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Birth of inflationary universes

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A cosmological model is proposed in which the Universe is created by quantum tunneling from "nothing" into a de Sitter space. The tunneling is described by a de Sitter–Hawking–Moss instanton. After the tunneling, the model evolves along the lines of the inflationary scenario. It is argued that at any time there exist parts of the Universe which are still in the de Sitter phase, while other parts have already recollapsed. This model does not have a big-bang singularity and does not require any initial or boundary conditions.

I. INTRODUCTION

The inflationary-universe scenario[1–3] gives a possible solution to three major cosmological puzzles. They are the following.

(1) Horizon puzzle.[4] The temperature of the Universe is nearly the same even in regions which, according to the standard scenario, have never been in causal contact.

(2) Flatness puzzle.[5] Our universe is almost flat, its density is within one order of magnitude of the critical density: $\rho/\rho_c \sim 1$. It is known, however, that small deviations from $\rho=\rho_c$ grow in time, and to have $\rho\sim\rho_c$ today, one must have $(\rho-\rho_c)/\rho_c \lesssim 10^{-58}$ at the Planck time.

(3) Monopole puzzle.[6] In the standard model, the number density of magnetic monopoles produced at the grand-unification phase transition is far too large to be consistent with observations.

The standard cosmological model assumes homogeneity, isotropy, and flatness of the Universe as initial conditions and offers no natural solution to the monopole puzzle. In the inflationary scenario the Universe passes through a de Sitter phase of exponential expansion: $a(t) \propto e^{Ht}$, where $a(t)$ is the cosmic scale factor. Such a phase can arise in a first-order phase transition with strong supercooling. As a result of inflation, all scales in the Universe are increased by the factor $Z=\exp(H\tau)$, where τ is the duration of the inflationary phase. Horizon, flatness, and monopole puzzles are solved if Z is sufficiently large.[1–3]

There remain, however, several fundamental cosmological problems which the inflationary scenario has not solved.

(1) What is the origin of the small density fluctuations which led to the formation of galaxies?

(2) Why is the cosmological constant so small today?

(3) What is the origin of the initial thermal state? In the present form of the inflationary scenario such a state is required if we want the Universe to be stuck in the false vacuum state.

(4) What is the big bang? In other words, what do we make of the cosmological singularity at $t=0$?

(5) Besides, there is another problem if we assume that the Universe is closed (which seems to be a more aesthetically appealing choice) and that the grand-unification scale is much smaller than the Planck mass. It is natural to expect that at about Planck time ($t\sim t_P$) the size and the energy density of the Universe are $O(1)$ in Planck units. But then the Universe will expand and recollapse in about one Planck time, its size will never much exceed the Planck length, and the stage of exponential expansion will never be reached. In order to cool down to temperatures $\sim 10^{14}$ GeV, the energy density at $t\sim t_P$ must be tuned to be near the critical density with an accuracy $\sim 10^{-10}$. This is just a milder version of the same flatness problem that we faced before.

Recently, there has been some progress on the problem of density fluctuations. The inflationary scenario predicts a scale-invariant spectrum of fluctuations.[7–10] Calculations based on gauge theories with a Coleman-Weinberg effective potential give $\delta\rho/\rho \gtrsim 1$, which is far too large. However, density fluctuations of desired magnitude can in principle be obtained using effective potentials of different shape.[8,9] If $\delta\rho/\rho$ turns out to be too small, the required density fluctuations can be produced by vacuum strings.[11]

No convincing solution has yet emerged for the cosmological-constant puzzle, and I will have nothing to say about it in this paper.

The purpose of this paper is to suggest a new version of the inflationary scenario in which the Universe is spontaneously created from *nothing*, and which avoids problems (3)–(5) in a natural way. This scenario does not require any changes in the fundamental equations of physics; it only gives a new interpretation to a well-known solution of Euclidean Einstein's equations.

