Analysis of preon models with a small number of flavors

Thomas G. Rizzo
Ames Laboratory and Department of Physics, Iowa State University, Ames, Iowa 50011
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We analyze a large number of four- and five-preon models along with several six-preon models. By imposing constraints such as cancellation of anomalies and reproduction of the usual quarks and leptons as composites we are able to eliminate many of these from further consideration. If we try to embed the electroweak interactions into the flavor group $G_{\rm PF}$, we are left with a single, five-preon model. The remaining model is inconsistent with grand unification and asymptotic freedom. We conclude that the simplest preon model must have at least six flavors and contain a single 3_C and three 1_C's.

I. INTRODUCTION

The idea that ordinary quarks and leptons may be composite objects[1] has now been accepted as a theoretical possibility although direct experimental evidence is lacking. Although the similarities between quarks and leptons in terms of their electroweak interactions may be explainable within the context of grand unified theories[2] (GUT's) the explanation for the apparent multifamily structure appears to be lacking. Horizontal symmetries[3] do not usually limit the number of families or explain why only (?) three exist and are only useful for calculating Cabibbo-type mixing angles.[4] The large number of "basic" fermionic states (six color-triplet quarks and six color-singlet leptons) cries out for an explanation. Although we are not (yet) in the situation of the early 1960's (when the quark model was proposed), in terms of the large number of states observed it seems unlikely that all these fermions are elementary. Theoretical prejudice is also swayed by observing that the Higgs bosons of the standard electroweak model,[5] which can be thought of as elementary fields as well, may also, in fact, be composites—condensates of fermion-antifermion pairs via hypercolor[6] and that electroweak symmetry breaking is dynamical.[6]

On the experimental side, present-day experiments using accelerators show no apparent quark or lepton substructure[7] strongly indicating that the binding scale $\Lambda_{\rm PC}$ (for precolor), must be >100 GeV at least. Other experiments such as those that establish upper limits on $\mu\to e\gamma$ (Ref. 8) and measure (g_e-2) (Ref. 9) tell us that $\Lambda_{\rm PC}$ must be at least 100 TeV and may be as large as 1000 TeV. If so, it is obvious that the binding of these more fundamental objects—preons— into quarks and leptons must be quite different than the binding of quarks into hadrons within quantum chromodynamics (QCD).[10]

In QCD we have several flavors of fundamental massless fermions (in the limit we turn off the electroweak interactions) which bind into bound states whose masses are of order Λ_c, the scale at which QCD becomes strong (i.e., $\alpha_s\sim 1$). This is due to flavor-chiral-symmetry breakdown giving masses to the baryons and vector mesons. The pseudoscalar mesons, on the other hand, would be massless Goldstone bosons except for the "small" quark masses produced via the electroweak interactions.

If quarks and leptons (which are essentially massless at a scale of order 100—1000 TeV) are preonic bound states then the preon dynamics must be such that preflavor chiral symmetry remains unbroken otherwise quark and lepton masses would be of order $\Lambda_{\rm PC}$. These chiral symmetries would then only be broken, very weakly, by the electroweak interactions and yield the light composite masses. These preflavor chiral symmetries thus protect the composites from getting masses on the scale of $\Lambda_{\rm PC}$. How do we ensure that chiral symmetry remains unbroken such that this miracle can occur?[7]

't Hooft[11] has proposed a set of conditions that would require massless composite bound states to occur in preon theories so long as these chiral symmetries remain unbroken. The first condition is that the massless composites as well as the preons must produce the same contribution to the Adler-Bell-Jackiw[12] (ABJ) anomalies in the currents of the unbroken chiral symmetries. The second, called the decoupling condition, would require that if one of the fermions making up a composite were to be given a large mass the remaining unbroken chiral symmetries would permit all composites containing this fermion also to get a large mass. The combination of these two conditions has proven to be so restrictive that finding suitable models for composite quarks and leptons has proven difficult.

Preskill and Weinberg[13] have recently pointed out that the use of the decoupling constraint may not be justified without further assumptions and have replaced this condition by what they call the "persistent-mass" condition. It says, essentially, that when any preon gets some mass the unbroken chiral symmetries permit all the composites containing this preon also to get some mass as well.

The preon theories which pass these criteria[14] are not at all simple and are quite complex in general and have a large number of fundamental fermion fields. (It is quite disheartening to need as many or more fundamental fermions in preon models than there are quark and leptons.) We propose here to go a simpler route and see what is the smallest number of fundamental fields that we can use. We will make some rather basic assumptions which appear reasonable—but may indeed be generalized; we will approach the problem from the spectroscopic point of view in our discussion below.

II. BASIC INGREDIENTS

(1) We will assume that quarks and leptons are bound states of three spin-$\frac{1}{2}$ preons which obey Fermi-Dirac statistics. This is sometimes referred to as the "valence" preon model[15] and is constructed in analogy with the simple quark model of baryons. We will ignore the possibility of scalar preons here although models containing such objects do exist.[16]

(2) For simplicity we will assume that the preons themselves transform in a simple way under $SU(3)_c$, i.e., as $\underline{1}$, $\underline{3}$, or $\underline{\overline{3}}$ of $SU(3)_c$; we will also demand that we must be able to construct (at least) composites of the expected variety: two color-singlet and two color-triplets states corresponding to a normal generation.

(3) These composite states must have charges $(0,-1)$ and $(\frac{2}{3},-\frac{1}{3})$, respectively, and have the normal values of baryon (B) and lepton (L) number. This restricts in turn the preon charges and forces a consistent labeling of $B+\alpha L$ (where α may be arbitrary) upon them.

(4) The gauge group responsible for binding preons into quarks and leptons, $G_{\rm PC}$, must be such that quarks and leptons are precolor singlets. We will assume that $G_{\rm PC}$ is either SU(N) or SO(N) below.

(5) We will take simplicity as our major guideline in looking at preon models and push it as far as it can go. A first question to ask is how many flavors of preons (preflavors) do we need to produce a normal generation regardless of the nature of $G_{\rm PC}$. It is obvious that a single color triplet of preons is insufficient to produce more than a single color-triplet and color-singlet composite. Hence, we need, at least, four flavors of preons; below we will consider models of four possible classes ($A,B,C,\ldots,$ will stand for preon labels):

$$\begin{aligned}
&\text{I: } A\sim\underline{3}_C,\ B\sim\underline{1}_C\ ,\\
&\text{II: } A\sim\underline{3}_C,\ B\sim\underline{1}_C,\ C\sim\underline{1}_C\ ,\\
&\text{III: } A\sim\underline{3}_C,\ B\sim\underline{3}_C\ ,\\
&\text{IV: } A\sim\underline{3}_C,\ B\sim\underline{\overline{3}}_C\ .
\end{aligned}\tag{2.1}$$

Given our restrictions above these are the only viable models with less than seven flavors of preons; model I has four flavors while II has five and III and IV both have six. We will assume that for every left-handed preon there also exists a right-handed preon (as in the case of quarks) such that for n equivalent representations of preons under $G_{\rm PC}$ our global flavor symmetry will be (at most)

$$G_{\rm PF}={\rm SU}(n)_L\times{\rm SU}(n)_R\times{\rm U}(1)_V\ ,\tag{2.2}$$

the axial symmetry is, of course, broken by ABJ anomalies. Some subgroup of $G_{\rm PF}$ can then be gauged (we hope) to give us the usual gauge interaction of ${\rm SU}(3)_C\times{\rm SU}(2)_L\times{\rm U}(1)_Y$. If there are various kinds of representatives of preons in $G_{\rm PC}$ then $G_{\rm PF}$ will be the product of groups like that in (2.2) plus possible extra U(1)'s.

