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Amplitude zeros

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We consider the general phenomenon of amplitude zeros, first discovered in the process $d\bar{u}\to W^-\gamma$. Using spin-0 particles, we investigate the circumstances for amplitude zeros to occur in the physical region and, as well, their location in phase space. A necessary condition is that all of the charges must be of the same sign or neutral. We analyze, in detail, both the massive and massless cases for radiative decays, photoproduction, and direct photon production.

I. INTRODUCTION

A few years ago Mikaelian reported[1] that the differential cross section for photoproduction of $W^\pm$ bosons, $\gamma d\to W^-u$, $\gamma u\to W^+d$, etc., reduced to a very simple expression if the anomalous-magnetic-moment parameter $\kappa=g-1$ for the W has the value assigned by gauge theories, namely $\kappa=1$. Later the crossed channels $d\bar{u}\to W^-\gamma$ and $u\bar{d}\to W^+\gamma$, where a similar simplification for $\kappa=1$ occurs, were investigated.[2] It was found that the angular distribution for $\kappa=1$, when compared with those for other values of κ, is very different. In studying these processes we discovered[3] that the *angular distribution for $\kappa=1$ vanishes* at a certain angle, and proposed using this peculiar behavior in $\bar{p}p$ or pp collisions as a means of measuring the magnetic moment of the W. More recently, it was pointed out[4] that the zero also occurs (for spin-0 quarks as well as the standard spin-$\frac{1}{2}$ quarks) in radiative decays of the W where, in this case, the energy distribution vanishes along a certain line in the Dalitz plot.

These zeros are quite remarkable—the lowest-order amplitude vanishes for each spin state and the position of the zero is independent of photon energy. (For massless quarks, it depends only on the quark charges.) The simplification (factorization) referred to above and the amplitude zero provide a check on the magnetic moments of both the W and the quarks[5] and the position of the zero enables a direct measure of fractional quark charges by real photons.[6]

It has been shown[7] that the tree diagrams for these reactions have a factorization property which is quite general. In this paper we will investigate the general conditions for a zero to occur in the physical region. Using spin 0 for the incoming and outgoing charged particles, and standard coupling to the photon, we will see that the zeros are essentially due to the complete destructive interference of the radiation patterns.[8]

In Sec. II we give the general requirements for an amplitude zero. In Sec. III we discuss the general radiative-decay process in both the massive and massless cases, treating in detail the decay into one charge, two charges, and, in two special cases, three charges. In Sec. IV we show that the conditions for an amplitude zero in photoproduction are essentially the same as those for radiative decays, so the results in Sec. III apply here as well. Section IV deals with direct photon production. Here again, we show that the zeros occur in the same place as in the corresponding radiative decay, the photon direction, however, now being determined. Section VI contains a brief summary of our results.

II. GENERAL REQUIREMENTS FOR A ZERO

Consider a process with one real photon and $n+1$ additional external particles Q and Q_i, $i=1,\ldots,n$ (Q and Q_i also denote their charges) with four-momentum k, P, and p_i, respectively. The masses of Q and Q_i are M and m_i. Now consider the amplitude for the set of tree graphs for this process, obtained by attaching the photon in all possible ways to the external lines. The following conditions must be satisfied in order to obtain an amplitude zero:

$$\frac{k\cdot P}{Q}=\frac{k\cdot p_1}{Q_1}=\frac{k\cdot p_2}{Q_2}=\cdots=\frac{k\cdot p_n}{Q_n}\ . \tag{1}$$

Assume the amplitude for this process contains, as a factor, the standard bremsstrahlung form (the photon polarization four-vector is ϵ), which is certainly true at the tree level for scalar charges:

$$A_0 = \left[\sum_{i=1}^{n} \frac{Q_i p_i}{k \cdot p_i} - \frac{QP}{k \cdot P} \right] \cdot \epsilon \ . \tag{2a}$$

One can rewrite A_0 in the symmetrized form

$$A_0 = \frac{1}{2k \cdot P} \sum_{i,j}^{n} f_{ij} g_{ij} \ , \tag{2b}$$

where

$$f_{ij} \equiv Q_i k \cdot p_j - Q_j k \cdot p_i$$

and (2c)

$$g_{ij} \equiv \left[\frac{p_i}{k \cdot p_i} - \frac{p_j}{k \cdot p_j} \right] \cdot \epsilon \ .$$

Now it can easily be seen that if the "zero conditions" [Eq. (1)] are satisfied, $A_0 = 0$ and the amplitude vanishes. (The above forms for A_0 describe the decay $Q \to Q_1 + Q_2 + \cdots + Q_n + \gamma$, but it is trivial to write A_0 for the other related processes.)