To my knowledge, the idea that our universe might be a vacuum fluctuation was first suggested by Albrow.[12] A more elaborate discussion was given by Tryon,[13] who argued that all strictly conserved quantum numbers of a closed universe might be equal to zero. If a universe of finite size is spontaneously created from the vacuum, the cosmological singularity problem is avoided. However, Tryon's work leaves us in the dark concerning the initial conditions at the moment of creation. Recently, Atkatz and Pagels[14] have discussed the possibility that the big bang was a quantum tunneling effect, in which the Universe tunneled through a barrier from some classically stable, static initial state. They have shown that such a tunneling event is possible only if the Universe is closed. A weak point of this picture (noticed by the authors themselves) is that the Universe could not stay in its initial state indefinitely long if that state is unstable with respect to quantum tunneling. Then we are faced with the questions of how did the Universe get in that state and what did it do before that. Brout *et al.*[15] have suggested that our universe has originated as a local quantum effect in a flat space-time. But again, if such an event has a finite probability per unit space-time volume, the intitial flat space could not have survived for an infinite time. The same difficulty is shared by the model proposed by Gott,[16] in which the Universe starts as a quantum effect in de Sitter space. One could think that in de Sitter space the problem is milder than in flat space-time: Analysis of bubble nucleation in an exponentially expanding universe shows[1,17] that the bubbles never completely fill space, and so at any moment of time there is still room for new bubbles to appear. We note, however, that in the full de Sitter space the phase of exponential expansion is preceded by a phase of exponential contraction [see Eq. (3.6)].

The model discussed in this paper is similar to Tryon's picture, but it goes further in that it gives a mathematical description of the tunneling process and determines the initial conditions at the moment of nucleation. This paper is an extended version of Ref. 18. Creation of universe from nothing has been discussed independently, by Grischuk and Zel'dovich,[19] however they have not suggested any description of the tunneling process.

This paper is organized as follows: Before we get to cosmic tunneling, we shall discuss two simple examples of quantum tunneling in Sec. II. Section III describes the birth of the inflationary universe by quantum tunneling from nothing. Section IV discusses the ultimate fate of the Universe: Will it recollapse or will it expand forever? Conclusions are summarized in Sec. V.

FIG. 1. A potential with a quasistable state at x_0.

II. EXAMPLES OF QUANTUM TUNNELING

We shall first consider a nonrelativistic particle of unit mass moving in a one-dimensional potential $U(x)$ of the form shown in Fig. 1. The classical equation of motion is

$$\ddot{x}+U'(x)=0 \ . \tag{2.1}$$

The point $x=x_0$ is a local minimum of the potential, and classically the particle can stay at $x=x_0$ indefinitely long. The origin for the potential is chosen so that $U(x_0)=0$. As we know, the state of the particle at $x=x_0$ is unstable with respect to quantum tunneling through the barrier. A semiclassical description of the tunneling is given[20] by the bounce solution of the Euclidean equation of motion [that is, of Eq. (2.1) with t changed to $\tau=-it$]. This solution is also called an instanton. The bounce solution starts with $x=x_0$ at $\tau\to-\infty$, bounces off the classical turning point at the end of the barrier ($x=x_1$), and returns back to $x=x_0$ at $\tau\to+\infty$. The tunneling probability is proportional to $\exp(-S)$, where

$$S=\int_{-\infty}^{\infty} d\tau \left[\frac{1}{2}\left[\frac{dx}{d\tau}\right]^2+U(x)\right] \tag{2.2}$$

is the Euclidean action of the bounce.

The decay probability per unit time, Γ, is related to the imaginary part of the energy of the quasistable state E_0:

$$\Gamma=2\,\mathrm{Im}E_0 \ . \tag{2.3}$$

It can be shown[20] that the bounce contribution to E_0

has an imaginary part only if the bounce solution is not a minimum but a saddle point of the Euclidean action. In other words, there should exist small perturbations of the bounce trajectory that decrease the action. If $x=\bar{x}(\tau)$ is the bounce solution, then the action for a perturbed trajectory $x=\bar{x}(\tau)+\xi(\tau)$ can be written as

$$S[x]=S[\bar{x}]+\frac{1}{2}\int \xi\delta^2 S[\bar{x}]\xi\, d\tau \ . \tag{2.4}$$

Here,

$$\delta^2 S[\bar{x}]=-\partial_\tau{}^2+U''(\bar{x}) \tag{2.5}$$

is the second variational derivative of S at the bounce solution. E_0 has an imaginary part only if the operator $\delta^2 S[\bar{x}]$ has a negative eigenvalue. It can be shown[20] that this is always the case for potentials of the form shown in Fig. 1.

