Static-bag-source meson field theory: Strong-coupling approximation

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In this paper we discuss quantum-mechanical strong-coupling calculations on a static-source meson field theory in which only the nucleon isospin degrees of freedom are coupled linearly to an isovector-scalar meson field. The nucleon source function is of the chiral bag type, namely a shell source. We perform numerical calculations in the strong-coupling approximation, namely for $g \gg (8\pi^2\mu R)^{1/4}$, where g is a dimensionless bare coupling constant, μ is the meson mass, and R is the bag size. We demonstrate that the corrections to our lowest-order approximation to the dressed nucleon wave function decrease as the coupling strength increases according to the above inequality. We find that there exist reasonable values of the parameters, say $g \sim 4$ and $R \sim 0.3$ fm, for which the lowest-order approximation to the dressed nucleon state of this model is good. We also find collective excitations of the dressed nucleon which are rotational and vibrational excitations in isospin space. The lowest-lying collective excitation is a rotational state which is identified as the Δ in this model. Furthermore, we generalize the model field theory by introducing a feature of the cloudy-bag-model Hamiltonian, namely, an intrinsic isobar source coupled to the meson field. We find that the strong-coupling approximation is realized for significantly smaller values of the bare coupling constant or larger values of the bag size when compared to the previous model. We also find a total bare-Δ component on the dressed nucleon state of 36%. Finally, we numerically solve the meson-nucleon scattering problem in the first model by employing the reaction theory of Feshbach.

I. INTRODUCTION

Over the past few years, there has been considerable interest in the role played by the mesonic degrees of freedom, in particular the pion, in determining the properties of the baryons, and the connection of this phenomenology to meson-nucleon scattering and the long-range part of the nucleon-nucleon force. Chiral bag models[1-5] are possibly relevant for establishing this connection. In these models, it is assumed that the meson degrees of freedom are approximately described by local fields which are coupled at the bag surface to quarks confined inside the bag. The form of the interaction is specified by an approximate symmetry of the strong interactions, namely, chiral symmetry. As interacting quantum field theories, the corresponding Hamiltonians of these models are extremely complicated to say the least. In order to make progress in applying these models phenomenologically, it is necessary to introduce approximations which one hopes to justify quantitatively. One particular issue, either at a classical or a quantum level, has been the bag radius of the nucleon. Aside from the importance of determining the value of this parameter phenomenologically, its value has important implications in regard to the strength of the meson-baryon interaction. This feature is of course familiar from static-source meson field theories, such as the Chew model.[6]

For these models, it is known that for sufficiently large bag-source sizes, say $R \gg 1/m_\pi$, where R is the bag radius and m_π the pion mass, the meson-baryon coupling is weak. Therefore, for sufficiently large source sizes perturbation theory in the meson-baryon interaction is expected to be valid. On the other hand, for small bag-source sizes, say $R \ll 1/m_\pi$, the coupling is strong[7] and nonperturbative calculation schemes are required. In addition to the computational differences, the role played by the meson degrees of freedom in both cases is quite different.

For example, in the case of the cloudy bag model (CBM),[2] a generalization of the static Chew model which includes an intrinsic isobar source interaction with the pion field, it is assumed that the bag-source size is sufficiently large,[8] $0.6 < R < 1$ fm, that truncation at the one-meson level in perturbation theory is valid, say for the nucleon self-energy and wave function. The lowest-order approximation to the nucleon state is a quark-shell-model state or bare nucleon state and the small corrections correspond to a bare nucleon plus a meson and a bare Δ plus a meson. A similar description applies to the classical-approximation scheme of Jaffe.[9]

On the other hand, the little-bag model (LBM),[3] developed by Brown and collaborators, assumes that the bag-source size is, say, of the order of the proton Compton wavelength. This model is motivated by a rather appealing picture of the nucleus which, in a certain sense, has an analogy in atomic and molecular physics. For a diatomic molecule in the Born-Oppenheimer approximation, the electronic motions are very rapid when compared to the relative motion between the atoms, so that the electrons and approximately fixed nuclei are a source for the average Coulomb field between the atoms. This Coulomb field then acts as a potential which determines the quantum-mechanical behavior of the relative motion of the atoms. In the case of hadronic and nuclear physics, for sufficiently small bag sizes the characteristic quark motion within the nucleon is very rapid when compared to the relative nucleon motion in the nucleus. Hence, the quarks are a source for the average pion field between the nucleons, which is the long-range part of the nucleon-nucleon force. This force with an additional short-range part can then be used to determine the relative motion of the nucleons. In this picture one would expect that the individual nucleon magnetic moments in the nucleus would be approximately the same as the free-space values, which accounts for the success of the shell model.

Aside from this appealing picture, the strong meson-baryon interaction for small bag-source sizes has important implications in regard to the physical description of the dressed nucleon state. In contrast to the CBM, for strong coupling the average number of mesons which dress the bare nucleon can be large and the bare-Δ-plus-meson-cloud component can be substantial. Furthermore, the meson cloud can be collectively excited such that excited states of the dressed nucleon exist. For example, in the static Chew model for strong coupling[7] the Δ is a collective rotational excitation of the nucleon in spin and isospin spaces and for sufficiently large coupling or small bag-source sizes the Δ can be stable, i.e., it lies below the meson-nucleon threshold. There are also collective vibrational excitations of the meson cloud, such as the N^*.

Both the CBM and LBM Hamiltonians[10] are examples of static-source meson field theories. In this paper, we consider a simple static-source model in the strong-coupling approximation[7] and we demonstrate that both quantum-mechanical and nonperturbative effects are important quantitatively for the determination of the dressed nucleon wave function and self-energy. Hence, computational schemes which are based on lowest-order perturbation theory are inaccurate for strong coupling.[3]

In Sec. II we define the Hamiltonian and elaborate on the strong-coupling formalism. In Secs. II A and II B we determine the eigenvalues and eigenfunctions of the unperturbed Hamiltonian. In Sec. III, we evaluate the effects of the perturbation and we present numerical results. In Sec. IV we generalize the Hamiltonian such that it has a feature of the CBM Hamiltonian, namely, an intrinsic isobar source coupled to the meson field. The same calculations are performed on this model; however the consequences are significantly different than the previous model. In Sec. V we solve the meson-nucleon scattering problem for the Hamiltonian defined in Sec. II.

II. STRONG-COUPLING FORMALISM

In this paper we perform strong-coupling calculations on an interacting quantum field theory in which the nucleon is described by a static-bag-source distribution and its isospin is coupled linearly to an isovector-scalar meson field. In the present investigation, we ignore the spin of the nucleon and the pseudoscalar nature of the meson degrees of freedom. However, the method of calculation is generalizable to the case of the CBM Hamiltonian or the LBM Hamiltonian and results on these models will be presented in a forthcoming paper. The model of interest is of the form

$$H = M_0 + \frac{1}{2}\int d^3r[\pi_i^2(\vec{r}) + \phi_i(\vec{r})(-\nabla^2+\mu^2)\phi_i(\vec{r})] - g\tau_i \int d^3r\, U(r)\phi_i(\vec{r}) , \tag{2.1}$$

where M_0 is the bare mass, g is a dimensionless bare coupling constant, the τ_i are the usual Pauli isospin matrices, μ is the meson mass which we take to be the pion mass, and the source function U is given by

$$U(r) = \frac{\delta(r-R)}{4\pi R^2} \tag{2.2}$$

which is of the chiral bag type and is such that its integral is unity. The sum over repeated indices is understood and the fields obey the usual commutation rules given by

$$[\pi_i(\vec{r}),\phi_j(\vec{r}\,')]=-i\delta_{ij}\delta^3(\vec{r}-\vec{r}\,') \ . \tag{2.3}$$

Note that the model has three free parameters, namely, M_0, g, and the source size R.

As a heuristic guide, we expand the field operators $\phi_i(\vec{r})$ in free-field creation and annihilation operators and evaluate the overlap of ϕ_i with the source function of Eq. (2.2), which gives

$$\int d^3r\, U(r)\phi_i(\vec{r})=\int_0^\infty \frac{k^2dk}{\sqrt{2}(2\pi)^3\omega(k)}j_0(kR)\int d\Omega_k[a_i(\vec{k})+a_i^\dagger(\vec{k})] \ . \tag{2.4}$$

The presence of the spherical Bessel function j_0 indicates that the most important region of integration is such that $k\sim 1/R$. Therefore for small source sizes, say $R \ll 1/\mu$, the high-momentum mesons have the most significant overlap with the source. Furthermore, the k dependence of the integrand implies that the overlap is large for $R \ll 1/\mu$, hence the effective coupling is strong. The overall coupling strength in Eq. (2.1) depends upon the choice of the bare coupling constant g; however given a g there exists a source size R such that the coupling is strong. The precise definition of the strong-coupling approximation will be presented in Sec. II B. Since the Hamiltonian of Eq. (2.1) involves linear coupling, we expect, for example, that as the coupling strength increases the average number of high-momentum mesons in the ground state of the system will increase. On the other hand for large source sizes, say $R \gg 1/\mu$, the coupling is weak and the average number of low momentum mesons is few in number.

For small source sizes, the heuristic argument presented above suggests a strong-coupling scheme in which a sufficient number of high-momentum mesons are included in the ground-state wave function and self-energy in first approximation and the lower-momentum mesons or weak-coupling mesons are treated as a perturbation. In order to assess the sensibility of this scheme, we require that the corrections to the wave-function normalization decrease as the coupling strength increases so that the lowest-order approximation to the wave function is good.

In order to achieve this goal, we separate the meson field and its canonical momentum into two parts, namely,

$$\phi_i(\vec{r})=F(r)q_i+\phi_i'(\vec{r}) \tag{2.5}$$

and

$$\pi_i(\vec{r})=F(r)p_i+\pi_i'(\vec{r}) \ , \tag{2.6}$$

where the first term in Eqs. (2.5) and (2.6) is the strong-coupling part and the second term is the weak-coupling part. $F(r)$ is a normalizable function such that

$$\int d^3r\, F^2(r)=1 \tag{2.7}$$

and the fields $\phi_i'(\vec{r})$ and $\pi_i'(\vec{r})$ satisfy constraint conditions, namely,

$$\int d^3r\, \phi_i'(\vec{r})F(r)=0 \tag{2.8}$$

and

$$\int d^3r\, \pi_i'(\vec{r})F(r)=0 \ . \tag{2.9}$$

It follows from the canonical commutation rules in Eq. (2.3), and Eqs. (2.7), (2.8), and (2.9) that q_i and p_i are canonical, namely,

$$[p_i,q_j]=-i\delta_{ij} \tag{2.10}$$

and $\pi_i'(\vec{r})$ and $\phi_i'(\vec{r})$ satisfy nonlocal commutation rules given by

$$[\pi_i'(\vec{r}),\phi_j'(\vec{r}\,')]=-i\delta_{ij}[\delta^3(\vec{r}-\vec{r}\,')-F(r)F(r')] \ . \tag{2.11}$$

From the definition of q_i and p_i, namely,

$$q_i=\int d^3r\, F(r)\phi_i(\vec{r}) \tag{2.12}$$

and

$$p_i=\int d^3r\, F(r)\pi_i(\vec{r}) \ . \tag{2.13}$$

It follows that $F(r)$ will be chosen such that for strong coupling it has a large overlap with the high-momentum components of the meson field $\phi_i(\vec{r})$ that couple to the source. For the moment we leave $F(r)$ unspecified and continue with the general development.