Let us start with models I and II above; first we ask how many color-triplet and color-singlet states can we construct out of these preons (excluding antiparticles in the singlet case). We find in the case of model I that there are 6 color triplets and 4 color singlets, while in model II there are 15 color triplets and 13 color singlets. For an arbitrary gauge group $G_{\rm PC}$, however, all of these states will not be precolor singlets. So we must now ask what kind of products of three representations in SU(N) and SO(N) lead to singlets. We will assume that both left- and right-handed preons are in the same representation of $G_{\rm PC}$.

(i) Consider R to be some complex representation of $G_{\rm PC}$ then there exist some R such that $R^3\sim\underline{1}_{\rm PC}$. A good example of this is the case where $G_{\rm PC}$ is ${\rm SU}(3)_{\rm PC}$ and R is either the $\underline{3}$ or $\underline{6}$.

(ii) Paralleling (i) there is the case where R is a real representation of $G_{\rm PC}$ and $R^3\sim\underline{1}_{\rm PC}$. Good examples here are the adjoint representations of SO(N) and SU(N) as well as any other real representations of SU(N) such as the $\underline{20}'$ of SU(4) and the $\underline{75}$ of SU(5). Note that in this case, because we have real representations all the color-triplet and color-singlet states for models I and II can be precolor singlets. Also, since we have real representations, the ABJ anomalies associated with $G_{\rm PC}$ all cancel.

(iii) In this case we have a complex representation

R of SU(N) such that $R^2\bar{R}\sim\underline{1}_{\rm PC}$. Examples of this case are the $\underline{10}$ and $\underline{15}$ of SU(3). There are four possible labelings of preons in this case:

1. $A,B\sim R,\ \ C\sim\bar{R}$,
2. $A\sim R,\ \ B,C\sim\bar{R}$,
3. $A\sim\bar{R},\ \ B,C\sim R$,
4. $A,B\sim\bar{R},\ \ C\sim R$.

Table I shows the combinations of preons A, B, and C which yield color singlets and triplets and which are also precolor singlets for cases (i) and (iii) above. We will discuss these models in detail below.

(iv) In this case we have preons in the fundamental representation $\underline{n}$, its complex conjugate $\bar{\underline{n}}$ and the adjoint $\underline{X}$; the product $\underline{n}\times\bar{\underline{n}}\times\underline{X}$ then can yield a precolor singlet. We can use any of the SU(N) groups in this case and we find six possible preon labelings:

1. $A,B\sim\underline{n},\ \ C\sim\underline{X}$,
2. $A\sim\bar{\underline{n}},\ \ B\sim\underline{n},\ \ C\sim\underline{X}$,
3. $A\sim\underline{X},\ \ B,C\sim\underline{n}$,
4. $A,B\sim\underline{X},\ \ C\sim\underline{n}$,
5. $A\sim\underline{n},\ \ B,C\sim\underline{X}$,
6. $A\sim\underline{n},B,C\sim\underline{X}$.

Table II shows the result of considering such a model for the color triplet and singlet states shown in the previous table. In cases 3, 5, and 6 there are either no color triplets or no color singlets; in 4 only one of each kind. In both cases 1 and 2 we find two color triplets and two color singlets but both color singlets have the same electric charge ($=Q_C$) and thus these models can be ruled out. Thus we dismiss models of this class (iv) from further consideration.

(v) Here we consider two complex representations

TABLE I. Color-triplet and -singlet states which are also precolor singlets for models I and II of types (i), (ii), and (iii). × labels a precolor-singlet state.

			(iii)			
	(i)	(ii)	$\underline{1}$	$\underline{2}$	$\underline{3}$	$\underline{4}$
Color triplets						
ABB	×	×		×	×	
$AB\bar{B}$		×	×	×	×	×
$A\bar{B}\bar{B}$		×	×			×
ABC	×	×	×	×	×	×
$AB\bar{C}$		×		×	×	
$A\bar{B}C$		×	×			×
$A\bar{B}\bar{C}$		×	×			×
ACC	×	×		×	×	×
$AC\bar{C}$		×	×	×	×	×
$A\bar{C}\bar{C}$		×	×			
$\bar{A}\bar{A}B$		×	×	×		×
$\bar{A}\bar{A}\bar{B}$	×	×			×	
$\bar{A}\bar{A}C$		×		×		
$\bar{A}\bar{A}\bar{C}$	×	×	×		×	×
$AA\bar{A}$		×	×	×	×	×
Color singlets						
AAA	×	×				
BBB	×	×				
$BB\bar{B}$		×	×	×	×	×
BBC	×	×	×			×
$BB\bar{C}$		×		×	×	
$B\bar{B}C$		×	×	×	×	×
BCC	×	×	×			×
$BC\bar{C}$		×	×	×	×	×
$B\bar{C}\bar{C}$		×		×	×	
CCC	×	×				
$CC\bar{C}$		×	×	×	×	×
$A\bar{A}B$		×	×	×	×	×
$A\bar{A}C$		×	×	×	×	×

TABLE II. Same as Table I except for models I and II of type iv.

	(iv)					
	$\underline{1}$	$\underline{2}$	$\underline{3}$	$\underline{4}$	$\underline{5}$	$\underline{6}$
Color triplets						
ABB			×			
$AB\bar{B}$						
$A\bar{B}\bar{B}$						
ABC		×	×			
$AB\bar{C}$		×	×			
$A\bar{B}C$	×					
$A\bar{B}\bar{C}$	×					
ACC			×	×		
$AC\bar{C}$						
$A\bar{C}\bar{C}$						
$\bar{A}\bar{A}B$						
$\bar{A}\bar{A}\bar{B}$						
$\bar{A}\bar{A}C$						
$\bar{A}\bar{A}\bar{C}$						
$AA\bar{A}$						
Color singlets						
AAA						
BBB						
$BB\bar{B}$						
BBC						
$BB\bar{C}$						
$B\bar{B}C$	×	×				
BCC						
$BC\bar{C}$				×		
$B\bar{C}\bar{C}$						
CCC						
$CC\bar{C}$						
$A\bar{A}B$					×	×
$A\bar{A}C$	×	×			×	×

in SU(N), the fundamental representation $\underline{n}$ and either the symmetric ($\underline{S}$) or antisymmetric ($\underline{A}$) second-rank tensor representation. We then can have

$$(\underline{n}\times\underline{n})_a\times\bar{\underline{A}}\sim 1_{\rm PC}$$

or

$$(\underline{n}\times\underline{n})_s\times\bar{\underline{S}}\sim 1_{\rm PC}\ ,$$

where the subscript a or s denotes the symmetric or antisymmetric combination of $n\times n$. Note that within SU(N), $\underline{A}$ has dimensionality $N(N-1)/2$ while $\underline{S}$ has dimensionality $N(N+1)/2$. There also exists the possibility that $\underline{A}$ or $\underline{S}$ may be real as in the case of the $\underline{6}$ of SU(4). Table III shows the eight possible labelings for A, B, and C and which composites are precolor singlets; R denotes the cases allowed when either $\underline{A}$ or $\underline{S}$ is real. The eight labelings are

1. $A,B\sim\underline{n},\ \ C\sim\underline{X}$,
2. $A\sim\bar{\underline{n}},\ \ B\sim n,\ \ C\sim\underline{X}$,
3. $A\sim\underline{X},\ \ B,C\sim\underline{n}$,
4. $A\sim\underline{X},\ \ B,C\sim\underline{n}$,
5. $A,B\sim\underline{X},\ \ C\sim\underline{n}$,
6. $A\sim\bar{\underline{X}},\ \ B\sim X,\ \ C\sim\underline{n}$,
7. $A\sim\underline{n},\ \ B,C\sim X$,
8. $A\sim\bar{\underline{n}},\ \ B,C\sim X$.