In addition, of course, we must also ensure energy-momentum conservation. Actually four-momentum and electric charge conservation always ensure that one of the n equations in Eq. (1) above is trivially satisfied. One can see from Eq. (1) that a *necessary condition* for a zero to exist is that all of the charges must be of the *same sign or neutral.* For this reason, in everything which follows, we will assume that we have *no opposite sign charges.* We will now consider in detail the three cases of interest.

III. RADIATIVE DECAYS

First, let us study the general radiative decay

$$Q \to Q_1 + Q_2 + \cdots + Q_n + \gamma \ . \tag{3}$$

For convenience, we define the following quantities:

$$X_i \equiv 2E_i / M \ ,$$
$$y \equiv 2E_\gamma / M \ , \tag{4}$$
$$\alpha_i \equiv \frac{1 - \beta_i \cos\theta_i}{2} \ ,$$

where E_i and E_γ are the energies of particle Q_i and the photon, respectively,

$$\beta_i = (1 - m_i^2 / E_i^2)^{1/2}$$

is the velocity of Q_i, and θ_i is the angle between the three-momentum of particle Q_i and the photon, all in the rest frame of Q. The zero conditions in Eq. (1) become

$$X_i \alpha_i = \frac{Q_i}{Q}, \quad i = 1, \ldots, n \ . \tag{5}$$

One can see now that these conditions are independent of E_γ. Energy conservation is simply

$$\sum X_i + y = 2 \ . \tag{6}$$

The condition for longitudinal (along the photon direction) momentum conservation is

$$\sum_{i=1}^{n} X_i \alpha_i = 1 \ . \tag{7}$$

We must, of course, ensure momentum conservation in the transverse directions as well. We can see, using Eq. (7), that one of Eqs. (5) is trivially satisfied because of charge conservation, as noted above.

For any *neutral particles,* $Q_i = 0$, we can see from Eqs. (4) and (5) that a *necessary condition* for an amplitude zero is $m_i = 0$ and $\theta_i = 0$, i.e, neutral particles must be massless and must travel along the photon direction.

Consider the case of massless particles, $m_i = 0$, $i = 1, \ldots, n$, but $M \neq 0$ (i.e., the extreme relativistic limit). We have $\beta_i \to 1$, $0 \le X_i \le 1$, $0 \le y \le 1$, and, of course, $0 \le \alpha_i \le 1$. Using Eqs. (5), (6), and (7), it is easy to verify that for $Q_i / Q > 0$, $i = 1, \ldots, n$, the amplitude vanishes at $X_i = Q_i / Q$ with $\alpha_i = 1$, $i = 1, \ldots, n$ ($y = 1$). In general, for the massless case, zeros will occur in a subspace region of the $X_1, \ldots, X_n$ space, while in the massive case, as we shall see, depending on the specific situation, an amplitude zero may or may not occur in the physical region. Finding all the zeros in the general case is, however, very complicated (for $n \ge 3$, the event need not be in a plane). We shall confine ourselves here to the cases $n = 1$, 2, and, for two special cases, $n = 3$).

$n = 1$. This case is very simple and the energy of Q and the photon are uniquely determined:

$$X_1 = 1 + \frac{m_1^2}{M^2} \tag{8}$$

and

$$y = 1 - \frac{m_1^2}{M^2} \ .$$

One can easily verify that the zero conditions [Eqs. (5), (6), and (7)], are satisfied. Thus the amplitude *identically vanishes* here. Of course, this process is forbidden anyway by angular momentum conservation.

