# Chiral color and partial-unification mass scales

Fayyazuddin
*Physics Department, King Saud University, Riyadh, Saudi Arabia*

Riazuddin
*Physics Department, University of Petroleum and Minerals, Dhahran, Saudi Arabia*
(Received 27 September 1982)

Partial-unification mass scales for the group $[\mathrm{SU}_c(4)]^2\times\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)$ are studied. We show that for this group the intermediate mass scale $m_R$ is given by $300\leq m_R\leq 10^7$ GeV for the partial-unification mass scale $M_X$ in the range $5\times10^{12}\geq M_X\geq 3\times10^{10}$ GeV. This is in contrast to other models for which $m_R\geq 10^9$ GeV.

## I. INTRODUCTION

The standard model for electroweak unification has built in the conservation laws $\Delta B=0$ and $\Delta L=0$. These conservation laws may not hold for unification groups higher than electroweak group $\mathrm{SU}_c(3)\times\mathrm{SU}_L(2)\times\mathrm{U}(1)$. In the grand unification groups[1] of SU(5) or SO(10), there is a complete desert from electroweak unification mass ($\simeq100$ GeV) to grand unification mass scale[2] of order $10^{15}$ GeV. These groups allow $\Delta B\neq0$ processes. The $\Delta B=1$ process manifests itself in proton decay; the experimental limit on proton decay, $\tau_p\geq10^{31}$ y, gives $M_X\geq10^{14}$ GeV. The $\Delta B=2$ processes such as $n$-$\bar{n}$ oscillations[3,4] are naturally suppressed and are not observable in these models.

It is of interest to consider intermediate mass scales that may manifest themselves in the $\Delta B=2$ and $\Delta L=2$ transitions. In the partial-unification group $\mathrm{SU}_c(4)\times\mathrm{Su}_L(2)\times\mathrm{SU}_R(2)$ of Pati and Salam,[5] $\Delta B=2$ and $\Delta L=2$ processes can occur.[4] However, the partial-unification mass scale[6,7] in this model comes out to be $\geq10^{12}$ GeV for $\sin^2\theta_W\leq0.25$. Such a large mass scale will naturally suppress $n$-$\bar{n}$ oscillations.

It is of interest to investigate models in which the partial-unification mass scale is lower than $10^{12}$ GeV. The natural mass scale for $n$-$\bar{n}$ oscillations is determined by the mass $m_R$ of right-handed vector bosons. The purpose of this paper is to show that by extending the color group $\mathrm{SU}_c(4)$ to the chiral color group

$$\mathrm{SU}_{cL}(4)\times SU_{cR}(4)\equiv[\mathrm{SU}_c(4)]^2$$

in the Pati-Salam group,[8,9] it is possible to bring down the partial-unification mass scale[10] to $10^9$ GeV. Within this model, it is shown that with the conventional value of $\sin^2\theta_W$, it is possible to have right-handed vector bosons of mass within the limits

$$300\leq m_R\leq10^7\ \mathrm{GeV}$$

for partial-unification mass scale

$$5\times10^{12}\geq M_X\geq3\times10^{10}\ \mathrm{GeV}\ .$$

With such a low value of $m_R$, it is possible to have observable $n$-$\bar{n}$ oscillations. This is in contrast to other models, in which $m_R$ comes out to be $\geq10^9$ GeV. In the end we also discuss the lepton masses, in particular, the relationship between neutrino masses and $m_R$.

## II. PARTIAL-UNIFICATION GROUP $\mathrm{SU}_c(4)\times\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)$

We first review the partial-unification mass scale for the group $\mathrm{SU}_c(4)\times\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)$. For the symmetry-breaking pattern

$$\mathrm{SU}_c(4)\times\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)\underset{M_X}{\rightarrow}\mathrm{SU}_c(3)\times\mathrm{SU}_L(2)\times\mathrm{U}(1)\underset{m_L}{\rightarrow}\mathrm{SU}_c(3)\times\mathrm{U}_{\mathrm{EM}}(1)\ ,$$

the partial-unification mass is given by[7]

$$l_n\frac{M_x}{m_L}=\frac{\alpha^{-1}(m_L)}{2(\beta'-\beta_2-C_1{}^2\beta_3)}\left[(1-2\sin^2\theta_W)-C_1{}^2\frac{\alpha(m_L)}{\alpha_s(m_L)}\right]\ ,\qquad(1)$$

