

Supersymmetry at ordinary energies. II. R invariance, Goldstone bosons, and gauge-fermion masses

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We explore the observable consequences of supersymmetry, under the assumption that it is broken spontaneously at energies of order 300 GeV. Theories of this sort tend automatically to obey a global R symmetry, which presents us with a choice among phenomenologically unacceptable alternatives. If the R symmetry is broken by scalar vacuum expectation values of order 300 GeV, there will be a semiweakly coupled light Goldstone boson, similar to an axion. If it is not broken by such vacuum expectation values but is broken by quantum-chromodynamic (QCD) anomalies, then there will be a light ninth pseudoscalar meson. If it is not broken by QCD anomalies, then the asymptotic freedom of QCD is lost at high energies, killing the hope of an eventual meeting of the electroweak and strong couplings within the regime of validity of perturbation theory. We also confront the problem of an uncomfortably light gluino. A general analysis of gaugino masses shows that the gluino mass is at most of order 1 GeV, and in many cases much less.

I. INTRODUCTION

This paper will continue the study of supersymmetry at ordinary energies that was begun in Ref. 1.

Our theoretical framework is as follows. We assume that supersymmetry survives down to energies of order 300 GeV, where, along with the electroweak gauge symmetry, it is spontaneously broken by vacuum expectation values (VEV's) of weakly coupled scalar fields. Where relevant, we assume that these VEV's are also responsible for quark and lepton masses. In order to avoid light scalars² and fast proton decay,¹ we pursue the suggestion of Fayet that the gauge group at low energies should contain in addition to the usual $SU(3) \times SU(2) \times U(1)$ at least an additional $U(1)$ factor, called here $\tilde{U}(1)$, with generator \tilde{Y} . However, most of our discussion would also apply in the alternative case³ where the gauge group is just $SU(3) \times SU(2) \times U(1)$ with light scalars avoided by having all quark, lepton, and associated scalar masses arise from radiative corrections, and we shall occasionally refer to $SU(3) \times SU(2) \times U(1)$ theories. We try here to avoid basing our considerations on any specific menu of superfields, but we have in mind a model including the following left chiral superfields:

	SU(3)	SU(2)	Y	\tilde{Y}
$Q_L = \begin{bmatrix} U_L \\ D_L \end{bmatrix}$	3	2	$-\frac{1}{6}$	1
U_R^*	$\bar{3}$	1	$\frac{2}{3}$	1
D_R^*	$\bar{3}$	1	$-\frac{1}{3}$	1
$L_L = \begin{bmatrix} N_L \\ E_L \end{bmatrix}$	1	2	$\frac{1}{2}$	1
E_R^*	1	1	-1	1
$H = \begin{bmatrix} H^0 \\ H^- \end{bmatrix}$	1	2	$\frac{1}{2}$	-2
$H' = \begin{bmatrix} H^{+'} \\ H^{0'} \end{bmatrix}$	1	2	$-\frac{1}{2}$	-2

plus additional chiral superfields like the $SU(3) \times SU(2) \times U(1)$ -neutral X with $\tilde{Y} = +4$ of Ref. 2, which was introduced to allow a spontaneous violation of supersymmetry.

One severe problem with this class of theories is that they are beset with triangle anomalies in gauge currents.¹ (For instance, with just the above superfields, there is a QCD anomaly in the \tilde{Y} current.) Furthermore, if enough new chiral scalar superfields

are introduced to cancel these anomalies, one tends to find that the scalar VEV's either do not break supersymmetry or do break charge or color gauge invariances.⁴ We will not deal with this problem here, but will simply assume that some set of chiral scalar superfields has been found which allow a realistic pattern of VEV's while at the same time canceling all anomalies in gauge currents. The considerations presented here will not be sensitive to the details of how this is done.

Our chief concern in this paper is with another problem: the phenomenological implications of a global symmetry known as R invariance. This symmetry is not put into these theories by hand, but is automatic in a wide class of supersymmetric models, including all those containing a gauge quantum number like \tilde{Y} whose values for left chiral superfields are restricted to 1 (mod 3). We do not know if this symmetry is broken by scalar VEV's of order 300 GeV, or by QCD anomalies, or by QCD condensates, or by some of these mechanisms, or by none of them, so we explore all of these alternatives. Our conclusion, summarized in the last section, is that each one of these alternatives leads to a severe conflict with experiment or with current theoretical ideas.

R invariance is introduced in its several forms in Sec. II. The possible mechanisms for breaking R invariance are outlined in Sec. III. Then in Sec. IV the properties of the Goldstone bosons associated with each of these mechanisms are considered. Section V deals with the masses of the gauge fermions, in the light of the previous analysis of R invariance.

II. R INVARIANCE

A large class of renormalizable supersymmetric theories automatically have a global symmetry of the type called R invariance. By an R invariance is meant any global $U(1)$ symmetry which acts non-trivially on the superfield coordinate θ , and therefore acts differently on the spinor and the scalar or vector components of a superfield. R symmetries were introduced by Salam and Strathdee⁵ and by Fayet⁶ as a means of imposing lepton conservation in supersymmetric models, and also in order to constrain the Lagrangian to rule out the possibility that the scalar fields could have vacuum expectation values which would leave supersymmetry unbroken. The discussion here will differ in that R symmetry is not imposed on the theory, but is found to arise in the theories that interest us whether we like it or not.

As one example of a large class of theories which automatically have an R symmetry, consider those renormalizable supersymmetric theories which are

prohibited by gauge symmetries from including any sort of super-renormalizable linear or bilinear F terms. [For instance, this is the case if there is a $U(1)$ gauge symmetry like that discussed by Fayet² and in I, for which the left chiral scalar superfields carry only the quantum numbers $1, -2, 4, -5, 7, \dots$] The Lagrangian of any such theory will contain only the kinetic terms and gauge couplings of the chiral superfields $S(x, \theta)$ [contained in the D terms $(S^* e^V S)_D$], plus the Yang-Mills F terms $(WW)_F$ (W is defined below in terms of V), and possible Fayet-Iliopoulos⁷ D terms $(V)_D$ involving the gauge vector superfields $V(x, \theta)$ alone, plus trilinear F terms $(S^3)_F$ and $(S^3)^*_F$. Any Lagrangian constructed from such ingredients will automatically be invariant under a global $U(1)$ transformation whose generator R_0 has the values $+1$ (-1) for θ_L (θ_R), $+\frac{2}{3}$ ($-\frac{2}{3}$) for all left (right) chiral superfields S (S^*), and 0 for all gauge vector superfields V .

To see this, note that the D term and F term of any function of superfields are the coefficients of $\theta_L^2 \theta_R^2$ and θ_L^2 , respectively, so if we arbitrarily assign the value $R_0 = +1$ to θ_L (and hence $R_0 = -1$ to $\theta_R \propto \theta_L^*$), then the D terms and F terms of any function have R_0 values equal to those of the function and the function minus 2, respectively. The functions $S^* e^V S$ and V obviously have $R_0 = 0$, so their D terms conserve R_0 . The left chiral spinor superfield W which contains the Yang-Mills curl is given schematically by

$$W \sim \left[\frac{\partial}{\partial \theta_R} + \theta_L \partial \right]^2 e^{-V} \left[\frac{\partial}{\partial \theta_L} + \theta_R \partial \right] e^V,$$

so it has $R_0 = +1$; W^2 has $R_0 = +2$; and so its F term conserves R_0 . Finally the function S^3 has $R_0 = 3 \times \frac{2}{3} = 2$, so again its F term conserves R_0 .

An R symmetry sometimes arises also in theories that do contain super-renormalizable F terms. For instance, if there are just two kinds of left chiral scalar superfields S_{\pm} which carry values ± 1 for some $U(1)$ gauge quantum number, then the only allowed renormalizable term $[f(S)]_F$ is $(S_+ S_-)_F$, and the Lagrangian is then invariant under an R_1 symmetry for which S_{\pm} both carry the R values $R_1 = 1$. Where not otherwise indicated the discussion here will be restricted to theories without super-renormalizable couplings, in which all left chiral scalar superfields have $R_0 = \frac{2}{3}$, but much of this discussion would also apply in more general circumstances.

