^{(0)}(p_a,q,\epsilon_a)\ . \tag{8.5}$$

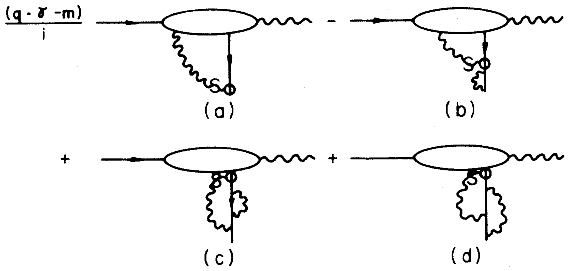


FIG. 17. Total contribution to $\partial G_\perp^{(0)}/\partial \ln p_a^+$ in the next-to-the-leading-logarithm approximation.

The contribution from the factorized diagrams on the other hand, may be expressed as a product of $F_\perp^{(0)}$, $i/(\not q - m)$, and $G_\perp^{(0)}$. Thus, up to the next-to-the-leading-logarithm level, the amplitude A^+ may be factored into a product of three parts, a part which depends on p_b but not on p_a, a part which is independent of p_a and p_b, and a part which depends on p_a but not on p_b. Then, according to an argument due to Mason,[5] the amplitude must have an exponential form in this approximation. We shall, however, give a more direct proof of the exponentiation, and also find out the trajectory function $\alpha(q)$ up to order g^4.

$\partial G_\perp^{(0)}/\partial \ln p_a^+$ is given by the sum of the diagrams in Figs. 9(b)–9(d), with $n=0$. Now, S^+k^-, S^-k^+, and $S_\perp \cdot k_\perp$ may be expressed in the form $(k^+\partial/\partial k^+ - k^-\partial/\partial k^-)f$, where f is a function of k. Then, integrating over k^+ and k^- by parts, we may show that the contribution to Fig. 9(c) from the one-loop graph vanishes for $n=0$. So does the two-loop diagram, which is obtained by adding a vacuum polarization bubble to the one-loop contribution to Fig. 9(c). The diagrams which contribute to $\partial G_\perp^{(0)}/\partial \ln p_a^+$ up to the next-to-leading-logarithm level, are shown in Fig. 17. Of these, the effect of Fig. 17(b) is to change the $-i(\not q - m)$ factor in Fig. 17(a) to $-iS_F^{-1}(q)$. Hence we may concentrate on Fig. 17(a), since Figs. 17(c) and 17(d) are already in the form const$\times G_\perp^{(0)}$.

First, we shall limit ourselves to the leading-logarithm approximation. In this approximation we must choose the $\frac{1}{2}S^-\gamma^+$ term from the vertex where the S gluon is attached to the fermion line inside the blob, because, if we choose the $\frac{1}{2}S^+\gamma^-$ or the $S_\perp \cdot \gamma$ term, it will constrain the fermion line to which it is attached, to be soft, and hence lose some logarithms. The contribution to Fig. 17(a) in the leading-logarithm approximation may then be written as

$$g\frac{\not q - m}{i}\int \frac{d^4k}{(2\pi)^4} S^-(k) \frac{(-i)}{k^2-\mu^2+i\epsilon} \times \frac{i[(q-k)\cdot\gamma + m]}{(q-k)^2 - m^2 + i\epsilon} G^{(1)}(p_a^+, q, \epsilon_a, k) . \quad (8.6)$$

In the leading-logarithm approximation we must choose the perpendicular component of $G^{(1)}$; thus we must choose the $(q-k)_\perp \cdot \gamma + m$ term from the fermion numerator, in order to make (8.6) a product of transverse γ matrices only. Also, we may take $G^{(1)}$ to be independent of k^+ in this approximation. Then in the region $|k^-| \ll m$, the integrand of (8.6) becomes an antisymmetric function of k_i^+ and the integral vanishes. This shows that in the leading-logarithm approximation, the integral (8.6) receives contribution only from the $|k^-| \sim m$ region. In this region, we may make a Grammer-Yennie decomposition of the S gluon. The G term does not contribute in the leading-logarithm approximation. The contribution from the K term may be written as

$$-ig^2(\not q - m)\left[\int \frac{d^4k}{(2\pi)^4} S^-(k) \frac{(-i)}{k^2-\mu^2+i\epsilon} \frac{i[(q-k_\perp)\cdot\gamma + m]}{(q-k)^2 - m^2 + i\epsilon} \frac{(-1)}{k^- - i\epsilon}\right] G_\perp^{(0)}(p_a^+, q, \epsilon_a)$$

$$\equiv \alpha^{(2)}(q) G_\perp^{(0)}(p_a^+, q, \epsilon_a) . \quad (8.7)$$

Thus,

$$\partial G_\perp^{(0)}/\partial \ln p_a^+ = \alpha^{(2)}(q) G_\perp^{(0)}(p_a^+, q, \epsilon_a) , \quad (8.8)$$

the solution to which is

$$G_\perp^{(0)}(p_a, q, \epsilon_a) = \exp\left[\alpha^{(2)}(q) \ln \frac{p_a^+}{m}\right] B_\perp^{(0)}(q, \epsilon_a) , \quad (8.9)$$

where $B_\perp^{(0)}$ is some constant, independent of p_a^+. In the lowest order in g, $B_\perp^{(0)}$ is equal to $(-ig\not\epsilon_a)$.

In the next-to-the-leading-logarithm level, there are various extra contributions to $\partial G^{(0)}/\partial \ln p_a^+$. First, let us find out the extra terms from (8.6). In the region $|k^-| \ll m$, there are two effects which may destroy the antisymmetry of the integrand under $k^+ \to -k^+$. We may choose the $\gamma^+ G_+^{(1)}$ term from $G^{(1)}$ and the $k^+\gamma^-$ term from the soft-fermion numerator. Also, now $G^{(1)}$ can no longer be considered to be independent of k^+. $G_+^{(1)}$ receives contributions from the regions of integration in the loop momentum space shown in Fig. 12.

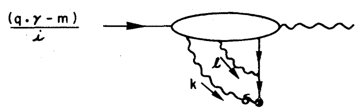


FIG. 18. Contribution to (8.6) from the $G_+^{(1)}$ part and the k^+ dependent part of $G_\perp^{(1)}$ in the next-to-the-leading-logarithm approximation. Here k and l are soft momenta.

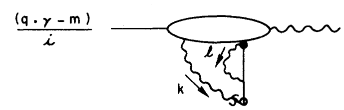


FIG. 19. Sum of all insertions of the K part of the l gluon in Fig. 18.

Also, as shown in Appendix B, in order to get a nonsuppressed contribution to

$$G^{(1)}(p_a,q,\epsilon_a,k^+,k^-,k_\perp)-G^{(1)}(p_a,q,\epsilon_a,k^+=0,k^-,k_\perp)\ ,$$

one of the internal loop momentum of $G^{(1)}$ must be soft. Thus, if in (8.6) we express $G^{(1)}(p_a,q,\epsilon_a,k)$ as

$$G_\perp^{(1)}(p_a,q,\epsilon_a,k^+=0,k^-,k_\perp)+[G_\perp^{(1)}(p_a,q,\epsilon_a,k^+,k^-,k_\perp)-G_\perp^{(1)}(p_a,q,\epsilon_a,k^+=0,k^-,k_\perp)]$$
$$+\tfrac{1}{2}\gamma^+G_+^{(1)}(p_a,q,\epsilon_a,k^+,k^-,k_\perp)\ , \qquad (8.10)$$

then the second and the third terms will receive contribution from the regions of integration shown in Fig. 18. When we substitute the first term in place of $G^{(1)}$ in (8.6), the integral receives contributions from the $k^-\sim m$ region only, since the k^+ integral vanishes by symmetry in the $|k^-|\ll m$ region.