Another example is the creation of electron-positron pairs in a constant electric field. For simplicity, we shall work in a $(1+1)$-dimensional space-time. The electron trajectory in a constant electric field E is a hyperbola,

$$x-x_0=\pm[\kappa^2+(t-t_0)^2]^{1/2} \ , \tag{2.6}$$

where $\kappa=m/eE$, m and e are electron mass and charge, respectively, and $x_0,t_0=\text{const}$. The classical turning points are at $x=x_0\pm\kappa$. The solution of the Euclidean equations of motion can be obtained from (2.6) by changing $t\to -i\tau$:

$$(x-x_0)^2+(\tau-\tau_0)^2=\kappa^2 \ . \tag{2.7}$$

This solution describes a circular trajectory; it is an example of a compact instanton (or bounce). The process of pair production described by the instanton (2.7) is symbolically represented in Fig. 2. AB and DE are classically allowed trajectories. AB describes an electron moving backward in time, that is, a positron. The semicircle BCD represents the instanton (2.7). The fact that the solution (2.7) is not defined for $\tau\to\pm\infty$ indicates that there are no electrons and positrons in the initial state, i.e., the pair is created from the vacuum.

The bounce action equals

$$S=\int\left\{m\left[1+\left[\frac{dx}{d\tau}\right]^2\right]^{1/2}-eEx\right\}d\tau \ . \tag{2.8}$$

Introducing a new variable θ according to $x-x_0=\kappa\cos\theta$, $\tau-\tau_0=\kappa\sin\theta$, we obtain $(eE>0)$

$$S=2\pi m\kappa-\pi eE\kappa^2 \ . \tag{2.9}$$

With $\kappa=m/eE$ this gives $S=\pi m^2/eE$, and the semiclassical probability of pair creation is

$$P\propto\exp(-\pi m^2/eE) \ , \tag{2.10}$$

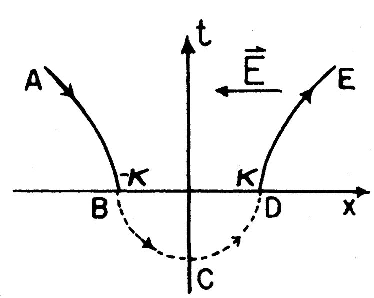

FIG. 2. A schematic representation of pair creation in a constant electric field. The dashed semicircle represents the "under-barrier" part of the trajectory. (Below the horizontal axis t is the Euclidean time.) The classical evolution starts at $t=0$.

in agreement with Schwinger's result. It is easily seen from Eq. (2.9) that S is decreased if we vary the radius of the circle κ. The bounce action is a maximum in the space of circular Euclidean trajectories of different radii, and thus the bounce is a saddle point of the action.[21]

III. THE BIRTH OF THE INFLATIONARY UNIVERSE

Suppose we have a gauge theory in which the symmetry is spontaneously broken when the Higgs field ϕ acquires a vacuum expectation value. The Higgs field of realistic grand-unified theories have several components, but for simplicity we shall consider a single scalar field ϕ with an effective potential $V(\phi)$. If $\phi=\sigma$ is the true minimum of the effective potential, then we require that $V(\sigma)\cong 0$, so that the cosmological constant is small today. Besides $\phi=\sigma$, $V(\phi)$ can have other extrema. If $\phi=\phi_0$ is such an extremum,

$$V'(\phi_0)=0 \ , \tag{3.1}$$

then $\phi=\phi_0=\text{const}$ is a solution of the classical field equation for ϕ,

$$\Box\phi+V'(\phi)=0 \ . \tag{3.2}$$

The vacuum energy density at $\phi=\phi_0$ will, in general, be nonzero (and positive):

$$\rho_v=V(\phi_0) \ . \tag{3.3}$$

Suppose now that the Universe starts in a vacuum state with $\phi=\phi_0$ and is described by a closed Robertson-Walker metric,

$$ds^2=dt^2-a^2(t)[d\chi^2+\sin^2\chi(d\theta^2+\sin^2\theta\, d\phi^2)] \ . \tag{3.4}$$