The scalar and spinor component fields \mathcal{S} and s_L of a left chiral scalar superfield S are the coefficients of 1 and θ_L in the expansion of $S(x, \theta)$, while the spinor and vector component fields λ_L and V^μ of a

real gauge superfield V are the coefficients of $\theta_R^2 \theta_L$ and $\theta_R \theta_L$ in the expansion of $V(x, \theta)$. Hence these component fields have the R_0 values:

$$\begin{aligned} \text{left chiral scalars } \mathcal{S}: R_0 &= \frac{2}{3} - 0 = \frac{2}{3}, \\ \text{left chiral spinors } s_L: R_0 &= \frac{2}{3} - 1 = -\frac{1}{3}, \\ \text{left gauge spinors } \lambda_L: R_0 &= 0 + 1 = 1, \\ \text{vector gauge fields } V_\mu: R_0 &= 0 + 0 = 0. \end{aligned} \quad (1)$$

Any such R symmetry is surely broken by the vacuum expectation values of the $R_0 = \frac{2}{3}$ Higgs scalars which break $SU(2) \times U(1)$ and give masses to the quarks and leptons. However, it is sometimes possible to combine this broken global R symmetry with a suitable broken gauge symmetry to obtain an unbroken global R symmetry. (We do not consider the possibility of combining R_0 with a broken *global* symmetry to obtain an unbroken R symmetry, because this would lead to consequences similar to those of breaking R —specifically, a semiweakly coupled Goldstone boson.) The neutral Higgs scalars whose vacuum expectation values give masses to the quarks of charge $\frac{2}{3}$ and $-\frac{1}{3}$ (and charged leptons) belong to left chiral superfields with opposite values for electroweak hypercharge and zero values for charge and color, so there is no way that the R_0 symmetry defined above could be combined with $SU(3) \times SU(2) \times U(1)$ generators to yield an unbroken symmetry. On the other hand, suppose there is an additional $U(1)$ gauge symmetry whose generator \tilde{Y} has equal values for the Higgs superfields (as in the models of Fayet² and Secs. IV–VI of I). To keep the same notation as in I, let us take this value as $\tilde{Y} = -2$. Then we can define a new global symmetry

$$\tilde{R} = R_0 + \frac{1}{3} \tilde{Y} \quad (2)$$

which has the value zero for the Higgs scalars, and is therefore not broken by their vacuum expectation values. Even so it is still an open question whether \tilde{R} conservation is broken by other vacuum expectation values, or by Adler-Bell-Jackiw (ABJ) anomalies, or by dynamical effects of the strong interactions, or by suppressed nonrenormalizable terms in an effective Lagrangian resulting from an \tilde{R} -noninvariant theory at a higher energy scale. All these possibilities will be considered in following sections.

Using the values for R_0 given above and taking $\tilde{Y} = 0$ and -2 for gauge and Higgs superfields, the \tilde{R} values of the component fields are as follows:

$$\begin{aligned} \text{Higgs scalars: } \tilde{R} &= 0, \\ \text{left-handed Higgs spinors: } \tilde{R} &= -1, \\ \text{left-handed gauge spinors: } \tilde{R} &= +1, \\ \text{gauge vector bosons: } \tilde{R} &= 0. \end{aligned} \quad (3)$$

If we suppose for simplicity that all left chiral quark and lepton superfields have equal \tilde{Y} values, then in order for them to couple in pairs to the Higgs superfields they would have to have $\tilde{Y} = +1$, and their component fields would thus have the \tilde{R} values

$$\begin{aligned} \text{quarks and leptons: } \tilde{R} &= 0, \\ \text{left chiral quark and lepton scalars: } \tilde{R} &= +1. \end{aligned} \quad (4)$$

Finally, in order to find a suitable supersymmetry-breaking solution it has generally been found necessary to introduce left chiral X superfields which can couple to pairs of Higgs superfields and hence have $\tilde{Y} = +4$ (see Ref. 2 and Appendix B of I). Their component fields would have \tilde{R} values

$$\begin{aligned} \text{left chiral } X \text{ scalars: } \tilde{R} &= +2, \\ \text{left chiral } X \text{ spinors: } \tilde{R} &= +1. \end{aligned} \quad (5)$$

It is convenient that the known particles of low mass, including quarks, leptons, gluons, and photons, all have $\tilde{R} = 0$. Hence \tilde{R} invariance if unbroken would rule out interactions in which exotic particles with $\tilde{R} \neq 0$ such as quark or lepton scalars or Higgs or gauge spinors or X scalars or spinors (including the Goldstone fermion) are produced singly in collisions of known low-mass particles.⁸ An unbroken \tilde{R} invariance would also severely restrict the mass matrices of these exotic particles, and prohibit their mixing with known low-mass particles. We will return to these masses in Sec. V but first it is necessary to study the mechanisms that might break \tilde{R} invariance.

III. MECHANISMS FOR \tilde{R} BREAKING

We can distinguish five different mechanisms which can either individually or jointly break R invariance in supersymmetric theories.

A. Intrinsic R_0 breaking

As explained in the previous section, R_0 invariance can only be broken in a renormalizable supersymmetric Lagrangian by super-renormalizable F terms of the form $(S_1 S_2)_F$ or $(S_3)_F$, where S_i are generic left chiral scalar superfields. These are not allowed if there is a $\tilde{U}(1)$ symmetry whose generator

\tilde{Y} has values equal to 1 (mod 3) (e.g., +1, -2, +4) for all left chiral superfields, as in the models of Fayet² and Secs. IV–VI of I. Bilinear F terms would be allowed if there were also left chiral superfields which belong to the complex conjugates of some of the representations of $SU(3) \times SU(2) \times U(1) \times \tilde{U}(1)$ furnished by quark, lepton, Higgs, etc., superfields; in particular these would have $\tilde{Y} = -1$ (mod 3). This would have the advantage of making it easy to cancel ABJ anomalies, and the disadvantage of making it easy to find sets of scalar vacuum expectation values which leave supersymmetry unbroken.

Alternatively it is possible to add left chiral superfields which belong to real representations of $SU(3) \times SU(2) \times U(1) \times \tilde{U}(1)$, and therefore cannot have renormalizable F -term interactions with "known" superfields, but which can have both bilinear and trilinear F -term interactions with each other. One possible addition of this sort is a superfield S_0 that is neutral under all gauge groups. This could have $(S_0^3)_F$, $(S_0^2)_F$, and $(S_0)_F$ interactions, which would break R_0 , but since other superfields would have no interaction whatever with S_0 , their own R_0 would still be conserved. A more interesting possibility is to add superfields which have no F -term interactions with known superfields, but belong to nontrivial real representations of gauge groups. R violation in the new F -term sector could then induce transitions of left gauge fermions ($R_0 = +1$) into their antiparticles, breaking R_0 for all particles that feel these gauge forces. For instance, a color octet $SU(2) \times U(1) \times \tilde{U}(1)$ -neutral chiral superfield E could have $(E^2)_F$ and $(E^3)_F$ interactions, leading through radiative corrections to a Majorana mass term for the gluino,⁹ which violates R_0 .

Of course, if the gauge group at ordinary energies were just $SU(3) \times SU(2) \times U(1)$ then it would be easy to break R_0 by including an interaction of the form $(HH')_F$, which generates a Majorana Higgs-fermion mass. However if the only left chiral superfields in addition to H and H' were quark superfields Q and lepton superfields L , with F -term interactions schematically of the form $(HH')_F$, $(QQH \text{ (or } H'))_F$ and $(LLH)_F$, then the Lagrangian would automatically be invariant under an R symmetry for which H and H' carry the values $R_1 = 1$ while Q and L carry the values $R_1 = \frac{1}{2}$ (and θ_L still carries the value $R_1 = 1$). To break this R symmetry in the Lagrangian would require the introduction of new superfields, as, for instance, a $SU(3) \times SU(2) \times U(1)$ -neutral left chiral superfield N with interactions $(NHH')_F$ and $(N^3)_F$, $(N^2)_F$, and/or $(N)_F$.

Although there are evidently many ways to break R_0 and \tilde{R} in the Lagrangian, they all have a distaste-

ful feature. One of the reasons for pursuing the possibility that supersymmetry survives down to ordinary energies is that it would help to solve the hierarchy problem. But (inverted hierarchies aside)¹⁰ supersymmetry is not doing this job if it allows super-renormalizable F terms—without fine tuning these would have coefficients in the Lagrangian of the order of the superhigh energy scale M ($\approx 10^{17}$ GeV?) or M^2 , giving some fields masses of order M . These fields would have to be integrated out in constructing the effective Lagrangian which describes physics at ordinary energies, and this effective Lagrangian would then contain no super-renormalizable F terms. Note that this argument against super-renormalizable F terms does not apply to the super-renormalizable Fayet-Iliopoulos D terms. There are general theorems¹¹ which prevent such terms from appearing to any finite order of perturbation theory in an effective low-energy theory which arises from an underlying grand unified theory based on a simple or semisimple gauge group. To get these D terms at all it is necessary to rely on uncertain nonperturbative effects,¹² but this is still better than having to fine-tune the Lagrangian to keep super-renormalizable F terms sufficiently small.