Here $\underline{X}$ is either $\underline{A}$ or $\underline{S}$. As can be seen from the table, models 3, 4, 7, and 8 are ruled out since there are either no color-triplet or color-singlet states; models 5 and 6 are ruled out since they each yield only a single $\underline{1}_c$ and $\underline{3}_c$ composite. Models 1 and 2 are allowed *if* $\underline{X}$ is a real representation only. We will come back to these cases below.

(vi) The last case we will consider is the case of two real representations in SO(N); specifically the cases involving the vector representation:

$$(\underline{n}\times\underline{n})_a\times\underline{A}\sim 1_{\rm PC}\ ,$$

$$(\underline{n}\times\underline{n})_s\times\underline{S}\sim 1_{\rm PC}\ ,$$

where $\underline{S}$ is $[N(N+1)/2]-1$ dimensional here, and also products involving the spinorial representations of SO(N):

$$(N\text{ odd})\ \ (\underline{SP}\times\underline{SP})_a\times\underline{n}\sim 1_{\rm PC}\ ,$$

$$(N\text{ even})\ \ (\underline{SP}\times\underline{SP})_s\times\underline{n}\sim 1_{\rm PC}\ .$$

Note the spinorial representation $\underline{SP}$ is $2^{(N-1)/2}$ ($2^{(N/2)-1}$) dimensional for N odd (even). There are two possible labelings of A, B, C in this case:

1. $A,B\sim\underline{f},\ \ C\sim\underline{X}$,
2. $A\sim X,\ \ B,C\sim\underline{f}$,

where f is either $\underline{n}$ or $\underline{SP}$ and $\underline{X}$ is either one of $\underline{A}$ or $\underline{S}$ when $\underline{f}$ is $\underline{n}$ and $\underline{X}$ is $\underline{n}$ when $\underline{f}$ is $\underline{SP}$. Table III shows the results of these two labelings; we see immediately that case 2 is out since it yields no color singlets. Case 1 will be examined below.

Let us now turn to an analysis of models of types I and II which have survived the brief comments of our survey.

III. ANALYSIS OF MODELS I AND II

Looking at Table I we immediately see that (iii) cannot be used for models of type I since, in this case both color-singlet states have the same charge. In case (i) we see that we have two color-singlet

TABLE III. Same as Table I except for models I and II for types v and vi. R denotes the cases allowed when either A or S is real.

	(v)								(vi)	
	1	2	3	4	5	6	7	8	1	2
Color triplets										
ABB			R	$\times$						$\times$
$AB\bar{B}$										$\times$
$A\bar{B}\bar{B}$			$\times$	R						$\times$
ABC	R	$\times$	R	$\times$					$\times$	$\times$
$AB\bar{C}$	$\times$	R							$\times$	$\times$
$A\bar{B}C$									$\times$	$\times$
$A\bar{B}\bar{C}$			$\times$	R					$\times$	$\times$
ACC			R	$\times$		$\times$				$\times$
$AC\bar{C}$										$\times$
$A\bar{C}\bar{C}$			$\times$	R	$\times$					$\times$
$\bar{A}\bar{A}B$							$\times$	R		
$\bar{A}\bar{A}\bar{B}$							R	$\times$		
$\bar{A}\bar{A}C$	$\times$	R					$\times$	R	$\times$	
$\bar{A}\bar{A}\bar{C}$	R	$\times$					R	$\times$	$\times$	
$AA\bar{A}$										
Color singlets										
AAA										
BBB										
$BB\bar{B}$										
BBC	R	$\times$							$\times$	
$BB\bar{C}$	$\times$	R							$\times$	
$B\bar{B}C$									$\times$	
BCC						$\times$				
$BC\bar{C}$										
$B\bar{C}\bar{C}$					$\times$					
CCC										
$CC\bar{C}$										
$A\bar{A}B$										
$A\bar{A}C$										

states for type I models AAA and BBB; it is easy to see that both of these cannot be spin-$\frac{1}{2}$ states. The total wave function for the composite can be written as

$$\Psi_{\rm comp} \sim \Psi_{\rm color} \times \Psi_{\rm precolor} \times \Psi_{\rm flavor} \times \Psi_{\rm Lorentz} \ .$$

Since the composite is a fermion we must produce a totally antisymmetric wave function $\Psi_{\rm comp}$ for both AAA and BBB—both of which are flavor symmetric, i.e., $\Psi_{\rm flavor}$ is symmetric. BBB involves no color degrees of freedom, hence, if it is antisymmetric in precolor it must be symmetric in the Lorentz (spin) degrees of freedom and vice versa. However it cannot be spin symmetric otherwise it would be spin $\frac{3}{2}$ so we thus demand it be precolor symmetric and Lorentz antisymmetric. AAA however is antisymmetric in color so if we also make it precolor symmetric and Lorentz antisymmetric we end up violating Fermi-Dirac statistics. Thus if AAA is spin $\frac{1}{2}$, BBB must be spin $\frac{3}{2}$ and vice versa; in the case of $SU(3)_{PC}$ both AAA and BBB must be antisymmetric in precolor forcing BBB to be spin $\frac{3}{2}$. Thus case (i) does not work for model I at all and for model II we see that both BBB and CCC are spin-$\frac{3}{2}$ states and are presumed heavy. In case (ii) both models I and II work but II is quite complex so we will only consider type I models in this case in any detail.

We are thus left with analyzing (ii) for model I and (i) and (iii) for model II. By examining Tables II and III we see that all types of (iv) are ruled out and only type II models are allowed for (v) and (vi). Let us first examine the preflavor symmetry groups for each of these cases; it is easy to see what these are by simply reading off the representation contents. We find the following global preflavor groups:

$$G_{\rm PF} = {\rm SU}(5)_L \times {\rm SU}(5)_R \times {\rm U}(1)_V \quad \text{for II(i), II(ii), and II(iii)},$$

$$G_{\rm PF} = {\rm SU}(4)_L \times {\rm SU}(4)_R \times [{\rm U}(1)]^3 \quad \text{for II(v) and II(vi)},$$

$$G_{\rm PF} = {\rm SU}(4)_L \times {\rm SU}(4)_R \times {\rm U}(1)_V \text{ for I(ii)} .$$

If we try to embed the usual $SU(3)_C \times SU(2)_L \times U(1)_Y$ electroweak group inside $G_{\rm PF}$ we fail except for $SU(5)_L \times SU(5)_R \times U(1)_V$ since $SU(4) \to SU(3) \times U(1)$. The $U(1)_V$ factor corresponds to globally conserved preon number. If we gauge only the $SU(5)_L \times SU(5)_R$ subgroup of $G_{\rm PF}$ and leave $U(1)_V$ as a global charge we can use the usual embedding

$${\rm SU}(5)_i \to {\rm SU}(3)_i \times {\rm SU}(2)_i \times {\rm U}(1)_i$$

so that $G_{\rm PF}$ can contain the subgroup

$${\rm SU}(3)_{L+R} \times {\rm SU}(2)_L \times {\rm SU}(2)_R \times {\rm U}(1)_{L+R} .$$

We then can identify $SU(3)_{L+R}$, the vectorial subgroup, as $SU(3)_C$; note that the weak-interaction gauge group we have produced appears to be the left-right-symmetric model.[17] This will be the case if both Q and $B-L$ can be written as a sum of $SU(5)_L \times SU(5)_R$ generators.