In Fig. 18, we must have an even number of γ^+ and γ^- matrices on the soft-fermion line. As seen in Sec. VII, in the region $|l^-|\ll m$, $|k^-|\ll m$, the integrand is antisymmetric under the transformation $k^+\to -k^+$, and hence the integral vanishes. Thus, if $|k^-|\ll m$, l^- must be of order m. But then it may be factorized, using the Grammer-Yennie technique. The G term does not contribute in the next-to-the-leading-logarithm approximation; the K term gives a contribution of the form shown in Fig. 19. The integrand is antisymmetric under the transformation $l^\pm\to -l^\pm$, $k^+\to -k^+$, in the $|k^-|\ll m$, $l^-\sim m$ region, and hence the integral vanishes. Thus the integral (8.6) receives contribution only from the $k^-\sim m$ region in the next-to-the-leading-logarithm approximation. In this region, we can decompose the S gluon into G and K parts. The K-gluon contribution is identical to (8.7). The G part constrains one of the internal loop momenta l of $G^{(1)}$ to be soft, and again, using the antisymmetry of the integrand under $k^\pm\to -k^\pm$, $l^+\to -l^+$ for $|l^-|\ll m$, we may bring the contribution in the form of Fig. 20(a).

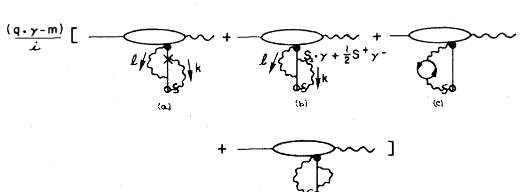


FIG. 20. Total extra contribution to $\partial G_\perp^{(0)}/\partial \ln p_a^+$ from Fig. 9(b) in the next-to-the-leading-logarithm approximation.

There are some extra contributions from Fig. 17(a) in the next-to-the-leading-logarithm approximation, which is not included in (8.6). This is the contribution where we choose the $\frac{1}{2}S^+\gamma^-+S_\perp\cdot\gamma$ term instead of the $\frac{1}{2}S^-\gamma^+$ term from the vertex, where the S gluon is attached to the fermion line inside the blob. This vertex constrains the fermion line, to which the S gluon is attached, to be soft. Using the symmetry property of the integrand, and the Grammer-Yennie decomposition, we may bring the contribution from this term into the form of Fig. 20(b). Also, we must include the one-loop self-energy diagrams for the soft gluon and the soft fermion [Figs. 20(c) and 20(d)]. Figure 20 then gives the net extra contribution to $\partial G_\perp^{(0)}/\partial \ln p_a^+$ in the first-non-leading-logarithm approximation from Fig. 17(a). As we have mentioned before, the effect of Fig. 17(b) is to change the $-i(\not q-m)$ factor in Eq. (8.6) to $-iS_F^{-1}(q)$. Thus, in the next-to-the-leading-logarithm approximation,

$$\partial G_\perp^{(0)}/\partial \ln p_a^+=[\alpha^{(2)}(q)+\alpha^{(4)}(q)]G_\perp^{(0)}\ , \qquad (8.11)$$

where $\alpha^{(4)}(q)$ is given by the sum of the contributions from Figs. 20(a)–20(d), Figs. 17(c) and 17(d), and a one-loop self-energy diagram multiplied by $\alpha^{(2)}(q)$ coming from Fig. 17(b). Thus,

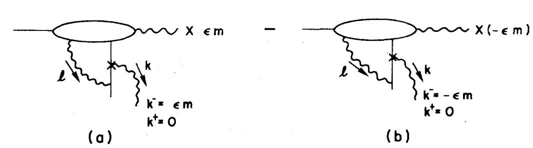


FIG. 21. Contribution from the first term on the right-hand side of (6.5) to $\partial\Gamma_\perp^{(1)}/\partial \ln p_a^+$.

$$
\begin{aligned}
&\alpha^{(2)}(q)+\alpha^{(4)}(q)\\
&\quad=[iS_F(q)]^{-1}\left[g^2\int\frac{d^4k}{(2\pi)^4}\left[\frac{(-i)}{k^2-\mu^2+i\epsilon}S^-(k)\right.\right.\\
&\qquad\qquad\left.+\frac{(-i)}{k^2-\mu^2+i\epsilon}S_\mu(k)(k^2g^{\mu-}-k^\mu k^-)i\Pi(k^2)\frac{(-i)}{k^2-\mu^2+i\epsilon}\right]\\
&\qquad\left.\times iS_{F\perp}(q-k)\frac{(-1)}{k^--i\epsilon}\right]\\
&\qquad+g^4(-i)(\not q-m)\int\frac{d^4k}{(2\pi)^4}\frac{d^4l}{(2\pi)^4}\frac{(-i)}{k^2-\mu^2+i\epsilon}\frac{(-i)}{l^2-\mu^2+i\epsilon}\frac{(-1)}{l^--i\epsilon}N^{-\mu}(l)\\
&\qquad\qquad\times\left[\frac{i}{(q-k)\cdot\gamma-m+i\epsilon}(-i\gamma_\mu)\frac{i}{(q-k-l)\cdot\gamma-m+i\epsilon}\right.\\
&\qquad\qquad\times\left[S^-(k)\frac{i}{k^--i\epsilon}(\tfrac{1}{2}k^+\gamma^-+k_\perp\cdot\gamma)-i[S_\perp(k)\cdot\gamma+\tfrac{1}{2}S^+(k)\gamma^-]\right]\\
&\qquad\qquad\left.\times\frac{i}{(q-l)\cdot\gamma-m+i\epsilon}\right]_\perp\\
&\qquad-g^4\int\frac{d^4k}{(2\pi)}\frac{d^4l}{(2\pi)^4}\left[\left[-iS^\mu(k)\gamma_\mu\frac{i}{(q-k)\cdot\gamma-m)}(-i\gamma_\nu)+(-i\gamma_\nu)\frac{i}{(q-l)\cdot\gamma-m}(-i)S^\mu(k)\gamma_\mu\right]\right.\\
&\qquad\qquad\left.\times\frac{i}{(q-k-l)\cdot\gamma-m}(-i\gamma_\sigma)\frac{i}{(q-k)\cdot\gamma-m}\right]_\perp\\
&\qquad\times\frac{(-i)}{k^2-\mu^2+i\epsilon}\frac{(-i)}{l^2-\mu^2+i\epsilon}N^{\nu\sigma}(l)\,,
\end{aligned}\tag{8.12}
$$

where $i\Pi(k^2)(k^2g^{\mu\nu}-k^\mu k^\nu)$ is the contribution from the one-loop vacuum polarization bubble. In writing the contribution to (8.12) from Fig. 20(c), we have used some symmetry properties of the integral. $iS_{F\perp}(q-k)$ is that part of the full one-loop fermion self-energy, which is proportional to the identity matrix or product of transverse γ matrices only. $[\;]_\perp$ means that in any of the terms enclosed in the square brackets, we must commute all the γ^-'s through the other γ matrices to the extreme right using Eqs. (2.7), and, at the end of this process, keep only those terms which contain products of transverse γ matrices only. For example, a term of the form $\gamma^- l_\perp\cdot\gamma\gamma^+$ is reduced to

$$-4l_\perp\cdot\gamma+l_\perp\cdot\gamma\gamma^+\gamma^-\tag{8.13}$$

and we pick up only the $(-4l_\perp\cdot\gamma)$ term. All the terms on the right-hand side of Eq. (8.12), except the last term, have explicit factors of $(\not q-m)$, and hence vanish at $\not q=m$. It can be shown, using the Ward identities, that the last term also has a factor of $(\not q-m)$, and hence vanishes at $\not q=m$. Thus the trajectory function vanishes at $\not q=m$.