The scale factor $a(t)$ can be found from the evolution equation

$$\dot{a}^2+1=\frac{8\pi G}{3}\rho_v a^2 , \tag{3.5}$$

where $\dot{a}=da/dt$. The solution of this equation is the de Sitter space,

$$a(t)=H^{-1}\cosh(Ht) \tag{3.6}$$

with

$$H=(8\pi G\rho_v/3)^{1/2} . \tag{3.7}$$

It describes a universe which is contracting at $t<0$, reaches its minimum size ($a_{\min}=H^{-1}$) at $t=0$, and is expanding at $t>0$. This behavior is analogous to that of a particle bouncing off a potential barrier at $a=H^{-1}$. (Here a plays the role of the particle coordinate.) We know that in quantum mechanics particles can tunnel through potential barriers. This suggests that the birth of the Universe might be a quantum tunneling event. Then the Universe has emerged having a finite size ($a=H^{-1}$) and zero "velocity" ($\dot{a}=0$); its following evolution is described by Eq. (3.6) with $t>0$.

We shall discuss this cosmic tunneling by analogy with the simple examples of Sec. II. The Euclidean version of Eq. (3.5) is

$$-\dot{a}^2+1=H^2a^2 , \tag{3.8}$$

and bounce solution is

$$a(\tau)=H^{-1}\cos(H\tau) . \tag{3.9}$$

This solution can also be obtained from Eq. (3.6) by changing $t\to -i\tau$. Equations (3.4) and (3.9) describe a four-sphere S^4. This is the well-known de Sitter instanton.[22,23] The solution (3.9) bounces at the classical turning point $a=H^{-1}$, which indicates that it describes a tunneling to the de Sitter space (3.6). However, it does not approach any initial state at $\tau\to\pm\infty$. In fact, S^4 is a compact space, and the solution (3.9) is defined only for $|\tau|\leq\pi/2H$. [Compare with Eq. (2.7).] The instanton (3.9) can be interpreted as describing the tunneling to de Sitter space (3.6) from *nothing*, where by nothing I mean a state with no classical space-time. Then the birth of the Universe is symbolically represented in Fig. 3.

"Nothing" is the realm of unrestrained quantum gravity; it is a rather bizarre state in which all our basic notions of space, time, energy, entropy, etc., lose their meaning. This does not mean, however, that cosmic tunneling cannot be described without complete understanding of quantum gravity. The curvature of the instanton solution (3.9) is

$$R=12H^2=32\pi\rho_v/m_P{}^2 , \tag{3.10}$$

and we expect quantum gravitational corrections to be small, as long as $\rho_v \ll m_P{}^4/32\pi$.

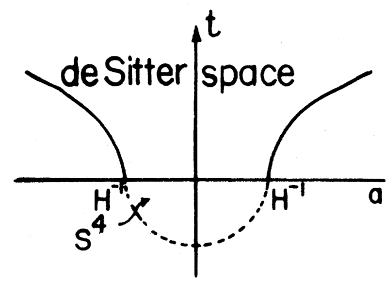

FIG. 3. A schematic representation of the nucleation of the inflationary universe.

The action of the de Sitter instanton equals

$$S=S_G+S_M , \tag{3.11}$$

where

$$S_G=-\frac{1}{16\pi G}\int R\sqrt{g}\, d^4x \tag{3.12}$$

and

$$S_M=\int [\tfrac{1}{2}(\partial_\mu\phi)^2+V(\phi)]\sqrt{g}\, d^4x \tag{3.13}$$

are the gravitational and the matter action, respectively. Using Eqs. (3.4), (3.9), and (3.10) we find[22]