This is not a conclusive argument. In particular, it is possible that Fayet-Iliopoulos terms cannot be generated nonperturbatively, and that like it or not we will have to rely on an O'Raifeartaigh mechanism¹³ to allow a spontaneous breakdown of supersymmetry at ordinary energies, which would necessarily require the appearance of super-renormalizable and hence R_0 -noninvariant terms in the Lagrangian. However in order for the superpotential naturally to have the form required for the O'Raifeartaigh mechanism, it is generally necessary to impose some sort of R symmetry. The consequences of any such R symmetry will be much the same as those of the R_0 symmetry which characterizes theories without super-renormalizable terms.

B. Vacuum expectation values

As we have seen, in $SU(3) \times SU(2) \times U(1)$ theories any R symmetry that is formed by combining R_0 with gauge symmetries is necessarily broken by the Higgs VEV's. This is not the case in $SU(3) \times SU(2) \times U(1) \times \tilde{U}(1)$ theories, where the Higgs VEV's leave the \tilde{R} of Eq. (2) unbroken. Nevertheless, it is possible that in the spontaneous breakdown of supersymmetry and $SU(2) \times U(1) \times \tilde{U}(1)$, there arise vacuum expectation values, perhaps of order 300 GeV, not only for Higgs scalars but also for other scalars with $\tilde{Y} \neq 2$. These scalars have $\tilde{R} \neq 0$, so their expectation values would

produce a large spontaneous breakdown of \tilde{R} in the tree approximation. One would not expect the $\tilde{R}=1$ scalar counterparts of the quarks and leptons to develop vacuum expectation values, since that would lead to an unsuppressed violation of color, charge, or lepton conservation, but the X scalars with $\tilde{Y}=+4$ and $\tilde{R}=2$ might perhaps develop large vacuum expectation values. Such vacuum expectation values would still leave unbroken an “ \tilde{R} parity,”⁸ $\exp(i\pi\tilde{R})$, which would rule out the production of single Higgs, gauge, or X spinors or quark or lepton scalars in collisions of known low-mass particles, and would prohibit the mass mixing of these exotic particles with known low-mass particles, though not constraining their mixing with each other. Our discussion below of the consequences of breaking \tilde{R} with large VEV’s applies to theories with or without an extra $\tilde{U}(1)$ gauge symmetry.

C. Strong ABJ anomalies

In quantum chromodynamics \tilde{R} conservation can be violated by strong ABJ anomalies¹⁴ in the presence of instantons.¹⁵ These anomalies arise from triangle diagrams in which an \tilde{R} current and two gluons are attached to a loop of colored fermions. To assess the anomalies, it is convenient to consider the R_0 currents rather than those of \tilde{R} ; since \tilde{Y} is a gauge symmetry its current must in any case be anomaly-free, so the anomalies of the R_0 and \tilde{R} currents are the same, and in considering R_0 instead of \tilde{R} we can lump together all chiral superfields, whatever their values of \tilde{Y} . Suppose that the left-handed colored fermions comprise one color octet of gauge fermions (“gluinos”) with $R_0=+1$ and some number of fermionic components of various left chiral superfields, with $SU(3)$ generator $t_{SU(3)}$ and $R_0=-\frac{1}{3}$. The anomaly is then

$$A \equiv \text{Tr}[R(t_{SU(3)})^2] = 3 - \frac{1}{3}\text{Tr}(t_{SU(3)})^2. \quad (6)$$

The trace of $(t_{SU(3)})^2$ is a half-integer, so A is a sixth-integer. The effect of this anomaly in a gluon instanton field of winding number ν is that the R_0 and \tilde{R} quantum numbers change by the amounts

$$\Delta\tilde{R} = \Delta R_0 = 2A\nu. \quad (7)$$

If the only colored chiral superfields were the three generations of quarks then $\text{Tr}(t_{SU(3)})^2 = 3 \times 4 \times \frac{1}{2}$, so there would be an anomaly with $A=1$. However, additional colored chiral superfields must be added in any case in order to cancel the QCD anomaly in the $\tilde{U}(1)$ current, so in general we can only conclude that $A < 1$. In particular, the octet left chiral superfield O_L , which was introduced in I to cancel the QCD anomaly in the $\tilde{U}(1)$ current

when there are three quark generations, contributes $+3$ to $\text{Tr}(t_{SU(3)})^2$, which together with a gluino and the three quark generations yields $A=0$.

The case of vanishing QCD anomaly, $A=0$, has an interesting and somewhat unpleasant special feature. Recall that the divergence of the R_0 current is in the same supermultiplet as the trace of the energy-momentum tensor,¹⁶ so if the one-loop QCD anomaly of the R_0 current vanishes, then so does the one-loop β function of QCD. This can easily be verified: for instance, one octet of Majorana fermions and their complex scalar superpartners plus the gluinos and three generations of quarks and their scalar superpartners just cancels the gluon contribution to the one-loop β function of QCD. Many of these spin-0 and spin- $\frac{1}{2}$ particles are expected to have masses of order m_W , so QCD would still appear to be asymptotically free at energies below m_W , but with a β function less negative than usually assumed. At higher energies the β function would arise only from two-loop and higher-order contributions. Detailed calculation shows that the two-loop terms¹⁷ have the wrong sign for asymptotic freedom, and all these terms are very small for couplings of order e . Thus with $A=0$ there would be no hope that the strong and electroweak couplings could become equal at any energy below the Planck mass.¹⁸

Although for $A \neq 0$ the ABJ anomaly would break the continuous \tilde{R} symmetry, Eq. (7) shows that it would not break the discrete symmetry of multiplication by

$$Z \equiv \exp(i\pi\tilde{R}/A). \quad (8)$$

Whether or not this is a useful symmetry depends on the value of the sixth-integer A . For $|A| = \frac{1}{2}$ or $\frac{1}{6}$, Z invariance would have no consequences for particles of integer \tilde{R} , all of which would have $Z=1$. For $|A|=1$ or $\frac{1}{3}$, Z would be the same as the \tilde{R} parity encountered above in Sec. IIIB, and would allow $|\Delta\tilde{R}|=1$ transitions. If $|A|$ were to have any value other than $1, \frac{1}{2}, \frac{1}{3}$, or $\frac{1}{6}$ then transitions with $|\Delta\tilde{R}|=1$ and $|\Delta\tilde{R}|=2$ would all be forbidden, so Z invariance would lead to the same constraints on the masses of gauge, Higgs, and X fermions and quark and lepton scalars as if \tilde{R} conservation were not violated by an ABJ anomaly.

For any value of A not equal to zero the \tilde{R} invariance of the Lagrangian would solve the strong CP problem in the same way as the $U(1)$ symmetry of Peccei and Quinn,¹⁹ with the difference that \tilde{R} was not invented specifically for this purpose, but is automatically encountered in a large class of supersymmetric theories. Unfortunately, as we shall see, the case $A \neq 0$ runs into a variety of familiar conflicts with experiment.

D. Other strong-interaction effects

Even if \tilde{R} conservation is unbroken by ABJ anomalies or by large scalar vacuum expectation values, it can still be spontaneously broken by dynamical effects of the strong interactions. However, here again we expect to retain an unbroken discrete symmetry. All strongly interacting fields that we have encountered here happen to satisfy the relation⁸

$$(-)^{2j}(-)^{3B}=(-)^{\tilde{R}} \quad (9)$$

with j the spin and B the baryon number. Hence any hadronic operator of zero spin and baryon number whose vacuum expectation value can break \tilde{R} conservation would have to have \tilde{R} even, and therefore must conserve the \tilde{R} parity $\exp(i\pi\tilde{R})$, with the same consequences as described in Sec. III B.

E. Suppressed nonrenormalizable effective interactions

Since \tilde{R} invariance was not imposed here as an *a priori* symmetry principle but was merely encountered as a more or less accidental consequence of renormalizability, supersymmetry, and gauge symmetries, we would not necessarily expect R invariance to be respected by the physics of much higher energy scales, and hence not by the very weak nonrenormalizable interactions in the effective interaction that describes physics at ordinary energies. These nonrenormalizable interactions were cataloged for $SU(3) \times SU(2) \times U(1) \times \tilde{U}(1)$ theories in Sec. IV of I. The lowest-dimensional allowed nonrenormalizable interactions had dimensionality $d=6$, but these are all of the form $(S^{*2}S^2)_D$ (where S is a generic left chiral superfield) and therefore conserve R_0 and hence \tilde{R} . However, there is a whole host of allowed $d=7$ operators of the form $(S^6)_F$ (including the F term of the square of whatever functions F term appears in the renormalizable part of the interaction), which all have $R_0=6 \times \frac{2}{3}-2=2$, and hence break R_0 and \tilde{R} invariance but not \tilde{R} parity. These interactions are suppressed by $d-4=3$ powers of whatever superheavy mass (10^{17} GeV?) characterizes the scale of the nonrenormalizable interactions, and probably by several powers of gauge couplings as well, so their effects are extremely small.