 

where

$$C_1^2=\tfrac{2}{3},\ \ \beta_2=\frac{1}{4\pi}(-\tfrac{22}{3}+\tfrac{2}{3}N_f),\ \ \beta_3=\frac{1}{4\pi}(-\tfrac{33}{3}+\tfrac{2}{3}N_f),\ \ \beta'=\frac{1}{4\pi}(\tfrac{10}{9}N_f)\ . \tag{2}$$

Here $N_f$ is the number of quark flavors, $\alpha$ is the fine-structure constant, and $\alpha_s=g_3{}^2/4\pi$ is the similar constant for the color group $SU_c(3)$.

For the symmetry-breaking pattern

$$\mathrm{SU}_c(4)\times\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)\underset{M_X}{\rightarrow}\mathrm{SU}_c(3)\times\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)\times\mathrm{U}_{B-L}(1)$$
$$\underset{m_R}{\rightarrow}\mathrm{SU}_c(3)\times\mathrm{SU}_L(2)\times\mathrm{U}(1)\underset{m_L}{\rightarrow}\mathrm{SU}_c(3)\times U_{\mathrm{EM}}(1)\ ,$$

the unification mass is given by

$$2\left[(\beta_1-C_1{}^2\beta_3)l_n\frac{M_X}{m_R}+(\beta'-\beta_2-C_1{}^2\beta_3)\ln\frac{m_R}{m_L}\right]=\left[(1-2\sin^2\theta_W)-C_1{}^2\frac{\alpha(m_L)}{\alpha_s(m_L)}\right]\alpha^{-1}(m_L)\ , \tag{3}$$

where

$$\beta_1=\frac{1}{4\pi}\left[\frac{1}{2}\left[\sum_f\tfrac{1}{4}Y^2\right]\frac{2}{3}N_f\right]\ . \tag{4}$$

Here $(\frac{1}{2}Y)$ is the generator associated with $\mathrm{U}_{B-L}(1)$. With $\alpha^{-1}(m_L)=128$, $\alpha_s(m_L)=0.14$, and $\sin^2\theta_W\le 0.25$, we get $M_X\ge 10^{12}$ GeV. On the other hand, for $\sin^2\theta_W\simeq 0.23$ and $m_L\simeq 80$ GeV, the partial-unification mass $M_X$ and intermediate mass scale $m_R$ are given by

$$\begin{aligned}&10^5\le m_R\le 10^{10}\ \mathrm{GeV}\ ,\\ &5.7\times10^{22}\ge M_X\ge 5.7\times10^{17}\ \mathrm{GeV}\ .\end{aligned} \tag{5}$$

It is clear from the above results, that if we require the partial-unification mass scale to be less than Planck mass ($10^{19}$ GeV), then $m_R$ should be $\ge 10^9$ GeV. This gives the natural mass scale for $n$-$\bar{n}$ oscillations too high to be observable. However, if we use the value of $\sin^2\theta_W$ in the range[11] $0.28\le\sin^2\theta_W\le 0.33$, $m_L=65-70$ GeV, and $m_R<300$ GeV, then Eq. (3) gives

$$2.6\times10^{14}\le\frac{M_X}{m_L}\le 1.5\times10^{19}\ . \tag{6}$$

## III. PARTIAL-UNIFICATION GROUP $[\mathrm{SU}_c(4)]^2\times\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)$

We now consider the extended partial-unification group

$$G=\mathrm{SU}_{cL}(4)\times\mathrm{SU}_{cR}(4)\times\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)$$
$$g_L=g_R=g\qquad g_{2L}=g_{2R}=g_2\ .$$

Note that, as in the previous case, we have two independent couplings at the partial-unification mass $M_X$, namely, $g$ and $g_2$; the equality of couplings within SU(4)'s and SU(2)'s indicated above, as is well known,[2,4,6–8,10] is due to the fact that the model considered is left-right symmetric. A higher unification group $G'$ of which $G$ is a subgroup would have related $g$ and $g_2$ also at the unification mass of $G'$ but our purpose is to consider the partial unification group in its own right whether or not it descends from a higher unification group.