$n = 2$. Since there are only two independent kinematical variables here, one has the following relationship:

$$X_2 \alpha_2 - X_1 \alpha_1 = \frac{X_2 - X_1}{y} + \frac{2\Delta}{y} \ , \tag{9}$$

where

$$\Delta = \frac{m_1^2 - m_2^2}{M^2} . \tag{10}$$

Using the zero conditions [Eq. (5)] we obtain

$$X_2 - X_1 + 2\Delta = \left[\frac{Q_2 - Q_1}{Q} \right] y , \tag{11}$$

and, hence, a line of zeros given by

$$Q_1(1-X_1) = Q_2(1-X_2) - Q\Delta . \tag{12}$$

Whether or not these zeros are in the physical region in a given situation can be determined by constructing the phase-space boundaries which, of course, depend on the masses. We have explicitly verified that transverse-momentum conservation is consistent with this solution. This is not surprising, however, since, as stated above, there are only two independent kinematical variables in this case (i.e., choosing X_1 and X_2 determines the event completely). Figure 1 shows two representative cases with $Q_1 < Q_2$, the solid line for the massless case and the dotted line for the massive case, $m_1 > m_2 > 0$. For the massless case we have the line given by

$$Q_1(1-X_1) = Q_2(1-X_2) \tag{13}$$

which is in the physical region, for any $Q_1/Q_2 \geq 0$ and which agrees with the result found by Grose and Mikaelian[4] for the decay $W^- \to d\bar{u}\gamma$. Notice that *for equal masses* $m_1 = m_2 \neq 0$, *the line of zeros is identical* to the one for the massless case [Eq. (13)]. Furthermore, *for the general massive case,* $m_1 \neq m_2$ and $m_i \neq 0$, $i = 1,2$, *the slope of the line is identical* to that in the massless (or equal-mass) case, i.e, Q_1/Q_2, but the line is shifted down by $(Q/Q_2)\Delta$, for $\Delta > 0$. In Fig. 1, to be specific, we have chosen $Q_1 > Q_2$ and $m_1 > m_2$, however, it is, of course, very easy to discuss the other cases as well. Finally, we would like to point out that, in the case of a massless neutral charge $m_1 = Q_1 = 0$, the zero line is $X_2 = 1$ and for $m_2 = Q_2 = 0$, it is $X_1 = 1$.

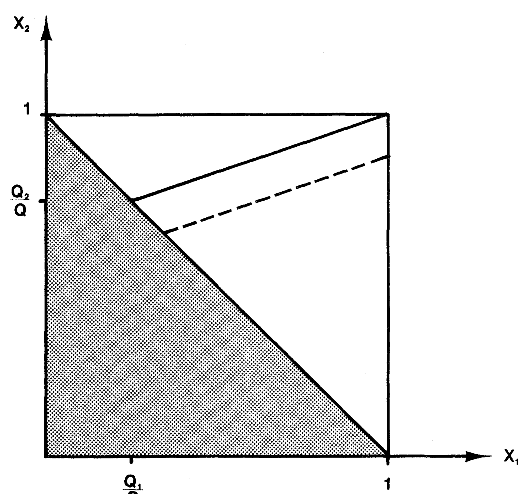

FIG. 1. The line of zeros is shown for the radiative decay $Q \to Q_1 + Q_2 + \gamma$, with $Q_1 \leq Q_2$. The solid line represents the case $m_1 = m_2$, while the dashed line is for $m_1 > m_2$.

n=3. As noted above, the situation is considerably more complicated here. Thus, we will confine ourselves to two special cases.

(i) Two neutral massless particles: $M_2 = Q_2 = m_3 = Q_3 = 0$. As discussed above we must have $\alpha_2 = \alpha_3 = 0$. (The neutral particles must be massless and must travel along the photon direction.) This implies that

$$\theta_1 = \pi$$

and (14)

$$\alpha_1 = \frac{1+\beta_1}{2}$$

and, hence, the zero occurs for

$$X_1 = 1 + \frac{m_1^2}{M^2} ,$$
$$X_2 + X_3 + y = 1 - \frac{m_1^2}{M^2} . \tag{15}$$

In the massless limit $m_1 = 0$, this becomes

$$X_1 = 1 = X_2 + X_3 + y . \tag{16}$$

We have explicitly verified (for $m_1 = 0$) that, under these conditions, the rate[9] for $\mu \to e\nu\bar{\nu}\gamma$ does indeed vanish. This could be of considerable interest in the case of radiative τ decays, e.g.,

$$\tau^- \to e\nu_\tau\bar{\nu}_e\gamma , \; \tau^- \to \mu\nu_\tau\bar{\nu}_\mu\gamma , \; \tau^- \to q_i q_j \nu_\tau \gamma ,$$

etc., where the presence or absence of the zero may provide a sensitive test[10] of the g value of the τ.