$$S=-3m_P{}^4/8\rho_v . \tag{3.14}$$

By analogy with "normal" quantum-mechanical tunneling, we can require that the instanton (3.9) should be a saddle point of the action. The behavior of S under small variations of the metric has been studied in Ref. 23, where it is shown that the operator $\delta_g{}^2S$ for the de Sitter instanton has no physically significant negative modes. (There are some negative modes, but their contributions cancel out in the functional integral of quantum gravity.) This implies that there should be a direction in the ϕ space, in which the matter action S_M decreases. Obviously, the gradient terms can only increase the action, and we can restrict ourselves to homogeneous perturbations of ϕ. For such perturbations

$$\delta^2S=V''(\phi_0)\sqrt{g} , \tag{3.15}$$

and the operator δ^2S has a negative mode if and only if $V''(\phi_0)<0$, that is, if $\phi=\phi_0$ is a local *maximum* of the effective potential. (It should be noted that this argument is based on a somewhat shaky foundation: δ^2S should have a negative mode in order to give an imaginary part to the energy of the decaying state. However, the energy of "nothing" is undefined.)

One can push the analogy with the tunneling of particles a little further and interpret $\exp(-S)$ as being proportional to the tunneling probability. (Needless to say, the tunneling probability for the

Universe is not a measurable quantity.) Then, if the effective potential has several maxima, most of the universes will nucleate at the maximum with the smallest value of ρ_v.

Poor understanding of quantum gravity does not allow us to go beyond a semiclassical analysis of the nucleation process. The hope is that this analysis will eventually be justified in the framework of full quantum theory of gravity. One can speculate that the relevant functional integral for the "nothing-nothing" amplitude is the integral over all compact Euclidean manifolds. Such an integral has been discussed by Hawking[24] in a different context. The action of the de Sitter instanton is smaller than that of all other compact gravitational instantons,[24] and one can expect that it gives a dominant contribution to the path integral.

The de Sitter instanton with the Higgs field at a maximum of the effective potential was first discussed by Hawking and Moss,[25] who gave it a different interpretation. They considered quantum tunneling from a local minimum of $V(\phi)$ at $\phi=0$ and interpreted the instanton (3.9) as describing a tunneling event in which ϕ jumps from $\phi=0$ to $\phi=\phi_0$ and the Hubble constant jumps from $8\pi GV(0)/3$ to $H=8\pi GV(\phi_0)/3$ simultaneously in the entire Universe. Note, however, that the maximal spatial section of the metric (3.9) has size H^{-1}, which is the new de Sitter horizon, and it is hard to see how it can tell what happens to the Universe outside the horizon. "Creation from nothing" appears to be a more natural interpretation of the Hawking-Moss instanton.

IV. AFTER CREATION

We now turn to the question of how the newly born Universe will evolve. The state with $\phi=\phi_0$ at a maximum of the effective potential is a point of unstable equilibrium, and the Higgs field will be driven away from ϕ_0 by quantum fluctuations.

To simplify the equations, we shall set $\phi_0=0$ and introduce the notation

$$\mu^2=-V''(0)\ . \tag{4.1}$$

Then at small values of ϕ, the effective potential has the form

$$V(\phi)\approx-\tfrac{1}{2}\mu^2\phi^2 \tag{4.2}$$

and ϕ satisfies a free field equation with a tachyonic mass,

$$(\Box-\mu^2)\phi=0\ . \tag{4.3}$$

We shall first concentrate on the more interesting case when $\mu\ll H$. The growth of quantum fluctuations can be characterized by the vacuum expectation value of the field operator squared, $\langle\phi^2\rangle$. This quantity is divergent and should be renormalized, as explained, e.g., in Refs. 26 and 27. Here we will be interested only in the finite time-dependent part of $\langle\phi^2\rangle$ and will subtract the constant infinite part so that $\langle\phi^2\rangle=0$ at $t=0$. Then for $\mu\ll H$, we have[27,28]

$$\langle\phi^2\rangle=\frac{3H^4}{8\pi^2\mu^2}\left[\exp\left[\frac{2\mu^2t}{3H}\right]-1\right]. \tag{4.4}$$