IV. R GOLDSTONE BOSONS

The existence of an R symmetry of the Lagrangian which is broken spontaneously or by QCD anomalies would require the appearance of a Goldstone or pseudo-Goldstone boson. The properties of this boson depend critically on which of the various

symmetry-breaking mechanisms discussed in the preceding section is in operation. Our task in dividing up the various possibilities is simplified by the observation that when R invariance is broken by a scalar VEV of order 300 GeV it is irrelevant whether it is also broken by QCD condensates with a scale of 300 MeV. Also, as we shall see, whatever the dominant mechanism for R -invariance breaking, a crucial consideration for the phenomenology is whether the QCD anomaly of the \tilde{R} current vanishes or not. On the basis of these remarks, it is useful to distinguish four materially different combinations of symmetry-breaking mechanisms.

A. Large scalar VEV's, vanishing QCD anomaly

Suppose that the Lagrangian has an R symmetry but R_0 and \tilde{R} and all other linear combinations of R_0 and gauge symmetries are spontaneously broken in the tree approximation by large vacuum expectation values of order $f_R \approx 300$ GeV. Then there must be a Goldstone boson (let us call it R^0) having semiweak derivative interactions with coupling constant $1/f_R$. If the R symmetry is exact and unbroken by QCD anomalies, then this Goldstone boson is strictly massless. If the symmetry is intrinsically broken only by $d=7$ operators in the effective Lagrangian, then the Goldstone boson has a squared mass inversely proportional to the cube of the superheavy mass scale M , and hence on dimensional grounds of order

$$m_{GB}^2 \approx f_R^5/M^3. \quad (10)$$

For $f_R=300$ GeV and $M=10^{17}$ GeV, this still gives a negligible mass, of order 10^{-10} eV.

If all scalar vacuum expectation values are of the same order, then the current to which this Goldstone boson couples will be a linear combination of the currents of \tilde{R} and of the ordinary weak hypercharge Y and the $\tilde{U}(1)$ generator \tilde{Y} , all with comparable coefficients. Hence this Goldstone boson will have semiweak couplings to ordinary quarks and leptons which are qualitatively similar to those predicted for the old axion,²⁰ though its mass is much less. Those experiments that have searched without success for axions through their interactions (but not their decays) thus provide evidence against this light R Goldstone boson as well.

For instance, in an $SU(3) \times SU(2) \times U(1) \times \tilde{U}(1)$ gauge theory the Goldstone boson field ϕ_{GB} has the effective interaction

$$\frac{1}{f_R} (J_R^\mu + cJ_Y^\mu + \tilde{c}J_{\tilde{Y}}^\mu)_{NP} \partial_\mu \phi_{GB}, \quad (11)$$

where NP denotes the part of the current excluding

the Goldstone-boson pole, and c and \tilde{c} are coefficients of order unity. If the complex scalar fields with nonvanishing vacuum expectation values in-

clude only Higgs fields $\mathcal{H}^0, \mathcal{H}^{0'}$ with $\tilde{Y} = -2$ and $Y = \pm 1$ and a field \mathcal{X}^0 with $Y = 0$ and $\tilde{Y} = +4$, then²¹

$$f_R = \frac{2\langle \mathcal{H} \rangle \langle \mathcal{H}' \rangle \langle \mathcal{X} \rangle}{(\langle \mathcal{H} \rangle^2 \langle \mathcal{H}' \rangle^2 + \langle \mathcal{H} \rangle^2 \langle \mathcal{X} \rangle^2 + \langle \mathcal{H}' \rangle^2 \langle \mathcal{X} \rangle^2)^{1/2}},$$

$$c = \frac{\langle \mathcal{X} \rangle^2 (\langle \mathcal{H}' \rangle^2 - \langle \mathcal{H} \rangle^2)}{(\langle \mathcal{H} \rangle^2 \langle \mathcal{H}' \rangle^2 + \langle \mathcal{H} \rangle^2 \langle \mathcal{X} \rangle^2 + \langle \mathcal{H}' \rangle^2 \langle \mathcal{X} \rangle^2)},$$

$$\tilde{c} = \frac{-\langle \mathcal{X} \rangle^2 (\langle \mathcal{H} \rangle^2 + \langle \mathcal{H}' \rangle^2)}{2(\langle \mathcal{H} \rangle^2 \langle \mathcal{H}' \rangle^2 + \langle \mathcal{H} \rangle^2 \langle \mathcal{X} \rangle^2 + \langle \mathcal{H}' \rangle^2 \langle \mathcal{X} \rangle^2)}.$$

Quarks and leptons contribute to J_Y^μ and $J_{\tilde{Y}}^\mu$, so that this Goldstone boson will in general have appreciable semiweak couplings to quarks and leptons, and probably would have been seen.

We can also consider the possibility that the scalar vacuum expectation values are of rather different magnitudes. Suppose, for instance, in the example above that $\langle \mathcal{X} \rangle$ is much larger than $\langle \mathcal{H} \rangle$ or $\langle \mathcal{H}' \rangle$. In this case the coefficients in (11) approach $\langle \mathcal{X} \rangle$ -independent limits:

$$f_R \rightarrow \frac{2\langle \mathcal{H} \rangle \langle \mathcal{H}' \rangle}{(\langle \mathcal{H} \rangle^2 + \langle \mathcal{H}' \rangle^2)^{1/2}},$$

$$c \rightarrow \frac{\langle \mathcal{H}' \rangle^2 - \langle \mathcal{H} \rangle^2}{\langle \mathcal{H}' \rangle^2 + \langle \mathcal{H} \rangle^2}, \quad \tilde{c} \rightarrow -\frac{1}{2}.$$

Since $\langle \mathcal{H} \rangle$ and $\langle \mathcal{H}' \rangle$ break $SU(2) \times U(1)$, neither can be much greater than 300 GeV, so the Goldstone boson is still semiweakly coupled. Also \tilde{c} and perhaps c are still of order unity. The semiweak couplings of the Goldstone bosons thus include interactions with quarks and leptons, which are experimentally ruled out by limits²² on ψ and T decay to photon plus axion.

The fact that even in the large- $\langle \mathcal{X} \rangle$ limit the Goldstone boson does not decouple from quarks and leptons can be understood as follows. Since $\langle \mathcal{X} \rangle$ breaks not only \tilde{R} but also the $\tilde{U}(1)$ gauge symmetry, if $\langle \mathcal{H} \rangle$ and $\langle \mathcal{H}' \rangle$ were zero the Goldstone boson associated with $\langle \mathcal{X} \rangle$ would be eliminated by the Higgs mechanism. The symmetry left in this limit is the gauge $SU(2) \times U(1)$ and the global $\tilde{R} - \frac{1}{2}\tilde{Y}$. The Goldstone boson here thus arises only from the smaller vacuum expectation values $\langle \mathcal{H} \rangle$ and $\langle \mathcal{H}' \rangle$ which break $\tilde{R} - \frac{1}{2}\tilde{Y}$, so it naturally is semiweakly coupled, and coupled to quarks and leptons. The only way to make this Goldstone boson have couplings weaker than semiweak would be to suppose that \tilde{R} is broken by large vacuum expectation values

of scalar fields that do not carry any gauge quantum numbers, but the familiar particles would not have any renormalizable interactions with such scalars, so their \tilde{R} quantum number would remain conserved.

On the other hand, suppose that $\langle \mathcal{X} \rangle$ is much less than $\langle \mathcal{H} \rangle$ and $\langle \mathcal{H}' \rangle$. This seems at first sight like an unpromising case, because f_R approaches $2\langle \mathcal{X} \rangle$, so this Goldstone boson couples more strongly than semiweakly. However, c and \tilde{c} also vanish as $\langle \mathcal{X} \rangle \rightarrow 0$, and like $\langle \mathcal{X} \rangle^2$ instead of $\langle \mathcal{X} \rangle$. Hence the direct couplings c/f_R and \tilde{c}/f_R of the Goldstone boson to quarks and leptons vanish like $\langle \mathcal{X} \rangle$ for $\langle \mathcal{X} \rangle \rightarrow 0$. The strongest limits come from beam-dump experiments²³ which give

$$[\sigma(pN \rightarrow R^0 X) \sigma(R^0 N \rightarrow X')]^{1/2} \lesssim 10^{-6} \text{ mb}.$$

The above estimates of R^0 quark couplings are consistent with this limit, assuming $\langle \mathcal{H} \rangle \sim \langle \mathcal{H}' \rangle$, if $\langle \mathcal{X} \rangle \lesssim 40$ GeV.²⁴ However, we must also take into account the indirect couplings of the Goldstone boson via gluons; this is done in Ref. 25 and in Sec. IV C below. Of course, in the limit of small $\langle \mathcal{X} \rangle$, \tilde{R} invariance is only slightly broken, and we recover the consequences of an unbroken \tilde{R} symmetry, to which we will come at the end of this section.