(ii) One neutral massless particle: $m_3 = Q_3 = 0$. We must have, as before, $\alpha_3 = 0$. Using Eqs. (5), (6), and (7) we find the following conditions:

$$X_1\alpha_1 = Q_1/Q, \quad X_2\alpha_2 = Q_2/Q ,$$

and (17)

$$X_3 + y = 2 - X_1 - X_2 .$$

This is precisely what we had for $n=2$. Thus, the zeros occur along the line given in Eq. (12) (which may or may not be in the physical region) where the photon energy is now shared with the neutral particle ($y \to y + X_3$). In the massless case, $m_1 = m_2 = 0$, we of course obtain Eq. (13) and so the line of zeros is in the physical region.

In general, one can see that for n particles Q_i, $i=1,\ldots,n$ with $n-m$ of them neutral and massless, $Q_i=m_i=0$, $i=m+1,\ldots,n$, the solution is identical to the $n=m$ case with charges Q_i, $i=1,\ldots,m$ and the neutral particles sharing the photon energy

$$\left[y\to y+\sum_{i=m+1}^{n}X_i\right].$$

IV. PHOTOPRODUCTION

In this section we will consider the general photoproduction process

$$\gamma+Q\to Q_1+Q_2+\cdots+Q_n\ . \tag{18}$$

We will describe this process in the center-of-momentum frame so that we can make use of Sec. III. Here we scale the energies by the center-of-momentum energy $E_{\text{c.m.}}$:

$$X_i\equiv 2E_i/E_{\text{c.m.}}\ ,$$

$$X\equiv 2E/E_{\text{c.m.}}=1+\rho\ ,$$

$$y\equiv 2E_\gamma/E_{\text{c.m.}}=1-\rho\ ,$$

$$\sum_{i=1}^{n}X_i=X+y=2\ ,$$

and

$$\rho\equiv M^2/s$$

with

$$s\equiv E_{\text{c.m.}}{}^2\ . \tag{19}$$

The zero conditions are again

$$X_i\alpha_i=Q_i/Q,\quad i=1,n \tag{20}$$

with α_i again given by Eq. (4). Longitudinal-momentum conservation is given by

$$\sum_{i=1}^{n}X_i\alpha_i=1 \tag{21}$$

as before, and, we must, of course, again also have transverse-momentum conservation. We see that the conditions for a zero amplitude in photoproduction are identical to those for radiative decays—the only difference being the energies here are scaled by $E_{\text{c.m.}}$ and the photon direction is, of course, fixed. So the results in Sec. III apply here as well. Note that in Sec. III energy conservation was given by

$$\sum_{i=1}^{n}X_i+y=2\ , \tag{6}$$

whereas here it is given by $\sum_{i=1}^{n}X_i=2$ corresponding to Eq. (6) with $y=0$. Let us now consider a specific case.

$n\,=\,2$. From the above considerations and Eqs. (11) or (12) we see that the zero condition is satisfied for the physical values

$$X_1=1+\rho_{12}$$

and

$$X_2=1-\rho_{12}\ ,$$

where

$$\rho_{12}\equiv\frac{m_1{}^2-m_2{}^2}{s}\ . \tag{22}$$

Using Eq. (20) we find the direction in which we have a zero amplitude is given by ($\cos\theta_2=-\cos\theta_1$)

$$\cos\theta_1=\frac{1}{\beta_1}\left[1-\frac{2Q_1}{Q(1+\rho_{12})}\right]$$
$$=\frac{1}{\beta_2}\left[\frac{2Q_2}{Q(1-\rho_{12})}-1\right]. \tag{23}$$