For $t\ll H/\mu^2$ this gives

$$\langle\phi^2\rangle\approx H^3t/4\pi^2\ . \tag{4.5}$$

Note that μ has dropped out of Eq. (4.5), and so the behavior of the fluctuations at $t\ll H/\mu^2$ is not much different from that for a massless field. Equation (4.5) has been derived in Refs. 27 and 28. This equation is important for what follows and we shall spend some time discussing it. A free scalar field theory in de Sitter space is stable if $m^2+\xi R>0$, where m and ξ are, respectively, the mass and the conformal parameter of the field. In this case, all modes of the field ϕ are decreasing functions of time (due to the cosmological expansion). For $m^2+\xi R<0$ the theory is unstable, and the modes of ϕ grow in time (when their wavelengths become sufficiently large). A massless, minimally coupled field, for which $m^2+\xi R=0$, is a borderline case. The modes of such a field are decreasing until their wavelengths become greater than the de Sitter horizon (H^{-1}) and remain constant afterward. As time goes on, more and more modes come out of the horizon, their contributions accumulate, and $\langle\phi^2\rangle$ grows.

The time dependence $\langle\phi^2\rangle\propto t$ can be pictured[9] as a Brownian motion of the field ϕ. As a result of quantum fluctuations, the magnitude of ϕ on the horizon scale changes by $\pm(H/2\pi)$ per expansion time (H^{-1}). Then the average "displacement" squared is $\langle\phi^2\rangle=(H/2\pi)^2N$, where $N=Ht$ is the number of "steps." In this picture, the values of ϕ at points separated by a distance l in the range $H^{-1}\ll l\ll H^{-1}\exp(Ht)$ made many Brownian steps together and started wandering away from one another only at $t=t_l$, where t_l is the time when the comoving scale l came out of the horizon,

$$t-t_l=H^{-1}\ln(Hl)\ .$$

Then the mean-square variation in the values of ϕ is

$$\langle(\delta\phi_l)^2\rangle=(H/2\pi)^2\ln(Hl)\ . \tag{4.6}$$

[This conclusion is confirmed[8,29] by a calculation of the two-point correlation function $\langle\phi(x)\phi(x')\rangle$.] Note that $\langle(\delta\phi_l)^2\rangle\ll\langle\phi^2\rangle$ for $l\ll H^{-1}\exp(Ht)$.

At $t \sim H/\mu^2$ the negative mass squared becomes important, and the evolution of ϕ can be pictured as a Brownian motion in a field of force which drives ϕ towards greater values. At some later time t_*, when ϕ grows sufficiently large ($\phi \sim \phi_*$), quantum fluctuations become unimportant, and the following evolution of ϕ is described by the classical solution of Eq. (4.3):

$$\phi \approx \phi_* \exp\left[\frac{\mu^2}{3H}(t-t_*)\right] . \tag{4.7}$$

The moment at which the value $\phi \sim \phi_*$ is reached may be different in different regions of space, as a result of quantum fluctuations [cf. Eq. (4.6)], and thus $t_* = t_*(\vec{x})$. This is the effect that gives rise to cosmological density perturbations in the inflationary universe.[7–10] The magnitude of ϕ_* can be roughly estimated as follows. Quantum fluctuations change the value of ϕ by $\sim H$ on a time scale $\sim H^{-1}$, and so the fluctuation of "velocity" $\dot{\phi}$ is $\delta\dot{\phi} \sim H^2$. On the other hand, from Eq. (4.7) the classical velocity is $\dot{\phi} = \mu^2\phi/3H$. Requiring that $\delta\dot{\phi} \sim \dot{\phi}$, we find $\phi_* \sim H^3/\mu^2$.

Equations (4.4) and (4.7) assume that the form of potential (4.1) extends to sufficiently large values of ϕ. Our discussion can easily be extended to other shapes of the effective potential, e.g., to the case where there is an additional $-\lambda\phi^4$ term or where there is only such term and $\mu = 0$. [In the latter case Eq. (4.5) applies up to[28,29] $t \sim \lambda^{-1/2}H^{-1}$ and $\phi_* \sim \lambda^{-1/3}H$. The evolution of ϕ is qualitatively unchanged if $\lambda \ll 1$.]

