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Colorless clusters in jets: Longitudinal-momentum distributions

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We discuss the application of two improved versions of the jet calculus (one by Bassetto, Ciafaloni, and Marchesini and another by Crespi and Jones). Modified parton "propagators" are computed and compared for the two cases. The momentum spectrum of colorless clusters with mesonic quantum numbers is computed.

I. INTRODUCTION

Over the past few years, considerable theoretical effort has gone into developing improved versions of the jet calculus originally proposed by Konishi, Ukawa, and Veneziano[1] (KUV). The thrust of these papers was originally to pick out (quark—antiquark—multiple-gluon) clusters guaranteed to be in the color-singlet state[2] with the idea that these formed a package of partons with finite mass already prepared by the perturbative evolution of planar QCD graphs.[3]

Recently, additional improvements have been made with the idea of also improving the infrared behavior of the equations[4] and it has been shown that this improvement is necessary to obtain adequate regularization of the mass of the produced colorless clusters.[5]

Improvement in a different direction was made by Crespi and Jones,[6] who showed how to include the momentum of the gluons with the momentum of the quark and antiquark in computing properties of the colorless clusters. Although some information of this sort can be obtained by methods alluded to in Ref. 2, the equations in Ref. 6 are the first to yield directly the momentum distribution of the entire colorless cluster rather than merely the momentum of the quarks and antiquarks contained in it.

The purpose of this paper is to examine the solutions of the equations in Refs. 4 and 6. We show the results of the calculations of propagators and the spectra of the calculable momentum distributions for the colorless clusters. Properties of the Bassetto-Ciafaloni-Marchesini (BCM) and Crespi-Jones (CJ) functions are compared, and their applicability to phenomenology discussed.

II. BASSETTO-CIAFALONI-MARCHESINI FORMALISM

The formalism of Ref. 2, as improved in Ref. 4, amounts to keeping track of the "first" quark emitted from an incident parton jet. The equations for the resulting quark distributions in various incoming partons are depicted graphically in Fig. 1; these take the form

$$
\begin{aligned}
k^2\frac{d}{dk^2}\Gamma_g^i(k^2,Q_0{}^2;x)=&V_g(k^2)\Gamma_g^i(k^2,Q_0{}^2;x)+\int_x^1\frac{dz}{z}\frac{\alpha(z(1-z)k^2)}{2\pi}\sum_J\frac{\hat{P}_g^{q\bar{q}}(z)}{N_F}\Gamma_j^i\left[\lambda(z)k^2,Q_0{}^2;\frac{x}{z}\right]\\
&+\int_x^1\frac{dz}{z}\frac{\alpha(z(1-z)k^2)}{4\pi}\hat{P}_g^{gg}(z)\Gamma_g^i\left[\lambda(z)k^2,Q_0{}^2;\frac{x}{z}\right]\\
&+\int_0^{1-x}\frac{dz}{1-z}\frac{\alpha(z(1-z)k^2)}{4\pi}\hat{P}_g^{gg}(z)\sigma_g(\lambda(z)k^2,Q_0{}^2)\Gamma_g^i\left[\lambda(1-z)k^2,Q_0{}^2;\frac{x}{1-z}\right],
\end{aligned}
\tag{1a}
$$

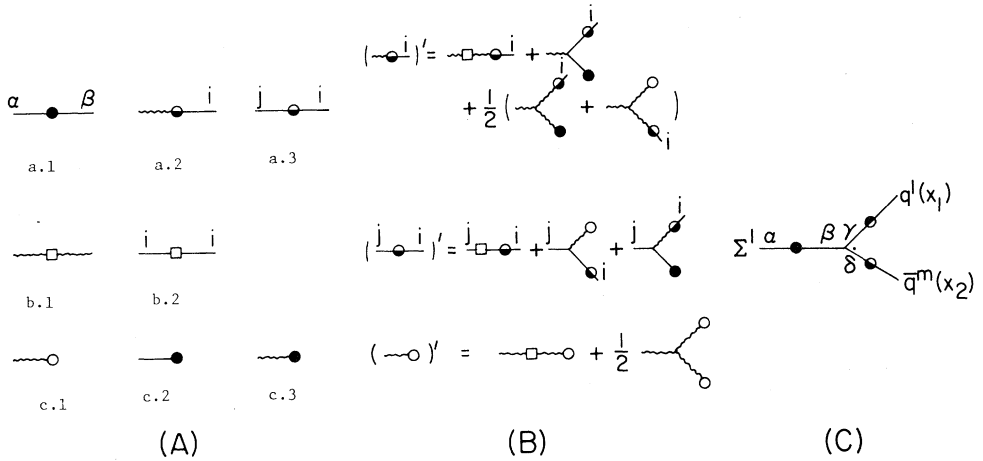

FIG. 1 (A) Notation. (a.1) KUV symbol for $D_{\beta\alpha}(Q^2,Q_0{}^2;x)$, the probability to find parton β with momentum fraction x at $Q_0{}^2$ if one starts with parton α at Q^2. (a.2) and (a.3) BCM symbols for $\Gamma_g^i(Q^2,Q_0{}^2;x)$ and $\Gamma_j^i(Q^2,Q_0{}^2;x)$. (b.1) Representation of the virtual potential for gluons

$$V_g(k^2)=-\int_\epsilon^{1-\epsilon}\frac{\alpha(z(1-z)k^2)}{2\pi}\left[\tfrac{1}{2}\hat{P}_g^{gg}(z)+P_g^{q\bar{q}}(z)\right] .$$

