Nonperturbative analysis of the effective potential in the O(N) model

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The effective potential $V(|\hat{\phi}|)$ of a general O(N)-symmetric scalar field theory is discussed. It is shown that the first and second derivatives of $V(|\hat{\phi}|)$ with respect to the magnitude $|\hat{\phi}|$ of the classical field cannot be negative. These properties imply that if spontaneous symmetry breaking occurs in such a model, then $V(|\hat{\phi}|)$ does not exist for a range of $|\hat{\phi}|$, and that at most one nontrivial minimum is present in the theory. This analysis does not depend upon the spacetime dimensionality of the system or upon the particulars of any perturbation series. The results of a Monte Carlo calculation of the derivative of the effective potential in a lattice version of the theory are also presented.

The concept of spontaneous symmetry breaking is of crucial importance in elementary particle physics. Our understanding of this phenomenon, however, is based largely upon arguments couched within the framework of perturbation theory. Unfortunately one can never be entirely certain whether a perturbation series reflects the fundamental nature of the theory or whether it includes artifacts of the perturbative method. A nonperturbative analysis of a problem previously addressed only with perturbative techniques in fact often leads to surprises.

One such surprise occurs in the study of a quantity known[1-6] as the effective potential in scalar field theories. The effective potential $V(\hat{\phi})$ is the energy per unit volume in a state in which the expectation values $\hat{\phi}$ of field operators are constrained to equal their (translationally invariant) classical values.[6] When the effective potential of ϕ^4 field theory is calculated by means of the (perturbative) loop expansion, the lowest-order-result exists for all $\hat{\phi}$ and equals the potential term $U(\hat{\phi})$ in the action of the theory. If spontaneous symmetry breaking takes place this lowest-order *approximate* result for $V(\hat{\phi})$ possess a double-well (or "wine-bottle") shape.

Nonperturbative arguments,[7] however, indicate that whenever spontaneous symmetry breaking occurs in this theory the *exact* effective potential *does not exist* for a range of $\hat{\phi}$. In addition it has been shown[7] that a double-well shape for $V(\hat{\phi})$ violates certain positivity constraints and can therefore never occur. Thus the loop expansion in ϕ^4 theory must break down for a range of $\hat{\phi}$ when spontaneous symmetry breaking occurs. These arguments are independent of the spacetime dimensionality of the system and are therefore not related to the known[8-11] triviality of continuum ϕ^4 theory in four and greater dimensions.

In this paper the arguments developed previously for single-component ϕ^4 theory are generalized to models containing an interacting field $\underline{\phi}$ whose action is O(N) symmetric. The major result of the analysis of the single-component theory—the nonexistence of the effective potential $V(|\hat{\underline{\phi}}|)$ for a range of $|\hat{\underline{\phi}}|$ when spontaneous symmetry breaking occurs—persists in this more general case. This result is independent of the presence of gauge fields. The derivative of the effective potential is also calculated for various values of N in a lattice version of the theory.

At this point it is useful to review the effective-potential formalism[1-6,12] for an N-component scalar field $\underline{\phi}$. The following remarks and their implications (as described above) do not depend on the regulator used. However, since the numerical analysis presented in the latter part of this paper is performed using a Euclidean lattice version of the theory, it is this lattice theory which is explicitly discussed below.

It is first necessary to define the generating functional $W\{\underline{J}\}$,

$$e^{-W\{\underline{J}\}} \equiv \int \mathscr{D}\phi\, e^{-S\{\underline{\phi},\underline{J}\}}\ , \tag{1a}$$

where $\underline{\phi}$ and $\underline{J}$ are N-component fields defined on each of the L lattice sites, and the action is given by the sum

$$S\{\underline{\phi},\underline{J}\} \equiv \tfrac{1}{2}\sum_{\langle ij\rangle}(\underline{\phi}_i-\underline{\phi}_j)^2 + \sum_i [U(|\underline{\phi}_i|)+\underline{J}\cdot\underline{\phi}]\ , \tag{1b}$$

with

$$|\underline{\phi}_i|^2 \equiv \sum_{a=1}^{N} \phi_{i,a}{}^2 \equiv \underline{\phi}_i \cdot \underline{\phi}_i \tag{1c}$$

and

$$\int \mathscr{D}\phi \equiv \prod_m \int d\phi_m \ .$$

The quantity $U(|\underline{\hat{\phi}}|)$ is an unspecified $O(N)$-symmetric potential term. The notation $\langle ij \rangle$ refers to all nearest-neighbor pairs on the lattice (summed once), and each component of the real $\{\underline{\phi}\}$ ranges from $-\infty$ to $+\infty$.