If $\mu \gg H$, then $\langle \phi^2 \rangle \sim H^2 \exp(2\mu t)$, and the growth of ϕ is probably too fast to have sufficient inflation. The same applies to $-\lambda\phi^4$ potential with $\lambda \gtrsim 1$.

When ϕ reaches the true minimum of the effective potential, the vacuum energy thermalizes and the universe heats up to a temperature $T \sim {\rho_v}^{1/4}$. In our model this is the maximum temperature the universe has ever had. At the time of thermalization, the curvature radius of the Universe is $a_0 \sim H^{-1}\exp(H\tau)$, where τ is the duration of the inflationary phase (for our purposes we will not need a rigorous definition of τ). If τ is large enough, a_0 can be much greater than the present horizon.

If thermalization occurs everywhere at about the same time, then we end up with a closed universe of size $\sim a_0$. Such a universe would reach a maximum size and then recollapse. It will now be argued that the actual evolution of our model is quite different. Right after the creation, the Higgs field can grow in positive or negative ϕ direction with equal probability. More generally, if ϕ is a multicomponent field, there will be several directions of growth. Some of these directions may lead to the true vacuum, while others may lead to a false vacuum, and then parts of the Universe will be permanently stuck in the de Sitter phase.

A more interesting case is when the effective potential is symmetric about $\phi = 0$, so that all possible directions of growth lead to the true minimum of V. Here we note that Eqs. (4.4) and (4.5) give the magnitude of ϕ^2 averaged over an ensemble of universes or, alternatively, over different regions of space. However, the actual value of ϕ may be quite different at some places, and it is possible that in some regions ϕ will be very small (say, $\phi^2 \lesssim H^2$) even at large values of t. The probability of having $\phi^2 \lesssim H^2$ at $t \gg H^{-1}$ is small, but improbable events happen if the number of trials is sufficiently large.

Now let us try to quantify this using the Brownian-motion picture for quantum fluctuations. Let $F(\phi,t)$ be the probability distribution function for ϕ. At small ϕ the slope of the effective potential is negligible, and the evolution of F is described by the diffusion equation

$$\frac{\partial F}{\partial t} = D\frac{\partial^2 F}{\partial \phi^2} \tag{4.8}$$

with the initial condition

$$F(\phi,0) = \delta(\phi) . \tag{4.9}$$

The solution is

$$F(\phi,t) = \tfrac{1}{2}(\pi D t)^{-1/2}\exp(-\phi^2/4Dt) ,$$

which gives

$$\langle \phi^2 \rangle = \int F(\phi,t)\phi^2 d\phi = 2Dt . \tag{4.10}$$

Comparing this with Eq. (4.5) we obtain

$$D = H^3/8\pi^2 . \tag{4.11}$$

To take account of the slope of the effective potential, we note that the classical time scale of variation of ϕ due to the slope is $\sim 3H/\mu^2$. (Here it is assumed[30] that $\mu \ll H$.) On the other hand, Eq. (4.10) gives the typical time $\sim \phi^2/2D \sim 4\pi^2\phi^2/H^3$. The slope becomes important when the two time scales become comparable, that is, when $\phi^2 \sim 3H^4/4\pi^2\mu^2$. [This also follows from a comparison of Eqs. (4.4) and (4.5).] For $\phi \gtrsim 3H^4/4\pi^2\mu^2$ the diffusion becomes biased in the direction of greater values of ϕ. The effect of the slope can be roughly estimated by considering a diffusion on a segment with absorbing boundaries at $\phi = \pm\Phi$, where

$$\Phi^2 \sim 3H^4/4\pi^2\mu^2 . \tag{4.12}$$

The corresponding boundary conditions are

$$F(\Phi,t)=F(-\Phi,t)=0\ . \tag{4.13}$$

The solution of Eqs. (4.8), (4.9), and (4.13) can be found in the form of an infinite series,

$$F(\phi,t)=\Phi^{-1}\sum_{m=1}^{\infty} b_m \cos\left[\frac{m\pi\phi}{2\Phi}\right] \times \exp\left[-\frac{m^2H^3t}{32\Phi^2}\right], \tag{4.14}$$