Substituting the variables defined in Eqs. (2.5) and (2.6) into Eq. (2.1) we have

$$\begin{aligned} H=&M_0+\tfrac{1}{2}(\underline{p}^2+\Omega^2\underline{q}^2-2g\lambda\underline{\tau}\cdot\underline{q})+\tfrac{1}{2}\int d^3r[\pi_i'^2(\vec{r})+\phi_i'(\vec{r})(-\nabla^2+\mu^2)\phi_i'(\vec{r})]\\ &+\int d^3r[(-\nabla^2+\mu^2)F(r)]\phi_i'(\vec{r})q_i-g\tau_i\int d^3r\, U(r)\phi_i'(\vec{r}) \ , \end{aligned} \tag{2.14}$$

where

$$\Omega^2=\int d^3r\, F(r)(-\nabla^2+\mu^2)F(r) \tag{2.15}$$

is the expectation value of the Klein-Gordon operator which we define as the collective frequency squared and

$$\lambda=\int d^3r\, U(r)F(r) \tag{2.16}$$

is the overlap of the source function with the basis function $F(r)$. The corresponding total-isospin operator $\underline{I}_T$ is given by

$$\underline{I}_T=\underline{T}+\underline{I}\ , \tag{2.17}$$

where the meson isospin $\underline{T}$ is given by

$$\underline{T}=\int d^3r\, \underline{\phi}'(\vec{r})\times\underline{\pi}'(\vec{r})\ , \tag{2.18}$$

the dressed-nucleon isospin $\underline{I}$ is given by

$$\underline{I}=\underline{t}+\frac{\underline{\tau}}{2}\ , \tag{2.19}$$

and the collective meson isospin $\underline{t}$, is given by

$$\underline{t}=\underline{q}\times\underline{p}\ . \tag{2.20}$$

This terminology will become clearer later on.

At this point it is useful to outline our strategy for the determination of approximate eigenfunctions and eigenvalues of the Hamiltonian H defined by Eq. (2.14). The terms which involve only the p_i and q_i variables comprise a collective Hamiltonian given by

$$H_c=\tfrac{1}{2}(\underline{p}^2+\Omega^2\underline{q}^2-2g\lambda\underline{\tau}\cdot\underline{q})\ . \tag{2.21}$$

We want to choose $F(r)$ such that the collective frequency Ω and the overlap λ increase as the source size R decreases. From the definition of Ω in Eq. (2.15) this implies that for such a choice the high-momentum components of the meson field $\phi_i(\vec{r})$ contribute to the collective Hamiltonian. Furthermore, the interaction energy, namely, the $\underline{\tau}\cdot\underline{q}$ term in Eq. (2.21), will be large in the strong-coupling approximation due to its dependence on λ and g.

A choice of $F(r)$ that satisfies the desired conditions is the field produced by the source function $U(r)$ which is given by

$$F(r)=N\hat{U}(r)\ , \tag{2.22}$$

where $\hat{U}(r)$ satisfies the equation

$$(-\nabla^2+\mu^2)\hat{U}(r)=\frac{\delta(r-R)}{4\pi R^2} \tag{2.23}$$

with the explicit solution given by

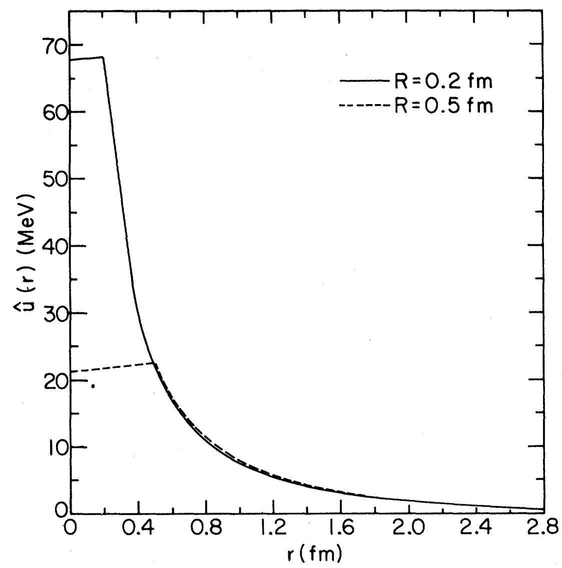

FIG. 1. A plot of the field $\hat{U}$ as a function of the distance r, for two choices of the bag size R.

$$\hat{U}(r)=\frac{1}{4\pi R}\left[e^{-\mu R}\frac{\sinh\mu r}{\mu r}\theta(R-r)+\sinh\mu R\frac{e^{-\mu r}}{\mu r}\theta(r-R)\right]. \tag{2.24}$$

The normalization constant N in Eq. (2.22) is determined by Eq. (2.7) and is given by

$$N^2=\frac{8\pi\mu^3R^2e^{2\mu R}}{(e^{\mu R}\sinh\mu R-\mu R)}\ . \tag{2.25}$$

The collective frequency Ω defined by Eq. (2.15) and overlap λ defined by Eq. (2.16) are given by

$$\Omega=\frac{\mu\sqrt{2}e^{\mu R/2}(\sinh\mu R)^{1/2}}{(e^{\mu R}\sinh\mu R-\mu R)^{1/2}} \tag{2.26}$$

and

$$\lambda=\frac{N}{4\pi R}e^{-\mu R}\frac{\sinh\mu R}{\mu R}\ . \tag{2.27}$$

In Fig. 1, we plot $U(r)$ in Eq. (2.24) as a function of r for various values of the source size R. From Eq. (2.24) for U and Fig. 1 we conclude that as $R\to 0$ the magnitude and curvature of $\hat{U}$ at $r=R$ increase and therefore the overlap λ and collective frequency Ω increase. Furthermore, the source function U approaches a point source and hence the normalization constant N approaches a constant. More specifically, from Eqs. (2.25), (2.26), and (2.27), we have in the limit $\mu R \ll 1$

$$N^2 \cong 8\pi\mu \tag{2.28}$$

and

$$\Omega \cong \mu \left[\frac{2}{\mu R} \right]^{1/2} \tag{2.29}$$

with

$$\lambda \cong \frac{1}{(2\pi)^{1/2}} \frac{\mu^{3/2}}{\mu R} . \tag{2.30}$$

These limiting values are consistent with our heuristic discussion.

The purely mesonic Hamiltonian given by

$$H' = \frac{1}{2} \int d^3r [\pi_i'(\vec{r})^2 + \phi_i'(\vec{r})(-\nabla^2+\mu^2)\phi_i'(\vec{r})] \tag{2.31}$$

commutes with the collective Hamiltonian H_c defined by Eq. (2.21) and therefore can be simultaneously diagonalized. We will discuss this further in Sec. II A.

Finally, there are the perturbation terms which are given by

$$\tilde{H} = N \int d^3r\, U(r)\phi_i'(\vec{r})q_i - g\tau_i \int d^3r\, U(r)\phi_i'(\vec{r}) ,$$

where we have set $F(r) = N \cdot \hat{U}(r)$ in Eq. (2.14). It is not at all obvious that these operators are small in the sense of perturbation theory. This is especially true of the second term which depends on g. We postpone further discussion of these operators until Sec. IV.

A. The meson Hamiltonian

In order to obtain the eigenvalues and eigenfunctions of H', we proceed as in free-field theory by specifying a basis which diagonalizes the Klein-Gordon operator, however this basis must be chosen orthogonal to $F(r)$ so that the constraint conditions defined by Eqs. (2.8) and (2.9) are satisfied. As we shall see, this implies that H' defined by Eq. (2.31) has a discrete energy eigenvalue, the meson vacuum energy, and a continuous spectrum of meson eigenvalues. The vacuum state and the meson states are not free-field states. Furthermore, the vacuum energy is not the free-field vacuum energy, however the meson energies are free meson energies.

To see this, we define the operator

$$h = (1 - |F\rangle\langle F|)(-\nabla^2+\mu^2)(1-|F\rangle\langle F|) \tag{2.32}$$

which is the projection of the Klein-Gordon operator onto a space orthogonal to $F(r)$. Note that h has a zero-energy bound-state solution, namely, $F(r)$. Furthermore, there are a set of scattering solutions of h given by

$$\psi_k(\vec{r}, {}^{\text{in}}_{\text{out}}) = f_k(\vec{r}) - C(k,\pm) \int d^3r' G_k(\vec{r},\vec{r}\,',\pm)F(r') , \tag{2.33a}$$

where $G_k(\pm)$ are free Green's functions with appropriate boundary conditions and

$$f_k(\vec{r}) = \frac{e^{i\vec{k}\cdot\vec{r}}}{(2\pi)^{3/2}} .$$

The constants $C(k,\pm)$ are given by

$$C(k,\pm) = \frac{\tilde{F}(k)}{\int d^3r\, d^3r' F(r) G_k(\vec{r},\vec{r}\,',\pm) F(r')} \tag{2.33b}$$

which guarantee that the scattering solutions defined by Eq. (2.33a) are orthogonal to $F(r)$ with $\tilde{F}(k)$ the Fourier transform of $F(r)$. Note that the scattering solutions imply that there is only s-wave scattering. This is consistent with the coupling term of the model Hamiltonian defined by Eq. (2.1). From the scattering solutions defined by Eq. (2.33a), we have the S matrix given by

$$S(\vec{k}\,',\vec{k}) = \delta^3(\vec{k}\,'-\vec{k}) - 2\pi i \delta(\omega(k)-\omega(k'))t(\vec{k}\,',\vec{k}) , \tag{2.34}$$

where $\omega(k) = (k^2+\mu^2)^{1/2}$ and

$$t(\vec{k}\,',\vec{k}) = t(k^2) = -\int d^3r\, \psi_k^*(r;\text{out})(-\nabla^2+\mu^2)F(r) \frac{\tilde{F}^2(k)}{2\omega(k)} . \tag{2.35}$$

For further details of the eigenfunctions of the operator h defined by Eq. (2.32) and related quantities the reader is referred to Appendix A.