B. Large VEV's and QCD anomalies

At the same time that \tilde{R} is broken by large scalar vacuum expectation values and (presumably) by suppressed nonrenormalizable effective interactions, it is possible for \tilde{R} to be broken also spontaneously by hadronic vacuum expectation values and/or intrinsically by QCD anomalies. The appearance of hadronic vacuum expectation values is irrelevant as long as \tilde{R} is also broken by much larger scalar-field vacuum expectation values. On the other hand, if there are QCD anomalies in the \tilde{R} current, then this current is not the right place to look for Goldstone

bosons. Instead we must consider a linear combination of the \tilde{R} current with the U(1) axial-vector current of the light quarks, the coefficients being chosen so that the QCD anomalies of the two currents cancel. The conservation of this new current is broken, spontaneously by the large vacuum expectation values that break \tilde{R} conservation, and intrinsically by the small masses of the light quarks. Hence there is a Goldstone boson here with a squared mass of order

$$m_{GB}^2 \approx m_{u,d} \Lambda_{QCD}^3 / f_R^2. \quad (12)$$

In fact, this is nothing but the old axion.

C. Hadronic VEV's, vanishing QCD anomaly

If the Lagrangian has an \tilde{R} symmetry which is not broken by large vacuum expectation values but is spontaneously broken by strong interaction effects, then there will be a strongly interacting Goldstone boson. If the \tilde{R} current is free of QCD anomalies then the spontaneous breakdown of \tilde{R} invariance leads to a true Goldstone boson of zero mass, except that $d=7$ \tilde{R} -violating terms in the effective Lagrangian would give it a mass of order

$$m_{GB}^2 \sim \Lambda_{QCD}^5 / M^3$$

for $\Lambda_{QCD} \simeq 300$ MeV and $M \simeq 10^{17}$ GeV, this gives an utterly negligible mass, of order 10^{-17} eV.

The \tilde{R} current does not contain quark terms, so it might be hoped that this Goldstone boson, R^0 , although strongly interacting might have escaped detection. This hope unfortunately proves illusory.²⁵ The point is that QCD forces can only bring about a spontaneous breakdown of \tilde{R} invariance if there exist \tilde{R} -nonneutral particles—e.g., gluinos—which escape getting masses of order m_W in the breakdown of supersymmetry and $SU(2) \times U(1)$, so that they are present in the effective theory that describes physics at energies where the QCD forces are strong. These strongly interacting \tilde{R} -nonneutral particles then mediate indirect interactions via gluons between the R^0 Goldstone boson and ordinary hadrons. Since the QCD anomaly of the \tilde{R} current is being assumed here to vanish, the R^0 coupling to quarks via a gluino loop connected through 2, 3, or 4 gluons [Fig. 1(a)] is canceled by whatever cancels the anomaly. Thus the dominant coupling of R^0 's to quarks is pairwise, as shown in Fig. 1(b).²⁶ We have just argued that the mass scale of the intermediate \tilde{R} -nonneutral state is of order of hadronic masses, and the coupling is $O(\alpha_{QCD}^2)$, so that the R^0 pairwise interaction with hadrons is semistrong. It may not be immediately obvious that massless neutral bosons pair-produced with cross sections less than a few

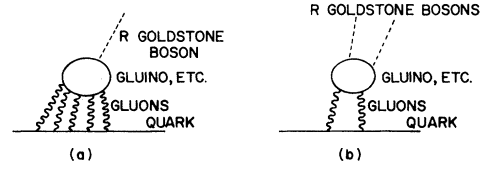


FIG. 1. Diagrams for R Goldstone boson coupling to quarks.

millibarns are experimentally excluded; certainly they would not have been observed in exclusive experiments, and conventional beam dumps with very long filters between the target and the detector are not sensitive to them because semistrongly interacting R^0 's would be absorbed before reaching the detector. In fact, however, beam-dump experiments at Brookhaven²⁷ and at a Fermilab beam-dump test run²⁸ had sufficiently short filters to be able to exclude such semistrongly produced Goldstone bosons.

D. QCD anomalies, no large scalar VEV's

Now suppose that the Lagrangian has an \tilde{R} symmetry that is not spontaneously broken by large scalar vacuum expectation values but is broken by QCD anomalies. This is of course only possible if there are colored fermions with $\tilde{R} \neq 0$ whose mass vanishes to all orders of perturbation theory: if all colored fermions were allowed by $SU(3)$ and \tilde{R} to get masses, then the colored left-handed fermion fields would have to form a real representation of $SU(3) \times \tilde{R}$, so the anomaly A would have to vanish. Such colored massless fermions pose a problem for the consistency of theories of this sort with experiment, to be discussed in Sec. V. For the present, we have to worry about a different problem that is bad enough, the problem of pseudo-Goldstone bosons.

Although the \tilde{R} current is not itself associated with a Goldstone boson, it may be combined with the U(1) axial-vector current of the light quarks to form an anomaly-free current $J_{\tilde{R}}^\mu$, which is intrinsically broken only by the small masses of the light u and d quarks. This \tilde{R}' current must also be spontaneously broken by dynamical effects of the strong interactions, because otherwise the theories would have to contain colorless hadrons which become massless in the limit m_u or $m_d \rightarrow 0$, and which therefore would have masses of only a few MeV. [This is intuitively plausible, and can be shown with greater rigor by adapting an argument of 't Hooft.²⁹ Although there is no \tilde{R}' - $SU(3)$ - $SU(3)$ triangle anomaly, since \tilde{R}' receives a contribution from quarks

there are a number of other nonvanishing triangle anomalies, such as $\tilde{R}'\text{-}\tilde{R}'\text{-}\tilde{R}'$, $\tilde{R}'\text{-}Q\text{-}Q$, and $\tilde{R}'\text{-}B\text{-}B$, with Q electric charge and B baryon number. If \tilde{R}' were exactly conserved and not spontaneously broken by the strong interactions, then these triangle anomalies would have to be reproduced by massless untrapped color-neutral hadrons carrying nonvanishing values of \tilde{R}' , Q , and B .] Since \tilde{R}' must be spontaneously broken by dynamical effects of strong interactions, it is irrelevant here whether also \tilde{R} is or is not spontaneously broken by the strong interactions. Also, since \tilde{R}' is intrinsically broken by the quark masses, it is irrelevant to us whether it is or is not also broken by suppressed nonrenormalizable terms in the effective Lagrangian.

The problem here is that the conserved anomaly-free \tilde{R}' current can be used in just the same way as the axial-vector $U(1)$ current of the quarks was used before the discovery of the effects of QCD instantons, to derive an unacceptable result³⁰: The spontaneous breakdown of the \tilde{R}' current leads to the same ninth pseudoscalar meson, with mass³¹

$$m_{GB} < \sqrt{3}m_\pi$$

that was the crux of the old $U(1)$ problem.

It is not obvious that the problem of the ninth pseudoscalar meson is avoided even if we add colored superfields with $\tilde{R} \neq 0$ (like the octet superfield of I) to cancel the QCD anomaly in the \tilde{R} current. The point³² is that if the scalar particles in such superfields have large masses (say $\approx m_W$) then at lower energies we have *two* approximately conserved \tilde{R} currents, one for the quarks and gluinos and the other for the fermion members (e.g., color-octet fermions O) of the new superfields. Only the sum of these two currents is anomaly-free; either one individually may be combined with the axial-vector current of the quarks to deduce the existence of a ninth pseudoscalar meson. To avoid this, the scalar components of the new superfield must be supposed to be fairly light, but fortunately they do not have to be lighter than a few GeV, as it is only necessary that the scalar exchange graphs which violate the separate \tilde{R} conservation for gluinos and quarks be larger than the very small up- and down-quark mass terms which give the pion its mass.

E. \tilde{R} unbroken, vanishing QCD anomaly

Of course, if \tilde{R} is not broken, neither spontaneously nor by QCD anomalies, then there will be no Goldstone boson to worry us. Ordinary particles have $\tilde{R}=0$, so their properties would be unconstrained by this assumption, while \tilde{R} -nonneutral particles would have to be massless or parity doubled.

At the constituent level this would be realized by massless gluinos, while at the composite level any R hadrons would have to be parity doubled, since massless R hadrons are unacceptable. (R hadrons are color-singlet hadrons containing a gluino in addition to quarks and/or gluons, and hence coming in a variety of possible charges and flavors.) It is amusing that the 't Hooft anomaly-matching argument,²⁹ which can often be used to demonstrate the necessity of certain massless composite fermions, is not applicable in this case, since the \tilde{R} anomaly is assumed here to vanish at the constituent level. The chief difficulty with this proposal is the absence of asymptotic freedom in QCD at high energies, which as discussed in Sec. III C above is a necessary consequence of the absence of an \tilde{R} anomaly. Another potential problem is the masslessness of the gluino, to be discussed below in Sec. V.