The solution to Eq. (8.11) is

$$
\begin{aligned}
&G_\perp^{(0)}(p_a^+,q,\epsilon_a)\\
&\qquad=\exp\left[[\alpha^{(2)}(q)+\alpha^{(4)}(q)]\ln\frac{p_a^+}{m}\right]B_\perp^{(0)}(q,\epsilon_a)\,.
\end{aligned}\tag{8.14}
$$

Similar solutions may also be obtained for $F_\perp^{(0)}$. The total contribution to the positive-signature amplitude is obtained from the sum of the contributions from Fig. 15(a), the corresponding diagram for the backward Compton scattering amplitude, and expression (8.5). It is given by

$$A^+=2\tilde{B}_\perp^{(0)}(q,\epsilon_b)\exp\left[[\alpha^{(2)}(q)+\alpha^{(4)}(q)]\ln\left[\frac{s}{m^2}\right]\right]$$
$$\times\left[iS_F(q)+g^2\int\frac{d^4k}{(2\pi)^4}P\left[\frac{1}{k^-}\right]\frac{1}{k^++i\epsilon}\frac{(q-k_\perp)\cdot\gamma+m}{(q-k)^2-m^2+i\epsilon}\frac{1}{k^2-\mu^2+i\epsilon}N^{+-}(k)\right]B_\perp^{(0)}(q,\epsilon_a)\ . \tag{8.15}$$

Next we shall turn to the negative-signature amplitude, which comes entirely from the graph in Fig. 15(b), and the corresponding graph for the backward Compton scattering amplitude. We shall first express the contribution in terms of $\Gamma_\perp^{(1)}$, $\Gamma_\perp^{(0)}$, $\Phi_\perp^{(1)}$, and $\Phi_\perp^{(0)}$, according to the prescription of Sec. V. We first define $\Gamma^{(1)}$ as

$$\Gamma^{(1)}=\int_{-\epsilon m}^{\epsilon m}dk_1^-G^{(1)}(p_a,q,\epsilon_a,k_1^-,k_{1\perp},k_1^+=0)\ , \tag{8.16}$$

i.e., we take M in (3.13) to be equal to ϵm. Here ϵ is an arbitrarily small, but fixed number. The final result must be independent of M, thus it is independent of ϵ. Hence, if we consistently ignore all terms which vanish as $\epsilon\to 0$, we shall get the correct final result. As we shall see, this causes a certain simplification in the intermediate stages. We define $\Phi^{(1)}$ in a similar way.

Now, it can be easily seen, using Eqs. (8.1), that the contribution to A^-, given in (8.4), from the region of integration where either k^+ of k^- or both are of order m, vanishes by symmetry. Thus we may limit the $k^\pm$ integrals in the region $|k^\pm|<\epsilon m$. Neglecting $O(\epsilon)$ terms, the integral (8.4) may be written as

$$2g^2\int\frac{d^2k_\perp}{(2\pi)^4}\Phi_\perp^{(1)}(p_b,q,\epsilon_b,k_\perp)\frac{(q-k_\perp)\cdot\gamma+m}{(\vec{q}-\vec{k}_\perp)^2+m^2}\Gamma_\perp^{(1)}(p_a,q,\epsilon_a,k_\perp)(\vec{k}_\perp{}^2+\mu^2)^{-1}\ . \tag{8.17}$$

Thus our remaining task is to find out $\Phi_\perp^{(1)}$ and $\Gamma_\perp^{(1)}$ in the leading-logarithm approximation. In order to find $\Gamma_\perp^{(1)}$ we go back to Eq. (6.5). As was shown in Sec. VI, the first term on the right-hand side of (6.5) may be analyzed by Grammer-Yennie decomposition. Contributions from the K parts cancel. The contributions from the G parts are given by Fig. 21. The "soft" line l shown in this figure now carries the plus component of $\sim m/\epsilon$, but we can still regard this as soft, since it does not carry momentum $\sim\sqrt{s}$. Contribution from these figures may be written in the form

$$\int\frac{d^4l}{(2\pi)^4}[f_1(l,k_\perp,k^+=0,k^-=\epsilon m)-f_1(l,k_\perp,k^+=0,k^-=-\epsilon m)]G^{(1)}(p_a^+,q,\epsilon_a,l)\ , \tag{8.18}$$

where f_1 is the contribution from the soft gluon and the fermion lines in Fig. 21. In the leading-logarithm approximation, we may ignore the dependence of $G^{(1)}$ on l^+ and choose the $G_\perp^{(1)}$ term from $G^{(1)}$. In the region $l^-\sim m$, we can factorize the l gluon from $G^{(1)}$ by $G-K$ decomposition. The contribution from the G term is nonleading, while the contribution from the K term vanishes, since the integrand is antisymmetric under the transformation $l^\pm\to -l^\pm$. Hence the integral receives contribution only from the $|l^-|\ll m$ region. We can limit the l^- integration range to $|l^-|<\epsilon m$. The contribution may be expressed as

$$\int\beta(q,k_\perp,l_\perp)\Gamma_\perp^{(1)}(p_a^+,q,\epsilon_a,l_\perp)d^2l_\perp\ , \tag{8.19}$$

where

$$\beta(q,k_\perp,l_\perp)=\frac{1}{(2\pi)^4}\int\frac{dl^+}{2}[f_1(l^+,l^-=0,l_\perp,k_\perp,k^-=\epsilon m)-f_1(l^+,l^-=0,l_\perp,k_\perp,k^-=-\epsilon m)]\ . \tag{8.20}$$

f_1 may be read out from Fig. 21. Substituting this in (8.20) and carrying out the l^+ integral we get

$$\beta(q,k_\perp,l_\perp)=\frac{g^2}{(2\pi)^3}\frac{1}{\vec{l}_\perp{}^2+\mu^2}\left[(q\cdot\gamma-m)\frac{1}{(q-l_\perp)\cdot\gamma-m}-[(q-k_\perp)\cdot\gamma-m]\frac{1}{(q-k_\perp-l_\perp)\cdot\gamma-m}\right]\ . \tag{8.21}$$

The contribution from the second term on the right-hand side of (6.5) is given by integrals of the graphs shown in Figs. 9(b)–9(d), over k^-. Of these, Fig. 9(d) does not contribute if we are interested in the leading-logarithm contribution to $\Gamma^{(1)}$. Contribution to Fig. (9c) vanishes in the region $k^+=0$, $|k^-|\ll m$, due to the same reason as the vanishing of the corresponding contribution to $\partial G^{(0)}/\partial\ln p_a^+$. Thus we are left with the contribution from Fig. 9(b). This may be analyzed in the same way as the corresponding contribution to

$\partial G^{(0)}/\partial \ln p_a^+$ in the leading-logarithm approximation. The result is

$$\alpha^{(2)}(q-k_\perp)\Gamma_\perp{}^{(1)}(p_a,q,\epsilon_a,k_\perp)\ . \tag{8.22}$$

Hence $\Gamma_\perp{}^{(1)}$ satisfies the equation

$$\begin{aligned}\partial\Gamma_\perp^{(1)}/\partial \ln p_a^+ &= \alpha^{(2)}(q-k_\perp)\Gamma_\perp^{(1)} + \int \beta(q,k_\perp,l_\perp)\Gamma_\perp^{(1)}(p_a,q,\epsilon_{a,}l_\perp)d^2l_\perp \\ &\equiv \int \psi(q,k_\perp,l_\perp)\Gamma_\perp^{(1)}(p_a,q,\epsilon_a,l_\perp)d^2l_\perp\ ,\end{aligned} \tag{8.23}$$

where

$$\psi(q,k_\perp,l_\perp)=\alpha^{(2)}(q-k_\perp)\delta(k_\perp-l_\perp)+\beta(q,k_\perp,l_\perp)\ . \tag{8.24}$$