If we gauge any subgroups of $SU(4)_L \times SU(4)_R \times [U(1)]^n$ the best we could do is reproduce QCD with several extra U(1) factors:

$${\rm SU}(4)_L \times {\rm SU}(4)_R \times [{\rm U}(1)]^n \to {\rm SU}(3)_C \times [{\rm U}(1)]^{n+2} .$$

Thus *if* we demand that the electroweak gauge group be a subgroup of $G_{\rm PF}$ then only models II(i), (ii), and (iii) are allowed. Note that if the global $U(1)_V$ is broken at the scale of $\Lambda_{\rm PC}$ (where it must be if it were broken at all) it would lead to a superlight superweakly coupled Goldstone boson similar to the Majoron[18] and the invisible axion.[19] This is along the lines of the work of Albright, Schrempp, and Schrempp.

Although they do not explicitly contain any SU(2) subgroups we will also examine models II(v), II(vi), and I(ii) below for the sake of completeness; we will see that in these models although both $SU(2)_L$ and $SU(2)_R$ are absent it is possible to define one or two of the U(1) groups as T_{3L} and/or T_{3R} so that the ordinary quarks and leptons can be put into doublets, triplets, etc., of weak isospin to a limited extent.

Models II(v) and II(vi) are quite similar in their particle content so we treat them together; keeping T_{3L} and/or T_{3R} in mind we see that we can arrange the color triplets and color singlets into multiplets:

$$B\bar{B}\,\bar{C},\ \begin{bmatrix} BBC \\ BB\bar{C} \end{bmatrix},\ \begin{bmatrix} ABC \\ AB\bar{C} \end{bmatrix},\ \begin{bmatrix} \bar{A}\,\bar{A}C \\ \bar{A}\,\bar{A}\,\bar{C} \end{bmatrix},\ \begin{bmatrix} A\bar{B}C \\ A\bar{B}\,\bar{C} \end{bmatrix}$$

if C distinguishes between "up" and "down," or

$$\begin{bmatrix} ABC \\ A\bar{B}C \end{bmatrix},\ \begin{bmatrix} AB\bar{C} \\ A\bar{B}\,\bar{C} \end{bmatrix},\ \begin{bmatrix} BB\bar{C} \\ B\bar{B}\,\bar{C} \end{bmatrix},$$

$$\bar{A}\,\bar{A}C,\ \ \bar{A}\,\bar{A}\,\bar{C},\ \ BBC$$

if B plays the role of "isospin" changer. In each case there are several alternatives depending upon the choice of (ν,e) and (u,d) among the various multiplets. For C carrying "isospin" six possible cases are shown in Table IV which gives the electric charges and values of $B+\alpha L$ for each of the preons (α is arbitrary in general). Note that in cases 1–3 $B\bar{B}\bar{C}$ is a $Q=-\frac{1}{2}$ state with $B=L=0$ while in cases 4–6 its charge is $Q=\frac{1}{2}$ with $B=L=0$ again.

In each case we find other exotic color triplets with the quantum numbers:

TABLE IV. Charge and $B+\alpha L$ assignments for model II types v and vi.

	1 $BBC=\nu$ $ABC=u$	2 $BBC=\nu$ $\bar{A}\,\bar{A}C=u$	3 $BBC=\nu$ $A\bar{B}C=u$	4 $\bar{B}\,\bar{B}C=\nu$ $ABC=u$	5 $\bar{B}\,\bar{B}C=\nu$ $\bar{A}\,\bar{A}C=u$	6 $\bar{B}\,\bar{B}=\nu$ $A\bar{B}C=u$
Q_A	$\frac{5}{12}$	$-\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{1}{12}$	$\frac{5}{12}$
Q_B	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
Q_C	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$(B+\alpha L)_A$	$\frac{1}{3}-\alpha/2$	$-\frac{1}{6}$	$\frac{1}{3}+\alpha/2$	$\frac{1}{3}+\alpha/2$	$-\frac{1}{6}$	$\frac{1}{3}-\alpha/2$
$(B+\alpha L)_B$	$\alpha/2$	$\alpha/2$	$\alpha/2$	$-\alpha/2$	$-\alpha/2$	$-\alpha/2$
$(B+\alpha L)_C$	0	0	0	0	0	0

$$(1)\quad \left.\begin{matrix} \bar{A}\bar{A}C:\ Q=-\frac{1}{3} \\ \bar{A}\bar{A}\bar{C}:\ Q=-\frac{4}{3}\end{matrix}\right\},\ B+\alpha L=-\tfrac{2}{3}+\alpha;\quad \left.\begin{matrix} A\bar{B}C:\ Q=\frac{7}{6} \\ A\bar{B}\bar{C}:\ Q=\frac{1}{6}\end{matrix}\right\},\ B+\alpha L=\tfrac{1}{3}-\alpha\ ;$$

$$(2)\quad \left.\begin{matrix} ABC:\ Q=\frac{1}{6} \\ AB\bar{C}:\ Q=-\frac{5}{6}\end{matrix}\right\},\ B+\alpha L=-\tfrac{1}{6}+\alpha/2;\quad \left.\begin{matrix} A\bar{B}C:\ Q=\frac{2}{3} \\ A\bar{B}\bar{C}:\ Q=-\frac{1}{3}\end{matrix}\right\},\ B+\alpha L=\tfrac{1}{6}-\alpha/2\ ;$$

$$(3)\quad \left.\begin{matrix} ABC:\ Q=\frac{1}{6} \\ A\bar{B}\bar{C}:\ Q=-\frac{5}{6}\end{matrix}\right\},\ B+\alpha L=\tfrac{1}{3}+\alpha;\quad \left.\begin{matrix} \bar{A}\bar{A}C:\ Q=\frac{2}{3} \\ \bar{A}\bar{A}\bar{C}:\ Q=-\frac{1}{3}\end{matrix}\right\},\ B+\alpha L=-\tfrac{2}{3}-\alpha\ ;$$

$$(4)\quad \left.\begin{matrix} \bar{A}\bar{A}C:\ Q=\frac{2}{3} \\ \bar{A}\bar{A}\bar{C}:\ Q=-\frac{1}{3}\end{matrix}\right\},\ B+\alpha L=-\tfrac{2}{3}-\alpha\ ;\quad \left.\begin{matrix} A\bar{B}C:\ Q=\frac{1}{6} \\ A\bar{B}\bar{C}:\ Q=-\frac{5}{6}\end{matrix}\right\},\ B+\alpha L=\tfrac{1}{3}+\alpha\ ;$$

$$(5)\quad \left.\begin{matrix} ABC:\ Q=\frac{2}{3} \\ AB\bar{C}:\ Q=-\frac{1}{3}\end{matrix}\right\},\ B+\alpha L=-\tfrac{1}{6}-\alpha/2;\quad \left.\begin{matrix} A\bar{B}C:\ Q=\frac{1}{6} \\ A\bar{B}\bar{C}:\ Q=-\frac{5}{6}\end{matrix}\right\},\ B+\alpha L=-\tfrac{1}{6}+\alpha/2\ ;$$

$$(6)\quad \left.\begin{matrix} \bar{A}\bar{A}C:\ Q=-\frac{1}{3} \\ \bar{A}\bar{A}\bar{C}:\ Q=-\frac{4}{3}\end{matrix}\right\},\ B+\alpha L=-\tfrac{2}{3}+\alpha;\quad \left.\begin{matrix} A\bar{B}C:\ Q=\frac{7}{6} \\ A\bar{B}\bar{C}:\ Q=\frac{1}{6}\end{matrix}\right\},\ B+\alpha L=\tfrac{1}{3}-\alpha\ .$$

Note that in cases 2–5 we obtain two sets of color triplets with $Q=\frac{2}{3},-\frac{1}{3}$; the second set, however, has exotic values of $B+\alpha L$ unless it is chosen wisely.