The existence of "in" and "out" scattering solutions defined by Eq. (2.33a) implies that there exists "in" and "out" meson creation and annihilation operators given by

$$\phi_i'(\vec{r}) = \int \frac{d^3}{\sqrt{2\omega(k)}} [\psi_k(\vec{r}, {}^{\text{in}}_{\text{out}}) a_i'(\vec{k}, {}^{\text{in}}_{\text{out}}) + \psi_k(\vec{r}, {}^{\text{in}}_{\text{out}}) a_i'^{\dagger}(\vec{k}, {}^{\text{in}}_{\text{out}})] . \tag{2.36}$$

It is straightforward to show that the in and out creation and annihilation operators are related by the S matrix defined by Eq. (2.34), namely,

$$a_i'(\vec{k},\text{in})=\int d^3k' S^*(\vec{k}',\vec{k})a_i'(\vec{k}',\text{out}) , \quad (2.37)$$

where the asterisk on S denotes complex conjugation and

$$a_i'^{\dagger}(\vec{k},\text{out})=\int d^3k' S^*(\vec{k},\vec{k}')a_i'^{\dagger}(\vec{k}',\text{in}) . \quad (2.38)$$

Equation (2.37) implies that the meson vacuum or ground state of H' is stable, i.e.,

$$|\,\text{vac}',\text{in}\rangle = |\,\text{vac}',\text{out}\rangle = |\,\text{vac}'\rangle$$

since $a_i'(\vec{k},\text{in})$ and $a_i'(\vec{k},\text{out})$ annihilate the same vacuum state. Note that the one-meson state contains scattering since

$$\langle \vec{k}',\text{out}\,|\,\vec{k},\text{in}\rangle = \langle \text{vac}'\,|\,a_j'(\vec{k}',\text{out})a_i'^{\dagger}(\vec{k},\text{in})\,|\,\text{vac}'\rangle = S(\vec{k}',\vec{k})\delta_{ji} \quad (2.39)$$

with $S(\vec{k}',\vec{k})$ given by Eq. (2.34). This result will become important when we discuss the full meson-nucleon scattering problem.

The ground-state or zero-point energy of the meson Hamiltonian H' is not the free-field vacuum energy. In Appendix B it is shown that this energy is given by

$$E'_{\text{vac}}=E_{\text{vac}}-E_{\text{shift}} , \quad (2.40)$$

where E_{vac} is the usual free-field vacuum energy given by

$$E_{\text{vac}}=\frac{3V}{2}\int \frac{d^3k}{(2\pi)^3}\omega(k)$$

with V the volume of all space and E_{shift} is given by

$$E_{\text{shift}}=\tfrac{3}{2}\int d^3r\, d^3r' F(r) f(\vec{r},\vec{r}\,')F(r') \quad (2.41)$$

with

$$f(\vec{r},\vec{r}\,')=\int \frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot(\vec{r}-\vec{r}\,')}\omega(k) . \quad (2.42)$$

Note that E_{shift} depends on the source size R through $F(r)$ and therefore contributes to the ground-state energy, while E_{vac} is a divergent and cutoff-dependent contribution. We will absorb E_{vac} into a redefinition of the bare mass M_0 appearing in Eq. (2.1).

Mathematically, E_{shift} is a consequence of the completeness relation for the scattering solutions defined by Eq. (2.33a), namely,

$$\int d^3k\, \psi_k(\vec{r},{}^{\text{in}}_{\text{out}})\psi_k^*(\vec{r},{}^{\text{in}}_{\text{out}})=\delta^3(\vec{r}-\vec{r}\,')-F(r)F(r') .$$

An intuitive understanding of the physical origin of E_{shift} follows from a description of the scattering process, which is described by h defined in Eq. (2.32), in a spherical region of space of dimension r_0, which is much greater than the range of the interaction potential. We require that the scattering wave function vanish at the boundary of this region, hence

$$\tilde{k}_{n,l}=k_{n,l}-\delta_l(\tilde{k}_{n,l})\cdot\delta_{l0/r_0} , \quad (2.43)$$

where δ_{l0} is a Kronecker Δ which indicates that there is only s-wave scattering in our problem, $\tilde{k}_{n,l}$ is the interacting momentum, $k_{n,l}$ is the free momentum given by

$$k_{n,l}=(n\pi+l\pi/2)/r_0$$

and $\delta_l(\tilde{k}_{n,l})$ is the phase shift. For sufficiently large r_0 the interacting energy is given by

$$\tilde{\omega}_{n,l}\cong\omega_{n,l}-\frac{k_{n,l}\delta_l(k_{n,l})}{r_0}\delta_{l0}+O\left[\frac{1}{r_0{}^2}\right] , \quad (2.44)$$

where $\tilde{\omega}_{n,l}=(\tilde{k}_{n,l}{}^2+\mu^2)^{1/2}$. Hence, the total zero-point energy of the interacting system is given by

$$E_{\text{zero point}}=\tfrac{3}{2}\sum_{n,l}\tilde{\omega}_{n,l} \cong E_{\text{vac}}-\tfrac{3}{2}\sum_{n}\frac{k_{n,0}\delta_0(k_{n,0})}{r_0} , \quad (2.45)$$

where E_{vac} is the free-field vacuum energy. In evaluating the sum over n in Eq. (2.45) we note that the number of levels in a momentum interval, $\Delta k_{n,0}$, centered about $k_{n,0}$ increases as r_0 increases. Therefore, the second term in Eq. (2.45) makes a finite contribution to the total zero-point energy. This is the physical origin of E_{shift}.

The multi-meson states are obtained in the usual manner by applying the relevant meson creation operators defined by Eq. (2.36) to the meson vacuum.

B. The collective Hamiltonian

The exact numerical solution of the eigenvalue equation for the collective Hamiltonian defined by Eq. (2.21) is straightforward. A choice of basis which spans the space is given by

$$\Psi(\vec{q})=R(q,\uparrow)\Phi(\Omega_q,\uparrow)+R(q,\downarrow)\Phi(\Omega_q,\downarrow) , \quad (2.46)$$

where Ω_q is the solid angle subtended by the collective meson variable $\vec{q}$. Note that Ψ implicitly depends on the isospin quantum numbers I and I_z, which is a consequence of the fact that the dressed-nucleon isospin defined by Eq. (2.19) is conserved by H_c. The collective meson isospin $\underline{t}$ defined by Eq. (2.20) is not conserved because of the presence of the $\underline{\tau}\cdot\underline{q}$ interaction. The isospinor wave functions $\Phi(\Omega_q,\uparrow)$ and $\Phi(\Omega_q,\downarrow)$ are, respectively, the stretched, $I=t+\frac{1}{2}$, and jacknife, $I=t'-\frac{1}{2}$, isospin coupling representations with the collective meson isospin, $t'=t-1$. Upon taking the matrix elements of H_c with respect to $\Phi(\uparrow)$ and $\Phi(\downarrow)$ we obtain a set of coupled equations for $R(\uparrow)$ and $R(\downarrow)$ which are numerically straightforward to solve.

Rather than adopting the above procedure, we will choose a basis which takes advantage of the simplicity of the strong-coupling approximation. The collective Hamiltonian is of the three-dimensional harmonic oscillator type, therefore we expect vibrational and angular excitations in isospin space. The fact that the interaction depends on the angle between the nucleon isospin $\underline{\tau}$ and the collective meson variable $\underline{q}$ has important consequences. For example, the states of lowest energy are such that $\underline{\tau}$ is parallel to $\underline{q}$. In analogy with the case of a one-dimensional harmonic oscillator with a linear coupling potential, this implies that the functional dependence of the eigenstates on the collective meson variable $\underline{q}$ is such that the radial component of $\underline{q}$ is displaced by a classical value. Consequently, the ground-state expectation value of q is nonzero. Furthermore, as the classical displacement increases, the average number of mesons which dress the bare nucleon increases.

The angular dependence of the $\underline{\tau}\cdot\underline{q}$ interaction further implies that there is a torque on $\underline{\tau}$ such that it precesses around $\underline{q}$. In addition, $\underline{q}$ is a dynamical variable which undergoes angular motion in isospin space. Hence, as the bare nucleon isospinor rotates in this space it drags around a meson cloud with an average number of mesons. As the classical displacement of q increases, the average number of mesons increases and therefore the moment of inertia of the meson cloud increases thereby decreasing the rotational energy of the system.

In order to make these intuitive arguments more precise we rewrite the collective Hamiltonian in the form

$$H_c=\tfrac{1}{2}\left[K(q)+\frac{t^2}{q^2}+\Omega^2q^2-2g\lambda\underline{\tau}\cdot\underline{q}\right], \tag{2.47}$$

where $K(q)$ is given by

$$K(q)=-\frac{1}{q^2}\frac{\partial}{\partial q}q^2\frac{\partial}{\partial q}$$

and we have made the centrifugal barrier of the collective meson isospin explicit. Consider the situation in which the $\underline{\tau}\cdot\underline{q}$ interaction term is large compared to the other terms in the Hamiltonian. As in the Paschen-Back effect, it is economical to choose a basis which diagonalizes the interaction term. Therefore, we choose a basis such that

$$\underline{\tau}\cdot\hat{\underline{q}}\,\chi_\eta(\Omega_q)=\eta\chi_\eta(\Omega_q)\,, \tag{2.48}$$

where the χ implicitly depend on I and I_z, $\hat{q}$ is the unit vector pointing in the direction of $\underline{q}$ given by

$$\hat{\underline{q}}=\sin\theta\cos\phi\,\hat{\underline{i}}+\sin\theta\sin\phi\,\hat{\underline{j}}+\cos\theta\,\hat{\underline{k}}$$

and η is the eigenvalue. It is trivial to prove that $\eta=\pm1$ and from theHermiticity of the $\underline{\tau}\cdot\hat{\underline{q}}$ operator that the χ's satisfy the orthogonality properties

$$\chi_\eta^\dagger(\Omega_q)\chi_{\eta'}(\Omega_q)=0 \tag{2.49}$$

for $\eta\neq\eta'$ and all angles Ω_q. In Appendix C we prove a projection theorem which implies that matrix elements of $\vec{\tau}$ and $\hat{q}$ in the χ basis are related such that

$$\langle I'I_z'\eta\,|\,\tau_i\,|\,II_z\eta\rangle=\eta\langle I'I_z'\eta\,|\,\hat{q}_i\,|\,II_z\eta\rangle\,. \tag{2.50}$$

This relation will turn out to be important later. Roughly speaking, the χ's are a basis for which $\underline{\tau}$ is either parallel or antiparallel to $\hat{\underline{q}}$. Equation (2.50) is the matrix element representation of this statement.