(b.2) Representation of the virtual potential for quarks

$$V_q(k^2)=-\int_\epsilon^{1-\epsilon}dz\frac{\alpha(z(1-z)k^2)}{2\pi}\left[\tfrac{1}{2}\hat{P}_q^{qg}(z)+\tfrac{1}{2}\hat{P}_q^{gq}(z)\right] .$$

(c.1) Representation of the probability that gluons go only to gluons, $\sigma(k^2,Q_0{}^2)$. (c.2) The probability that a quark decays in some (any) way = 1. (c.3) The probability that a gluon decays in any way = 1. (B) Graphical depiction of Eqs. (1a)–(1c). (C) Formation of colorless clusters using the Γ's.

$$\frac{k^2d}{dk^2}\Gamma_j^i(k^2,Q_0{}^2;x)=V_q(k^2)\Gamma_j^i(k^2,Q_0{}^2;x)+\int_x^1\frac{dz}{z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_q^{gq}(z)\Gamma_g^i\left[\lambda(z)k^2,Q_0{}^2;\frac{x}{z}\right]$$
$$+\int_0^{1-x}\frac{dz}{1-z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_q^{gq}(z)\sigma_g(\lambda(z)k^2;Q_0{}^2)\Gamma_j^i\left[\lambda(1-z)k^2;Q_0{}^2;\frac{x}{1-z}\right] , \quad (1b)$$

$$k^2\frac{d}{dk^2}\sigma_g(k^2,Q_0{}^2)=\sigma_g(k^2,Q_0{}^2)V_g(k^2)+\tfrac{1}{2}\int dz\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_g^{gg}(z)\sigma_g(\lambda(z)k^2,Q_0{}^2)\sigma_g(\lambda(1-z)k^2,Q_0{}^2) . \quad (1c)$$

Here Γ_g^i is the probability of finding quark i in a gluon jet, Γ_j^i is the probability of finding quark i in the quark-j jet, and σ is the probability that gluons emit only gluons. We have implemented the infrared procedure of Ref. 4. Note that we give the equations for specific outgoing quarks (antiquarks), because in any phenomenological application one must keep track of the quantum numbers of the quarks. The originators of these equations, with more global concepts in mind, have always summed over the final-state quarks implicitly.

The distribution of quarks and antiquarks in colorless [color-singlet (CS)] clusters in parton jet a then assumes the form [see Fig. 1(C)]

$$\left.\frac{k^2d\sigma}{\sigma\, dk^2dx_1dx_2}\right|_{CS}=\sum_{\beta\gamma\delta}{}'\int_{x_1+x_2}\frac{dx}{x^2}D_a^\beta(Q^2,k^2;x)$$
$$\times\int_{x_1/x}^{1-x_2/x}\frac{dz}{z(1-z)}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_\beta^{\gamma\delta}(z)\Gamma_\gamma^{q_l}\left[\lambda(z)k^2,Q_0{}^2;\frac{x_1}{xz}\right]$$
$$\times\Gamma_\delta^{\bar{q}_m}\left[\lambda(1-z)k^2,Q_0{}^2;\frac{x_2}{x(1-z)}\right] \quad (2)$$

and if one wishes the x distribution of quarks and antiquarks within the colorless clusters, this is simply obtained by

$$\frac{1}{\sigma}\frac{d\sigma_\alpha}{dx}=\int dx_1 dx_2 \frac{d\sigma_\alpha}{\sigma_\alpha dx_1 dx_2}\Bigg|_{\rm CS}\delta(x-x_1-x_2)\ . \tag{3}$$

While it may be possible to solve these equations directly in x space, we have to date found it convenient to deal with moments of the distributions. To be clear about our exact procedure, we list the moment equations in Appendix A; where applicable these are the same equations obtained by Ref. 5. The equations are solved subject to the initial conditions

$$\begin{aligned}&\Gamma_i^i(Q_0{}^2,Q_0{}^2;x)=\delta(1-x)\ ,\\ &\Gamma_j^i(Q_0{}^2,Q_0{}^2;x)=0\ ,\quad i\neq j\ ,\\ &\Gamma_g^i(Q_0{}^2,Q_0{}^2;x)=0\ ,\\ &\sigma(Q_0{}^2,Q_0{}^2)=1\ .\end{aligned} \tag{4}$$

We have used the asymptotic form for the strong coupling,

$$\alpha_s=\frac{1}{b\ln(Q^2/\Lambda^2)}\ ,\quad b=\frac{11N_c-2N_f}{12\pi}\ ,$$

with the parameter $\Lambda=0.2\ \mathrm{GeV}^2$. The initial point $Q_0{}^2$ has been set alternatively at 0.062 GeV^2 (where $\alpha_s=\pi$) and at 0.2 GeV^2 (a value we found convenient for the production of light mesons in our previous phenomenology using the KUV jet calculus).