The expectation value $\underline{\hat{\phi}}_i$ in the presence of sources $\{\underline{J}\}$ is given by

$$\underline{\hat{\phi}}_i \equiv \langle \underline{\phi}_i \rangle_J = \frac{\partial W\{\underline{J}\}}{\partial \underline{J}_i} \ . \tag{2}$$

The effective action $\Gamma\{\underline{\hat{\phi}}\}$ is equal to the Legendre transform of $W\{\underline{J}\}$,

$$\Gamma\{\underline{\hat{\phi}}\} \equiv W\{\underline{J}\} - \sum_i \langle \underline{\hat{\phi}}_i \rangle \cdot \underline{J}_i \ . \tag{3}$$

The right-hand side of Eq. (3) is rendered an explicit function of the $\{\underline{\hat{\phi}}\}$ by using Eq. (2). The effective potential is then determined from the effective action by setting all the $|\underline{\hat{\phi}}_i|$ equal to some value $|\underline{\hat{\phi}}|$,

$$V(|\underline{\hat{\phi}}|) = \frac{1}{L}\Gamma\{|\underline{\hat{\phi}}_i| = |\underline{\hat{\phi}}|\} \ . \tag{4}$$

Note, however, that if Eq. (2) cannot be inverted to give $\underline{J}$ as a (real) function of $\underline{\phi}$, the effective potential cannot rigorously be defined. This point is central to the following analysis.

Equations (2) and (3) can be used to derive a result which is Legendre dual to Eq. (2), namely,

$$\frac{\partial \Gamma\{\underline{\hat{\phi}}\}}{\partial \underline{\hat{\phi}}_i} = -\underline{J}_i \ . \tag{5}$$

As the effective potential in an $O(N)$-symmetric field theory must depend only on the magnitude $|\underline{\hat{\phi}}|$, Eqs. (4) and (5) yield

$$|\underline{\hat{\phi}}|\, V' \equiv |\underline{\hat{\phi}}| \frac{dV(|\underline{\hat{\phi}}|)}{d|\underline{\hat{\phi}}|} = -\underline{\hat{\phi}} \cdot \underline{J} \ . \tag{6}$$

Spontaneous symmetry breaking occurs whenever the left-hand side of Eq. (6) *approaches* zero for nonzero $|\underline{\hat{\phi}}|$. Equation (6) is also used below to determine the derivative of the effective potential (with respect to $|\underline{\hat{\phi}}|$) directly, given J and the expectation values $\langle \underline{\phi} \rangle_J$.

For the case of a single-component ϕ^4 theory (i.e., when $N=1$) a relation which crucially delimits the form of the effective potential follows[7] from Eq. (5), namely,

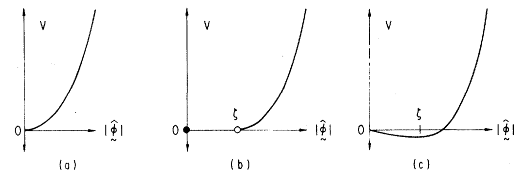

FIG. 1. (a) Qualitative form of the effective potential $V(|\underline{\hat{\phi}}|)$ when spontaneous symmetry breaking does not take place. (b) Qualitative form of the effective potential $V(|\underline{\hat{\phi}}|)$ when spontaneous symmetry breaking takes place. (c) Qualitative form of the classical approximation to the effective potential $V(|\underline{\hat{\phi}}|)$ when spontaneous symmetry breaking takes place.

$$V'' \equiv \frac{d^2V(|\underline{\hat{\phi}}|)}{d|\underline{\hat{\phi}}|^2} \geq 0 \ . \tag{7}$$