V. GAUGINO MASSES

There are three distinct cases to be considered when discussing the masses of gauginos, the fermionic partners of gauge bosons: tree level masses, radiatively induced masses with R invariance unbroken, and radiatively induced masses with spontaneously broken R invariance.

A. Tree-level masses

Although spontaneous supersymmetry breaking permits tree-level mass splittings within chiral supermultiplets and massive gauge supermultiplets, this is not possible within multiplets containing gauge bosons of an *unbroken* gauge group. An explicit mass term for such a gaugino would violate supersymmetry since by the assumption of unbroken gauge invariance the gauge boson is massless. Furthermore, the Higgs mechanism does not produce such masses in tree approximation, since by supersymmetry gauginos couple only to fields having a nonzero charge under the corresponding gauge group, so mass generation could only occur by mixing of a gaugino with a charged fermion, which is impossible because the charged scalar superpartner has zero VEV. Thus the gluinos and photinos have zero mass at tree level even when supersymmetry is spontaneously broken.

What about the fermionic partners of massive gauge bosons, such as w^\pm , z^0 , and \tilde{z}^0 ? As is well known, spontaneous supersymmetry breaking never produces pure gaugino ($\lambda\lambda$) mass terms in tree approximation, but off-diagonal mass terms (λs) connecting gauginos with chiral fermions can be produced in this way.

The isodoublet Higgs superfields H, H' contain

charged Higgs fermions h^- and $h^{+'}$ (having $\tilde{R} = -1$) that can pair with the charged gauginos w^+ and w^- (having $\tilde{R} = +1$), thereby giving them tree-approximation masses of order m_W , whether or not \tilde{R} is spontaneously broken. Such particles are phenomenologically acceptable, and we need not discuss radiative corrections to their masses. However, if \tilde{R} is not spontaneously broken and if there are more Higgs doublet superfields than just one H and one H' , then there will be more than just one pair of charged Higgs fermions with $\tilde{R} = -1$ and some charged Higgs fermions will be left in the tree approximation with zero mass. Even if \tilde{R} is broken by QCD anomalies or condensates, the mass of the left-over charged Higgs fermions will be very small, much less than 1 GeV. (See the discussion in Sec. VC below.) Charged Higgs fermions with mass below about 16 GeV are ruled out by e^+e^- colliding beam experiments,³³ so theories with extra Higgs superfields and \tilde{R} unbroken by large VEV's are definitely untenable.

The case of neutral colorless gauginos and Higgs fermions is more complicated. The gauge bosons Z^0 , \tilde{Z}^0 , and γ have gaugino partners z^0 , \tilde{z}^0 , and photino, of which the photino remains massless in tree approximation, while z^0 and \tilde{z}^0 in general get masses of order m_W by mixing with Higgs fermions h^0, h'^0 and with the fermion member x^0 of the X chiral superfield. However, one linear combination of the $z^0, \tilde{z}^0, h^0, h'^0$, and x^0 must provide the Goldstone fermion and hence remain strictly massless.

If there are more than two neutral Higgs superfields and only one X superfield then R invariance, if unbroken, would keep all but two of the neutral Higgs fermions massless. Even if strictly massless, these extra Higgs fermions would behave at low energies like neutrinos with only neutral-current interactions, and so might have escaped detection.

B. Radiatively induced R -conserving gluino and photino masses

The issue of radiative corrections to gaugino masses is really only consequential for those particles which would have zero mass in the absence of radiative corrections, the photino and gluinos. While such masses may be compatible with gauge and supersymmetry invariances, as noted above gauginos carry $\tilde{R} = R_0 = +1$, so that diagonal mass terms $\lambda\lambda$ for the photino and gluinos violate R invariance. \tilde{R} -conserving off-diagonal mass terms $\lambda\psi$ are possible if there are $\tilde{R} = -1$ fermions, ψ , with the same conserved quantum numbers as the photino and gluinos. However, in fact, such off-diagonal terms are often excluded by the discrete symmetry $V \rightarrow V, \phi \rightarrow -\phi$, where V and ϕ are the

superfields containing λ and ψ , respectively. For instance, gluinos cannot mix with the color-octet fermions O introduced in I because, given the quark quantum numbers, the only O couplings allowed are O^+e^+V and OOX which respect $V \rightarrow V, O \rightarrow -O$. In Fayet's example⁹ of gluino masses arising from off-diagonal terms he was obliged to introduce not only an additional chiral octet but also two new heavy quark fields with nonstandard quantum numbers and super-renormalizable couplings, explicitly breaking R invariance. (In fact the true vacuum of this model breaks color conservation and does not break supersymmetry.) It is doubtful that, in a model sufficiently complex to include all known particles, the additional fields necessary to generate off-diagonal gluino masses can be added while maintaining supersymmetry breaking.

In the minimal model with $SU(2) \times U(1) \times \tilde{U}(1)$ broken by two Higgs doublets, H and H' , there are two neutral Higgs fermions with $\tilde{R} = -1$. However, as we have seen, there are four neutral $\tilde{R} = +1$ fermions in the three neutral gauge multiplets and the X chiral multiplet, so that in the end there are two massless fermions (the photino and the Goldstone fermion) and two massive partners for the Z^0 and \tilde{Z}^0 . Thus for there to be enough degrees of freedom to give the photino an off-diagonal \tilde{R} -conserving mass an additional neutral chiral multiplet, N , must be introduced. It should be an electroweak isosinglet so that no massless charged fermion is introduced. However, if it is an isosinglet and has $\tilde{R} = -1$, its only couplings are its gauge couplings and NNX , unless nonstandard quarks or leptons are also added. Consequently the discrete symmetry $V_\gamma \rightarrow V_\gamma$ and $N \rightarrow -N$ would prevent the photino from mixing with ψ_N , even at the quantum level.

Thus, in summary, the photino and gluinos will be massless when \tilde{R} invariance is unbroken unless exotic quarks or leptons are introduced.

C. Radiatively induced R -breaking gluino and photino masses

Having disposed of R -invariant gluino and photino mass terms we now examine the situation when spontaneous R -invariance breaking permits a Majorana mass $\lambda\lambda$. There are basically two cases: R breaking from a nonzero VEV such as $\langle \mathcal{S} \rangle$, or dynamical R breaking from QCD condensation giving a nonzero value to $\langle \lambda_{\text{gl}} \lambda_{\text{gl}} \rangle$ or $\langle \mathcal{Q} \mathcal{Q}^c \rangle$.

The underlying supersymmetry, R invariance, and gauge symmetries of the Lagrangian, even though spontaneously broken, result in many cancellations among the diagrams which can generate gaugino masses. Thus a method of identifying the structure associated with nonvanishing contributions is useful.

This can be accomplished³⁴ by considering higher order corrections as generating terms in an effective Lagrangian which necessarily has the same set of invariances as the original Lagrangian had before the fields which develop VEV's were shifted. A somewhat unconventional use is being made here of an effective Lagrangian: it is not a matter of eliminating very massive fields from a low-energy effective interaction, because all of the fields we are considering are of a relatively low ($\lesssim 300$ GeV) mass scale; rather it is being used as a device to expose the quite nontrivial constraints on possible mass terms imposed by the combined supersymmetry and internal symmetries. A term in \mathcal{L}_{eff} capable of generating a $\lambda\lambda$ mass term must have the following properties³⁴:

- (i) It must be supersymmetric and gauge invariant and respect all global symmetries.
- (ii) It must be bilinear in the gauge superfield V and of such a form that the gaugino field λ enters without derivatives.
- (iii) It must be at least linear in a supersymmetry breaking VEV such as $\langle \tilde{D} \rangle$ or an $\langle F \rangle$, since unbroken supersymmetry would require the gluinos and photinos to be strictly massless.
- (iv) It must be linear in a $\Delta R = 2$ VEV (e.g., $\langle \lambda_g \lambda_g \rangle$, $\langle \mathcal{D} \mathcal{D}^c \rangle$, $\langle \mathcal{X} \rangle$, or $\langle F_H \rangle$).
- (v) The term must not contain any additional fields with nontrivial quantum numbers under the unbroken symmetries, since to generate the mass-term structure $\lambda\lambda$ all fields in \mathcal{L}_{eff} except V must be given VEV's.