We consider the following three cases of symmetry breaking[8]:

*Case 1:*

$$[\mathrm{SU}_c(4)]^2\times\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)\underset{M_X}{\rightarrow}\mathrm{SU}_c(3)\times\mathrm{SU}_L(2)\times\mathrm{U}(1)\underset{m_L}{\rightarrow}\mathrm{SU}_c(3)\times\mathrm{U}_{\mathrm{EM}}(1)\ . \tag{7}$$

For this case, we get back Eq. (1) for the unification mass scale:

*Case 2(a):*

$$[\mathrm{SU}_c(4)]^2\times\mathrm{SU}_L(2)\times\mathrm{SU}_R(2)\underset{M_X}{\rightarrow}[\mathrm{SU}_c(3)]^2\times\mathrm{SU}_L(2)\times\mathrm{U}(1)\underset{m_L}{\rightarrow}\mathrm{SU}_c(3)\times\mathrm{U}_{\mathrm{EM}}(1)\ . \tag{8}$$

*Case 2(b):*

$$[\mathrm{SU}_c(4)]^2\times \mathrm{SU}_L(2)\times \mathrm{SU}_R(2) \underset{M_X}{\rightarrow} [\mathrm{SU}_c(3)]^2\times \mathrm{SU}_L(2)\times \mathrm{SU}_R(2)\times \mathrm{U}_{B-L}(1)$$
$$\underset{m_R}{\rightarrow} [\mathrm{SU}_c(3)]^2\times \mathrm{SU}_L(2)\times \mathrm{U}(1) \underset{m_L}{\rightarrow} \mathrm{SU}_c(3)\times \mathrm{U}_{\mathrm{EM}}(1) \ . \tag{9}$$

*Case 3(a):*

$$[\mathrm{SU}_c(4)]^2\times \mathrm{SU}_L(2)\times \mathrm{SU}_R(2) \underset{M_X}{\rightarrow} [\mathrm{SU}_c(3)]^2\times \mathrm{SU}_L(2)\times \mathrm{U}(1) \underset{m_L}{\rightarrow} [\mathrm{SU}_c(3)]^2\times \mathrm{U}_{\mathrm{EM}}(1) \ . \tag{10}$$

*Case 3(b):*

$$[\mathrm{SU}_c(4)]^2\times \mathrm{SU}_L(2)\times \mathrm{SU}_R(2) \underset{M_X}{\rightarrow} [\mathrm{SU}_c(3)]^2\times \mathrm{SU}_L(2)\times \mathrm{SU}_R(2)\times \mathrm{U}_{B-L}(1)$$
$$\underset{m_R}{\rightarrow} [\mathrm{SU}_c(3)]^2\times \mathrm{SU}_L(2)\times \mathrm{U}(1) \underset{m_L}{\rightarrow} [\mathrm{SU}_c(3)]^2\times \mathrm{U}_{\mathrm{EM}}(1) \ . \tag{11}$$

For the cases 3(a) and 3(b), we have, respectively,

$$2(\beta'-\beta_2-2C_1{}^2\beta'_3)\ln\frac{M_X}{m_L}=\alpha^{-1}(m_L)\left[(1-2\sin^2\theta_W)-2C_1{}^2\frac{\alpha(m_L)}{\alpha_s(m_L)}\right] , \tag{12}$$

$$2\left[(\beta_1-2c_1{}^2\beta'_3)\ln\frac{M_X}{m_R}+(\beta'-\beta_2-2C_1{}^2\beta'_3)\ln\frac{m_R}{m_L}\right]=\alpha^{-1}(m_L)\left[(1-2\sin^2\theta_W)-2C_1{}^2\frac{\alpha(m_L)}{\alpha_s(m_L)}\right] . \tag{13}$$

Here

$$\beta'_3=\frac{1}{4\pi}\left[-\tfrac{33}{3}+\tfrac{1}{3}N_f\right] .$$

For the cases 2(a) and (b), replace $2C_1{}^2$ by $C_1{}^2$ on the right-hand side of Eqs. (12) and (13).