When spontaneous symmetry breaking does not occur, V' is always bounded away from zero for nonzero $\underline{\hat{\phi}}$. In this case Eq. (7) places no real constraint on the form of the effective potential and it is qualitatively much like its classical form [Fig. 1(a)]. In the broken-symmetry case, however, the effective potential of single-component ϕ^4 theory looks[7] qualitatively like the plot in Fig. 1(b). Note two novel features of this plot: (i) The effective potential is not defined when $|\underline{\hat{\phi}}|$ is less than or equal to the value ζ at which $V(|\underline{\hat{\phi}}|)$ attains its minimum,[13] except for a single point at $|\underline{\hat{\phi}}|=0$; (ii) the effective potential evaluated at a nontrivial minimum equals the effective potential at the origin, where it is set equal to zero [compare the classical picture, Fig. 1(c)].

Both of these features persist in the general $O(N)$ theory. In order to demonstrate this fact it is first necessary to define the matrix

$$\omega_{ab} \equiv \frac{-\partial^2 W(|\underline{J}|)}{\partial J_a \partial J_b} \tag{8a}$$

and its inverse

$$\gamma_{ab} \equiv [\omega^{-1}]_{ab} = \frac{\partial^2 V(|\underline{\hat{\phi}}|)}{\partial \hat{\phi}_a \partial \hat{\phi}_b} \ . \tag{8b}$$

In Eqs. (8) the indices a and b refer to each of the N vector components of a translationally invariant current $\underline{J}$ and the corresponding expectation values $\underline{\hat{\phi}}$. The matrix ω can be expressed in terms of expectation values:

$$\omega_{ab} = \langle \delta\phi_a \delta\phi_b \rangle_J \ , \tag{9a}$$

$$\delta\phi_a \equiv \phi_a - \langle \phi_a \rangle_J \ . \tag{9b}$$

The matrix ω is positive semidefinite (that is, none of its eigenvalues are negative). This positivity condition may be derived by noting that if the classical potential energy of a particle at x^a in N dimensions is given by

$$C(\underline{x})=\frac{1}{2}\langle(\underline{x}\cdot\delta\underline{\phi})^2\rangle_J\ , \tag{10}$$

then the squared eigenfrequencies of the resulting harmonic motion are given by the eigenvalues of the matrix ω. Since motion in the potential, Eq. (10), is bounded or at most unconstrained, none of the eigenvalues of ω are negative.

When the inverse matrix γ exists (i.e., when the effective potential and its first two derivatives can be defined), it too must therefore be positive semidefinite. The matrices ω and γ can then be written explicitly in terms of the effective potential,

$$\omega_{ab}=\delta_{ab}\frac{|\hat{\underline{\phi}}|}{V'}+e_a e_b\left[\frac{1}{V''}-\frac{|\hat{\underline{\phi}}|}{V'}\right], \tag{11a}$$

$$\gamma_{ab}=\delta_{ab}\frac{V'}{|\hat{\underline{\phi}}|}+e_a e_b\left[V''-\frac{V'}{|\hat{\underline{\phi}}|}\right], \tag{11b}$$

where

$$e_a\equiv\frac{\hat{\phi}_a}{|\hat{\underline{\phi}}|}\ , \tag{11c}$$

and Eq. (11a) is understood to be written as a function of $\underline{J}$.

The effect of the applied current $\underline{J}$ is to *decrease* the average action of the system. The only term in the action which is not explicitly O(N) symmetric is the coupling of the $\underline{\phi}$ field to the applied current $\underline{J}$. Thus when $\underline{J}$ is nonzero the N-component vector $\langle\underline{\phi}\rangle_J$ always points in a direction such that the expectation value of the source term in the action is minimized, i.e.,

$$\underline{J}\cdot\langle\underline{\phi}\rangle_J=-|\underline{J}||\langle\underline{\phi}\rangle_J|\le 0\ , \tag{12a}$$

and thus from Eq. (6),

$$V'=|\underline{J}|\ge 0\ . \tag{12b}$$

Since γ is positive semidefinite its determinant cannot be negative,

$$\mathrm{Det}\,\gamma=V''\left[\frac{V'}{|\hat{\underline{\phi}}|}\right]^{N-1}\ge 0\ , \tag{13a}$$

and therefore

$$V''\ge 0\ , \tag{13b}$$

provided $V'\neq 0$. Of course the presence of spontaneous symmetry breaking requires that V' *approach* zero arbitrarily closely for some nonzero $|\hat{\underline{\phi}}|$. Note that the arguments used to derive the relations (12b) and (13b) are independent of whether or not gauge fields are present in the path integral.