Finding terms in \mathcal{L}_{eff} with the required properties has been done in Ref. 34 and we simply take over the result here. For the minimal $SU(3) \times SU(2) \times U(1) \times \tilde{U}(1)$ model the lowest-order contributions to gluino and photino masses not resulting from R -breaking QCD condensates correspond to pieces in \mathcal{L}_{eff} of the form

$$(W^\alpha W_\alpha \tilde{W}_{\dot{\alpha}} \tilde{W}^{\dot{\alpha}} H^\dagger H)_{\theta\theta\bar{\theta}\bar{\theta}} \supset \lambda\lambda \langle \tilde{D} \rangle^2 \langle F_H \rangle \langle \mathcal{X}^* \rangle \quad (13a)$$

(and $H \rightarrow H'$) or

$$(W^\alpha W_\alpha \tilde{W}_{\dot{\alpha}} \tilde{W}^{\dot{\alpha}} X^\dagger X)_{\theta\theta\bar{\theta}\bar{\theta}} \supset \lambda\lambda \langle \tilde{D} \rangle^2 \langle F_X \rangle \langle \mathcal{X}^* \rangle. \quad (13b)$$

In fact these are equivalent when the equations of motion are used to eliminate the F 's:

$$\langle F_H \rangle = -g_X^* \langle \mathcal{X} \rangle^* \langle \mathcal{X}' \rangle^* \quad (14)$$

and

$$\langle F_X \rangle = -g_X^* \langle \mathcal{X} \rangle^* \langle \mathcal{X}' \rangle^*. \quad (15)$$

Although we will argue below that the most impor-

tant component of a gluino mass is likely to have the structure of (13), it is interesting to note that there can be a mass term independent of $\langle \tilde{D} \rangle$ coming from

$$(W^\alpha W_\alpha X^\dagger X (XHH')^\dagger)_{\theta\theta\bar{\theta}\bar{\theta}} \supset \lambda\lambda |\langle F_X \rangle|^2 \langle \mathcal{X}^* \rangle \langle \mathcal{X}' \rangle \langle \mathcal{X}'' \rangle. \quad (16)$$

The lowest-order diagrams yielding these structures are shown in Fig. 2 and lead to the order-of-magnitude estimates:

$$m_{\text{gl}} \sim \frac{\alpha_{\text{QCD}}}{2\pi} \frac{\tilde{g}^2 \langle \tilde{D} \rangle^2 g_H^2 \langle \mathcal{X}^* \rangle \langle F_H \rangle}{M^6} \quad (17)$$

from (13) [Fig. (2a)] and

$$m_{\text{gl}} \sim \frac{\alpha_{\text{QCD}}}{16\pi^3} \frac{g_U^2 g_D^2 g_X^3 |\langle F_X \rangle|^2 \langle \mathcal{X}^* \rangle \langle \mathcal{X}' \rangle \langle \mathcal{X}'' \rangle}{M^6} \quad (18)$$

from (16) [Fig. (2b)]. For photino masses replace α_s by α_{QED} times the quark or lepton charges. In (17) and (18) M is the largest mass scale in the diagrams and is presumably to be identified with the spin-0 quark masses. g_U and g_D are the Yukawa couplings giving quark masses so that $g_U \langle \mathcal{X} \rangle = m_u$. Evidently the heaviest quark family makes the most important contribution to m_{gl} . Since generally Yukawa couplings are small compared to gauge couplings we will take (17) to be the most important contribution to m_{gl} in the following estimates. More complicated terms in \mathcal{L}_{eff} , containing more fields, can only be generated at the expense of higher powers of the coupling constants and thus we neglect them relative to these.

Before proceeding to bound and estimate these expressions we note several features. First, as expected, they vanish in the \tilde{R} -conserving limit $\langle \mathcal{X} \rangle \rightarrow 0$. Second, there is no contribution linear in $\langle \tilde{D} \rangle$. This might not have been anticipated and is an illustration of the utility of the \mathcal{L}_{eff} approach.

These expressions for m_{gl} are more amenable to approximate evaluation and bounding than one

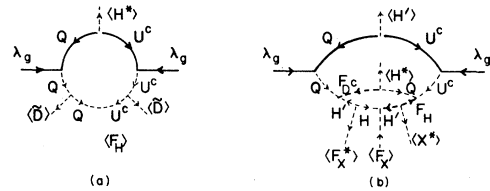


FIG. 2. Illustrative diagrams for radiatively induced gluino masses when \tilde{R} is broken by large VEV's.

might at first imagine, due to the dependence of the spin-0 quark masses on $\tilde{g}\langle\tilde{D}\rangle$ and $g_X\langle\mathcal{X}\rangle$. In general the spin-0 partners of the left and right chiral quarks are not degenerate in mass: in fact, their average $(\text{mass})^2$ is

$$\bar{M}^2 = \frac{1}{2}\tilde{g}\langle\tilde{D}\rangle, \quad (19)$$

assuming the quark mass m is negligible in comparison, and the mass splitting is

$$\Delta M^2 = 4g_Q\langle F_H\rangle, \quad (20)$$

where g_Q is the Yukawa coupling of the quark to its Higgs boson H , i.e.,

$$m_Q = g_Q\langle\mathcal{X}\rangle. \quad (21)$$

Since a negative mass² for either spin-0 quark would cause a spontaneous breakdown of color, charge, and baryon number we know that

$$\frac{1}{2}|\tilde{g}\langle\tilde{D}\rangle| > 2|g_Q\langle F_H\rangle|,$$

or, using Eq. (14),

$$|\tilde{g}\langle\tilde{D}\rangle| > 4|g_X g_Q\langle\mathcal{X}'\rangle\langle\mathcal{X}\rangle|. \quad (22)$$

This is in accord with the prejudices that $\langle\tilde{D}\rangle \sim \langle\mathcal{X}\rangle^2$, that $0 < \langle\mathcal{X}'\rangle < \langle\mathcal{X}\rangle$ (the latter being also required by the Goldstone-boson analysis of Sec. IV A) and that gauge couplings be larger than Yukawa couplings.

In principle several experimental facts can further constrain the quantities we need, although in fact the constraints at present are not stronger than the guesses we would make based on the considerations mentioned above. These are the following.

(i) The bound on axion coupling. Following the discussion of Sec. IV A we have the rough upper bound $\langle\mathcal{X}\rangle \lesssim 40$ GeV.

(ii) The experimental lower limit on the masses of spin-0 quarks, $M > 16$ GeV.³³ With (19)–(21) and $\langle\mathcal{X}\rangle \approx \langle\mathcal{X}'\rangle$ this gives $2g_X\langle\mathcal{X}\rangle m_Q < \frac{1}{2}\tilde{g}\langle\tilde{D}\rangle + m_Q^2 - (16\text{GeV})^2$.

(iii) Limits on parity violation in the strong interactions. If the masses of the scalar and pseudo-scalar quarks (s_q and t_q) are not degenerate, then the radiative corrections to the quark-gluon interaction coming from diagrams with gluinos and spin-0 quarks (see, e.g., Fig. 3) do not conserve parity. Suzuki³⁵ has recently analyzed this and finds

$$\left| \left[\frac{\ln((M_{s_u}/m_{gl})^2)}{M_{s_u}^2} - (s \leftrightarrow t) - (u \leftrightarrow d) \right] \right| < \frac{1}{(380 \text{ GeV})^2} \frac{1}{\alpha_{\text{QCD}}^2}. \quad (23)$$

The logarithms in Eq. (23) are of order 1 or larger, so that

$$[(M_{s_u}^2 - M_{t_u}^2) - (u \leftrightarrow d)]/M^4 \leq \frac{1}{380 \text{ GeV}^2} \frac{1}{\alpha_{\text{QCD}}^2}. \quad (24)$$

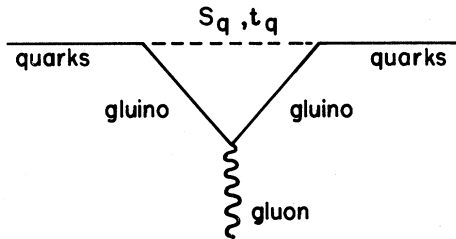


FIG. 3. Diagram responsible for parity violation in the quark-gluon coupling when the spin-0 partners of the left and right-handed quarks are not degenerate in mass.