It is straightforward to establish the relationship between the Φ's of Eq. (2.46) and the χ's defined by Eq. (2.48) by noting that $\underline{\tau}\cdot\hat{\underline{q}}$ is an operator which satisfies the following relation:

$$\Phi(\uparrow)=\underline{\tau}\cdot\hat{\underline{q}}\,\Phi(\downarrow)$$

and therfore

$$\chi_{\eta=\pm1}=\frac{1}{\sqrt{2}}[\Phi(\uparrow)\pm\Phi(\downarrow)]\,.$$

In the discussion to follow we refer to the states with $\eta=+1$ as the low-lying states and with $\eta=-1$ as the high-lying states. The only term in H_c which induces transitions between the low-lying and high-lying states is the centrifugal barrier. We now project H_c onto the subspace of states diagonal in η so that the Schrödinger equation for that part of the collective Hamiltonian is given by

$$\frac{1}{2}\left[K(q)+\frac{I(I+1)+0.25}{q^2}+\Omega^2q^2-2g\lambda q\right]Q_{I,n}^{(+)}(q)=\epsilon_{I,n}^{(+)}Q_{I,n}^{(+)}(q)$$

for the low-lying states and

$$\frac{1}{2}\left[K(q)+\frac{I(I+1)+0.25}{q^2}+\Omega^2q^2+2g\lambda q\right]Q_{I,n}^{(-)}(q)=\epsilon_{I,n}^{(-)}Q_{I,n}^{(-)}(q) \tag{2.51a}$$

for the high-lying states, where $Q_{I,n}^{(\pm)}$ are radial wave functions for $\eta=\pm1$ and the subscript n is a vibrational quantum number. We will treat the off-diagonal elements in η of the centrifugal barrier as a perturbation in the next section. It is convenient to plot the potentials in Eqs. (2.51a), which are defined by

$$V^{(\pm)}(q)=\frac{\Omega^2q^2}{2}\mp g\lambda q \tag{2.51b}$$

and the centrifugal barrier. This is presented in Fig. 2. The low-lying potential $V^{(+)}$ has a minimum when

$$q_0=g\lambda/\Omega^2 \tag{2.52}$$

which is the classical displacement. We now consider the situation when the classical displacement q_0 is much greater than the quantum fluctuation in q, namely, we define a quantity x_0 such that

$$x_0=q_0(2\Omega)^{1/2}\gg 1\ . \tag{2.53}$$

Using the limiting forms of Ω and λ defined by Eqs. (2.29) and (2.30) this becomes

$$g\gg(8\pi^2\mu R)^{1/4}=\bar{R}\ , \tag{2.54}$$

where $\mu R\ll 1$. This is the definition of the strong-coupling approximation.

While it is straightforward to numerically solve Eq. (2.51a) for the radial eigenfunctions and eigenvalues, it is possible to infer several results from Fig. 2. For example, when the classical displacement is much larger than the quantum fluctuation in q, the low-lying radial wave functions are localized about q_0. Since the centrifugal barrier effects the radial wave functions in the neighborhood of the origin, then for large displacements the low-lying wave functions approach the wave functions of a one-dimensional harmonic oscillator with a linear coupling potential. Therefore, the ground-state expectation value of q is approximately q_0 and the ground-state energy is approximately given by

$$\epsilon_{1/2,0}^{(+)}\cong\frac{\Omega}{2}+\frac{1}{2q_0{}^2}-\frac{g^2\lambda^2}{2\Omega^2}\ ,$$

where the first term is the zero-point energy, the second is the centrifugal energy, and the last term is the potential energy at its minimum. Furthermore, the high-lying potential, $V^{(-)}$, is steeply sloped in the neighborhood of the origin so that the high-lying radial wave functions are localized within this neighborhood. Hence the overlap between the low-lying and high-lying radial wave functions is approximately given by

$$\langle Q_{1/2,0}^{(+)}\,|\,Q_{1/2,0}^{(-)}\rangle\cong N_0e^{-x_0{}^2}\ , \tag{2.55}$$

where N_0 is a factor of order 1 and x_0 is defined by Eq. (2.53).

The energy gap between the low-lying and high-lying states is roughly given by the minimum value of the low-lying potential, $V^{(+)}$, namely,

$$E_{\text{gap}}=\epsilon_{1/2,0}^{(+)}-\epsilon_{1/2,0}^{(-)}\cong-\frac{g^2\lambda^2}{2\Omega^2}\ . \tag{2.56}$$

The energy spacing between the ground state and the first rotational excitation is approximately given by

$$\delta E\cong\frac{3}{2}\frac{1}{q_0{}^2}\ , \tag{2.57}$$

where $q_0{}^2$ is related to the moment of inertia of the meson cloud. Note that for the sufficiently large displacements the rotationally excited levels in isospin space become degenerate with the ground state. Finally, the first low-lying vibrational level is of the order of the collective frequency, Ω, above the ground state.

To summarize, the eigenfunctions of the diagonal projection in η of the collective Hamiltonian, H_c, and the meson Hamiltonian, H', are of the form

$$|\,II_zn(\eta=\pm1)\alpha'^{\text{in}}_{\text{out}}\rangle=Q_{I,n}^{(\pm)}(q)\chi_{II_z(\eta=\pm1)}\,|\,\alpha'^{\text{in}}_{\text{out}}\rangle\ , \tag{2.58}$$

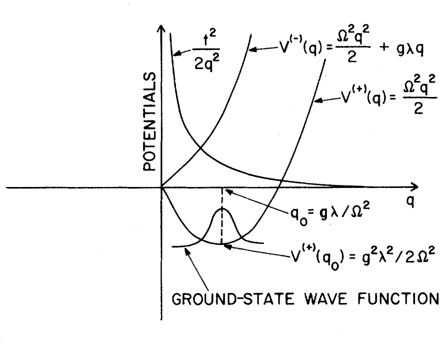

FIG. 2. A plot of the potentials $V^{(\pm)}$, as a function of the radial component of the collective meson variable q.

where $Q_{I,n}^{(\pm)}$ are solutions of Eqs. (2.51a), n is a vibrational quantum number, the $\mathcal{X}$'s are eigenfunctions of the operator $\underline{\tau}\cdot\hat{\underline{q}}$, and $|\alpha'^{\text{in}}_{\text{out}}\rangle$ are eigenfunctions of H' with α' the meson quantum numbers. The eigenvalues of the states in Eq. (2.58) are given by

$$E^{(0)}_{In(\eta)n'}=M_0+\epsilon^{(\eta)}_{I,n}-E_{\text{shift}}+\sum_{i=1}^{n'}\omega(k_i)\ , \qquad (2.59)$$

where $\epsilon^{(\eta)}_{I,n}$ are the eigenvalues of Eqs. (2.51a), E_{shift} is defined by Eq. (2.41), and $\omega(k_i)$ is the usual meson energy with n' the number of mesons in the state $|\alpha'^{\text{in}}_{\text{out}}\rangle$. For strong coupling, the low-lying collective energy eigenvalues are approximately given by

$$\epsilon^{(\eta=1)}_{In}\cong\Omega_0(n+\tfrac{1}{2})+\frac{[I(I+1)+0.25]}{2q_0^{\,2}}-\frac{g^2\cdot\lambda^2}{2\Omega^2} \qquad (2.60)$$

and upon substituting the limiting forms for Ω and λ defined by Eqs. (2.29) and (2.30) we have

$$\epsilon^{(\eta=1)}_{In}\cong\mu\left[\frac{2}{\mu R}\right]^{1/2}(n+\tfrac{1}{2})+\frac{4\pi\mu[I(I+1)+0.25]}{g^2}-\frac{g^2}{8\pi R}\ . \qquad (2.61)$$

In the next section we discuss the perturbation corrections and present specific numerical results.

III. PERTURBATION THEORY

Returning to the form of the Hamiltonian defined by Eq. (2.14) we have

$$H=H_0+H_{\text{pert}}\ , \qquad (3.1)$$

where

$$H_0=M_0+\tfrac{1}{2}\left[K(q)+\frac{t_{\text{d}}^{\,2}}{q^2}+\Omega^2q^2-2g\lambda\underline{\tau}\cdot\underline{q}\right]+H'$$

is the unperturbed Hamiltonian with eigenvalues given by Eq. (2.59) and

$$H_{\text{pert}}=\frac{1}{2}\frac{t_{\text{od}}^{\,2}}{q^2}+N\int d^3r\,U(r)\phi_i'(\vec{r})q_i-g\tau_i\int d^3r\,U(r)\phi_i'(\vec{r})$$

is the perturbation. The subscripts d and od on t^2 refer to the parts of t which are diagonal and off-diagonal in η, respectively.

Since the radial component of $\underline{q}$ fluctuates about q_0, which is given by

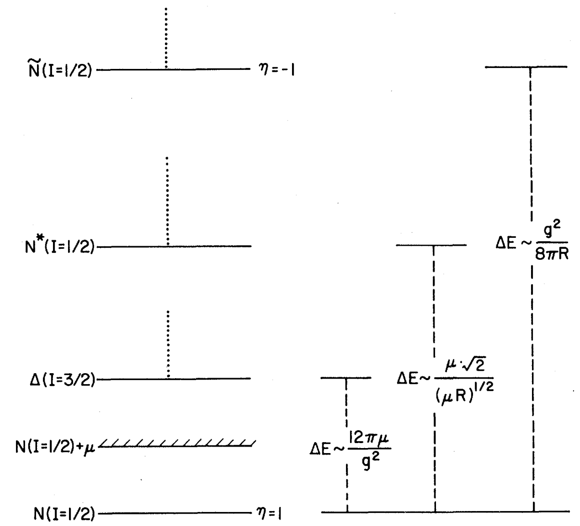

FIG. 3. Schematic representation of the energy levels of the unperturbed Hamiltonian H_0. Indicated are the parameters which characterize the energy spacing of the levels relative to the ground state in the strong-coupling approximation.

$$q_0=\frac{g}{N}$$

for our choice of F, we add and subtract a term in H_{pert} such that

$$H_{\text{pert}}=\frac{1}{2}\frac{t_{\text{od}}^{\,2}}{q^2}+N\int d^3r\,U(r)\phi_i'(\vec{r})\hat{q}_i(q-q_0)-g\cdot(\tau_i-\hat{q}_i)\int d^3r\,U(r)\phi_i'(\vec{r})\ . \qquad (3.2)$$

In order to facilitate the discussion of the perturbative effects of H_{pert} we refer to the energy-level diagram in Fig. 3. Note that there are no first-order energy corrections. The centrifugal term connects the low-lying and high-lying states. In doing so, this operator conserves the dressed-nucleon isospin I and I_z. It also conserves the meson number, n'. However, it can induce transitions between the low-lying and high-lying states with a change in the vibrational quantum number, n. Roughly speaking, the second-order corrections of the centrifugal term have large energy denominators of the order of E_{gap} defined by Eq. (2.56) and numerators which involve the overlap of the radial functions $Q^{(+)}_{1/2,0}$ and $Q^{(-)}_{1/2,n}$ which are small in the strong-coupling approximation.