The solution to (8.23) is

$$\begin{aligned}\Gamma_\perp^{(1)}&(p_a^+,q,\epsilon_a,k_\perp)\\ &= \int \Bigg[\delta(k_\perp-l_\perp)+\psi(q,k_\perp,l_\perp)\ln(p_a^+/m)\\ &\qquad + \sum_{n=2}^{\infty}\frac{1}{n!}\int d^2k_{1\perp}\cdots d^2k_{(n-1)\perp}\psi(q,k_\perp,k_{1\perp})\psi(q,k_{1\perp},k_{2\perp})\cdots\psi(q,k_{(n-1)\perp},l_\perp)\ln^n(p_a^+/m)\Bigg]\\ &\qquad \times B_\perp^{(1)}(q,\epsilon_a,l_\perp)d^2l_\perp\ ,\end{aligned} \tag{8.25}$$

where $B_\perp^{(1)}$ is an unknown constant to be determined from the boundary conditions. The term inside the square brackets is the generalized exponential defined in Sec. VII. Now, at the tree level,

$$G_\perp^{(1)}=\frac{(-1)}{k^- - i\epsilon}(-ig\not\epsilon_a)\ . \tag{8.26}$$

Hence,

$$\Gamma_\perp^{(1)}=\int_{-\epsilon m}^{\epsilon m} G_\perp^{(1)}dk^- = (-i\pi)(-ig\not\epsilon_a)+O(g^3)\ . \tag{8.27}$$

Comparing (8.27) with (8.25), we get

$$B_\perp^{(1)}(q,\epsilon_a,l_\perp)=(-i\pi)(-ig\not\epsilon_a)+O(g^3)\ . \tag{8.28}$$

We may similarly find the expression for $\Phi_\perp^{(1)}$. It is given by

$$\begin{aligned}\Phi_\perp^{(1)}(p_b^-,q,\epsilon_b,k_\perp)=(-ig\not\epsilon_b)(i\pi)\int d^2l_\perp \Bigg[&\delta(l_\perp-k_\perp)+\tilde\psi(q,l_\perp,k_\perp)\ln\frac{p_b^-}{m}\\ &+\sum_{n=2}^{\infty}\frac{1}{n!}\int d^2k_{1\perp}\cdots d^2k_{(n-1)\perp}\tilde\psi(q,l_\perp,k_{1\perp})\\ &\times\tilde\psi(q,k_{1\perp},k_{2\perp})\cdots\tilde\psi(q,k_{(n-1)\perp},k_\perp)\ln^n(p_b^-/m)\Bigg]\ ,\end{aligned} \tag{8.29}$$

$\tilde\psi$ being the analog of ψ for $\Phi^{(1)}$. In fact, it can be shown that $\tilde\psi$ is obtained from ψ by using the relation

$$\tilde\psi(q,l_\perp,k_\perp)=\gamma^0\psi^+(q,k_\perp,l_\perp)\gamma^0\ . \tag{8.30}$$

Equations (8.17), together with Eqs. (8.25) and (8.29) give us the total contribution to the negative-signature channel. However, we may further simplify the result by claiming that the final result should be Lorentz invariant. Then the result may be expressed as a power series in $\ln s$. Since

$$(\ln s)^n=\sum_{r=0}^{n}\binom{n}{r}\ln^r\left[\frac{p_a^+}{m}\right]\ln^{(n-r)}\left[\frac{p_b^-}{m}\right] \tag{8.31}$$

the coefficient of $\ln^n s$ must be equal to the coefficient of $\ln^n(p_a^+/m)$ in (8.17). Thus we get

$$A^- = -2\pi^2 g^2 \epsilon_b \int \frac{d^2k_\perp}{(2\pi)^4} \frac{1}{\vec{k}_\perp^2+\mu^2} \frac{(q-k_\perp)\cdot\gamma+m}{(\vec{q}-\vec{k}_\perp)^2+m^2} \times \left[1+\sum_{n=1}^{\infty}\frac{1}{n!}\ln^n\frac{s}{m^2}\int d^2l_{1\perp}\cdots d^2l_{n\perp}\psi(q,k_\perp,l_{1\perp})\psi(q,l_{1\perp},l_{2\perp})\cdots\psi(q,l_{(n-1)\perp},l_\perp)\right]\epsilon_a \; . \tag{8.32}$$

IX. CONCLUSION

In this paper we have calculated, without using any tranverse-momentum cutoff, the contribution to the fermion exchange reactions in the Regge limit in the leading- and the next-to-the-leading-logarithm approximation. We have also developed a systematic way to calculate the higher-order corrections. We have separated the contribution to the positive- and the negative-signature channels. Up to the next-to-the-leading-logarithm approximation, the contribution to the positive-signature channel is given by a single Regge-pole term. The contribution is given in Eq. (8.15), with $\alpha^{(2)}(q)+\alpha^{(4)}(q)$ defined in (8.12). $\tilde{B}_\perp^{(0)}$ and $B_\perp^{(0)}$ are two constants, independent of s. The quantities $S^\mu(k)$, which appear in the definition of α, are given in Eq. (4.18).

The total contribution to the negative-signature channel is given in Eq. (8.32), with $\psi(q,k_\perp,l_\perp)$ defined in Eqs. (8.24) and (8.21).

If we take only the $\alpha^{(2)}(q-k_\perp)\delta(k_\perp-l_\perp)$ term from ψ, (8.32) reduces to

$$-2\pi^2 g^2 \epsilon_b \left[\int \frac{d^2k_\perp}{(2\pi)^4}\exp[\alpha(q-k_\perp)\ln\frac{s}{m^2}] \times \frac{(q-k_\perp)\cdot\gamma+m}{(\vec{q}-\vec{k}_\perp)^2+m^2}\frac{1}{\vec{k}_\perp^2+\mu^2}\right]\epsilon_a \tag{9.1}$$

which corresponds to a contribution from the Reggeized fermion-gluon exchange diagram. The term β, on the other hand, represents the four-point reggeon vertex for external gluons carrying momentum $k_\perp$ and $l_\perp$, and the external Reggeized fermions carrying momenta $(q-k_\perp)$ and $(q-l_\perp)$, respectively.[9]

Our result for $\alpha^{(2)}(q)$ agrees with the standard expression that exists in the literature,[1-5] after the integrations over the plus and the minus components of momenta are performed in (8.12). The imaginary part of the contribution to the positive-signature amplitude agrees with the results of McCoy and Wu.[4] We have also expanded our result for A^- up to four-loop order and found that they agree with the results of explicit calculation of McCoy and Wu. We, however, also find the real part of the first non-leading-logarithm contribution, which gives the $O(g^4)$ correction to the exponent $\alpha(q)$ and shows that the positive-signature amplitude Reggeizes in the next-to-the-leading-logarithm approximation. This is a completely new result.

We have also investigated the contribution to the amplitude beyond the next-to-the-leading-logarithm approximation. It turns out that in the νth non-leading-logarithm approximation ($\nu=0\rightarrow$ leading logarithm), the amplitude may be expressed as

$$\sum_{n=0}^{\nu}\sum_{n'=0}^{\nu}\int d^2k_{1\perp}\cdots d^2k_{n\perp}d^2k'_{1\perp}\cdots d^2k'_{n'\perp}\Phi_\perp^{(n')}(p_b^-,q,\epsilon_b,k'_{1\perp},\ldots,k'_{n'\perp}) \times \hat{S}^{(n,n')}(q,k_{1\perp},\ldots,k_{n\perp},k'_{1\perp},\ldots,k'_{n'\perp})\Gamma_\perp^{(n)}(p_a^+,q,\epsilon_a,k_{1\perp},\ldots k_{n\perp}) \; . \tag{9.2}$$