If we now specialize to the case $m_2=0$, $m_1\neq 0$ we find the zero direction is given by

$$\cos\theta_1=\frac{1+m_1{}^2/s-2Q_1/Q}{1-m_1{}^2/s}\ . \tag{24}$$

In order for the zero amplitude to be in the physical region, we must, of course, have $-1\le\cos\theta_1\le 1$. This leads to the two conditions

$$Q_1/Q\le 1\ , \tag{25}$$

which is automatically satisfied due to our assumption of no opposite-sign charges, and

$$Q_1/Q\ge m_1{}^2/s\ . \tag{26}$$

In the zero mass case $m_1=0$ we find

$$\cos\theta_1=1-2Q_1/Q\ , \tag{27}$$

which, under the conditions assumed, is always in the physical region ($0\le Q_1/Q\le 1$).

We have explicitly verified that the process[1] $\gamma q\to Wq'$ has a zero amplitude in the direction given by Eqs. (24) and (27). Of course, if the quarks have the standard charge assignment, the conditions of our assumption and Eq. (25) are not satisfied, and there is no zero in the physical region. For integral-charged quarks or leptons, however, the zero can occur in the physical region. For example, the process $\gamma e^- \to W^- \nu_e$ (or $\gamma e^+ \to W^+ \nu_e$) has an amplitude zero in the direction

$$\cos\theta_1 = -1 \,, \tag{28}$$

independent of $m_W{}^2/s$.

V. DIRECT PHOTON PRODUCTION

In this section we will consider the general direct-photon-production process

$$Q + Q' \to Q_1 + Q_2 + \cdots + Q_n + \gamma \,. \tag{29}$$

As in Sec. IV, we will describe this process in the center-of-momentum frame so that we can again make use of Sec. III. As before we scale the energies by $E_{\text{c.m.}}$:

$$X_i \equiv 2E_i / E_{\text{c.m.}} \,,$$

$$X \equiv 2E / E_{\text{c.m.}} = 1 + \rho' \,,$$

$$X' \equiv 2E' / E_{\text{c.m.}} = 1 - \rho' \,,$$

$$y \equiv 2E_\gamma / E_{\text{c.m.}} \,,$$

$$\sum_{i=1}^{n} X_i + y = X + X' = 2 \,,$$

and

$$\rho' \equiv \frac{M^2 - M'^2}{s}$$

with

$$s \equiv E_{\text{c.m.}}{}^2 \,. \tag{30}$$

Longitudinal-momentum conservation is again given by

$$\sum_{i=1}^{n} X_i \alpha_i = 1 \tag{31}$$

and again, of course, transverse momentum must also be conserved. The zero conditions are

$$\frac{X'\alpha'}{Q'} = \frac{X\alpha}{Q} = \frac{X_i \alpha_i}{Q_i} \,, \quad i = 1, \ldots, n \tag{32}$$

with α_i again given by Eq. (4) and the obvious notation

$$\alpha' = \frac{1 - \beta' \cos\theta'}{2}$$

and

$$\alpha = \frac{1 - \beta \cos\theta}{2}$$

with $\cos\theta' = -\cos\theta$,

$$\beta' = \left[1 - \frac{M'^2}{E'^2} \right]^{1/2} \tag{33}$$

and

$$\beta = \left[1 - \frac{M^2}{E^2} \right]^{1/2} .$$

Equations (4) and (33) lead to the result

$$\frac{X\alpha}{Q} = \frac{X'\alpha'}{Q'} = \frac{1}{Q_T} = \frac{X_i \alpha_i}{Q_i} \,, \quad i = 1, \ldots, n$$

where

$$Q_T = Q + Q' \tag{34}$$

and, hence, the amplitude zero direction is given by

$$\cos\theta = \left[\frac{Q'}{\beta} - \frac{Q}{\beta'} \right] \Big/ Q_T \,. \tag{35}$$

This determines the photon direction relative to the incident beam direction. The other conditions, Eqs. (30), (31), and (32), are identical to the conditions given in Sec. III for radiative decays [Eqs. (5), (6), and (7)] with $Q \to Q_T$. Thus the zeros here occur in the same place as in the corresponding radiative decay (quantities here scaled by $E_{\text{c.m.}}$) but with the photon direction now determined by Eq. (35).