The next term, which is linear in the meson field

ϕ_i' and the unit vector $\hat{q}_i$ of the collective meson variable, has no matrix elements between the low-lying and high-lying states. This is a consequence of the orthogonality property of the χ's defined by Eq. (2.49). However, the matrix elements of this operator involve states which differ by one meson. Furthermore, the presence of the operators $\hat{q}_i$ and $q-q_0$ implies that this term can induce rotational excitations in isospin space and vibrational excitations. Symbolically, this term can induce the following transitions from the ground state:

$$N \rightarrow N + \text{one meson} ,$$

$$N \rightarrow \Delta(I=\tfrac{3}{2}) + \text{one meson} ,$$

$$N \rightarrow N^*(I=\tfrac{1}{2}) + \text{one meson} ,$$

etc. As we pointed out earlier in the strong-coupling approximation the low-lying radial wave functions $Q_{I,n}^{(+)}$ approach the wave functions of a one-dimensional harmonic oscillator with a linear coupling potential. This implies that the matrix elements which correspond to the above transitions are small since they involve radial matrix elements of $q-q_0$. Furthermore, the vibrational excitations, such as the $N^*(I=\frac{1}{2})$, are characterized by a large energy spacing above the ground state of the order of Ω which results in large energy denominators in perturbation theory.

The final term of Eq. (3.2) has matrix elements between the low-lying and high-lying states only. This is a consequence of the projection theorem defined by Eq. (2.50). The transition matrix elements of this term differ by one meson and involve the overlap of the low-lying and high-lying radial wave functions. Again, the energy spacing is of the order of E_{gap} so that the second-order energy corrections are small in the strong-coupling approximation.

In order to make these intuitive arguments more precise, we present numerical results for the nucleon self-energy which is defined as

$$\epsilon_{1/2,0}^{(1)} - E_{\text{shift}} + E^{(2)} , \tag{3.3}$$

where $E^{(2)}$ is the second-order energy correction, in Fig. 4(a), the change in the nucleon wave-function normalization in Fig. 4(b), and the difference between the Δ and nucleon energies in Fig. 4(c). Each curve is determined for a fixed value of the bare coupling constant g, and plotted as a function of $\overline{R}$ defined by Eq. (2.54). In evaluating the second-order energy corrections and the change in the wave-function normalization, we have included contributions from the first three excited states for each of the terms of H_{pert} defined by Eq. (3.2). The difference between Δ and nucleon masses is a lowest-order calculation.

It is clear from the figures that the wave function[11] improves as the coupling strength increases according to the inequality defined by Eq. (2.54). Furthermore, it is possible to find values of the bare coupling constant g and the source size R such that the Δ is stable, i.e., it is lower in energy than the meson-nucleon threshold. This is due to the fact that in the strong-coupling approximation the Δ is a collective rotational excitation in isospin space of the meson cloud which dresses the bare nucleon. As the coupling strength increases, the average number of mesons which dress the bare nucleon increases. This increases the moment of inertia of the meson cloud and therefore decreases the rotational energy of the system in isospin space. If the coupling is not too strong then the Δ is unstable and it couples to the meson-nucleon continuum. We will return to the description of the physical Δ in this model when we discuss the meson-nucleon scattering problem in Sec. V.

IV. MODIFIED HAMILTONIAN WITH ISOBAR SOURCE

It is possible to generalize the model Hamiltonian defined by Eq. (2.1) so as to include a feature of the CBM Hamiltonian, namely, an intrinsic isobar source coupled to the meson field.[2] We assume that the nucleon and Δ have only isospin degrees of freedom which are coupled linearly to an isovector-scalar meson field such that

$$H = M_0 + \tfrac{1}{2}\int d^3r [\pi_i^2(\vec{r}) + \phi_i(\vec{r})(-\nabla^2+\mu^2)\phi_i(\vec{r})] - g\tau_i \int d^3r\, U(r)\phi_i(\vec{r}) , \tag{4.1}$$

where M_0 is now a diagonal 6×6 matrix with degenerate eigenvalues and the matrices τ_i are now given by

$$\tau_i = \begin{bmatrix} \tau_i^{NN} & a\tau_i^{N\Delta} \\ a\tau_i^{\Delta N} & b\tau_i^{\Delta\Delta} \end{bmatrix} , \tag{4.2}$$

where the matrices $\tau_i^{\alpha\beta}$ are defined in Ref. 12. The coupling constants a and b have the SU(4)-quark-model values given by

$$a = (\tfrac{72}{25})^{1/2};\ b = \tfrac{4}{5} . \tag{4.3}$$

All remaining quantities of Eq. (4.1) are the same as

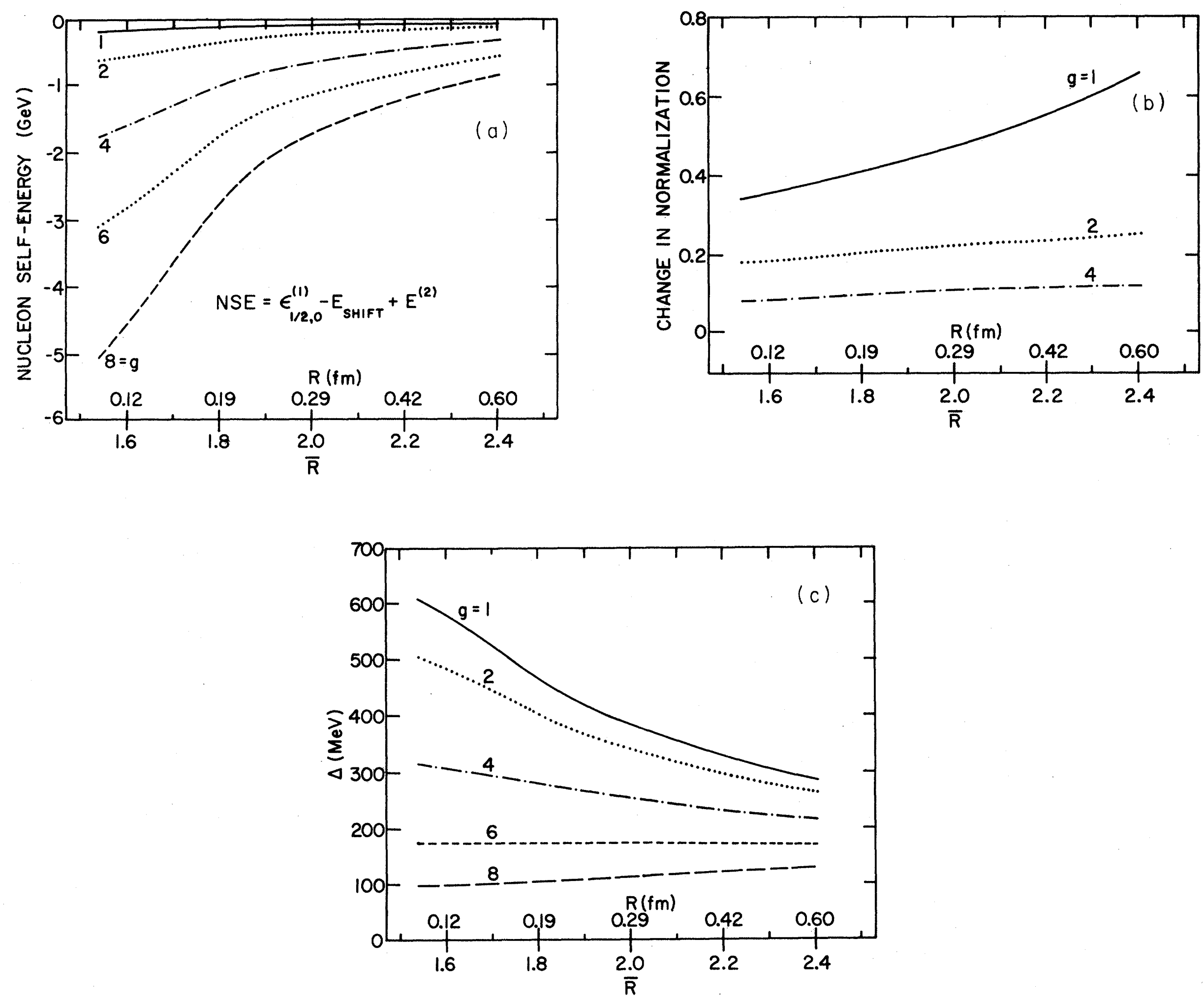

FIG. 4. (a) A plot of the nucleon self-energy (NSE) as a function of $\bar{R}$ the bare coupling constant g. For convenience, we have included the corresponding bag-source size R, for each $\bar{R}$. The NSE depends on three contributions, namely, the collective energy, $\epsilon^{(1)}_{1/2,0}$, E_{shift} and the second-order energy correction due to H_{pert}. (b) A plot of the change in the wave-function normalization for the nucleon state as a function of $\bar{R}$ and the bare coupling constant g. The change in normalization is computed to second order in H_{pert}. (c) A plot of the lowest-order approximation to the Δ-nucleon mass difference, as a function of $\bar{R}$ and the bare coupling constant g.

previously defined.