$\hat{S}^{(n,n')}$ is a calculable function of its arguments and has a well-defined perturbation expansion. The functions $\Gamma_\perp^{(n)}$ are given by

$$\Gamma_\perp^{(n)}(p_a^+,q,\epsilon_a,k_{1\perp},\ldots k_{n\perp}) = \sum_{n'=0}^{\nu}\int E^{(n,n')}(p_a^+,q,k_{1\perp},\ldots,k_{n\perp},k'_{1\perp},\ldots,k'_{n'\perp})B_\perp^{(n')}(q,\epsilon_a,k'_{1\perp},\ldots,k'_{n'\perp})d^2k'_{1\perp}\cdots d^2k'_{n'\perp} \; , \tag{9.3}$$

where

$$E^{(n,n')}(p_a^+,q,k_{1\perp},\ldots,k_{n\perp},k'_{1\perp},\ldots,k'_{n'\perp})$$
$$=\delta_{nn'}\prod_{i=1}^{n}\delta(k_{i\perp}-k'_{i\perp})+A^{(n,n')}(q,k_{1\perp},\ldots,k_{n\perp},k'_{1\perp},\ldots,k'_{n'\perp})\ln\frac{p_a^+}{m}$$
$$+\sum_{r=2}^{\infty}\frac{\ln^r(p_a^+/m)}{r!}\sum_{n_1=1}^{\nu}\sum_{n_2=1}^{\nu}\cdots\sum_{n_{r-1}=1}^{\nu}\int\prod_{i=1}^{r-1}(d^2k_{1\perp}^{(i)}d^2k_{2\perp}^{(i)}\cdots d^2k_{n_i\perp}^{(i)})A^{(n,n_1)}(q,k_{1\perp},\ldots,k_{n\perp},k_{1\perp}^{(1)},\ldots,k_{n_1\perp}^{(1)})$$
$$\times A^{(n_1,n_2)}(q,k_{1\perp}^{(1)},\ldots,k_{n_1\perp}^{(1)},k_{1\perp}^{(2)},\ldots,k_{n_2\perp}^{(2)})\cdots$$
$$\times A^{(n_{r-1},n')}(q,k_{1\perp}^{(r-1)},\ldots,k_{n_{r-1}\perp}^{(r-1)},k'_{1\perp},\ldots,k'_{n'\perp})\ . \quad (9.4)$$

The functions $B_\perp^{(n')}$ are undetermined constants of integration, independent of p_a^+. We have well-defined prescriptions for calculating the functions $A^{(n,n')}$ in perturbation theory. The p_a^+ independent functions $B_\perp^{(n')}$'s may be found, up to the required accuracy, by doing a complete explicit calculation of $\Gamma_\perp^{(n)}$'s up to the $(\nu-n)$-loop order and then comparing the result with expression (9.3).

Similarly, $\Phi_\perp^{(n)}$ is given by

$$\Phi_\perp^{(n)}(p_b^-,q,\epsilon_b,k_{1\perp},\ldots,k_{n\perp})$$
$$=\sum_{n'=0}^{\nu}\int\tilde{B}_\perp^{(n')}(q,\epsilon_b,k'_{1\perp},\ldots,k'_{n'\perp})\tilde{E}^{(n',n)}(p_b^-,q,k'_{1\perp},\ldots,k'_{n'\perp},k_{1\perp},\ldots,k_{n\perp})d^2k'_{1\perp}\cdots d^2k'_{n'\perp}\ , \quad (9.5)$$

where

$$\tilde{E}^{(n,n')}(x,q,k'_{1\perp},\ldots,k'_{n'\perp},k_{1\perp},\ldots,k_{n\perp})=\gamma^0E^{(n,n')\dagger}(x,q,k_{1\perp},\ldots,k_{n\perp},k'_{1\perp},\ldots,k'_{n'\perp})\gamma^0\ . \quad (9.6)$$

Equations (9.2)–(9.6) give us the asymptotic form of the amplitude under consideration. We can, however, state the final result in a simpler form by using the fact that the result must be Lorentz covariant, and hence must be a function of the product $s=p_a^+p_b^-$, rather than of p_a^+ and p_b^- individually. The coefficient of $\ln^n(s/m^2)$ in an expansion in powers of $\ln(s/m^2)$ must then be given by the coefficient of $\ln^n(p_a^+/m)\ln^0(p_b^-/m)$ [more generally, the coefficient of $\binom{n}{r}\ln^{n-r}(p_a^+/m)\ln^r(p_b^-/m)$, for any r]. Thus the full amplitude is given by

$$\sum_{n=0}^{\mu}\sum_{n'=0}^{\nu}\int d^2k_{1\perp}\cdots d^2k_{n\perp}d^2k'_{1\perp}\cdots d^2k'_{n'\perp}\hat{B}_\perp^{(n)}(q,\epsilon_b,k_{1\perp},\ldots,k_{n\perp})$$
$$\times E^{(n,n')}(s/m,q,k_{1\perp},\ldots,k_{n\perp},k'_{1\perp},\ldots,k'_{n'\perp})B_\perp^{(n')}(q,\epsilon_a,k'_{1\perp},\ldots,k'_{n'\perp})\ , \quad (9.7)$$

where

$$\hat{B}_\perp^{(n)}(q,\epsilon_b,k_{1\perp},\ldots,k_{n\perp})=\sum_{n'=0}^{\nu}\int d^2k'_{1\perp}\cdots d^2k'_{n'\perp}\tilde{B}_\perp^{(n')}(q,\epsilon_b,k'_{1\perp},\ldots,k'_{n'\perp})\hat{S}^{(n,n')}(q,k_{1\perp},\ldots,k_{n\perp},k'_{1\perp},\ldots,k'_{n'\perp})\ . \quad (9.8)$$

Thus, in Eq. (9.7), $\hat{B}_\perp^{(n)}$ and $B_\perp^{(n')}$ are constants, independent of s. The s dependence is contained solely in the function

$$E^{(n,n')}(s/m,q,k_{1\perp},\ldots,k_{n\perp},k'_{1\perp},\ldots,k'_{n'\perp})\ ,$$

whose functional dependence on s/m may be obtained from Eq. (9.4). For the νth non-leading-logarithm approximation, we need to keep at most terms up to order $g^{2\nu}$ in the expansion of A. Hence (9.4) combined with (9.7), gives us the coefficient of any arbitrary power of $\ln(s/m^2)$ in the νth non-leading-logarithm approximation, after we calculate A to order $g^{2\nu}$ using the prescription given in the text.

We have also applied the method, used in the text, to calculate the fermion-fermion scattering. The results are given in Appendix C. In the leading-logarithm approximation, the contribution to the

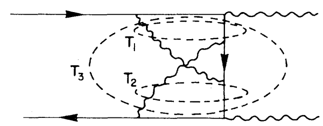


FIG. 22. Examples of tulips and gardens.

odd-C-parity amplitude, in which we have an odd number of gluon exchange in the t channel, is given by Eq. (C9), while the contribution to the even-C-parity amplitude, in which we have an even number of gluon exchange in the t channel, is given in Eq. (C10). The functions σ and ξ, which appear in these equations, are defined with the help of Eqs. (C6)—(C8) and Fig. 29.

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APPENDIX A: EXPRESSING THE AMPLITUDE AS A CONVOLUTION OF $G^{(n)}$, S, AND $F^{(n')}$

In this appendix we shall show how the total amplitude may be described as a sum of terms of the form (3.11). We shall follow an analysis by Collins and Soper.[15] For a given Feynman graph G, we define a "tulip" to be a subdiagram T, such that the full diagram may be topologically decomposed into the form shown in Fig. 3(a), with the subdiagram T as its central soft exchange part. T_1, T_2, and T_3 are the possible tulips in Fig. 22. We also define a "garden" to be a nested set of tulips $\{T_1,\ldots,T_n\}$ such that $T_j \subset T_{j+1}$ for $j=1,\ldots,n-1$. Thus, in Fig. 22, the sets $\{T_1\}$, $\{T_2\}$, $\{T_3\}$, $\{T_1,T_3\}$, and $\{T_2,T_3\}$ are possible examples of gardens. For any tulip T, we define an operation $S(T)$, which we call soft approximation, which multiplies each of the propagators of the gluons coming out of T by $M_c{}^2/(M_c{}^2+\vec{k}^2)$, where k is the momentum carried by the gluon and M_c is some large but fixed mass parameter ($M_c \gg m$). The operation $S(T)$ also picks up only the negative polarizations for the gluons attached to the upper jet, and positive polarizations for the gluons attached to the lower jet. The operation of $S(T)$ on a Feynman graph constrains the lines coming out of T (and hence also the lines within T) to be soft ($|\vec{k}| \lesssim M_c$). These soft lines attach to the upper and the lower blob through γ^+ and γ^- vertices, respectively. Hence the contribution to $S(T)G$ may be expressed in the form (3.11).