If B carries isospin then if we identify $\nu=BB\bar{C}$ and $e^-=B\bar{B}\bar{C}$ we find $Q_B=Q_C/2=\frac{1}{2}$, $(B+\alpha L)_B=0$, and $(B+\alpha L)_C=-\alpha$ with α arbitrary. If $u=ABC$, then $Q_A=-\frac{5}{6}$ and $(B+\alpha L)_A=\frac{1}{3}+\alpha$; if however, $u=AB\bar{C}$ then $Q_A=\frac{7}{6}$, $(B+\alpha L)_A=\frac{1}{3}-\alpha$. This case is also full of exotics; for example, if $u=ABC$ then

$$\left.\begin{matrix} AB\bar{C}:\ Q=\frac{7}{6} \\ A\bar{B}\bar{C}:\ Q=-\frac{1}{6}\end{matrix}\right\},\ B+\alpha L=2\alpha+\tfrac{1}{3};\quad \left.\begin{matrix} \bar{A}\bar{A}C:\ Q=\frac{8}{3} \\ \bar{A}\bar{A}\bar{C}:\ Q=\frac{5}{3}\end{matrix}\right\},\ B+\alpha L=-3\alpha-\tfrac{1}{3}\ ;$$

$$BBC:\ Q=2,\ \ L=-1\ ;$$

or, if $u=AB\bar{C}$ then

$$\left.\begin{matrix} ABC:\ Q=\frac{7}{3} \\ A\bar{B}C:\ Q=\frac{4}{3}\end{matrix}\right\},\ B+\alpha L=\tfrac{1}{3}-2\alpha;\quad \left.\begin{matrix} \bar{A}\bar{A}C:\ Q=-\frac{4}{3} \\ \bar{A}\bar{A}\bar{C}:\ Q=-\frac{10}{3}\end{matrix}\right\},\ B+\alpha L=-\tfrac{2}{3}+\alpha\ ;$$

$$BBC:\ Q=2,\ \ L=-1\ .$$

If we were to go further with these models we would next have to demand that both TrQ and Tr$(B-L)=0$ if both Q and $B-L$ are to be combination generators of $G_{\rm PF}$. It is easy to see that the simple condition Tr$Q=0$, which should be satisfied even in the $SU(4)_L\times SU(4)_R\times U(1)$ case if the U(1) of electromagnetism were a subgroup of $G_{\rm PF}$ is not satisfied for the model I(ii). The condition Tr$Q=0$ plus the fact that one lepton must be neutral leads to the unphysical result that all color triplets and singlets are neutral. We conclude then that we cannot embed $SU(3)_C\times U(1)_{\rm EM}$ inside $SU(4)_L\times SU(4)_R$ (so that there are no anomalies) within the context of this model. Model I(ii) is thus quite an unrealistic model of preon structure.

On the other hand, for models II(v) and II(vi) we can find embeddings for $SU(3)_C \times U(1)_{EM}$ at least since it is relatively easy to satisfy the $\mathrm{Tr}Q=0$ condition. It may also be possible to embed $B-L$ or some other combination of B and L as well although this possibility will not be pursued here.

Let us now turn to models II(i), II(ii), and II(iii) and demand $\mathrm{Tr}Q=0$ and $\mathrm{Tr}(B-L)=0$; the later condition is necessary in this case if the $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ electroweak subgroup is to be identified with the left-right-symmetric model. Let us first examine model II(i) which is the simplest of the three; the condition $\mathrm{Tr}Q=0$ is easily satisfied in this case. Remembering that at least one of the color singlets must be neutral (i.e., the neutrino) we find the following possible relationships between the charges:

(1) $Q_B=0$,

(2) $Q_A=0$,

(3) $Q_B=3Q_A$,

(4) $Q_B=-6Q_A$,

(5) $Q_B=-3Q_A$,

and $3Q_A+Q_B+Q_C=0$ in all five cases. Turning to the color triplets we find possibilities (1), (2), and (5) to be, perhaps, relevant—each of these lead to at least one, or possibly two, pair of u- and d-like states for $Q_A=\pm\frac{1}{3}$ or $Q_B=\pm\frac{1}{3}$. Next we try to enforce $\mathrm{Tr}(B-L)=0$; we get information on the $B-L$ values for A and B (C's value is fixed by the trace condition) by demanding that one of the color-triplet states be the u and another the d both of which have $B-L=\frac{1}{3}$. This gives us conditions on the two unknowns and we thus fix the $B-L$ values of A, B, and C. Then we calculate the $B-L$ values for the color-singlet states; we of course must find two states (with charges 0 and -1) with the same value of $B-L$, i.e., -1. Remembering that if AAA is spin $\frac{1}{2}$ then BBB and CCC must be spin $\frac{3}{2}$ and vice versa we find no two states with $B-L=-1$. Thus we can conclude that model II(i) does not allow an embedding of $U(1)_{B-L}$ into $SU(5)_L \times SU(5)_R$.

Model II(ii) has the largest number of states and is attractive in that it only involves real representations, hence, no anomalies in G_{PC}. Noting that at least one of the color singlets must be neutral and using the vanishing of the trace of Q we find only six possible charge relationships: 1—3 and 5 above along with either $Q_B=-Q_A$ or $Q_B=-2Q_A$. Given the large number of states we must simplify somehow—we will do this by assuming that B and C form a weak $SU(2)_L$ [or $SU(2)_R$] doublet with A as a weak singlet; then $Q_B=Q_C+1$ and we obtain the following multiplets for the color triplets:

$$AA\bar{A},\quad \begin{bmatrix}\bar{A}\,\bar{A}\,\bar{C}\\ \bar{A}\,\bar{A}\,\bar{B}\end{bmatrix},\quad \begin{bmatrix}\bar{A}\,\bar{A}B\\ \bar{A}\,\bar{A}C\end{bmatrix},$$

$$\begin{bmatrix}A\bar{C}\,\bar{C}\\ A\bar{B}\,\bar{C}\\ A\bar{B}\,\bar{B}\end{bmatrix},\quad \begin{bmatrix}ABB\\ ABC\\ ACC\end{bmatrix},\quad \begin{bmatrix}AB\bar{C}\\ AB\bar{B},AC\bar{C}\\ A\bar{B}C\end{bmatrix},$$

and the following multiplets for the color singlets:

$$\begin{bmatrix}BBB\\ BBC\\ BCC\\ CCC\end{bmatrix},\quad \begin{bmatrix}A\bar{A}B\\ A\bar{A}C\end{bmatrix},\quad \begin{bmatrix} & B\bar{C}\,\bar{C} & \\ B\bar{B}B & & C\bar{C}B\\ & \text{or} & \\ B\bar{B}C & & C\bar{C}C\\ & \bar{B}\,\bar{B}C & \end{bmatrix},\quad AAA\ .$$

Given that there are only two doublets of $\underline{3}_C$'s and one must be the (u,d) doublet we immediately find the charges $Q_A=-\frac{1}{3}$, $Q_B=1$, $Q_C=0$, and thus $u=\bar{A}\,\bar{A}\,\bar{C}$ and $d=\bar{A}\,\bar{A}\,\bar{B}$.