The method of calculation for the Hamiltonian defined by Eq. (4.1) remains unchanged, however the consequences are quite different. To see this, consider the eigenvalue equation

$$\underline{\tau}\cdot\hat{\underline{q}}\chi_{\eta_I}(\Omega_q)=\eta_I\chi_{\eta_I}(\Omega_q)\ , \tag{4.4}$$

where the χ implicitly depend on the dressed-nucleon isospin which is now given by

$$\underline{I}=\underline{t}+\begin{bmatrix} \dfrac{\underline{\tau}^{NN}}{2} & 0 \\ 0 & \dfrac{\underline{t}^{\Delta\Delta}}{2} \end{bmatrix}$$

with $\underline{t}$ defined by Eq. (2.20). Note that the eigenvalue η_I depends on I. It is convenient to expand the χ eigenfunctions in a basis which couples the

TABLE I. Tabulation of the probability amplitudes defined by $|\Psi\rangle=\sum_M C(M)|M\rangle$ for various components of the dressed nucleon and Δ states for the modified Hamiltonian with an isobar source. $N^{(0)}=$ bare nucleon, $\Delta^{(0)}=$ bare Δ, and $t=$ collective meson isospin.

Amplitude	$\eta^{(1)}_{1/1}$	$\eta^{(2)}_{1/2}$	$\eta^{(3)}_{1/2}$	$\eta^{(4)}_{1/2}$		
$C(N^{(0)};t=0)$	0.57	0.43	0.43	-0.57		
$C(N^{(0)};t=1)$	-0.57	0.43	-0.43	-0.57		
$C(\Delta^{(0)};t=1)$	-0.43	-0.57	0.57	-0.43		
$C(\Delta^{(0)};t=2)$	0.43	-0.57	-0.57	-0.43		
Amplitude	$\eta^{(1)}_{3/2}$	$\eta^{(2)}_{3/2}$	$\eta^{(3)}_{3/2}$	$\eta^{(4)}_{3/2}$	$\eta^{(5)}_{3/2}$	$\eta^{(6)}_{3/2}$
$C(N^{(0)};t=0)$	0.57	-0.43	0	0	-0.43	0.57
$C(N^{(0)};t=1)$	-0.57	-0.43	0	0	-0.43	-0.57
$C(\Delta^{(0)};t=0)$	0.30	-0.40	-0.5	-0.5	-0.40	0.30
$C(\Delta^{(0)};t=1)$	-0.13	-0.18	0.67	-0.67	-0.18	-0.13
$C(\Delta^{(1)};t=2)$	-0.30	0.40	-0.5	-0.5	0.40	-0.30
$C(\Delta^{(0)};t=3)$	0.40	0.54	0.22	0.22	0.54	0.40

bare nucleon and Δ isospinors to the collective-meson-isospin wave functions. For $I=\frac{1}{2}$, χ is a linear combination of four basis functions. Two of these couple the bare nucleon isospinor to $t=0$ and $t=1$ and the remaining two couple the bare Δ isospinor to $t=1$ and $t=2$. There are four eigenvalues $\eta_{1/2}$, given by

$$\eta_{1/2}(1)=2.0422\ ,\quad \eta_{1/2}(2)=0.842\,22\ ,\quad \eta_{1/2}(3)=-0.842\,22\ ,\quad \eta_{1/2}(4)=-2.0422\ . \tag{4.5}$$

The lowest-lying states have the eigenvalue $\eta_{1/2}(1)$. A straightforward computation indicates that the dressed nucleon state has a total bare-nucleon-plus-meson-cloud component of 64% and a total bare-Δ-plus-meson-cloud component of 36%. The detailed proportions are presented in Table I. For $I\geq\frac{3}{2}$, the χ's are a linear combination of six basis functions. Two of these involve the bare-nucleon components and the remaining four involve the bare-Δ components. There are six η_I eigenvalues given by

$$\eta_I(1)=2.0422\ ,\quad \eta_I(2)=0.842\,22\ ,\quad \eta_I(3)=0.6000\ ,\quad \eta_I(4)=-0.6000\ ,\quad \eta_I(5)=-0.842\,22\ ,\quad \eta_I(6)=-2.0422\ . \tag{4.6}$$

In contrast to the previous model, the classical displacement q_0, is larger by approximately a factor of 2. This is a consequence of the value of $\eta_I(1)$. The strong-coupling condition defined by Eq. (2.54) is now replaced by the inequality

$$g \gg \sqrt{4}\pi\left[\frac{\mu R}{2}\right]^{1/4}/\eta_I(1)=\bar{R}'\ . \tag{4.7}$$

Therefore the strong-coupling approximation for this model is realized for larger R or smaller g than in the previous model. This is to be expected since the isobar-source terms of the Hamiltonian defined by Eq. (4.1) add more attraction when compared to the previous Hamiltonian defined by Eq. (2.1).

The perturbation theory computations for the model Hamiltonian defined by Eq. (4.1) proceed in the same manner as outlined in Sec. III. We restrict the unperturbed eigenfunctions to the subspace of states with eigenvalues $\eta_I(1)$ and $\eta_I(2)$. This is a valid approximation since states with $\eta_I(i)$ and $i\geq 3$ are much higher in energy. The detailed numerical results are presented in Figs. 5(a)–5(c). These results are to be compared to the results in Figs. 4(a)–4(c).

V. MESON-NUCLEON SCATTERING

In this section, we perform meson-nucleon scattering calculations for the model Hamiltonian defined by Eq. (2.1). The method is sufficiently general so that various models can be studied, such as the CBM or the LBM. We shall adopt the elegant and lucid formalism of Feshbach.[13]

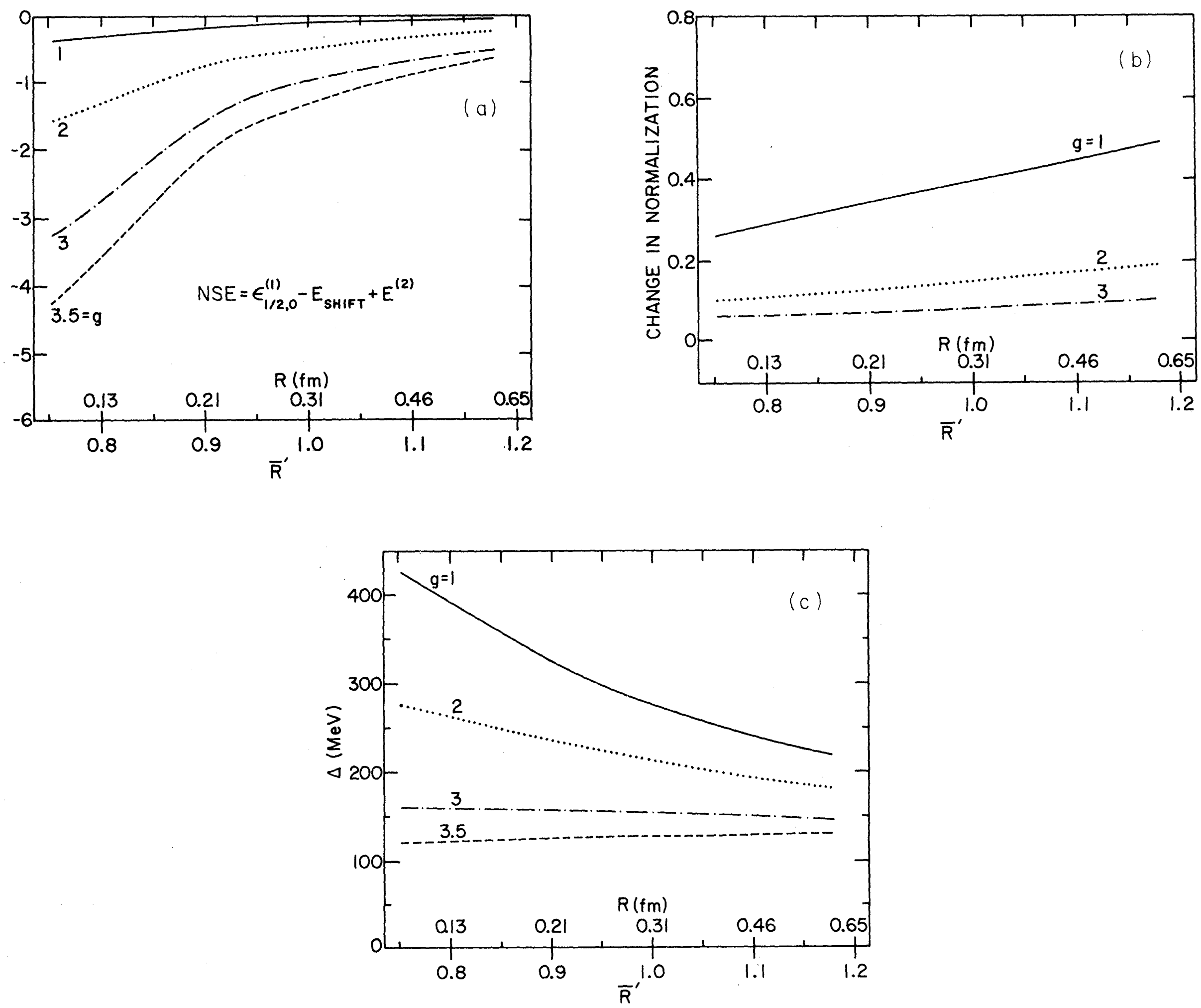

FIG. 5. (a) A plot of the NSE for the modified model Hamiltonian as a function of $\bar{R}'$ and the bare coupling constant g. (b) A plot of the change in the wave-function normalization for the nucleon state as a function of $\bar{R}'$ and the bare coupling constant g. (c) A plot of the lowest-order approximation to the Δ-nucleon mass difference as a function of $\bar{R}'$ and the bare coupling constant g.