Using (20), (14), $\langle\mathcal{X}\rangle \approx \langle\mathcal{X}'\rangle$, (21), and $m_u - m_d = 5$ MeV, this becomes (with $\alpha_{\text{QCD}} \approx 1$ to be conservative, because the momentum transfers are low)

$$g_X\langle\mathcal{X}\rangle \lesssim (\tilde{g}\langle\tilde{D}\rangle)^2 \times 10^{-4} \text{ GeV}^{-3}. \quad (25)$$

(iv) Limits on flavor-changing “neutral currents” arising from gluino exchange: When the spin-0 quark flavor mixing is not perfectly aligned with the quark mixing, strong flavor-changing “neutral currents” are generated unless the mass splitting between spin-0 quark flavors is very small compared to the spin-0 masses. Suzuki³⁶ has obtained the limit $M_c^2 - M_u^2 < (5 \times 10^{-6})M^3$. Assuming $\langle F_H \rangle \ll \langle \tilde{D} \rangle$, this translates to $m_c^2 - m_u^2 \sim (1.5 \text{ GeV})^2 < (5 \times 10^{-6})(\frac{1}{2}\tilde{g}\langle\tilde{D}\rangle)^{3/2}$ giving $\tilde{g}\langle\tilde{D}\rangle \gtrsim 10^4 \text{ GeV}^2$.

Now we can return to the problem of estimating m_{gl} from (17). Using $M^2 \sim \frac{1}{2}\tilde{g}\langle\tilde{D}\rangle$ and (14), assuming $\langle\mathcal{X}\rangle \sim \langle\mathcal{X}'\rangle$

$$m_{\text{gl}} \sim \frac{\alpha_{\text{QCD}}}{2\pi} \frac{8g_X \langle \mathcal{L} \rangle m_t^2}{\tilde{g} \langle \tilde{D} \rangle}. \quad (26)$$

We can use (ii) to get a (probably quite weak) bound on $g_X \langle \mathcal{L} \rangle m_t / \tilde{g} \langle \tilde{D} \rangle$ if we assume

$$m_t^2 - (16 \text{ GeV})^2 \ll \frac{1}{2} \tilde{g} \langle \tilde{D} \rangle,$$

giving [with $m_t \sim 30 \text{ GeV}$ and $\alpha_s \approx 0.1$, since the loops giving Eq. (17) probe α_{QCD} at short distances]

$$m_{\text{gl}} < \frac{\alpha_{\text{QCD}}}{\pi} m_t \lesssim 1 \text{ GeV}. \quad (27)$$

Or we can use (25) and (26) to give

$$m_{\text{gl}} < 8\alpha_{\text{QCD}}/2\pi (\tilde{g} \langle \tilde{D} \rangle m_t^2) \times 10^{-4} \text{ GeV}^{-3}.$$

With $\langle \tilde{D} \rangle \sim (300 \text{ GeV})^2$ this is not as good as (27). The best bound is obtained by using (i) for $\langle \mathcal{L} \rangle$, (iv) for $\tilde{g} \langle \tilde{D} \rangle$, and assuming $g_X \leq 1$; it gives

$$m_{\text{gl}} < \frac{8\alpha_{\text{QCD}}}{2\pi} \frac{40m_t^2}{10^4} \text{ GeV}^{-1} \sim 400 \text{ MeV}, \quad (28)$$

again, for $m_t \sim 30 \text{ GeV}$. The limit on the photino mass corresponding to Eq. (27) is $m_{\tilde{\gamma}} \sim 4 \text{ MeV}$. It should be emphasized that these are only relatively model-independent upper bounds which can be established using experimental constraints, and which we have only roughly estimated. In fact, plausible guesses for the VEV's $\langle \mathcal{L} \rangle$ and $\langle \tilde{D} \rangle$ and the couplings g_X and \tilde{g} lead to much lower values of the gluino and photino masses.

These estimates have assumed that R -invariance breaking results from a nonzero value of an $R=2$ scalar VEV $\langle \mathcal{L} \rangle$. While this VEV may vanish, it is natural to expect that strong color forces can generate the $\Delta R=2$ condensates $\langle \lambda_{\text{gl}} \lambda_{\text{gl}} \rangle$ and/or $\langle \mathcal{Q} \mathcal{Q}^c \rangle$ in the same way that $\langle \psi_q \psi_q^c \rangle$ forms. The lowest-order diagram of this sort is shown in Fig. 4 and comes from a term in \mathcal{L}_{eff} such as

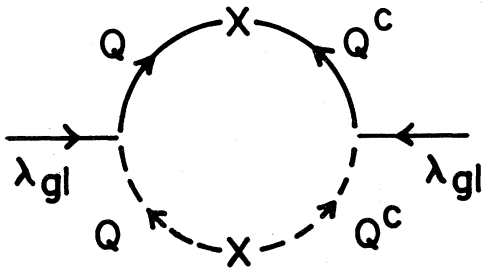


FIG. 4. Lowest-order diagram for gluino mass when R invariance is broken dynamically by a nonzero $\langle \mathcal{Q} \mathcal{Q}^c \rangle$.

$$[D^2(Q^\dagger e^\nu Q) \bar{D}^2(Q^c{}^\dagger e^{-\nu} Q^c)]_{\theta\theta\bar{\theta}\bar{\theta}}.$$

It leads to the order-of-magnitude estimate

$$m_{\text{gl}} \sim \frac{\alpha_{\text{QCD}}}{2\pi} \frac{\langle \mathcal{Q}^c{}^* \mathcal{Q}^* \rangle \langle \psi_q \psi_{qc} \rangle}{M^4}.$$

Taking all QCD condensates of the scale $\Lambda_{\text{QCD}} \sim 300 \text{ MeV}$ and $M \gtrsim 16 \text{ GeV}$ reveals that the resulting gluino mass is much less than an MeV. Mere dimensional analysis with no recourse to any detailed discussion of \mathcal{L}_{eff} is sufficient to arrive at this qualitative conclusion, that dynamical R breaking by QCD forces leads to tiny gluino masses: when $\langle \lambda \lambda^c \rangle$ is responsible, gauge invariance requires quark superfields to enter in pairs and condensation requires both a quark and its charge conjugate so there will be at least a factor $\Lambda_{\text{QCD}}^4/M^3 \lesssim 10^{-5} \text{ GeV}$ and when $\langle \lambda_{\text{gl}} \lambda_{\text{gl}} \rangle$ condensation is responsible there is minimally a factor $\Lambda_{\text{QCD}}^3/M^2 \lesssim 10^{-4} \text{ GeV}$. (Evidently a supercolor³⁷ dynamical breaking scheme could generate much larger gluino masses.)

We have seen that within the framework of $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \times \tilde{\text{U}}(1)$, gluino and photino masses cannot be large, barring complicated schemes with exotic quarks or huge supersymmetry-breaking scales. Since a small ($\sim 15 \text{ eV}$) photino mass is very attractive for astrophysics³⁸ this result is nice. However, a small gluino mass is only barely consistent with experiments putting lower limits on R -hadron masses. R hadrons, composed of a gluino and quarks or gluons, should be pair-produced in strong interactions at a rate only inhibited by their mass. Since glueballs are thought to have masses of order 1.5 GeV this could be a reasonable guess for the mass of an R hadron assuming massless gluinos. Experiments rule out R -hadron masses of less than 1.5 or 2 GeV ,^{8,39} so light gluinos are only barely tolerable.⁴⁰ If R -hadron masses are in this range they should be readily observed at Tevatron energies, and if they are not it will provide still more evidence against this class of theories.

VI. CONCLUSIONS

Our conclusions are pessimistic. Using supersymmetry to solve the gauge hierarchy problem has been a very appealing if as yet unrealized possibility. However, the present analysis has revealed the seriousness of the difficulties inherent in such an approach as far as the phenomenological consequences are concerned. Supersymmetric theories tend to have an R symmetry, which if broken spontaneously or by QCD anomalies leads to phenomenologically unacceptable Goldstone bosons, either axionlike or mesonlike. If we instead arrange that the R current

is *not* broken by QCD anomalies, then, whether or not it is spontaneously broken, we lose the asymptotic freedom of QCD above a few hundred GeV, ruling out any sort of grand unification of strong with electroweak forces below the Planck scale.¹⁸ If we suppose that the R symmetry is not spontaneously broken by vacuum expectation values of order 300 GeV, we encounter another feature which may come into conflict with experiment: the gluino would be very light. This problem may well be with us even if R symmetry is spontaneously broken by large vacuum expectation values, since our estimates give a gluino mass in any case of order 1 GeV or less.

We can try to avoid the difficulties raised by R invariance by introducing R -noninvariant terms in the Lagrangian. However, this is difficult in existing models: for instance, R invariance is automatic in $SU(3) \times SU(2) \times U(1) \times \tilde{U}(1)$ models unless we add new superfields. Furthermore the violation of R invariance requires the introduction of superrenormalizable F terms in the Lagrangian, which

seems to us to vitiate the use of low-energy supersymmetry to solve the hierarchy problem of grand unified theories.

It must be stressed that our analysis has been mainly directed at a rather conventional theoretical framework, in which supersymmetry is broken in the tree approximation by scalar vacuum expectation values of order 300 GeV. It is possible that more innovative models, with supercolor, an inverted hierarchy, or supergravity, may avoid the difficulties we have found.

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