Features (i) and (ii) of the single-component theory are easily generalized to the O(N)-symmetric theory by use of the relations (12b) and (13b). One method of demonstrating these facts is to consider the theory on a finite lattice, where spontaneous symmetry breaking cannot strictly take place (and V' is therefore strictly positive if $|\hat{\underline{\phi}}|$ is nonzero). All derivatives of $W\{\underline{J}\}$ are then analytic, so all (positive) values of $|\hat{\underline{\phi}}|$ are present and the effective potential exists for all $\hat{\underline{\phi}}$.

If spontaneous symmetry breaking is to occur in the infinite-system limit for a certain parameter set, the effective potential for the corresponding finite system has no minimum for nonzero $|\hat{\underline{\phi}}|$. When the infinite-system limit is taken, by assumption V' approaches zero at some nonzero value of $|\hat{\underline{\phi}}|$, denoted by ζ. The effective potential $V(|\hat{\underline{\phi}}|)$ must also approach zero at ζ in this limit since V'' is positive and V is set equal to zero at $|\hat{\underline{\phi}}|=0$. As the value of ζ is independent of the way $\underline{J}$ approaches zero [the effective potential is O(N) symmetric], the effective potential ceases to exist to the left of ζ in the infinite-system limit. Thus the effective potential in the broken-symmetry case must approach the qualitative form [Fig. 1(b)].

Similar "finite-system" arguments can be used to show that if a minimum of V exists for nonzero $|\hat{\underline{\phi}}|$ it must be unique. Such a result is in contrast to previous speculation[5] by Coleman, Jackiw, and Politzer on the existence of multiple minima in the large-N limit of the model.

Although feature (ii) implies that any nontrivial minimum of V must be a global minimum, neither it nor any other part of the above analysis indicates whether spontaneous symmetry breaking indeed occurs. Rather, *implications* of the presence of a nontrivial vacuum in the theory were discussed. The following Monte Carlo results are intended to provide some evidence for the existence of spontaneous symmetry breaking in a lattice O(N) model. The above analysis only utilized the O(N) symmetry of the potential term $U(|\hat{\underline{\phi}}|)$. For the following numerical analysis, the site potential is specified as

$$U(|\underline{\phi}_i|)=\frac{\lambda}{N}(|\underline{\phi}_i|^2-f)^2\ . \tag{14}$$

An important feature of the Monte Carlo calculation is the method used to "update" the lattice in order to bring it into equilibrium. Each component a of the field ϕ_i at site i is updated by first generating a new field $\phi_{i,a}^{\mathrm{new}}$ by

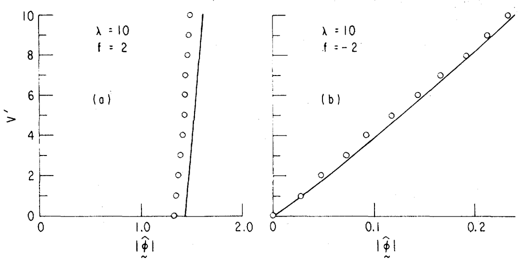

FIG. 2. (a) Plot of V' vs $|\underline{\hat{\phi}}|$ for $N=2$, $\lambda=10$, and $f=2$. The open circles show the result of a Monte Carlo calculation, while the solid line is a plot of the augmented classical approximation. (b) Same as (a), but with $f=-2$.

$$\phi_{i,a}^{\text{new}}=\phi_{i,a}+(2r-1)\Delta \ , \tag{15}$$

where r is a random number distributed uniformly between zero and unity and Δ is a parameter chosen empirically ($\Delta \sim 1-10$ in the present calculation). Acceptance of the generated value $\phi_{i,a}^{\text{new}}$ is governed by the Metropolis algorithm.[14] Undesirable correlations are avoided by separating each of the measurements of the expectation values $\langle \underline{\phi} \rangle$ by ten updates of the entire lattice. The plotted values are the average of ten such separated measurements. Also, the entire lattice is first allowed to equilibrate for 100 iterations before any measurements are taken. The derivative V' is evaluated by measuring the expectation values $\langle \underline{\phi} \rangle_J$ and applying Eq. (6).