Following Feshbach, we define a set of open-channel projection operators P and a set of closed-channel projection operators Q, which provide a representation of the Schrödinger equation of the form

$$\left[H_{PP}+H_{PQ}\frac{1}{E^{(+)}-H_{QQ}}H_{QP}-E\right]P\,|\,\Psi^{(+)}\rangle=0\ , \tag{5.1}$$

where the effects of the "closed channels" are included in an "effective potential" defined by

$$U_{PP}^{\text{eff}}=H_{PQ}\frac{1}{E^{(+)}-H_{QQ}}H_{QP} \tag{5.2}$$

with the total energy given by

$$E=M_N+\omega(k)\ .$$

The physical nucleon mass M_N is obtained from Eq. (2.59) by making an appropriate choice of the bare mass M_0. For the explicit construction of the projection operators, we choose the basis states defined by Eq. (2.58). More specifically, we restrict the total energy E in Eq. (5.1) such that only elastic and charge-exchange meson-nucleon scattering can occur. Therefore, the open-channel projection operators are given by

$$P^{(\text{in,out})}=\sum_{I_zT_z}\int d^3k\,|N,I_z,\vec{k},T_z(^{\text{in}}_{\text{out}})\rangle\langle N,I_z,\vec{k},T_z(^{\text{in}}_{\text{out}})|\ , \tag{5.3}$$

where the meson-nucleon states are obtained from Eq. (2.58) and the closed-channel projection operators are given by

$$Q^{(\text{in,out})}=1-P^{(\text{in,out})} \tag{5.4}$$

which include states such as the nucleon, the Δ, the nucleon plus two mesons, etc. It follows from the completeness of the in and out scattering states defined by Eq. (2.58), that the in and out projection operators are equivalent, namely,

$$P^{(\text{in})}=P^{(\text{out})}\ .$$

In what follows, we suppress the in and out labels on the projection operators. Using the eigenfunctions of H_{QQ} given by Eq. (2.58), we obtain an expansion of the effective potential U^{eff} defined by Eq. (5.2) of the form

$$U_{PP}^{\text{eff}}=\sum_{\alpha}H_{PQ}\frac{|\alpha\rangle\langle\alpha|}{E-E_{\alpha}^{(0)}}H_{QP}+\sum_{\beta}\int dE^{(0)'}H_{PQ}\frac{|\beta,E^{(0)'}\rangle\langle\beta,E^{(0)'}|}{E^{(+)}-E^{(0)'}}H_{QP}\ , \tag{5.5}$$

where the first summation is over discrete states starting in energy with the nucleon state and the second summation is over the continuum states which starts at the two-meson-plus-nucleon threshold.

We now assume that the total energy E is such that the nucleon and Δ states in the discrete summation are the most important. This implies that $Q\cong Q^N+Q^\Delta$ where Q^N and Q^Δ are the nucleon and Δ projection operators, respectively. Hence, we neglect the two-meson cut, etc. The effective potential is now Hermitian and of the form

$$U_{PP}^{\text{eff}}\cong U_{PP}^{N}+U_{PP}^{\Delta}\ , \tag{5.6}$$

where the nucleon-pole contribution is given by

$$U_{PP}^{N}=H_{PQ}^{N}\frac{1}{E-M_N}H_{QP}^{N}\ , \tag{5.7}$$

where $H_{PQ}^N=PHQ^N$ and the Δ contribution is given by

$$U_{PP}^{\Delta}=H_{PQ}^{\Delta}\frac{1}{E-M_\Delta}H_{QP}^{\Delta}\ , \tag{5.8}$$

where $H_{PQ}^\Delta=PHQ^\Delta$. At this level of approximation, the only operator in H_{pert} defined by Eq. (3.2) which contributes to the scattering is given by

$$H_s=N\int d^3r\,U(r)\phi_i'(\vec{r})\underline{\hat{q}}_i(q-q_0)\ . \tag{5.9}$$

This is a consequence of the fact that H_s is the only operator in H_{pert} which has nonvanishing matrix elements within the subspace of low-lying states.

At this point it is useful to describe the various scattering contributions. As we pointed out earlier, the meson-nucleon state defined by Eq. (2.58) contains scattering information. This is a consequence of the diagonalization of H' in Sec. II A which implied that there exists in and out meson creation and annihilation operators. The one meson in state described the elastic scattering of a meson from a background potential which is related to the field produced by the source. The elastic t matrix for this process is given by Eq. (2.35). In addition to this background scattering, there is the so-called "direct-reaction" part that arises in our approximation from the nucleon-pole contribution to the effective potential defined by Eq. (5.2). Unlike the previous contribution, the nucleon-pole term contributes to elastic and charge-exchange scattering. Since the nucleon state defined by Eq. (2.58) couples to the off-energy-shell meson-nucleon states via H_s defined by Eq. (5.9), then the nucleon mass, M_N, will acquire an additional self-energy contribution with real and imaginary parts. Finally, there is the resonant part of the scattering which is a consequence of the Δ contribution to the effective potential. Again, the fact that the Δ couples to the meson-nucleon continuum implies, that the Δ mass M_Δ, acquires an additional real part and a width.

To see this more clearly, in Appendix D we determine the total T matrix given by

$$T(I_z',T_z',\vec{k}';I_z,T_z,\vec{k})=\delta_{I_z'I_z}\delta_{T_z'T_z}t(\vec{k}',\vec{k})+T_d(I_z',T_z',\vec{k}';I_z,T_z,\vec{k})+T_R(I_z',T_z',\vec{k}';I_z,T_z,\vec{k})\ , \tag{5.10}$$

where $t(\vec{k}',\vec{k})$ is given by Eq. (2.35), the direct-reaction T matrix T_d is given by

$$T_d(I_z',T_z',\vec{k}';I_z,T_z,\vec{k})=\langle N,I_z',T_z',\vec{k}'\text{out}\,|\,U_{PP}^{N}\,|\,\Phi,\vec{k},\text{in}\rangle \tag{5.11}$$

and the resonant T matrix, T_R, is given by

$$T_R(I_z',T_z',\vec{k}';I_z,T_z,\vec{k})=\langle \Phi,\vec{k}',\text{out}\,|\,H_{PQ}^{\Delta}\frac{1}{E-M_\Delta-H_{QP}^{\Delta}\dfrac{1}{E^{(+)}-H_{PP}-U_{PP}^{N}}H_{PQ}^{\Delta}}H_{QP}^{\Delta}\,|\,\Phi,\vec{k},\text{in}\rangle \tag{5.12}$$

with the scattering wave function $|\Phi\rangle$ is given by

$$|\Phi,\vec{k},(^{\text{in}}_{\text{out}})\rangle=|N,I_z,T_z,\vec{k},(^{\text{in}}_{\text{out}})\rangle+\frac{1}{E^{(\pm)}-H_{PP}}H_{PQ}^{N}\frac{1}{E-M_N-H_{QP}^{N}\dfrac{1}{E^{(+)}-H_{PP}}H_{PQ}^{N}}H_{QP}^{N}\,|N,I_z,T_z,\vec{k},(^{\text{in}}_{\text{out}})\rangle\ . \tag{5.13}$$

From the form of U_{PP}^N defined by Eq. (5.7) and its dependence on H_s defined by Eq. (5.9), it is clear that T_d defined by Eq. (5.11) contributes to elastic- and charge-exchange scatterings. Furthermore, the form of the energy denominator in Eq. (5.13), demonstrates that the nucleon mass M_N, acquires an additional self-energy with real and imaginary parts. Finally, we note the characteristic resonant energy denominator of T_R defined by Eq. (5.12) and that the width of the resonance depends on the off-energy-shell meson-nucleon scattering via U_{PP}^N.

For the model Hamiltonian defined by Eq. (2.1), there is only s-wave scattering and therefore the T matrix only depends on k. As a special application of our formulas, we consider elastic scattering for the state $I_z=\frac{1}{2}$ and $T_z=1$, which corresponds to scattering in the 3 channel. We now discuss in sequence the various terms which contribute to Eq. (5.10). In Appendix A it was shown that the partial-wave amplitude for the background scattering t matrix, $t(k^2)$, defined by Eq. (2.35) is of the form

$$\eta_0=e^{i\delta_0(k)}\sin\delta_0(k)=\frac{2\pi^2kN^2\tilde{U}^2(kR)}{\omega^2(k)-\dfrac{N^2}{4\pi R}f(k,R)}\ , \tag{5.14}$$

where N is defined by Eq. (2.24), $f(k,R)$ is defined by Eq. (A8), and $\tilde{U}$ is the Fourier transform of the source function defined by Eq. (2.2). In the low-energy limit the s-wave phase shift is approximately given by

$$\delta_0(k)\cong-\frac{2k}{\mu} \tag{5.15}$$

which is similar to the hard-sphere scattering result. This result implies that the background scattering contribution from $t(k^2)$ is repulsive and that the cross section is finite in the limit $k\to 0$.

For the 3 channel, the direct-reaction part, T_d, defined by Eq. (5.11) is zero. This is a consequence of isospin conservation which implies that

$$H_{QP}^N\,|N,\tfrac{1}{2},1,\vec{k},(^{\text{in}}_{\text{out}})\rangle=0\ . \tag{5.16}$$

Using this result, Eq. (5.14) becomes

$$|\Phi,\vec{k},(^{\text{in}}_{\text{out}})\rangle=|N,\tfrac{1}{2},1,\vec{k},(^{\text{in}}_{\text{out}})\rangle$$

which simplifies the evaluation of T_R defined by Eq. (5.12). We will not present a detailed evaluation of T_R which is straightforward, but tedious. Instead, we will concentrate on the physical description of the interplay between the background and resonance scattering.

In Figs. 6(a)–6(d) we plot the contributions to elastic scattering in the 3 channel from background scattering, resonance scattering, and both contributions taken together for various values of the bare coupling constant g, and source size R. Since the scattering is s wave, the background scattering which is a consequence of $t(k^2)$ defined by Eq. (2.35) is large at low energy and decreases to zero at high energy. Furthermore, this scattering is repulsive. On the other hand, the resonant scattering which is a consequence of T_R defined by Eq. (5.12) increases with increasing energy until it reaches a maximum, when the resonant phase shift passes through $\pi/2$. As a function of the bare coupling constants g and R, note that as the coupling increases, the width of the resonant peak decreases, and the position of the peak moves to lower energy. These features are consistent with our previous results, namely, that the Δ becomes more stable as the coupling strength increases. When the background and resonant scatterings are taken together as in Eq. (5.10), the repulsive background scattering overwhelms the resonant scattering such that the total s-wave phase shift does not pass through $\pi/2$. However, the bump which appears in the cross section is due to the presence of a resonance. Finally, in Fig. 7 we plot the width of the Δ mass as a function of the meson laboratory energy for various values of the bare coupling constant.