This procedure is not entirely without difficulties, since the assumptions of asymptoticity made in the derivation are clearly not valid for the region of integration near $Q_0{}^2$. If one examines, for instance, the equations for the zeroth moment of the quark propagators, $\tilde{\Gamma}(k^2)=\int_0^1\Gamma(k^2,x)dx$,

$$k^2\frac{d\tilde{\Gamma}}{dk^2}=\left[\tilde{\Gamma}-\frac{1}{N_F}\right]\left\{\frac{-C_F}{\pi}\int_{Q_0{}^2}^{k^2}\frac{dk'^2}{k'^2}\alpha(k'^2)[1-\sigma(k'^2)]+\frac{3}{4\pi}C_F\alpha(k^2)[1-\sigma(k^2)]\right\} \tag{5}$$

(true for both Γ_i^i and Γ_j^i) one sees that the densities will move in the correct direction (that of decrease of $\tilde{\Gamma}_i^i$ and increase of $\tilde{\Gamma}_j^i$) only if the quantity in curly brackets is negative. However, this quantity is only negative above $Q^2=0.146\ \mathrm{GeV}^2$ for $Q_0{}^2=0.062\ \mathrm{GeV}^2$, and above $Q^2=0.61\ \mathrm{GeV}^2$ for $Q_0{}^2=0.2\ \mathrm{GeV}^2$. The probability density $\tilde{\Gamma}_j^i(k^2)$ therefore initially becomes negative (albeit only by a very small amount) prior to becoming positive. Similar problems occur at low Q^2 for moments of the colorless cluster distribution (different moments have problems in different ranges of Q^2).

One could presumably cure this problem by altering the shape of the strong coupling in the nonasymptotic region. Alternatively one could adopt a different set of approximations in going from the equations in x space (where only positive quantities appear) to the moment equations. The BCM recipe of actually using the $\lambda(z)$ function only where the $\hat{P}$ is actually singular is an approximation which is straightforward to implement; but it is not exact. In this paper we take the point of view that the whole procedure is expected to work only for relatively large values of Q^2 anyway; we thus disregard any results at energies where *any* of the computed moments become negative.

A. The propagators

In Fig. 2 we present the results for the BCM propagators Γ_i^i, Γ_j^i $(i\neq j)$, and Γ_g^i as a function of x for various Q^2. These curves are prepared using Yndurain's method[7] with moments from $n=1$ to 8. We see that, as expected, the gluon and unfavored propagators are peaked near small x at all Q^2 whereas the favored propagator (which begins as a δ function at $x=1$ at $Q_0{}^2$) retains its peaking at large x while building up the wee region.

In this regard, we must point out that Eq. (5.10) of Ref. 4 is incorrect, because it gives the impression that both the gluon and quark propagators possess the $x=1$ δ function. The physical meaning of the various Γ's makes it clear that this cannot be true; only the propagator which initially has a δ function can retain it.

Along these lines, we can check the coefficient of the δ function as stated by Eq. (1b). If we assume that the Γ_i^i function has a contribution like $\theta(k^2)\delta(1-x)$, then the coefficient must obey the equation

$$k^2\frac{d\theta}{dk^2}=-\theta\left[\frac{C_F}{\pi}\int\frac{dk'^2}{k'^2}\alpha(k'^2)-\frac{3}{4\pi}C_F\alpha(k^2)\right]\ .$$

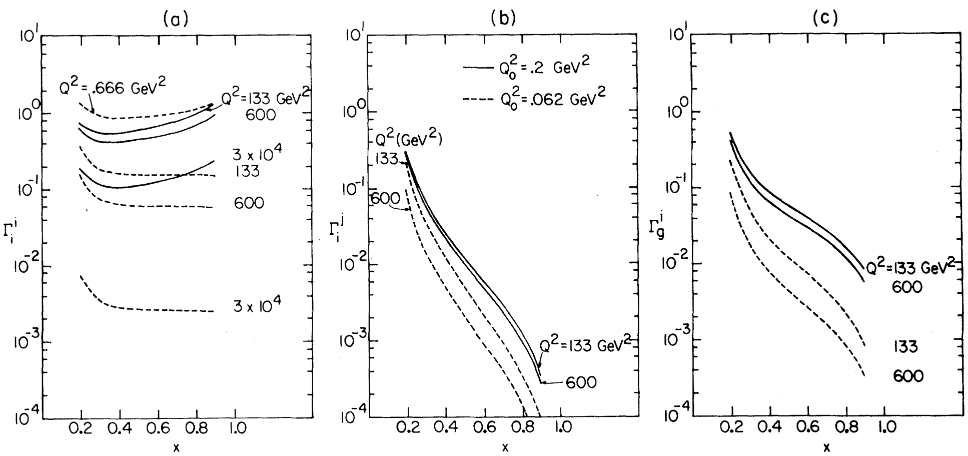

FIG. 2. Solution of Eqs. (1a) and (1b) for various values of initial and final Q^2. (a) Γ_i^i, (b) Γ_i^j, $i \neq j$, and (c) Γ_g^i.