Following Ref. 15, we shall express the total contribution to a given Feynman graph G as

$$G=\sum_{\substack{\text{inequivalent}\\ \text{gardens}}}(-1)^{N+1}S(T_1)\cdots S(T_n)G+G_R\ , \tag{A1}$$

where the symbol $S(T_1)\cdots S(T_n)G$ has the following meaning. First make soft approximation for the lines coming out of the largest tulip T_n, belonging to the garden. Then, for T_{n-1}, if some of the lines coming out of T_{n-1} are identical to those coming out of T_n, we leave them untouched, but for the rest of the lines, we again make the soft approximation, and so on. Two gardens are said to be equivalent if the soft approximation is identical for both of them; this happens if they have identical sets of boundaries. N is the maximum number of tulips in a garden. G_R is defined by Eq. (A1). Owing to the presence of the soft subtraction terms, the contribution to G_R from an RD of the form Fig. 3(a) comes from the regions of integration where $|\vec{k}| \gtrsim M_c$ for all the gluons coming out of the blob S. As a result, the contribution to G_R is suppressed by a power of m/M_c. Since M_c is an arbitrary parameter, and the final result must be independent of M_c, we can consistently ignore all terms, carrying a power of m/M_c, in our calculation. Hence the contribution from G_R may be ignored.

The first term on the right-hand side of (A1) may be expressed as

$$\sum_T\left[\sum_{\substack{\text{inequivalent}\\ \text{gardens with}\\ T_n=T}}(-1)^{N+1}S(T_1)\cdots S(T_{n-1})\right]S(T)G\ . \tag{A2}$$

$S(T)G$ may be expressed in the form (3.11). The factor $\sum(-1)^{N+1}S(T_1)\cdots S(T_n)$ makes internal subtractions inside the tulip T, i.e., they modify the function S in (3.11), but they do not affect the form of (3.11). Hence we can see that the total contribution to the amplitude may be expressed as a sum of terms of the form (3.11).

APPENDIX B: ANALYSIS OF $k_i{}^+\partial G^{(n)}/\partial k_i{}^+$

In this appendix, we shall show that the contribution to $\partial G^{(n)}/\partial k_i{}^+$ may be expressed in the form given in (4.20). To do this we use somewhat similar techniques as used in Appendix A. For a given

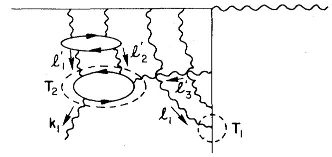

FIG. 23. Subdiagrams T_1 and T_2 in a typical graph contributing to $G^{(n)}$.

graph G contributing to $G^{(n)}$, let T_1 be the vertex at which the soft-fermion line carrying momentum $q-\sum k_i$ leaves $G^{(n)}$. If the gluon line k_i is attached to a fermion loop, then let T_2 be the fermion loop to which the gluon is attached (see Fig. 23). We define the soft approximation $\bar{S}(T_1)G$ by multiplying the Feynman integrand by $M_c^2/(M_c^2+\vec{l}_1^{\,2})$, where l_1 is the momentum of the gluon line attached to T_1, and M_c is a fixed mass, large compared to m. Similarly $\bar{S}(T_2)G$ is defined by multiplying the Feynman integrand by $\prod_{j=1}^{p}[M_c^2/(M_c^2+\vec{l}_j^{\,\prime 2})]$, $l'_1,\ldots,l'_p$ being the momenta of the gluon lines coming out of T_2. $\bar{S}(T_1)\bar{S}(T_2)G$ is defined by multiplying the integrand by a factor of

$$\prod_{j=1}^{p}[M_c^2/(M_c^2+\vec{l}_j^{\,\prime 2})][M_c^2/(M_c^2+l_1^2)] \, .$$

We express the contribution from the graph G as

$$\bar{S}(T_1)G+G_R \tag{B1}$$

or

$$\bar{S}(T_1)G+\bar{S}(T_2)G-\bar{S}(T_1)\bar{S}(T_2)G+G'_R \tag{B2}$$

depending on whether k_i is attached to the open fermion line or a fermion loop. G_R and G'_R in Eqs. (B1) and (B2) have their integrands multiplied by

$$1-M_c^2/(M_c^2+\vec{l}_1^{\,2}) \tag{B3}$$

and

$$1-\frac{M_c^2}{M_c^2+\vec{l}_1^{\,2}}-\prod_{j=1}^{p}\left[\frac{M_c^2}{M_c^2+\vec{l}_j^{\,\prime 2}}\right]+\frac{M_c^2}{M_c^2+\vec{l}_1^{\,2}}\prod_{j=1}^{p}\left[\frac{M_c^2}{M_c^2+\vec{l}_j^{\,\prime 2}}\right], \tag{B4}$$

respectively. Expression (B3) receives contribution from the region $\vec{l}_1^{\,2}\gtrsim M_c^2$, whereas (B4) receives contribution from the region $\vec{l}_1^{\,2}\gtrsim M_c^2$ and at least one of the $\vec{l}_j^{\,\prime 2}\gtrsim M_c^2$.

But we have seen from the analysis of Sec. IV that $k_i^+\partial G/\partial k_i^+$ receives contribution from the region where either l_1 or all the momenta l'_j are soft. Hence the contributions to $k_i^+\partial G_R/\partial k_i^+$ and $k_i^+\partial G'_R/\partial k_i^+$ are suppressed by a power of (m/M_c) and may be ignored. The contribution to $G^{(n)}$ from the other terms, containing soft approximation, may be written as a sum of terms of the form

$$\int f(k_1,\ldots,k_n,k'_1,\ldots,k'_{n'})\times G^{(n')}(p_a,q,\epsilon_a,k'_1,\ldots,k'_{n'})d^4k'_1\cdots d^4k'_{n'}, \tag{B5}$$

where the function f denotes the contribution from the soft lines. f may contain factors of $\delta(\sum_{j\epsilon A}k_j-\sum_{j\epsilon B}k'_j)$, A and B being two subsets of momenta $k_1,\ldots,k_n$ and $k'_1,\ldots,k'_{n'}$, respectively. For example, the contribution from the term $S(T_1)G$ in Fig. 23 has the form

$$\int f_1(k_1,l_1)\delta(k'_1-k_1)G^{(2)}(p_a,q,\epsilon_a,l_1,k'_1)\times d^4l_1d^4k'_1 \, , \tag{B6}$$

whereas the contribution from the term $S(T_2)G$ in Fig. 23 has a form

$$\int f_2(k_1,l'_1,l'_2,l'_3)\delta(k_1-l'_1-l'_2-l'_3)\times G^{(3)}(p_a,q,\epsilon_a,l'_1,l'_2,l'_3)d^4l'_1d^4l'_2d^4l'_3 \, . \tag{B7}$$

f_1 and f_2 in (B6) and (B7) are the contributions from those lines in $S(T_1)G$ and $S(T_2)G$, respectively, which are constrained to carry momenta $\lesssim M_c$, due to the presence of the soft approximation factors. If we now take the general expression (B5) and differentiate with respect to k_i^+, we get

$$\int \frac{\partial f}{\partial k_i^+}(k_1,\ldots,k_n,k'_1,\ldots,k'_{n'})\times G^{(n')}(p_a,q,\epsilon_a,k'_1,\ldots,k'_{n'})d^4k'_1\cdots d^4k'_{n'} \, . \tag{B8}$$