In the case of equal-mass incident particles $M = M'$, Eq. (35) becomes

$$\cos\theta = \frac{1}{\beta} \left[1 - \frac{2Q}{Q_T} \right] . \tag{36}$$

In the massless limit $M = M' = 0$ this becomes

$$\cos\theta = \left[1 - \frac{2Q}{Q_T} \right] . \tag{37}$$

This is the result found in the original discovery[3] of the zero, where, for $d\bar{u} \to W^- \gamma$, $Q = Q_d = -\frac{1}{3}$, and

$$\cos\theta' = -\cos\theta = -\tfrac{1}{3} \,. \tag{38}$$

It has also been directly verified for $d\bar{u} \to W^- \gamma$ that Eqs. (35) and (36) give the correct zero direction. It can be seen directly from Eqs. (35) and (36) that contrary to the massless case, in the case of massive incident particles, the zero direction does depend on the incident energies (as does the question of whether or not the zeros are in the physical region). For example, in $d\bar{u} \to W^- \gamma$, for $M = M' \neq 0$, Eq. (38) becomes

$$\cos\theta' = -\frac{1}{3\beta} \tag{39}$$

and the zero is in the physical region if and only if

$\beta \geq \frac{1}{3}$.

It is interesting to note from Eq. (35) that *for equal charge-to-energy ratios, $Q'/X'=Q/X$*, the amplitude zero, if it occurs at all, must occur at $\cos\theta=0$. That is, the photon direction must be perpendicular to the incident beam directions, *independent of charges, energies, and masses.*

If we now take the nonrelativistic limit for the incident particles, β and $\beta' \rightarrow 0$ and α and $\alpha' \rightarrow \frac{1}{2}$, independent of θ and θ'. The conditions for an amplitude zero [Eq. (34)] include $Q/M=Q'/M'$ as well as the conditions on the final particles. It should be emphasized that the zero here, if it does occur in the physical region, is *independent of photon direction (and, of course, photon energy); i.e., the amplitude identically vanishes.*

In particular, if we now specialize to the nonrelativistic collision $Q+Q' \rightarrow Q+Q'+\gamma$ we find that the zero conditions [see Eqs. (34) and (12)] are satisfied if and only if $Q/M=Q'/M'$, i.e., the amplitude identically vanishes in nonrelativistic collisions of particles with equal charge-to-mass ratios. Thus the zero conditions [Eq. (1)] are a generalization of the nonrelativistic result from classical electromagnetism that electric dipole radiation vanishes in collisions of particles with the same charge-to-mass ratio.[11,12]

n=2. An interesting example here is the pure QED process[11] $e^- + e^- \rightarrow e^- + e^- + \gamma$. From Eqs. (13) and (36) we find that this process has an amplitude zero for $X_1=X_2$ and $\cos\theta=0$ *independent of m_e and $E_{\text{c.m.}}$*.

Other interesting examples occur in quark-quark and quark-antiquark scattering:

$$
\begin{aligned}
&q+q \rightarrow q+q+\gamma\,,\\
&q+\bar{q} \rightarrow q+\bar{q}+\gamma\,,
\end{aligned}
\tag{40}
$$

subject, of course, to the same-sign-charge requirement. For example, the process $u+\bar{d} \rightarrow u+\bar{d}+\gamma$ has an amplitude zero at $\cos\theta=-1/3\beta$ and $X_2=2X_1-1$, where θ is the angle between the γ and the incident u direction and X_1 (X_2) is the scaled energy of the final u ($\bar{d}$). Although very interesting, zeros in these processes would, of course, be very difficult to observe experimentally.

VI. DISCUSSION AND SUMMARY

Using spin 0 for the incoming and outgoing charged particles, we have shown that the amplitude zeros are essentially due to the complete destructive interference of the radiation patterns. We have treated the general radiative-decay process in both the massive and massless cases and have given, in detail, the zero-amplitude solution for the decay into one charge, two charges, and, in two special cases, three charges.