This is the same equation obeyed by the Sudakov form factor,[4] thus verifying the coefficient of the δ function in the favored-quark propagator given in Eq. (5.10) of Ref. 4.

B. Colorless clusters

Next we compute the x distribution of Eq. (2): the amount of momentum carried by the quarks and antiquarks in the colorless clusters. Again we take moments of Eq. (2); the moment equations are listed in Appendix A. These moment equations, obtained by the BCM approximation technique, again produce negative values at some lower values of Q^2. We ignore this region.

By using the $n=1$ through 8 moments, we produce the distributions shown in Fig. 3. These are summed over all quark and antiquark pairs. The fraction of jet momentum in the $q+\bar{q}$ (i.e., the $n=1$ moment) is also shown. One sees that less than one-half of the momentum of the jet is going into quarks and antiquarks in colorless clusters at $Q^2=600$ GeV^2 for $Q_0{}^2=0.062$ GeV^2 (which produces asymptotic results at much lower values of Q^2 than does $Q_0{}^2=0.2$ GeV^2) even though (see Sec. III below) more than 90% of the momentum is going into colorless clusters at this energy.

This is no doubt the reason why phenomenologists using the KUV jet calculus[8] are forced to resort to conversion of the gluons present into $q\bar{q}$ pairs in order to obtain agreement with the momentum spectra of produced mesons and baryons. Clearly, therefore, any attempts to recombine only the quarks and antiquarks within colorless clusters into observed particles are doomed to failure.

III. CRESPI-JONES APPROACH

The approach of Ref. 6 is to define new "propagators" H which measure the momentum of the first quark plus that of the gluons omitted on the "open" side of the lines in Fig. 1. These functions obey the equations depicted in Fig. 4:

$$\begin{aligned} k^2\frac{d}{dk^2}H_g^i(k^2,x) = {} & V_g(k^2)H_g^i(k^2;x) + \int_x^1 \frac{dz}{z}\frac{\alpha(z(1-z)k^2)}{2\pi}\sum_J \frac{\hat{P}_g^{q\bar{q}}(z)}{N_F}H_g^i\left[\lambda(z)k^2;\frac{x}{z}\right] \\ & + \int_x^1 \frac{dz}{z}\frac{\alpha(z(1-z)k^2)}{4\pi}\hat{P}_g^{gg}(z)H_g^i\left[\lambda(z)k^2;\frac{x}{z}\right] \\ & + \int_0^x \frac{dz}{1-z}\frac{\alpha(z(1-z)k^2)}{4\pi}\hat{P}_g^{gg}(z)\sigma_g(\lambda(z)k^2;Q_0{}^2)H_g^i\left[\lambda(1-z)k^2;\frac{x-z}{1-z}\right], \end{aligned} \tag{6a}$$

$$k^2\frac{d}{dk^2}H_j^i(k^2;x)=V_q(k^2)H_j^i(k^2;x)+\int_x^1\frac{dz}{z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_q^{gq}(z)H_g^i\left[\lambda(z)k^2;\frac{x}{z}\right]$$
$$+\int_x^1\frac{dz}{z}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_q^{gq}(z)\sigma_g(\lambda(z)k^2;Q_0^2)H_j^i\left[\lambda(1-z)k^2;\frac{x-z}{1-z}\right]. \tag{6b}$$

The moment equations corresponding to these are given in Appendix B.

Now the expression for colorless clusters

$$\left.\frac{1}{\sigma_\alpha}\frac{d\sigma_\alpha}{dx_1dx_2}\right|_{\mathrm{CS}}=\sum_{\beta\gamma\delta}{}'\int_{Q_0^2}^{Q^2}\frac{dk^2}{k^2}\int_{x_1+x_2}^1\frac{dx}{x^2}D_\alpha^\beta(Q^2,k^2;x)$$
$$\times\int_{x_1/x}^{1-x_2/x}\frac{dz}{z(1-x)}\frac{\alpha(z(1-z)k^2)}{2\pi}\hat{P}_\beta^{\gamma\delta}(z)H_\gamma^{q_l}\left[\lambda(z)k^2;\frac{x_1}{xz}\right]$$
$$\times H_\delta^{\bar{q}_m}\left[\lambda(1-z)k^2;\frac{x_2}{x(1-z)}\right] \tag{7}$$

is such that $x=x_1+x_2$ is the total momentum carried by the colorless cluster, including the gluons. The moments of this in x are also given in Appendix B.

For these equations, as mentioned in Ref. 6, the moment equations are coupled; the equation for the k^2 variation of the nth moment also involves all lower moments. However, provided we are not overly ambitious regarding the number of moments to be calculated, a standard differential-equation solving package such as RKF45 or DO2PAF can be used successfully to simultaneously integrate the equations for the individual propagators and the colorless clusters.