Figures 2 and 3 show the derivative V' of the unrenormalized effective potential[15] plotted as a function of its argument $|\underline{\hat{\phi}}|$ for $N=2$ and 10, respectively, for a 4^4 lattice. Each pair (a) and (b) of figures displays this derivative for sets of parameters λ and f for which spontaneous symmetry breaking apparently does and does not take place, respectively. The effective potential can be obtained from these figures by integrating with respect to $|\underline{\hat{\phi}}|$. Note that in accord with the above discussion the effective potential apparently does not exist for a range of $|\underline{\hat{\phi}}|$ when V' approaches zero for nonzero $|\underline{\hat{\phi}}|$.

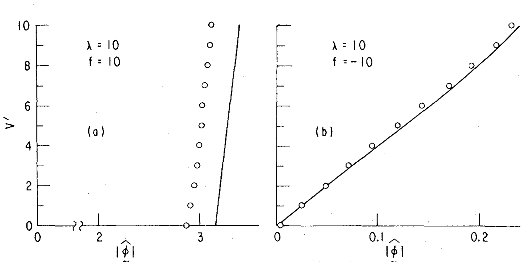

FIG. 3. (a) Plot of V' vs $|\underline{\hat{\phi}}|$ for $N=\lambda=f=10$. The legend is as for Fig. 2(a). (b) Same as (a), but with $f=-10$.

The solid line in these figures is constructed by applying a graphical procedure similar to one used in Ref. 7 for single-component ϕ^4 theory. The method is implemented by taking a perturbative formula for the effective potential and removing those regions where the positivity constraints, Eqs. (12b) and (13b), are violated. This prescription yields an *augmented* perturbative formula which exhibits several of the characteristics of the exact result lacking in the perturbation series. This augmented formula is a generalized "Maxwell construction." Note that the usual thermodynamic Maxwell construction is strictly valid only for *finite* systems.[7]

The classical (i.e., lowest-order loop expansion) formula for the effective potential,

$$V'_{\text{classical}}(|\underline{\hat{\phi}}|)=\frac{4\lambda}{N}|\underline{\hat{\phi}}|(|\underline{\hat{\phi}}|^2-f) \ , \tag{16}$$

is used here as the input to the graphical procedure. The result is an augmented classical approximation for $V'(|\underline{\hat{\phi}}|)$ which is equal to

$$V'_{\text{aug}}(|\underline{\hat{\phi}}|)=V'_{\text{classical}}(|\underline{\hat{\phi}}|) \ , \tag{17}$$

when $|\underline{\hat{\phi}}|^2$ is greater than f. The augmented classical approximation is undefined when $|\underline{\hat{\phi}}|^2$ is less than f. The agreement between this augmented classical approximation and the Monte Carlo results[15] is excellent, as can be seen in Figs. 2 and 3.

What has the above nonperturbative analysis taught us? First, we have learned that the effective potential of a scalar field theory with O(N) symmetry cannot have a multiple-well (wine-bottle) form since its first and second derivatives are nonnegative. This restriction may be a bit surprising, since the lowest-order term in a loop-expansion calculation is equal to the potential term in the action, which indeed can have such a structure when spontaneous symmetry breaking occurs.

The loop expansion for $V(|\underline{\hat{\phi}}|)$ is not expected to converge for small $|\underline{\hat{\phi}}|$ however. Each additional term in this perturbation series generally includes[3] more powers of $\ln|\underline{\hat{\phi}}|$, and so for sufficiently small $|\underline{\hat{\phi}}|$ each higher term in the expansion is larger than its previous term. In the broken-symmetry case the enormous qualitative difference between the nonperturbative result for the effective potential, Fig. 1(b), and the leading term in the loop expansion, Fig. 1(c), may be attributed to the breakdown of this perturbation series.

In addition, the above nonperturbative analysis yielded some facts about the nature of the effective potential in O(N) theories. Since these facts do not appear to be widely known, they are listed below.