We now consider case (3) in some detail, in particular, case 3(b), which is of interest for $n$-$\bar{n}$ oscillations. First from Eq. (12), we have for $\sin^2\theta_W\simeq 0.23$, $M_X\simeq 2\times 10^9$ GeV. On the other hand for[11] $\sin^2\theta_W\simeq 0.28$ ($m_R\simeq 200$ GeV) and $\sin^2\theta_W \simeq 0.31$ ($m_R\simeq 100 GeV$), we get $M_X\simeq 2.1\times 10^{10}$ GeV and $M_X\simeq 1.1\times 10^8$ GeV, respectively, for $m_L\simeq 70$ GeV.

For the case 3(b) and for $\sin^2\theta_W\simeq 0.23$, the partial-unification mass $M_X$ and intermediate mass $m_R$, as obtained from Eq. (13), are given by

$$\begin{aligned} &300\le m_R\le 10^7 \text{ GeV} ,\\ &5.1\times 10^{12}\ge M_X\ge 2.8\times 10^{10} \text{ GeV} . \end{aligned} \tag{14}$$

From the above results, it is clear that for case 3(b), it is possible to accommodate right-handed vector bosons of mass as low as 300 GeV. In this model the natural mass scale for $n$-$\bar{n}$ oscillations comes out to be low, so as to make them observable.

Note that in some cases above where $M_X$ is as low as $10^{12}$ GeV, there is no trouble with any of proton decay modes leading to $\tau_p<10^{31}$ y since the proton is stable in the partial-unification groups considered above. There are no gauge mediators which can induce proton decay simply because no mediators connect $u$ quarks with $u^c$ or $d^c$ and/or $d$ quarks with $u^c$ or $d^c$. For the Higgs-scalar mediators when the symmetry-breaking pattern is as in case (i) of the next section, the proton is stable because of a discrete symmetry.[4] Proton decay may occur due to mediators outside the partial-unification group. If this group has descended from some higher grand unification group, than these mediators have acquired masses at the primary stage of the symmetry breaking of the full group to the partial-unification group. As such, these mediators would have acquired much higher mass than the mediators of the partial-unification group. Our analysis of partial-unification mass scale in no way requires that the partial-unification group descend from some higher symmetry group.

## IV. LEPTON MASSES

The left-handed and right-handed fermions belong to representations ($\bar{4}$,1,2,1) and (1,$\bar{4}$,1,2) of the group:

$$\begin{aligned} F_L&\equiv\begin{bmatrix} u_1 & u_2 & u_3 & \nu_e\\ d_1 & d_2 & d_3 & e^- \end{bmatrix}_L ,\\ F_R&\equiv\begin{bmatrix} u_1 & u_2 & u_3 & N_e\\ d_1 & d_2 & d_3 & e^- \end{bmatrix}_R . \end{aligned} \tag{15}$$

For the case of Majorana neutrinos,[12] $N_e$ is distinct from $\nu_e$. For Dirac neutrinos, we also have two singlets[13] $\nu_{eR}$ and $N_{eL}$. The symmetry-breaking pattern envisaged in Sec. III can be implemented in two ways:

*Case (i).* We introduce a set of Higgs fields as follows:

$$\Sigma \equiv (\bar{4},4,1,1),\ \ \langle \Sigma_{44} \rangle = c\ ,$$

$$S_R \equiv (1,10,1,3),\ \ S_L(10,1,3,1)\ ,$$

$$\langle S^{+}_{R,44} \rangle = \frac{v}{\sqrt{2}},\ \ \langle S^{+}_{L,44} \rangle \simeq 0\ ,$$

$$\mathcal{M} \equiv (\bar{4},4,2,2)\ ,$$

$$\mathcal{M} \rightarrow \frac{1}{\sqrt{2}} \kappa I \otimes \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix}\ .$$