A. Propagators

In Fig. 5 we show the x and Q^2 dependence of the favored, unfavored, and gluon propagators into quarks. Note that the favored propagator has a δ function at $x=1$ which is much stronger than that in the Γ function. This is reasonable physically because this is basically the probability that the initial quark emits only gluons in its evolution. The coefficient of the δ function, $Z(k^2)$, obeys the equation

$$k^2\frac{dZ(k^2)}{dk^2}=-Z(k^2)\left\{\frac{C_F}{\pi}\int_{Q_0^2}^{k^2}\frac{dk'^2}{k'^2}\alpha(k'^2)[1-\sigma(k'^2)]-\frac{3\alpha(k^2)}{4\pi}C_F[1-\sigma(k^2)]\right\}. \tag{8}$$

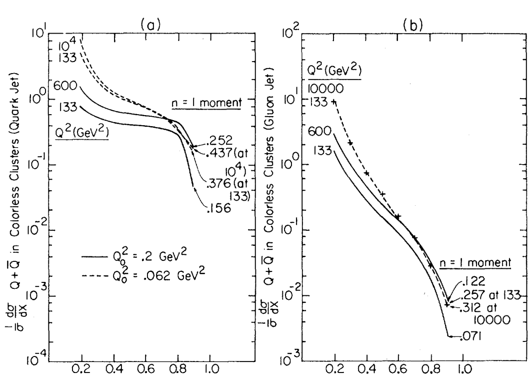

FIG. 3. Momentum distribution of the quarks plus antiquarks in colorless clusters, as calculated from Eqs. (1) and (3). All colorless clusters are summed. Notice that the quarks in colorless clusters carry less than half the momentum, even at very large Q^2. (a) Quark jet and (b) gluon jet.

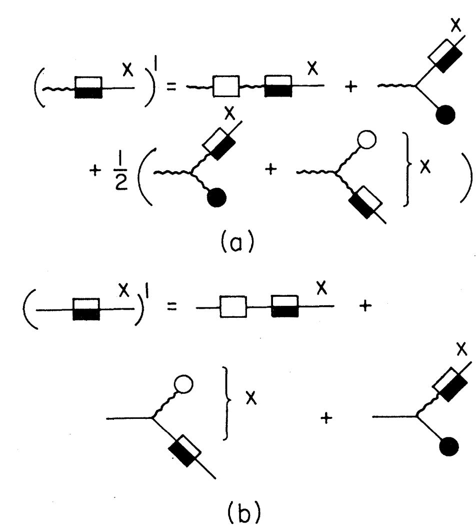

FIG. 4. Graphical depiction of Eqs. (6a) and (6b) (the equation for σ remains the same).

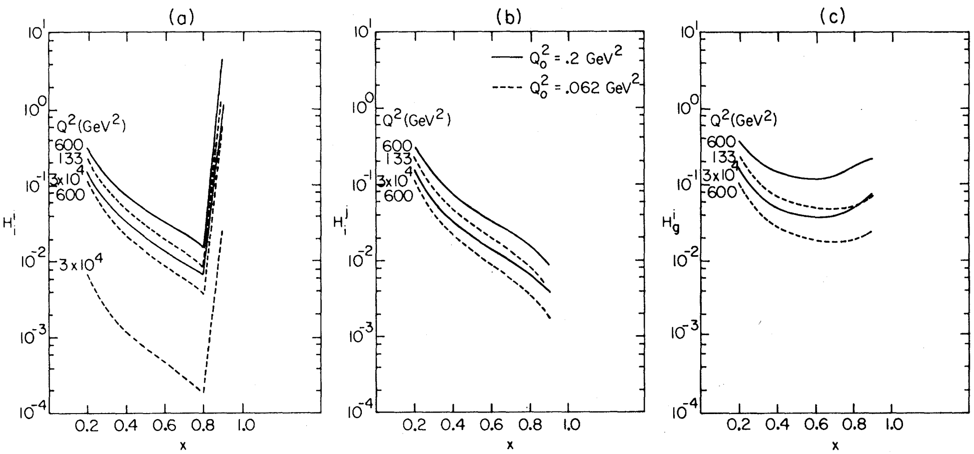

FIG. 5. Behavior of the Crespi-Jones functions computed from Eqs. (6).

This equation is exactly the equation for the function $\phi(k^2)$ used by BCM (Refs. 2 and 5) for the probability density of what they call the first quark. This falls off considerably more slowly with Q^2 than the Sudakov form factor, so the δ function in the H_i^i functions is noticeable for a very long range of k^2, unlike the δ function in the Γ_i^i function. Another consequence of this δ function in the favored quark propagator is the fact that the gluon propagator does not vanish at $x=1$.

B. Colorless clusters

The x distribution of the colorless clusters is shown in Fig. 6(a) for quark jets. Notice that the strong δ function in the favored propagator "reflects" into a noticeable peak at $x=1$ even for relatively high Q^2. The interpretation of this peak will depend on the mass of the colorless clusters involved. If this mass is relatively high, these clusters will be expected to decay via nonperturbative effects into several normal hadrons, which will appear at smaller x.