(a) The effective potential in the O(N) model does not exist for a range of $|\underline{\hat{\phi}}|$ if spontaneous symmetry breaking occurs. This result is in accord with the speculations and physical arguments given in Ref. 5.

(b) At most one nontrivial minimum exists in this quantity. This result is contrary to the speculations[5] of Coleman, Jackiw, and Politzer.

(c) The effective potential is zero at a nontrivial minimum. This occurrence is made possible by the nonexistence of V for a range of $|\hat{\underline{\phi}}|$, and made necessary by Eqs. (12b) and (13b) above. Note that the effective potential is set equal to zero at $|\hat{\underline{\phi}}|=0$.

The above conclusions are quite different from those one might have gleaned from a perturbative analysis[5,12] of the problem. Perturbative methods alone thus are not adequate for a complete treatment of spontaneous symmetry breakdown. Perhaps the most striking feature of the above nonperturbative analysis—the nonexistence of the effective potential for a range of $|\hat{\underline{\phi}}|$—is, however, easy to understand. Such a situation arises because it is in general impossible to realize all values of the classical field $\langle \underline{\phi} \rangle_J$ by the application of a translationally invariant current $\underline{J}$. Thus, there is a "forbidden region" of metastable field configurations in the space of the $\langle \phi \rangle_J$. An example of a similar phenomenon occurs[17] in magnetic systems, where certain values of the magnetization cannot be produced by any constant applied magnetic field. The concept of metastability is also very useful in thermodynamics.[18]

It is often possible to perform an analytic continuation of the effective potential into unphysical regions. The continuation typically possesses an imaginary part, which can be identified[5] with the decay probability of the unstable field configuration per unit space-time volume. Indeed, when the dynamical evolution of field configurations is important the concept of a complex generalization of the effective potential may be relevant. This concept could be used in an approximation scheme similar to time-dependent perturbation theory, where transitions to unstable excited configurations may be induced by a non-Hermitian contribution to the Hamiltonian (corresponding to complex $\underline{J}$).

A more correct analysis of the time evolution of a "prepared state" might involve the use of a current $\underline{J}(x_\mu)$ which is not spacetime translationally invariant. Such a current can be used to prepare a particular state at a given time, and the dynamical evolution of this state could then be studied via the effective action, Eq. (3).

Although much of the above analysis [in particular, the positivity constraints, Eqs. (12b) and (13b)] is independent of the presence of gauge fields, interesting phenomena[3] arise upon their inclusion. Extensions of the above research to problems involving gauge fields (such as the Abelian Higgs model[19]) are therefore underway.

Note added in proof. Since this paper was submitted several aspects of this problem have been discussed in detail.[20] It has also been suggested[21] that a microcanonical simulation[22] for a finite system may be useful in resolving problems of metastability in the effective potential.

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[1]J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962).

[2]G. Jona-Lasinio, Nuovo Cimento 34, 1970 (1964).

[3]S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

[4]R. Jackiw, Phys. Rev. D 9, 1686 (1974); J. S. Kang, *ibid.* 10, 3455 (1974); S. Coleman, in *Laws of Hadronic Matter,* edited by A. Zichichi (Academic, New York, 1975).

[5]S. Coleman, R. Jackiw, and H. D. Politzer, Phys. Rev. D 10, 2491 (1974).

[6]K. Symanzik, Commun. Math. Phys. 16, 48 (1970).

[7]D. J. E. Callaway and D. J. Maloof, Phys. Rev. D 27, 406 (1983). See also Ref. 6 and J. Iliopoulos, C. Itzykson, and A. Martin, Rev. Mod. Phys. 47, 165 (1975).

[8]K. G. Wilson, Phys. Rev. B 4, 3184 (1971); K. G. Wilson and J. Kogut, Phys. Rep. 12C, 75 (1974); G. A. Baker and J. Kincaid, Phys. Rev. Lett. 42, 1431 (1979); R. Schrader, Phys. Rev. B 14, 172 (1976); G. A. Baker and S. Krinsky, J. Math. Phys. 18, 590 (1977).

[9]C. M. Bender, F. Cooper, G. S. Guralnik, R. Roskies, and D. H. Sharp, Phys. Rev. D 23, 2976 (1981); 23, 2999 (1981).