Here $I$ is a unit $2 \times 2$ matrix in the space of $\mathrm{SU}_L(2) \times \mathrm{SU}_R(2)$ and $M$ is a $3 \times 3$ matrix representing a set of scalar mesons belonging to representation $(\bar{3},3)$ of $\mathrm{SU}_{cL}(3) \times \mathrm{SU}_{cR}(3)$ color group. In this pattern of symmetry breaking,

$$M_X \sim gc,\ \ m_R \sim gv,\ \ m_L \sim g\kappa\ , \tag{16}$$

for the case 3(b):

$$c \gg v \gg \kappa\ .$$

The leptons are liberated and they acquire masses proportional to $\kappa$. The quarks are still confined and massless. They acquire masses by soft breaking[9] of residual symmetry $[\mathrm{SU}_c(3)]^2$ to $\mathrm{SU}_{cL+R}(3)$. In this way both axial gluons and quarks acquire their masses (for details see Ref. 9). In this case of symmetry breaking, we get a $\Delta L = 2$ Majorana mass term[4,12] and $n$-$\bar{n}$ oscillations.[4] If all the Yukawa couplings including that of $S_R$ are of the same order, then neutrino masses are given by[4,12,13]

$$m_{\nu e} \simeq m_e \left[ \frac{m_L}{m_R} \right],\ \ m_{Ne} \simeq m_e \left[ \frac{m_R}{m_L} \right]\ . \tag{17}$$

In this case [called (i)(a)], using the experimental limit $m_{\nu e} < 30$ eV, we have[13]

$$m_{Ne} > 8.3\ \mathrm{GeV},\ \ m_R > 1.4 \times 10^6\ \mathrm{GeV}\ . \tag{18}$$

If, on the other hand, the Yukawa coupling of $S_R$ is of order $g$, the gauge coupling constant

$$m_{\nu e} \simeq {m_e}^2 / m_R,\ \ m_{Ne} \simeq m_R\ . \tag{19}$$

In this case [called (i)(b)], for $m_R > 300$ GeV,

$$m_{\nu e} < 0.87\ \mathrm{eV}\ . \tag{20}$$

*Case (ii).* The symmetry breaking is implemented by the following Higgs system:

$$\Sigma \equiv (\bar{4},4,1,1),\ \ \langle \Sigma_{44} \rangle = c\ ,$$

$$\Phi_R \equiv (1,\bar{4},1,2),\ \ \Phi_L \equiv (\bar{4},1,2,1)\ ,$$

$$\langle \Phi_R \rangle = \frac{v}{\sqrt{2}},\ \ \langle \Phi_L \rangle \simeq 0\ ,$$

$$\mathcal{M} \equiv (\bar{4},4,2,2)\ ,$$

$$\mathcal{M} \rightarrow \tfrac{1}{2} \kappa I \otimes \begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix}\ .$$

In this case no $\Delta L = 2$ Majorana mass term is possible; $\Delta B = 2$ and $\Delta L = 2$ processes are forbidden. The neutrinos are Dirac neutrinos with masses as given by[13] Eqs. (17) and (18) or Eqs. (19) and (20).

We conclude that for chiral color, for partial-unification mass scale

$$5 \times 10^{12} \geq M_X \geq 3 \times 10^{10}\ \mathrm{GeV}$$

the intermediate mass scale is given by

$$300 \leq m_R \leq 10^7\ \mathrm{GeV}\ .$$

However, the $K_L$-$K_S$ mass difference[14] gives a constraint on the mass $m_R$,

$$m_R \geq 1.6 \times 10^3\ \mathrm{GeV}\ .$$

For Majorana neutrinos, for the case (i)(a) the $n$-$\bar{n}$ oscillations are expected to be observable for

$$m_{\nu e} > 1\ \mathrm{eV}\ ,$$

i.e., for

$$m_R < 4 \times 10^7\ \mathrm{GeV}\ .$$

For the case (i)(b), on the other hand, $n$-$\bar{n}$ oscillations will be observable for $m_{\nu e}$ in the range

$$0.87 \geq m_{\nu e} \geq 2.6 \times 10^{-4}\ \mathrm{eV}$$

or for

$$300 \leq m_R \leq 10^6\ \mathrm{GeV}\ .$$

## ACKNOWLEDGMENTS

A part of this work was done when one of us (F) was at Brookhaven National Laboratory and Fermi National Accelerator Laboratory. He wants to thank these institutions for their hospitality.

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