Computation of the masses of the colorless clusters awaits solution of Eqs. (15) of Ref. 6 [the generalization of Eqs. (6) above including transverse momentum]. However, it is straightforward to compute the mass of the "parent" parton for the colorless cluster [i.e., parton β in Fig. 1(c)]. In fact, we can perform this calculation for the moments

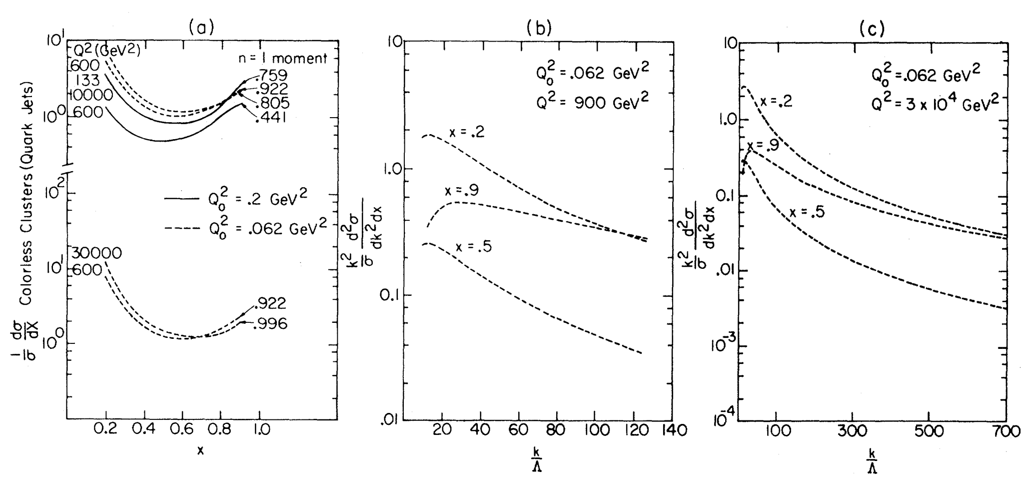

FIG. 6. Spectra of the colorless clusters in a quark jet (summed over *all* colorless clusters). (a) $(1/\sigma)d\sigma/dx$ as computed from Eqs. (6) and (7). (b) $k^2 d\sigma/\sigma\, dk^2 dx$ at $Q^2=900$ GeV^2. (c) same as (b) except at $Q^2=30\,000$ GeV^2.

and thus compute the mass spectrum of parent partons producing colorless clusters at a particular x value. These computations are shown in Figs. 6(b) and 6(c). We see that the spectrum of parent partons for the colorless clusters at $x=0.9$ is much broader than is the case for clusters produced at lower x values, and it peaks at a larger mass. Because the mass of the parent parton gives an upper bound on the masses of colorless clusters descended from it, this suggests that the large-x $q\bar{q}$ gluon clusters may also have large mass.

In Figs. 7 and 8 we show the distributions for gluon jets and for those colorless clusters in quark jets which do not contain the leading quark. In both these cases, the x distribution of the colorless clusters as a whole drops near $x=1$ as we would expect in the absence of the leading quark. The mass distributions of the parent partons for colorless clusters at various x have very similar shape in k^2 to those of the distributions discussed previously—the clusters with $x=0.9$ have a broader parent-parton distribution than do those for $x=0.2$ and 0.5. However, since very few of these $x=0.9$ clusters are present, one would expect that the average mass of clusters for gluon jets would be smaller than that for quark jets and that the average mass of the "unfavored" clusters would be less than that of the favored clusters.

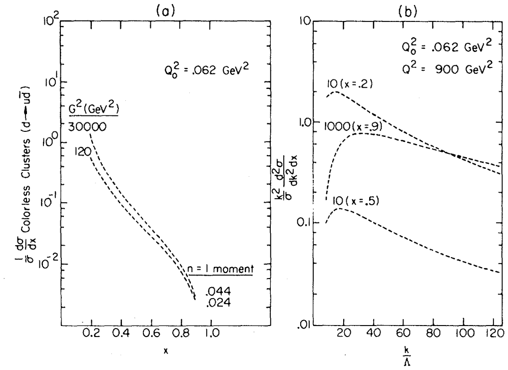

FIG. 8. Spectra of $u\bar{d}$ colorless clusters in a d-quark jet (this gets rid of the leading-quark effect). (a) $(1/\sigma)d\sigma/dx$ from Eqs. (6) and (7). (b) $(k^2/\sigma)d\sigma/dk^2dx$ at $Q^2=900$ GeV^2.

IV. SUMMARY AND CONCLUSIONS

We conclude that the Crespi-Jones equations can be solved reliably with standard methods, and that, if one takes $Q_0{}^2$ small, asymptotic behavior sets in at currently measurable values of Q^2. Complete comparison with the data will require knowledge of the mass spectrum of the colorless clusters, which has been discussed by other authors (Refs. 5 and 9), but not yet calculated within the framework of the Crespi-Jones approach. This work is in progress.

From our study of the masses of the parent partons, we see that the colorless $q\bar{q}$ gluon clusters obtained by this method are likely to be hadron precursors rather than hadrons themselves.