We have also shown how to find an amplitude zero, and whether or not it occurs in the physical region, for photoproduction and direct photon production. In each case, we have discussed some specific examples.

Recently we received a paper by Brodsky and Brown,[11] in which the following theorem is given: "Let T_G denote a tree graph with n external lines labeled by particle four-momenta p_i, charges Q_i, and masses m_i. The external and internal lines can be scalar, Dirac, or vector particles (spin ≤ 1). The vertices of T_G are taken to correspond to local interactions involving any number of fields with constant or single derivative couplings, and the derivative couplings must be of gauge-theory form. In particular, the photon-particle couplings, which are central to the theorem, must correspond to the same gyromagnetic ratio, $g=2$, for all spinning particles. *Theorem:* If M_γ is the single-photon emission amplitude which is the sum generated by making photon attachments (four-momentum q) in all possible ways onto T_G, then $M_\gamma=0$ if the ratios $Q_i/p_i \cdot q$ are all equal"; i.e., Eq. (1) is satisfied. The authors further state that neutral particles can be included, provided they are massless and either spin 0 or spin $\frac{1}{2}$. Thus we have the spin independence of the amplitude zeros and, hence, the generality and usefulness of the results presented in this paper.

Brodsky and Brown also state that the zeros persist if one includes radiation from internal lines (no closed loops). Let us first consider radiative decays (Sec. III), where the process in Eq. (3) is now modified to

$$
Q \rightarrow Q_1 + (Q_{\text{int}} \rightarrow Q_2 + Q_3 + \cdots + Q_n) + \gamma\,,
\tag{41}
$$

where Q_{int} represents a virtual particle of mass m and, of course,

$$
Q_{\text{int}} = Q - Q_i\ .
\tag{42}
$$

Otherwise, the notation is the same as before. The total amplitude for this process, obtained by attaching the photon in all possible ways to the external lines *and the internal line,* is

$$A_{\rm tot} \propto -i\epsilon\cdot \left\{ \left[\frac{QP}{k\cdot P} - \frac{Q_1 p_1}{k\cdot p_1} - \frac{Q_{\rm int}(P-p_1)}{k\cdot(P-p_1)} \right] \Big/ [(P-p_1-k)^2-m^2] - \left[\sum_{i=2}^{n} \frac{Q_i p_i}{k\cdot p_i} - \frac{Q_{\rm int}(P-p_1)}{k\cdot(P-p_1)} \right] \Big/ [(P-p_1)^2-m^2] \right\} . \qquad (43)$$

It can easily be verified that under the previous zero conditions [Eqs. (1) or (5)], the quantity in each square bracket in Eq. (43) vanishes and, hence, $A_{\rm tot}=0$. Thus, amazingly, *the zeros persist at the same location in phase space, independent of the mass of the internal particle.*

We can, in a similar way, extend the discussion of Sec. V for direct photon production. Let us consider the process

$$Q+Q' \to (Q_{\rm int} \to Q_1+Q_2+\cdots+Q_n)+\gamma , \qquad (44)$$

where $Q_{\rm int}$ again represents a virtual particle of mass m and, otherwise the notation is the same as before, with, of course,

$$Q_{\rm int}=Q+Q'=Q_T . \qquad (45)$$

The total amplitude for this process, again including radiation from the internal line, is

$$A_{\rm tot} \propto -i\epsilon\cdot \left\{ \left[\frac{QP}{k\cdot P} + \frac{Q'P'}{k\cdot P'} - \frac{Q_T(P+P')}{k\cdot(P+P')} \right] \Big/ [(P+P'-k)^2-m^2] - \left[\sum_{i=1}^{n} \frac{Q_i p_i}{k\cdot p_i} - \frac{Q_T(P+P')}{k\cdot(P+P')} \right] \Big/ [(P+P')^2-m^2] \right\} . \qquad (46)$$

Again, it can easily be verified that under the previous zero conditions [Eq. (34)], the quantity in each square bracket in Eq. (46) vanishes and, hence, $A_{\rm tot}=0$. Thus, again we have the remarkable result that the zeros persist at the same place, independent of the mass of the internal particle.

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