Now we impose the $\mathrm{Tr}(B-L)$ constraint; we find that $(B-L)_B=(B-L)_C=1$, $(B-L)_A=-\frac{2}{3}$, and thus it is the antiparticles of one of the lepton doublets which corresponds to (ν,e^-) as is also indicated by their charges. Also note that the usual $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ charge relationship is satisfied $Q=T_{3L}+T_{3R}+(B-L)/2$. Model II(ii) obviously provides a possible embedding of both $U(1)_{EM}$ and $U(1)_{B-L}$ within the $SU(5)_L \times SU(5)_R$ symmetry group. Table V shows a complete listing of all the composite states in this model.

We now turn to model II(iii) which has four subcases and try to impose $\mathrm{Tr}Q=0$ and incorporate B and C into a weak isodoublet. In cases 1, 3, or 4 we find either that u and d are members of an extended weak multiplet such as a triplet or that quarks of the right charge do not exist. In case 2 we find a viable possibility with $Q_A=-\frac{1}{3}$, $Q_B=0, Q_C=-1$ such that $(A,\bar{B},\bar{C})$ form a $\underline{5}_L$ or $\underline{5}_R$ under $SU(5)_L \times SU(5)_R$. Next we must check that $B-L$ is also traceless for case 2; we find that $(B-L)_B=(B-L)_C=-1,(B-L)_A=-\frac{2}{3}$ leading to a traceless $B-L$. Hence model II(iii) case 2 provides an embedding of both $U(1)_{EM}$ and $U(1)_{B-L}$ and, hence, the left-right-symmetric model as does model II(ii). Table VI shows a complete listing of all the composite states in this model; they may be grouped in the following manner:

TABLE V. Charge, $B-L$, and weak-isospin T_3 assignments for the composite states of model II(ii) which satisfy $\mathrm{Tr}Q=\mathrm{Tr}(B-L)=0$.

	Q	$B-L$	T_3
Color triplets			
$\bar{A}\bar{A}\bar{C}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$
$\bar{A}\bar{A}\bar{B}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$
$\bar{A}\bar{A}B$	$\frac{5}{3}$	$\frac{7}{3}$	$\frac{1}{2}$
$\bar{A}\bar{A}C$	$\frac{2}{3}$	$\frac{7}{3}$	$-\frac{1}{2}$
$A\bar{C}\bar{C}$	$-\frac{1}{3}$	$-\frac{8}{3}$	1
$A\bar{B}\bar{C}$	$-\frac{4}{3}$	$-\frac{8}{3}$	0
$A\bar{B}\bar{B}$	$-\frac{7}{3}$	$-\frac{8}{3}$	-1
ABB	$\frac{5}{3}$	$\frac{4}{3}$	1
ABC	$\frac{2}{3}$	$\frac{4}{3}$	0
ACC	$-\frac{1}{3}$	$\frac{4}{3}$	-1
$AB\bar{C}$	$\frac{2}{3}$	$-\frac{2}{3}$	1
$A(B\bar{B}-C\bar{C})/\sqrt{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0
$A\bar{B}C$	$-\frac{4}{3}$	$-\frac{2}{3}$	-1
$A(B\bar{B}+C\bar{C})/\sqrt{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0
$AA\bar{A}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0
Color singlets			
$\bar{C}\bar{C}\bar{C}$	0	-3	$\frac{3}{2}$
$\bar{B}\bar{C}\bar{C}$	-1	-3	$\frac{1}{2}$
$\bar{B}\bar{B}\bar{C}$	-2	-3	$-\frac{1}{2}$
$\bar{B}\bar{B}\bar{B}$	-3	-3	$-\frac{3}{2}$
$\bar{A}A\bar{C}$	0	-1	$\frac{1}{2}$
$\bar{A}A\bar{B}$	-1	-1	$-\frac{1}{2}$
$\bar{B}B\bar{C}$	0	-1	$\frac{1}{2}$
$\bar{B}B\bar{B}$	-1	-1	$-\frac{1}{2}$
$\bar{C}C\bar{C}$	0	-1	$\frac{1}{2}$
$\bar{C}C\bar{B}$	-1	-1	$-\frac{1}{2}$
AAA	-1	-2	0
$\bar{B}\bar{B}C$	-2	-1	$-\frac{3}{2}$
$B\bar{C}\bar{C}$	1	-1	$+\frac{3}{2}$

TABLE VI. Same as Table V but for model II(iii).

	Q	$B-L$	T_3
Color triplets			
ABB	$-\frac{1}{3}$	$-\frac{8}{3}$	1
ABC	$-\frac{4}{3}$	$-\frac{8}{3}$	0
ACC	$-\frac{7}{3}$	$-\frac{8}{3}$	-1
$\bar{A}\bar{A}B$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$
$\bar{A}\bar{A}C$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$
$AB\bar{C}$	$\frac{2}{3}$	$-\frac{2}{3}$	1
$A(B\bar{B}-C\bar{C})/\sqrt{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0
$A\bar{B}C$	$-\frac{4}{3}$	$-\frac{2}{3}$	-1
$AA\bar{A}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0
$A(B\bar{B}+C\bar{C})/\sqrt{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0
Color singlets			
$\bar{B}\bar{B}B$	0	1	$\frac{1}{2}$
$\bar{B}\bar{B}C$	-1	1	$-\frac{1}{2}$
$A\bar{A}B$	0	-1	$\frac{1}{2}$
$A\bar{A}C$	-1	-1	$-\frac{1}{2}$
$BC\bar{C}$	0	-1	$\frac{1}{2}$
$CC\bar{C}$	-1	-1	$-\frac{1}{2}$
$B\bar{C}\bar{C}$	2	1	$\frac{1}{2}$
$B\bar{C}\bar{B}$	1	1	$-\frac{1}{2}$

$$\begin{bmatrix} B\bar{C}\bar{C} \\ B\bar{C}\bar{B} \\ B\bar{B}\bar{B} \\ C\bar{B}\bar{B} \end{bmatrix}, \quad \begin{bmatrix} BA\bar{A} \\ CA\bar{A} \end{bmatrix}, \quad \begin{bmatrix} BC\bar{C} \\ CC\bar{C} \end{bmatrix}$$

for the leptons and

$$\begin{bmatrix} ABB \\ ABC \\ ACC \end{bmatrix}, \quad AA\bar{A}, \quad \begin{Bmatrix} AB\bar{C} \\ AC\bar{C}, AB\bar{B} \\ A\bar{B}C \end{Bmatrix}, \quad \begin{bmatrix} \bar{A}\bar{A}B \\ \bar{A}\bar{A}C \end{bmatrix}$$

for the quarks. We obviously identify the $(\bar{A}\bar{A}B, \bar{A}\bar{A}C)$ doublet with (u,d) and either lepton doublet with (ν,e).