Meanwhile, we point out that the values of the spectra of Fig. 6 at small x show promise when compared roughly with data on fragmentation functions, and there is every reason to hope that a fruitful phenomenology can be developed with these techniques.

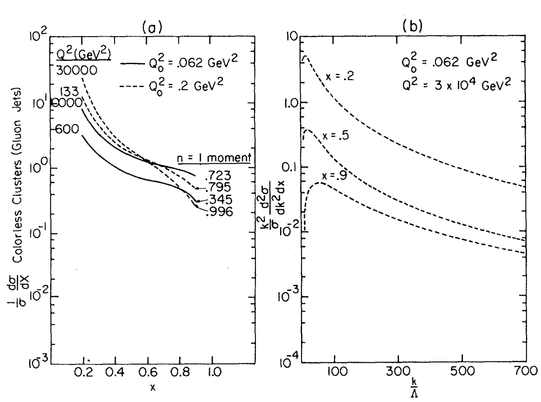

FIG. 7. Spectra of colorless clusters in a gluon jet. (a) $d\sigma/\sigma dx$ as computed from Eqs. (6) and (7). (b) $(k^2/\sigma)d\sigma/dk^2dx$ at $Q^2=30\,000$ GeV^2.

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APPENDIX A

In this appendix we give explicitly the moment equations derived from Eqs. (1) and (3) using the BCM method of regularization near $z=0$. We use the notation

$$\widetilde{\Gamma}(k^2;n)=\int_0^1 x^n\Gamma(k^2,Q_0{}^2;x)dx \ .$$

Also, we will label the moment equation corresponding to Eq. (1a) as (1a′), etc.:

$$\Sigma(k^2)=\frac{1}{\pi}\int_{Q_0{}^2}^{k^2}\frac{dk'^2}{k'^2}\alpha(k'^2)[1-\sigma(k'^2)] \ .$$

To be explicit about the favored and unfavored quark propagators, we use $\widetilde{\Gamma}_i^j$ only for $i\neq j$:

$$k^2\frac{d}{dk^2}\widetilde{\Gamma}^i_g(n)=-\widetilde{\Gamma}^i_g(n)\left[\frac{C_A}{2}\Sigma(k^2)-\frac{b\alpha_s}{2}(1-\sigma)-\frac{\alpha}{4\pi}A_g^{gg}(n)(1+\sigma)\right]$$
$$+[\widetilde{\Gamma}^i_i(n)+(N_f-1)\widetilde{\Gamma}^j_i(n)]\left[\frac{\alpha}{2\pi}A_g^{q\bar{q}}(n)\right] , \quad (1a')$$

$$k^2\frac{d}{dk^2}\widetilde{\Gamma}^{i,j}_i(n)=-\widetilde{\Gamma}^{i,j}_i(n)\left[C_F\Sigma(k^2)-\frac{3}{4}\frac{C_F\alpha_s}{\pi}(1-\sigma)-\frac{\alpha_s}{2\pi}A_q^{qg}(n)\sigma\right]+\widetilde{\Gamma}^i_g(n)\left[\frac{\alpha_s}{2\pi}A_q^{gq}(n)\right] . \quad (1b')$$

For incident quark jets:

$$\frac{k^2d\widetilde{\sigma}(n)}{\sigma\, dk^2}=D_{ii}(n)Q(n)+(N_f-1)D_{ij}(n)Q(n)+N_fD_{ij}(n)Q_B(n)+\frac{D_{gi}(n)}{2}G(n) , \quad (3')$$

where

$$G(n)=2N_f\widetilde{\Gamma}^i_g(n)\left\{\frac{(1-\sigma)\sigma_s}{2\pi}\left[A_g^{gg}(n)-\left[\frac{11C_A-2N_f}{6}\right]\right]+C_A\Sigma(k^2)\right\}$$
$$+\sum_{n_1=1}^{n-1}\frac{n!}{(n-n_1)!n_1!}\frac{\alpha_s}{2\pi}\hat{P}_g^{gg}(n_1,n-n_1)N_F{}^2\widetilde{\Gamma}^i_g(n-n_1)\widetilde{\Gamma}^i_g(n_1) ,$$

$$Q(n)=[\widetilde{\Gamma}^i_i(n)+(N_F-1)\widetilde{\Gamma}^j_i(n)]\left\{\frac{(1-\sigma)\alpha_s}{2\pi}[A_q^{qg}(n)-\tfrac{3}{2}C_F]+C_F\Sigma(k^2)\right\}+N_F\widetilde{\Gamma}^i_g(n)\frac{\alpha_s}{2\pi}A_q^{gq}(n)$$
$$+\sum_{n_1=1}^{n-1}\frac{n!}{(n-n_1)!n_1!}\frac{\alpha_s}{2\pi}\hat{P}_q^{qg}(n_1,n-n_1)N_F\widetilde{\Gamma}^i_g(n-n_1)[\widetilde{\Gamma}^i_i(n_1)+(N_f-1)\widetilde{\Gamma}^j_i(n_1)] ,$$