[10]B. Freedman, P. Smolensky, and D. Weingarten, Phys. Lett. 113B, 481 (1982), and references cited therein.

[11]M. Aizenmann, Phys. Rev. Lett. 47, 1 (1981); J. Fröhlich, Nucl. Phys. B200 [FS4], 281 (1982).

[12]The effective potential of the O(N) model has been stud-

ied in various approximation schemes. See, e.g., Refs. 4 and 5 above, and S. Weinberg, Phys. Rev. D 9, 3357 (1974); L. Dolan and R. Jackiw, *ibid.* 9, 3320 (1974); H. J. Schnitzer, *ibid.* 10, 1800 (1974); R. G. Root, *ibid.* 10, 3322 (1974); M. Kobayashi and T. Kugo, Prog. Theor. Phys. 54, 1537 (1975); R. W. Haymaker, Phys. Rev. D 12, 1178 (1975); 13, 968 (1976); L. F. Abbott, J. S. Kang, and H. J. Schnitzer, *ibid.* 13, 2212 (1976); R. L. Kobes and A. Z. Capri, *ibid.* 25, 1039 (1982); T. Appelquist and U. Heinz, *ibid.* 24, 2169 (1981); 25, 2620 (1982); U. Heinz, *ibid.* 25, 2717 (1982); and references cited therein.

[13]The fact that $V(|\hat{\phi}|)$ is not defined precisely *at* the point $|\hat{\phi}|=\zeta$ is a consequence of the fact that the expectation value $\langle\phi\rangle_J$ is a single-valued function of J (see Ref. 7 for a further discussion of this point), and is equal to zero when J equals zero. Because the point $|\hat{\phi}|=\zeta$ is missing, it is possible to expand in quadratic fluctuations about any point $|\underline{\phi}|>\zeta$ (recall that between any two points there lies a third point). Such an expansion is necessary in order to do perturbation theory in a nontrivial vacuum. Note that $\langle\phi\rangle_J$ may approach a nonzero *limit* for infinitesimal J and still vanish when J equals zero.

[14]N. Metropolis, A. W. Rosenbluth, M. N. Rosenbluth, A. H. Teller, and E. Teller, J. Chem. Phys. 21, 1087 (1953).

[15]Because no quantitative comparison between lattice and continuum results is being made here, the effective potential has not been renormalized. The qualitative results of this paper [including the positivity constraints, Eqs. (12b) and (13b)] should not be affected by the renormalization procedure. The necessary positivity of the field-strength renormalization constant can be demonstrated by general arguments (Ref. 16).

[16]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965), p. 140.

[17]See, e.g., D. Amit, *Field Theory, the Renormalization Group, and Critical Phenomena* (McGraw-Hill, London, 1978), p. 93.

[18]An excellent discussion of metastable states appears in H. B. Callen, *Thermodynamics* (Wiley, New York, 1960), Chap. 5.

[19]A preliminary Monte Carlo analysis of this model appears in D. J. E. Callaway and L. J. Carson, Phys. Rev. D 25, 531 (1982); and in K. C. Bowler, G. S. Pawley, B. J. Pendleton, D. J. Wallace, and G. W. Thomas, Phys. Lett. 104B, 481 (1981).

[20]Y. Fujimoto, L. O'Raifeartaigh, and G. Parravicini, Nucl. Phys. B212, 268 (1983); R. W. Haymaker and J. Perez-Mercader, Phys. Rev. D 27, 1948 (1983); C. M. Bender and F. Cooper, Los Alamos Report No. LA-UR-83-222 (unpublished); R. D. C. Miller, CERN Report No. TH.3484-CERN, 1982 (unpublished); F. Cooper and B. Freedman, Los Alamos report, 1983 (unpublished); W. A. Bardeen and M. Moshe, Fermilab report, 1983 (unpublished); P. Ginsburg and H. Levine, Harvard Report No. HUTP-82/A034 (unpublished).

[21]U. M. Heller and N. Seiberg, Institute for Advanced Study report, 1983 (unpublished).

[22]D. J. E. Callaway and A. Rahman, Phys. Rev. Lett. 49, 613 (1982); and Argonne Report No. ANL-HEP-PR-83-04 (unpublished).