This model, although involving a successful embedding into $SU(5)_L \times SU(5)_R$ has nonvanishing anomalies since we are dealing with SU(N) complex representations. Anomaly-cancellation requirements, if strictly adhered to, would then kill this model and we are thus left with model II(ii), which involve either SO(N) or real, anomaly-free representations of SU(N). Model II(ii) will be dealt with further after a discussion of models III and IV.

IV. MODELS OF TYPES III AND IV

A. Type III model labelings

In these models both A and B are $\underline{3}_C$'s and we will assume that they are in a representation R or $\bar{R}$

under G_{PC}. There are three possible cases:

(i) $A,B \sim R$, $R^3 \sim \underline{1}_{\text{PC}}$, with R complex,
(ii) Same as (i) with R real,
(iii) $A \sim R$, $B \sim \bar{R}$, $R^3 \sim 1_{\text{PC}}$.

Table VII shows the results of these labelings; we see immediately that case (i) is ruled out since there are no color triplets. Cases (ii) and (iii) both seem to work and we will return to them below.

B. Type IV model labelings

In these models $A \sim \underline{3}_C$ while $B \sim \bar{3}_C$; there are a large number of possible labelings some of which are

$$\left.\begin{array}{l} 1.\ A,B \sim \underline{R} \\ 2.\ A \sim \underline{R},\ \ B \sim \underline{\bar{R}} \\ 3.\ A \sim \underline{R},\ \ B \sim \underline{R} \end{array}\right\},\quad R^3 \sim \underline{1}_{\text{PC}}\ ,$$

$$\left.\begin{array}{l} 4.\ A,B \sim \underline{R} \\ 5.\ A \sim \underline{R},\ \ B \sim \underline{\bar{R}} \\ 6.\ A \sim \underline{\bar{R}},\ \ B \sim \underline{\bar{R}} \end{array}\right\},\quad R^2\bar{R} \sim \underline{1}_{\text{PC}}\ ,$$

$$7.\ A,B \sim \underline{R}\ (R \text{ real}),\quad R^3 \sim \underline{1}_{\text{PC}}\ .$$

Table VIII shows these models and we see that cases 2, 3, and 5 are ruled out since they do not have either color singlets or triplets; models 1, 4, and 6 appear acceptable. It should be noted that 1 is the Harari-Seiberg[20] model that has been widely discussed in the literature so it will not be discussed further here.

C. Analysis

Let us now look at the two possible models of type III; in both cases there is an explicit symmetry between preons A and B. In case (iii) we are thus free to choose $u = \bar{A}BB$ and $d = \bar{B}AA$ from which we find

	Q	$B-L$
A	0	$\frac{1}{3}$
B	$\frac{1}{3}$	$\frac{1}{3}$

and thus $e^- = \bar{B}\,\bar{B}\,\bar{B}$ and $\nu = \bar{A}\,\bar{A}\,\bar{A}$ so that $B-L$ is a conserved quantum number at this level. In case (ii) we must again have $Q_A = 0$ and $Q_B = \pm\frac{1}{3}$; we thus would find

$$Q(AA\bar{A}) = Q(B\bar{B}A) = 0\ ,$$

$$Q(BA\bar{A}) = Q(BB\bar{B}) = Q_B\ ,$$

$$Q(\bar{A}BB) = 2Q_B;\ \ Q(\bar{B}AA) = -Q_B\ .$$

If we choose $Q_B = \frac{1}{3}$ we find the same quantum numbers as in (iii), so

$$u = \bar{A}BB,\ \ \nu = \bar{A}\,\bar{A}\,\bar{A}\ ,$$

$$d = AA\bar{B},\ \ \bar{e} = \bar{B}\,\bar{B}\,\bar{B}\ ,$$

so that the color triplets $BA\bar{A}$ and $BB\bar{B}$ have $B = \frac{1}{3}$ and $Q = \frac{1}{3}$ while $AA\bar{A}$ and $AB\bar{B}$ are $B = \frac{1}{3}, Q = 0$ states. The binding of these exotic quarks into hadrons (color singlets) along with the usual quarks leads to fractionally charged states which should be observable since they are also precolor singlets.

Now let us turn to type IV models; let us consider case 4 first. We see immediately that the color-triplet states come in two different charge varieties:

$$\left.\begin{array}{c} AB\bar{B} \\ AA\bar{A} \end{array}\right\},\ Q_A;\quad \left.\begin{array}{c} \bar{B}B\bar{B} \\ \bar{B}A\bar{A} \end{array}\right\},\ Q_{\bar{B}}\ ;$$

TABLE VII. Color-triplet and color-singlet states which are also precolor singlets (labeled by ×) for models of type III.

	(i)	(ii)	(iii)
Color triplets			
$\bar{A}AA$		×	
$\bar{A}AB$		×	
$\bar{A}BB$		×	×
$\bar{B}BB$		×	
$\bar{B}BA$		×	
$AA\bar{B}$		×	×
Color singlets			
AAA	×	×	×
BBB	×	×	×

TABLE VIII. Same as Table VII but for models of type IV.

	$\underline{1}$	$\underline{2}$	$\underline{3}$	$\underline{4}$	$\underline{5}$	$\underline{6}$
Color triplets						
AAB	×				×	×
$AB\bar{B}$				×	×	×
$AA\bar{A}$				×	×	×
$B\bar{B}\,\bar{B}$				×	×	×
$A\bar{A}\,\bar{A}$				×	×	×
$\bar{A}\,\bar{B}\,\bar{B}$	×				×	×
Color singlets						
AAA	×	×	×			×
BBB	×	×	×			×
$AA\bar{B}$		×	×	×		×
$A\bar{B}\,\bar{B}$		×	×	×		×

with $A \leftrightarrow \bar{B}$ symmetry. We can choose $Q_A = \frac{2}{3}$ and $Q_B = \frac{1}{3}$ and $(B-L)_A = -(B-L)_B = \frac{1}{3}$; this implies $\nu = \bar{A}BB$ and $e^- = \bar{A}\,\bar{A}B$ in this model. We thus generate two ordinary "families" of quarks and a single lepton family within this scheme.

In case 6 we can either choose (a) $u = AAB, d = \bar{A}\,\bar{B}\,\bar{B}$ such that $Q_A = \frac{1}{3}, Q_B = 0$ or (b) choose the charges as in case 4 above; either choice leads to exotic states. In case (a) we find $\nu = BBB$ and $e^- = \bar{A}\,\bar{A}\,\bar{A}$ since $(B-L)_A = -(B-L)_B = \frac{1}{3}$; we also have the states

$$\left.\begin{matrix} AB\bar{B} \\ AA\bar{A} \end{matrix}\right\}, \quad Q = \tfrac{1}{3}, \quad B = \tfrac{1}{3}; \quad \left.\begin{matrix} BB\bar{B} \\ BA\bar{A} \end{matrix}\right\}, \quad Q = 0, \quad B = -\tfrac{1}{3}\ ;$$

$$\bar{A}\,\bar{A}B, \quad Q = -\tfrac{2}{3}, \quad L = 1; \quad \bar{A}BB, \quad Q = -\tfrac{1}{3}, \; L = 1\ .$$

In case (b) we find the exotic states

$$AAB, \quad Q = \tfrac{5}{3}, \quad B = \tfrac{1}{3}; \quad \bar{A}\,\bar{A}\,\bar{A}, \quad Q = -2, \quad L = 1\ ,$$

$$\bar{A}\,\bar{B}\,\bar{B}, \quad Q = -\tfrac{4}{3}, \quad B = \tfrac{1}{3}; \quad BBB, \quad Q = +1, \quad L = 1$$

besides those found in case 4 above.