If f does not contain a δ function involving k_i, the above expression has the form given in (4.20). If, however, f has a $\delta(\sum_{j\epsilon A}k_j-\sum_{j\epsilon B}k'_j)$ factor, where the set A contains the momentum k_i, then besides getting terms of the form (4.20), we also get a $\delta'(\sum_{j\epsilon A}k_j-\sum_{j\epsilon B}k'_j)$ factor in the integral. One of the k'_j integrals may then be integrated by parts, and the integral may be expressed as integrals of

$$\partial G^{(n')}(p_a,q,\epsilon_a,k'_1,\ldots,k'_n)/\partial k'^{\,+}_i \, .$$

The $G^{(n')}$'s which appear in (B8), however, have less number of loops than the original $G^{(n)}$. We may analyze the contribution to $\partial G^{(n')}/\partial k'^{\,+}_i$ in the same way as before. We may iterate the process; at each stage we encounter integrals of the form (4.20) and integrals of $\partial G^{(n')}/\partial k'^{\,+}_i$, with less number of loops than the previous stage. Proceeding this way, we shall finally reach the tree-level contribution to $G^{(n)}$, for which $\partial G^{(n)}/\partial k'^{\,+}_i$ vanishes. Hence the total

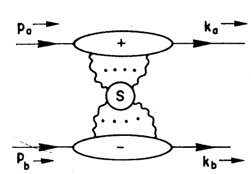

FIG. 24. Regions of integration contributing to the fermion-fermion scattering amplitude.

contribution to $\partial G^{(n)}/\partial k_i^+$ may be expressed completely in terms of integrals of the form (4.20).

APPENDIX C: GLUON EXCHANGE PROCESSES

In this appendix, we shall show how we can use the formalism, developed in this paper, to study the gluon exchange processes in the Regge limit; for example, the near-forward scattering of two fermions. If we work in the Coulomb gauge, we may conclude from power counting that the contribution to the amplitude comes from the regions of integration shown in Fig. 24. We define the functions $\bar{G}^{(n)}(p_a,q,k_1,\ldots,k_n)$ and $\bar{F}^{(n)}(p_b,q,k_1,\ldots,k_n)$ as the Green's functions shown in Fig. 25. We define

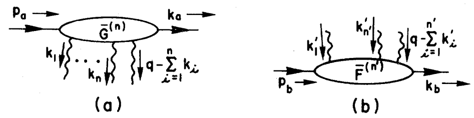

FIG. 25. Green's functions $\bar{G}^{(n)}$ and $\bar{F}^{(n')}$.

$$\bar{\Gamma}^{(n)}(p_a,q,k_{1\perp},\ldots,k_{n\perp})=\int_{-M}^{M}dk_1^-\cdots\int_{-M}^{M}dk_n^-\bar{G}^{(n)}(p_a,q,k_1^-,\ldots,k_n^-,k_1^+=0,\ldots,k_n^+=0,k_{1\perp},\ldots,k_{n\perp}) \quad \text{(C1)}$$

and the corresponding function $\bar{\Phi}^{(n)}$ as integrals of $\bar{F}^{(n)}$. Following the procedure of Sec. VI, we may decompose the amplitude $\bar{\Gamma}^{(n)}$ as

$$\frac{\gamma^+}{2}\bar{\Gamma}_+^{(n)}+\frac{\gamma^-}{2}\bar{\Gamma}_-^{(n)}+\bar{\Gamma}_\perp^{(n)}\,, \quad \text{(C2)}$$

where $\bar{\Gamma}_+^{(n)},\bar{\Gamma}_-^{(n)}$ and $\bar{\Gamma}_\perp^{(n)}$ are products of transverse γ matrices only. Only the $(\gamma^+/2)\bar{\Gamma}_+^{(n)}$ term contributes in the leading power in s. Following the procedure of Sec. V, we may now express the full amplitude as

$$\int \bar{\Phi}_-^{(n')}(p_b,q,k'_{1\perp},\ldots,k'_{n'\perp})\bar{S}^{(n,n')}(q,k_{1\perp},\ldots,k_{n\perp},k'_{1\perp},\ldots,k'_{n'\perp})\bar{\Gamma}_+^{(n)}(p_a,q,k_{1\perp},\ldots,k_{n\perp}) \times d^2k'_{1\perp}\cdots d^2k'_{n'\perp}d^2k_{1\perp}\cdots d^2k_{n\perp}\,. \quad \text{(C3)}$$

$\partial\bar{\Gamma}_+^{(n)}/\partial \ln p_a^+$ is given by an equation of the form (6.5). The second term in (6.5), which is given by Figs. 9(b)—9(d), has no counterpart in the expression for $\partial\bar{\Gamma}^{(n)}/\partial \ln p_a^+$. This is because the only external off-shell lines in $\bar{G}^{(n)}$ are the external soft gluons. In Abelian gauge theory, the sum of all insertions of the S gluon with propagator $(S^\mu k^\nu+S^\nu k^\mu)/(k^2-\mu^2+i\epsilon)$, to such a graph, is the graphs analogous to Fig. 9(a), which are canceled by the change in the wave-function renormalization constants for the external fermions. Thus we are left with the first term on the right-hand side of (6.5). This may be analyzed by Grammer-Yennie decomposition. The K term vanishes when we sum over all insertions. The G part of the ith gluon, when attached to the fermion line, makes that part soft. Power counting indicates that in the case of fermion-fermion scattering, the open fermion lines must always carry collinear momenta, thus soft-fermion lines can come only from the fermion loops. The contribution to $\partial\bar{\Gamma}_+^{(n)}/\partial \ln p_a^+$ then comes from diagrams of the form shown in Fig. 26. This shows that $\partial\bar{\Gamma}^{(0)}/\partial \ln p_a^+=0$, i.e., $\bar{\Gamma}^{(0)}$ does not have any logarithm of p_a^+. (We remind the reader that $\bar{\Gamma}^{(0)}$ is the two-fermion—one-gluon vertex.)

Before proceeding further, we shall make the following classification of diagrams. The sum of all the diagrams, in which the two fermion lines exchange an odd number of gluons (carrying charge parity -1) in the t channel, will be denoted by A^-, whereas the sum of diagrams where the two fermions exchange a charge parity of $+1$ (even number of gluons) will be denoted by A^+.

In the case of the fermion exchange amplitude, the one-particle exchange amplitude gave the leading-logarithm contribution. Here, however, the one-gluon exchange amplitude does not give any logarithm, since $\partial\bar{\Gamma}^{(0)}/\partial \ln p_a^+=0$. We thus have to

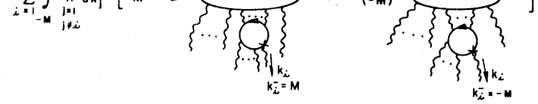

FIG. 26. Contribution to $\partial\bar{\Gamma}_+^{(n)}/\partial \ln p_a^+$.

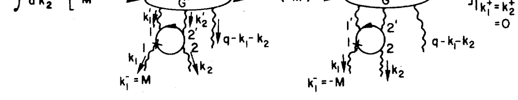

FIG. 27. A typical contribution to $\partial\bar{\Gamma}_+^{(2)}/\partial \ln p_a^+$.

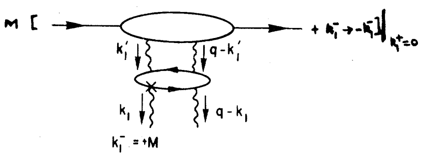

FIG. 28. A typical contribution to $\partial\overline{\Gamma}_+^{(1)}/\partial \ln p_a^+$.