$$Q_B(n)=[\widetilde{\Gamma}^i_i(n)+(N_f-1)\widetilde{\Gamma}^j_i(n)]\left\{\frac{(1-\sigma)\alpha_s}{2\pi}[A_q^{qg}(n)-\tfrac{3}{2}C_F]+C_F\Sigma(k^2)\right\}+N_F\widetilde{\Gamma}^i_g(n)\frac{\alpha_s}{2\pi}A_q^{gq}(n)$$
$$+\sum_{n_1=1}^{n-1}\frac{n!}{(n-n_1)!n_1!}\frac{\alpha_s}{2\pi}\hat{P}_q^{gq}(n_1,n-n_1)N_f\widetilde{\Gamma}^i_g(n_1)[\widetilde{\Gamma}^i_i(n-n_1)+(N_f-1)\widetilde{\Gamma}^j_i(n-n_1)] .$$

The corresponding equation for incident gluon jets is

$$\frac{k^2}{\sigma}\frac{d\widetilde{\sigma}}{dk^2}(n)=2N_fD_{ig}(n)Q(n)+D_{gg}(n)\tfrac{1}{2}G(n) .$$

APPENDIX B

In this appendix we give explicitly the moment equations we solved to obtain Figs. 5 and 6. (The figures were obtained by inverting eight moments using Yndurain's method with no "correction term.")

$$\Xi(k^2)=\int_{Q_0{}^2}^{k^2}\frac{dk'^2}{k'^2}\alpha(k'^2)\sigma(k'^2) ,$$

$$k^2\frac{d}{dk^2}\widetilde{H}^i_g(n)=V_g(k^2)\widetilde{H}^i_g(n)+\frac{\alpha_s}{2\pi}A_g^{q\bar{q}}(n)[\widetilde{H}^i_i(n)+(N_f-1)\widetilde{H}^j_i(n)]$$

$$+\widetilde{H}^{\,i}_{g}(n)\left\{\frac{\alpha_s}{4\pi}\left[A^{gg}_{g}(n)+\frac{-11C_A+2N_f}{6}\right]+\frac{2C_A}{4}[\Sigma(k^2)+\Xi(k^2)]\right\}$$

$$+\frac{\sigma}{4\pi N_f}\left\{\alpha_s(1-\sigma)\left[A^{gg}_{g}(n)+\frac{-11C_A+2N_f}{6}\right]+2C_A\pi\Sigma(k^2)\right\}$$

$$+\widetilde{H}^{\,i}_{g}(n)\left\{\frac{\alpha_s\sigma}{4\pi}\left[A^{gg}_{g}(n)+\frac{-11C_A+2N_f}{6}\right]+\frac{C_A}{2}\Xi(k^2)\right\}$$

$$+\sum_{n_1=1}^{n-1}\frac{n!}{n_1!(n-n_1)!}\frac{\alpha_s\sigma}{4\pi}\widetilde{\Gamma}^{\,i}_{g}(n_1)\hat{P}^{gg}_{g}(n-n_1,n_1)\,, \tag{6a$'$}$$

$$k^2\frac{d}{dk^2}\widetilde{H}^{\,i,j}_{i}(n)=V_q(k^2)\widetilde{H}^{\,i,j}_{i}(n)+\frac{\alpha_s}{2\pi}\widetilde{H}^{\,i}_{g}(n)A^{gq}_{q}(n)+\frac{\sigma\alpha_s}{2\pi}\widetilde{H}^{\,i,j}_{i}(0)A^{gq}_{q}(n)$$

$$+\sum_{n_1=1}^{n-1}\frac{n!}{n_1!(n-n_1)!}\frac{\sigma\alpha_s}{2\pi}\widetilde{H}^{\,i,j}_{i}(n_1)\hat{P}^{gq}_{q}(n-n_1,n_1)$$

$$+\frac{\widetilde{H}^{\,i,j}_{i}(n)}{2\pi}\left[\alpha_s\sigma A^{qg}_{q}(n)-\tfrac{3}{2}C_F\alpha_s\sigma+2C_F\pi\Xi(k^2)\right].$$

The equations for the moments of the colorless clusters are the same as (3′) except that $\widetilde{\Gamma}$ is replaced by $\widetilde{H}$.

These equations look very messy. However, because of the momentum-conservation sum rules [Eqs. (8a) and (8b) of Ref. 6]

$$\int_0^1\frac{1}{\sigma}\frac{d\sigma}{dx}\bigg|_{\rm CS}x\,dx=1-\sum_i\widetilde{H}^{\,i}_{j}(Q^2,1)\quad\text{for quarks}$$

and

$$\int_0^1\frac{1}{\sigma}\frac{d\sigma}{dx}\bigg|_{\rm CS}x\,dx=1-\sigma-2\sum_i\widetilde{H}^{\,i}_{g}(Q^2,1)\quad\text{for gluons,}$$

we have an automatic check on our solution routine. A typical difference between the left- and right-hand sides of these equations was 0.2×10^{-7}; we therefore regard our programs as correct.

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