All models of types III and IV naturally have troubles when it comes to unification; all of these models have a global preflavor symmetry $G_{\rm PF} = {\rm SU}(6)_L \times {\rm SU}(6)_R \times {\rm U}(1)_V$ and thus if we try to embed QCD and the electroweak interactions into ${\rm SU}(6)_L \times {\rm SU}(6)_R$ such as to have no anomalies [as we have done with the ${\rm SU}(5)_L \times {\rm SU}(5)_R$ model above] there are some obvious problems. Since Q must be a linear combination of ${\rm SU}(6)_L \times {\rm SU}(6)_R$ generators we must again have ${\rm Tr}Q = 0$ which implies $Q_A = \pm Q_B$. It then becomes impossible to simultaneously have both neutral and charged composites as long as there are only bound states of an odd number of preons. We thus conclude that a six-preon model must contain a $\underline{3}_C$ and three $\underline{1}_C$'s if it is to work at all. A similar problem arises when we try to embed the $B-L$ generator, i.e., $(B-L)_A = \pm (B-L)_B$ which we cannot handle consistently. Since the $\underline{3}_C$ plus $\underline{1}_C$ model is merely an extension of the five-preon model II(ii) which apparently works as well, we will not consider it further here but in a subsequent publication.

V. UNIFICATION CONSTRAINTS

If we want to unify the precolor forces together with the color and electroweak forces at some mass scale, $10^{14} \le M_U \le 10^{19}$ GeV and we want the precolor forces to become strong in the 100–1000 TeV energy range this imposes severe limitations on the group structure of $G_{\rm PC}$ (Ref. 21) as well as the representation content. We already know that since model II(ii) involves triple products of real representations which yield precolor singlets, $R^3 \sim \underline{1}_{\rm PC}$, our possible choices of representations are quite limited. One set of possible representations satisfying the above criterion are the adjoint representations of either SU(N) or SO(N).

The one-loop β function can always be put into the form (neglecting scalars)

$$\beta = \tfrac{11}{3} C_2(G) - \tfrac{2}{3} S_2(F)$$

with $C_2(G) = N$ for SU(N) and $2(N-2)$ for SO(N). $S_2(F)$ depends on the choice of fermion representation. For the case at hand [model II(ii)] we have five four-component preons each of which given an identical contribution to $S_2(F)$ which we shall call T. Thus we have

$$\beta_{\rm PC} = \tfrac{11}{33} N - \tfrac{20}{3} T \quad [{\rm SU}(N)]$$

or

$$\beta_{\rm PC} = \tfrac{22}{3}(N-2) - \tfrac{20}{3} T \quad [{\rm SO}(N)]\ .$$

Now at energies right above $\Lambda_{\rm PC}$ the fermions contributing to all the β functions are preons. Since $\alpha_{\rm PC}(\Lambda_{\rm PC}) \gg \alpha_c(\Lambda_{\rm PC})$ we must have (at least) $\beta_{\rm PC} > \beta_c$ otherwise no simple unification will take place at all independently of M_U. Now, above $\Lambda_{\rm PC}$, $\beta_c = \frac{31}{3}$ so obviously we must have $\beta_{\rm PC} > \frac{31}{3}$. We can easily see that we cannot put our preons in the adjoint representations in either case since then $\beta_{\rm PC}$ would in fact be negative. Thus we must look for other representations which would satisfy these constraints; the elimination of SU(N) as a candidate group is rather easy. Obviously, for T to be as small as possible we must pick the dimensions of the SU(N) representations as small as possible. Thus we must consider only the $\underline{6}$ and $\underline{20}'$ of SU(4), the $\underline{75}$ of SU(5), the $\underline{20}$ and $\underline{175}$ of SU(6), etc.

The $\underline{6}$ of SU(4) and $\underline{20}$ of SU(6) can be eliminated right away since they do not satisfy $R^3 \sim \underline{1}$. The $\underline{20}'$ of SU(4) has $T = 8$ while the $\underline{75}$ of SU(5) has $T = 25$; since T obviously grows faster than N for real SU(N) representations we must conclude that SU(N) must be eliminated from the possible $G_{\rm PC}$ since it leads to nonasymptotically free theories. Our only possibilities lie in SO(N) with nonadjoint representations which satisfy $22N > 75 + 20T$.

An examination of the representations of SO(N), however, reveals[22] that the value of T is too large for any representation which satisfies $R_3 \sim \underline{1}$. For example, some of the second-rank symmetric tensors in SO(N) [with $T = 2(N+2)$] and satisfy $R_3 \sim \underline{1}$; T is even larger for higher dimensional representations. Hence, we conclude that for models of type II(ii) not only can we not unify $G_{\rm PC}$ with the usual gauge groups but, in addition, that $G_{\rm PC}$ is not even asymptotically free.

VI. CONCLUSIONS

Our major conclusion is that there do not exist any four- or five-flavor preon models which satisfy all the constraints one would like to impose on any realistic model such as grand unification.

We started out very simply by assuming that we have a collection of several spin-$\frac{1}{2}$ preons which can form color-triplet and -singlet composites. We next demanded that there be at least two $\underline{3}_C$'s and two $\underline{1}_C$'s and that a normal electric-charge assignment be possible. This eliminated a large number of candidate models. If we then assumed that the preflavor group $G_{\rm PF}$ is sufficiently large to contain an SU(2)-type subgroup so that the electroweak interactions are a gauged subgroup of $G_{\rm PF}$ we eliminate all four-preon models.

For the five- and six-preon models that remain we try to embed the strong and electroweak interactions into anomaly-free $\mathrm{SU}(n)_L \times \mathrm{SU}(n)_R$ $(n=5,6)$ so that no anomalies are present. This leads to the conditions that $\mathrm{Tr}Q = \mathrm{Tr}(B-L) = 0$ since they are formed from linear combinations of $G_{\rm PF}$ generators. All six-flavor preon models with either two $\underline{3}_C$'s or a $\underline{3}_C$ and a $\overline{\underline{3}}_C$ are ruled out by this constraint, leaving open only the possibility of a single $\underline{3}_C$ plus three $\underline{1}_C$'s in six-preon models—an avenue we have not explored.

The only remaining model which incorporates (u,d) and (ν,e) into SU(2) doublets is then model II(ii) which only involves real representations of $G_{\rm PC}$ and is thus anomaly free automatically. A further demand that $G_{\rm PC}$ be either asymptotically free or that one can grand unify precolor with the other interactions leads to the elimination of this model as a candidate. Therefore, the simplest possible models should be at least as large as the six-flavor $\underline{3}_C$ + three $\underline{1}_C$ model. It is not yet quite clear if such models of this type could be found which are asymptotically free and consistent with grand unification.

We thus find that the requirements that the usual quarks and leptons appear in the correct representations with the correct charges, together with cancellation of $G_{\rm PC}$ and $G_{\rm PF}$ anomalies, asymptotic freedom of $G_{\rm PC}$, and the embedding of the usual interactions into $G_{\rm PF}$ lead to inconsistencies for all four- and five-flavor models examined. We conclude that the smallest viable preon model must then have at least six preons in it.

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