where $b_m=1$ for m odd and 0 for m even. The probability of having $\phi^2 \lesssim H^2$ at large t is

$$p\sim(H/\Phi)\exp(-H^3t/32\Phi^2)\ . \tag{4.15}$$

The number of horizon-size regions with $\phi^2 \lesssim H^2$ is $N\sim p\mathcal{N}$, where

$$\mathcal{N}\sim\exp(3Ht) \tag{4.16}$$

is the total number of horizon-size regions in the Universe. ($\mathcal{N}$ corresponds to the number of trials.) For $\mu \ll H$, we see from Eqs. (4.15), (4.16), and (4.12) that $N \gg 1$. This means that even at the present time there are parts of the Universe which still expand exponentially. The evolution of each horizon-size region with $\phi^2 \lesssim H^2$ repeats the evolution of the whole inflationary universe, which itself had size $\sim H^{-1}$ at the time of creation. In this cosmological model the Universe has a beginning but has no end. Parts of the Universe recollapse and develop singularities, while other parts are still in the inflationary phase.

V. CONCLUSIONS

In this paper we have discussed a cosmological model in which the Universe is spontaneously created by quantum tunneling from nothing into a de Sitter space. Here nothing means a state without any classical space-time. The tunneling is described by the de Sitter–Hawking–Moss instanton, in which the Higgs field ϕ is at a local maximum of the effective potential, $\phi=\phi_0$. The Universe is closed and has size H^{-1} at the time of creation (H is the de Sitter Hubble constant). Throughout the paper we have assumed that $H \ll m_P$, so that quantum gravity corrections are unimportant. The case of $H\sim m_P$ cannot, at present, be analyzed quantitatively, but one can expect that the qualitative picture of the quantum nucleation from nothing will still apply.

After the creation the Universe evolves along the lines of the inflationary scenario. Quantum fluctuations drive the Higgs field away from $\phi=\phi_0$. When ϕ reaches the true minimum of the effective potential, the vacuum energy thermalizes and the Universe heats up to a temperature $T\sim(m_PH)^{1/2}$. In our model this is the maximum temperature the Universe has ever had.

Quantum fluctuations near $\phi=\phi_0$ can be pictures as a Brownian motion of the field ϕ. The magnitude of ϕ on the horizon scale fluctuates by an amount $\sim H$ per expansion time ($\sim H^{-1}$). When ϕ gets far enough from ϕ_0, the slope of the effective potential becomes important, and ϕ starts "rolling down." The probability of having $(\phi-\phi_0)^2 \lesssim H^2$ at large t is very small. However, the number of horizon-size regions grows like exp($3Ht$), and the number of regions with $(\phi-\phi_0)^2 \lesssim H^2$ is increasing with time.

Each horizon-size region with $(\phi-\phi_0)^2$ repeats the evolution of the whole Universe, which itself had size $\sim H^{-1}$ at the time of creation. At any time t there exist parts of the Universe which are still in the de Sitter phase, while other parts have already recollapsed and developed singularities. Thus, in this model, the Universe has a beginning, but it has no end.

Most of the problems discussed in this paper belong to what Steinhardt[31] has called "metaphysical cosmology" (or metacosmology), which is the branch of cosmology totally decoupled from observations. (This does not mean, however, that such problems do not allow a rational analysis.) The advantages of the scenario presented here are of an aesthetic nature. It gives a cosmological model which does not have a singularity at the big bang (there still are final singularities) and does not require any initial or boundary conditions. The structure and evolution of the Universe(s) are totally determined by the laws of physics.

We note also that the model discussed here may open some new theoretical possibilities. In the "standard" inflationary and noninflationary models the Universe starts at infinite temperature and reaches its present state through a series of phase transitions. At $T\rightarrow\infty$ one normally has the phase of maximum symmetry. My point is that there are particle models in which, in the course of expansion and cooling, the Universe gets stuck in a wrong phase (such are some supersymmetric models[32]). In the creation-from-nothing type of scenario such models should not be ruled out, since the Universe can nucleate at *any* maximum of the effective potential, and the temperature of the Universe has never been higher than $\sim\rho_v{}^{1/4}$.

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