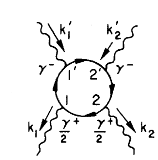

FIG. 29. The Green's function λ.

turn to $\overline{\Gamma}^{(2)}$ and $\overline{\Phi}^{(2)}$ in order to get the leading-logarithm contribution to A^-. In the leading-logarithm approximation, we must try to get as few soft loops in $\partial\overline{\Gamma}^{(2)}/\partial \ln p_a^+$ as possible. As a result, typical contributions to $\partial\overline{\Gamma}^{(2)}/\partial \ln p_a^+$ come from graphs of the form shown in Fig. 27. Similarly the leading-logarithm contribution to A^+ comes from $\overline{\Gamma}^{(1)}$ and $\overline{\Phi}^{(1)}$. $\partial\overline{\Gamma}^{(1)}/\partial \ln p_a^+$ may be analyzed in an exactly similar way and is given by the contribution from the graphs of the form shown in Fig. 28 in the leading-logarithm approximation.

The analysis of the contributions from Figs. 27 and 28 may be carried out exactly in the same way as for the fermion-exchange amplitude. The results are

$$\partial\overline{\Gamma}_+^{(2)}(p_a^+,q,k_{1\perp},k_{2\perp})/\partial \ln p_a^+ = \int d^2k'_{1\perp}d^2k'_{2\perp}\sigma(q,k_{1\perp},k_{2\perp},k'_{1\perp},k'_{2\perp})\overline{\Gamma}_+^{(2)}(p_a^+,q,k'_{1\perp},k'_{2\perp}) , \tag{C4}$$

$$\frac{\partial\overline{\Gamma}_+^{(1)}}{\partial \ln p_a^+}(p_a^+,q,k_{1\perp}) = \int d^2k'_{1\perp}\xi(q,k_{1\perp},k'_{1\perp})\overline{\Gamma}_+^{(1)}(p_a^+,q,k'_{1\perp}) , \tag{C5}$$

where

$$\begin{aligned}\sigma(q,k_{1\perp},k_{2\perp},k'_{1\perp},k'_{2\perp}) = &\frac{1}{(2\pi)^4}\{(-1)/[(\vec{k}'^{\,2}_{1\perp}+\mu^2)(\vec{k}'^{\,2}_{2\perp}+\mu^2)]\}(M/2)\\ &\times\int \frac{dk'^+}{2}[\tau(M,k'^+,k_{1\perp},k_{2\perp},k'_{1\perp},k'_{2\perp})+\tau(M,k'^+,k_{1\perp},q-k_{1\perp}-k_{2\perp},k'_{1\perp},k'_{2\perp})\\ &\qquad+\tau(M,k'^+,k_{2\perp},q-k_{1\perp}-k_{2\perp},k'_{1\perp},k'_{2\perp})] ,\end{aligned} \tag{C6}$$

$$\xi(q,k_{1\perp},k'_{1\perp}) = \frac{1}{(2\pi)^4}\frac{(-1)}{(\vec{k}'^{\,2}_{1\perp}+\mu^2)[(\vec{q}-\vec{k}'_{1\perp})^2+\mu^2]}\left[\frac{M}{2}\right]\int\frac{dk'^+}{2}\int d^2k'_{2\perp}\tau(M,k'^+,k_{1\perp},q-k_{1\perp},k'_{1\perp},k'_{2\perp}), \tag{C7}$$

where M is an arbitrary mass $\sim m$ and

$$\begin{aligned}&\tau(M,x,k_{1\perp},k_{2\perp},k'_{1\perp},k'_{2\perp})\\ &\quad=[\lambda(k_1^+=k_2^+=0,k_1^-=-k_2^-=M,k_{1\perp},k_{2\perp},k_1'^-=k_2'^-=0,k_1'^+=-k_2'^+=x,k'_{1\perp},k'_{2\perp})\\ &\qquad+\lambda(k_1^+=k_2^+=0,k_1^-=-k_2^-=-M,k_{1\perp},k_{2\perp},k_1'^-=k_2'^-=0,k_1'^+=-k_2'^+=x,k'_{1\perp},k'_{2\perp})]\\ &\qquad\times\delta^{(2)}(k'_{1\perp}+k'_{2\perp}-k_{1\perp}-k_{2\perp}) ,\end{aligned} \tag{C8}$$

λ being the truncated four-photon Green's function shown in Fig. 29. ξ and σ may easily be seen to be independent of M, by rescaling the positive and the negative momenta by M and M^{-1}, respectively.

Equations (C4) and (C5) may be solved to obtain a series expansion for $\overline{\Gamma}_+^{(2)}$ and $\overline{\Gamma}_+^{(1)}$, respectively, exactly in the way we solved Eq. (7.9). Similar solutions may be obtained for $\overline{\Phi}_-^{(1)}$ and $\overline{\Phi}_-^{(2)}$. Putting together all the results, we find the following expressions for the contribution to the odd- and the even-charge- parity amplitudes respectively,

$$\begin{aligned}A_{\rm odd} = &\frac{1}{2}\left[\frac{i}{3!}\right](2\pi)^4 g^6\overline{u}(k_a)\gamma^+u(p_a)\overline{u}(k_b)\gamma^-u(p_b)\\ &\times\int\frac{d^2k_{1\perp}}{(2\pi)^4}\frac{d^2k_{2\perp}}{(2\pi)^4}\Big[(\vec{k}_{1\perp}^{\,2}+\mu^2)^{-1}(\vec{k}_{2\perp}^{\,2}+\mu^2)^{-1}[(\vec{q}-\vec{k}_{1\perp}-\vec{k}_{2\perp})^2+\mu^2]^{-1}\\ &\qquad\times\Big[1+\sum_{n=1}^{\infty}\frac{1}{n!}\ln^n\frac{s}{m^2}\int\prod_{i=1}^{n}(d^2l_{1\perp}^{(i)}d^2l_{2\perp}^{(i)})\sigma(q,k_{1\perp},k_{2\perp},l_{1\perp}^{(1)},l_{2\perp}^{(1)})\\ &\qquad\times\sigma(q,l_{1\perp}^{(1)},l_{2\perp}^{(1)},l_{1\perp}^{(2)},l_{2\perp}^{(2)})\cdots\sigma(q,l_{1\perp}^{(n-1)},l_{2\perp}^{(n-1)},l_{1\perp}^{(n)},l_{2\perp}^{(n)})\Big]\Big] ,\end{aligned} \tag{C9}$$

$$A_{\text{even}} = -\pi^2 g^4 \bar{u}(k_a)\gamma^+ u(p_a)\bar{u}(k_b)\gamma^- u(p_b) \times \int \frac{d^2k_\perp}{(2\pi)^4} \left\{ (\vec{k}_\perp{}^2+\mu^2)^{-1}[(\vec{q}-\vec{k}_\perp)^2+\mu^2]^{-1} \times \left[1+\sum_{n=1}^{\infty} \frac{1}{n!} \ln^n \left[\frac{s}{m^2} \right] \int d^2 l_\perp^{(1)} \cdots d^2 l_\perp^{(n)} \xi(q,k_\perp,l_\perp^{(1)}) \times \xi(q,l_\perp^{(1)},l_\perp^{(2)}) \cdots \xi(q,l_\perp^{(n-1)},l_\perp^{(n)}) \right] \right\} \tag{C10}$$

Corrections to Eqs. (C9) and (C10) due to the nonleading logarithms may be evaluated exactly in the same way as for the fermion exchange amplitude. Hence we shall not discuss them here. Also, if we want to be consistent, we should add to the expression (C9) for A^- the one-gluon exchange amplitude, which does not have any logarithm of s, but which still contributes in the leading power in s. It has the form

$$f(q)\bar{u}(k_a)\gamma^+ u(p_a)\bar{u}(k_b)\gamma^- u(p_b) \ . \tag{C11}$$

$f(q)$ is a function of q, but is independent of s.

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