For the case of the CBM or LBM the above description will change since the scattering proceeds through a p wave. The threshold factors for p-wave

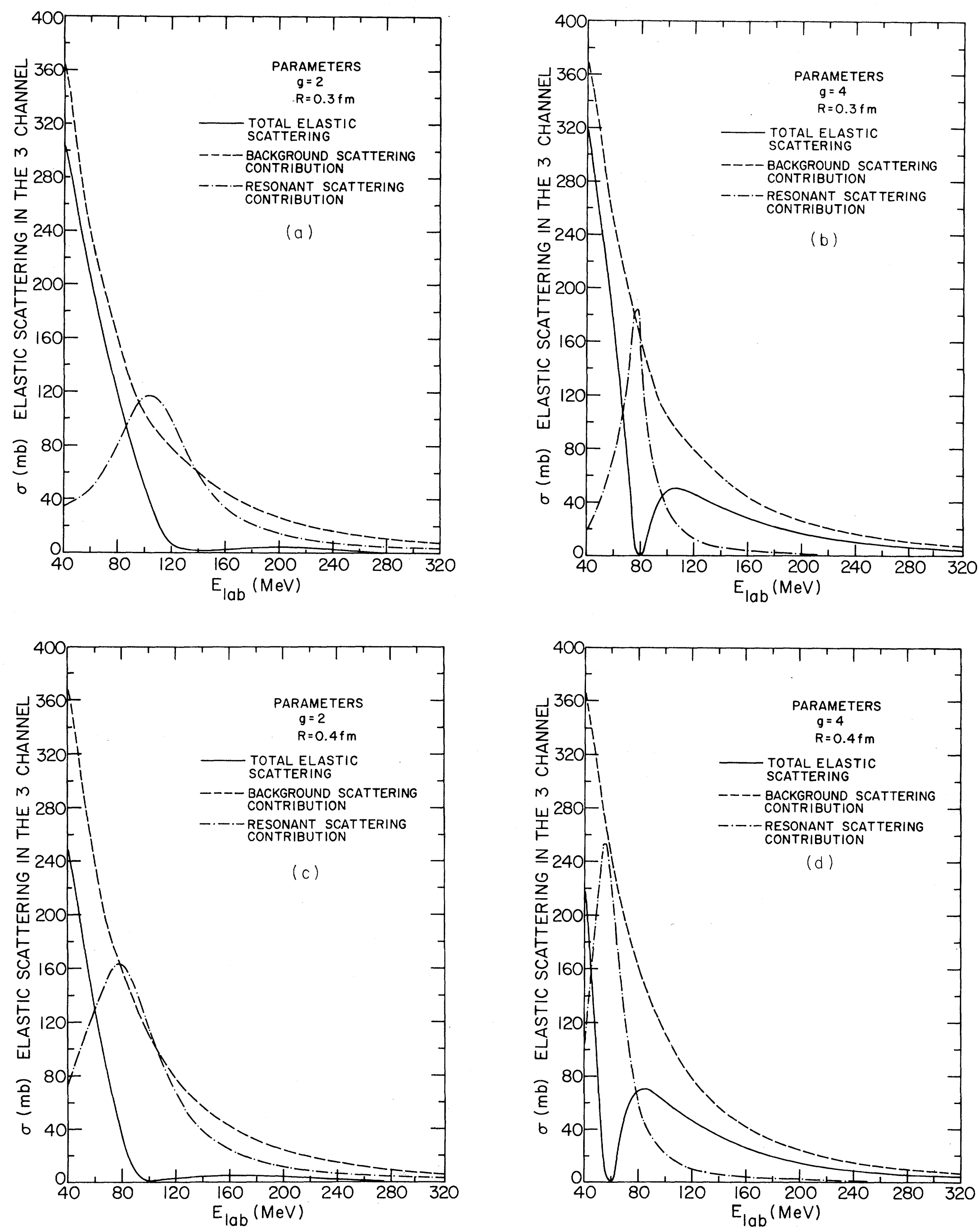

FIG. 6. Plots of the elastic meson-nucleon cross section for the model Hamiltonian defined by Eq. (2.1) in the total-isospin $\frac{3}{2}$ and z-component $\frac{3}{2}$ channel as a function of the meson energy. The solid curves correspond to the total T-matrix contribution, the dashed curves correspond to the background-scattering contribution, and the dash-dot curves correspond to the resonant-scattering contribution.

scattering will suppress the background scattering at low energy so that the resonance scattering will sit on top of the background.

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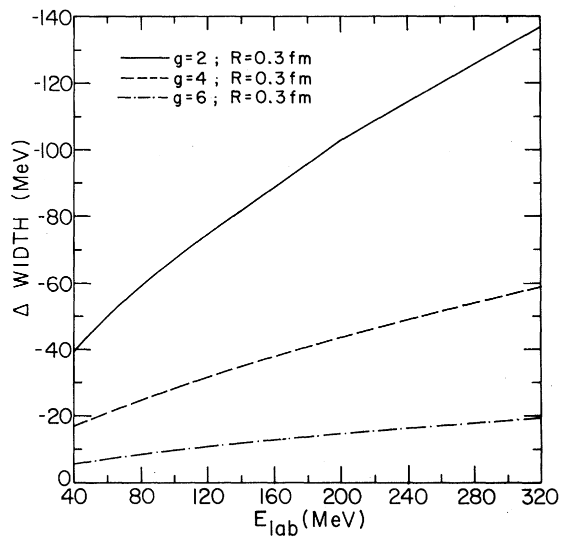

FIG. 7. A plot of the Δ width obtained from the resonant T matrix T_R, as a function of the meson laboratory energy for several values of the bare coupling constant g.

APPENDIX A

In this appendix we determine the eigenfunctions of the operator h defined by Eq. (2.32) and the pertinent scattering information. The relevant eigenvalue equation is given by

$$(-\nabla^2+\mu^2)\psi(\vec{r})-F(r)\int d^3r'[(-\nabla'^2+\mu^2)F(r')]\psi(\vec{r}\,')=\omega^2\psi(\vec{r})\,, \tag{A1}$$

where we have used the orthogonality condition, namely,

$$\int d^3r\, F(r)\psi(\vec{r})=0\,. \tag{A2}$$

Equation (A1) has no bound-state solutions which are orthogonal to $F(r)$, however it has two sets of scattering solutions with eigenvalues $\omega^2=k^2+\mu^2$. The scattering solutions with appropriate boundary conditions are given by Eq. (2.23a), where $C(k,\pm)$ in Eq. (2.33a) is defined by

$$C(k,\pm)=\int d^3r'[(-\nabla'^2+\mu^2)F(r')]\psi_k(\vec{r}\,',{}^{\rm in}_{\rm out}) \tag{A3}$$

and G_k in Eq. (2.33a) satisfies

$$[-\nabla^2+\mu^2-\omega^2(k)]G_k(\vec{r},\vec{r}\,',\pm)=-\delta^3(\vec{r}-\vec{r}\,')\,. \tag{A4}$$

Upon substituting Eq. (2.33a) into Eq. (A3) and using Eq. (A4) and the normalization condition for F which is given by Eq. (2.7), it is straightforward to show that $C(k,\pm)$ is given by Eq. (2.33b). This implies that Eq. (A2) is satisfied. For the choice of $F(r)$ defined by Eq. (2.22), a straightforward computation gives

$$\psi_k(\vec{r},{\rm in})=\frac{e^{i\vec{k}\cdot\vec{r}}}{(2\pi)^{3/2}}-\frac{\tilde{U}(kR)\Gamma_k(r,{\rm in})}{[\Gamma_k(R,{\rm in})-1/N^2]}\,, \tag{A5}$$

where $\tilde{U}$ is the Fourier transform of the source function defined by Eq. (2.2), N^2 is given by Eq. (2.25), and

$$\Gamma_k(r,{\rm in})=\Theta(R-r)\Gamma_k(r,{\rm in})_<+\Theta(r-R)\Gamma_k(r,{\rm in})_> \tag{A6}$$

with

$$\Gamma_k(r,{\rm in})_>=h_0(kr)I_1(k,r)+j_0(kr)I_2(k,r)$$

and

$$\Gamma_k(r,{\rm in})_<=h_0(kr)I_3(k,r)+j_0(kr)I_4(k,r)\,,$$

where h_0 and j_0 are the spherical Hankel and Bessel functions, respectively, and

$$I_1(k,r)=\frac{\mu R j_0(kR)}{\omega^2(k)}-\frac{e^{-\mu r}\sinh\mu R}{\omega^2(k)}\left[\cos kr+\frac{\mu}{k}\sin kr\right],$$

$$I_2(k,r)=\frac{\sinh\mu R}{k\omega^2(k)}(k-i\mu)e^{(ik-\mu)r}\,,$$

$$I_3(k,r)=\frac{e^{-\mu R}}{\omega^2(k)}\left[\frac{\mu}{k}\sin kr\cosh\mu r-\cos kr\sinh\mu r\right],$$

$$I_4(k,r)=\frac{e^{-\mu R}}{k\omega^2(k)}[e^{ikr}(k\sinh\mu r+i\mu\cosh\mu r)-i\mu e^{(ik+\mu)R}]\,.$$

Equation (A6) takes on a particular simple form when $r=R$, namely,

$$\Gamma_k(R,\text{in})=\frac{f_k(R,\text{in})}{\omega^2(k)4\pi R}\ , \tag{A7}$$

where

$$f_k(R,\text{in})=e^{ikR}j_0(kR)-e^{-\mu R}i_0(\mu R) \tag{A8}$$

with i_0 is a modified spherical Bessel function. The partial-wave amplitude can be obtained from the asymptotic behavior of Eq. (A5) or from Eq. (2.35). In either case the result is

$$\eta_0(k)=e^{i\delta_0(k)}\sin\delta_0(k)=\frac{2\pi^2\cdot k\cdot N^2\tilde{U}^2(kR)}{\omega^2(k)-\dfrac{N^2}{4\pi R}f_k(R,\text{in})}\ . \tag{A9}$$

From the form of $\eta_0(k)$ defined by Eq. (A9) and $f_k(R,\text{in})$ defined by Eq. (A8), we conclude that the partial-wave amplitude has a double pole when $k=i\mu$. However a careful examination of the scattering wave functions defined by Eq. (A5) demonstrates that the wave functions do not have these poles. Therefore they do not correspond to eigenvalues of the operator h defined by Eq. (2.32). The analytic structure of $\eta_0(k)$ is such that it has poles in the lower half of the complex k plane, namely, $\text{Im}k<0$. However the poles are much too far from the physical region of the scattering to have any sizable effect.

APPENDIX B

In this appendix we determine the ground-state or vacuum energy of the meson Hamiltonian H' defined by Eq. (2.31) subject to the constraints defined by Eqs. (2.8) and (2.9). The constraint conditions are satisfied for the expansion of $\phi_i'(\vec{r})$ and $\pi_i'(\vec{r})$ in the basis of scattering wave functions, $\psi_k(\vec{r},{}^{\text{in}}_{\text{out}})$, defined by Eq. (2.33a). For the discussion to follow, we will suppress the "in" and "out" labels since they are of no consequence. It follows from the eigenvalue equation of Eq. (A1) and the orthogonality condition of Eq. (A2), that the scattering wave functions defined by Eq. (2.33a) diagonalize the Klein-Gordon operator, namely,

$$\int d^2r\,\psi_{k'}^*(\vec{r})(-\nabla^2+\mu^2)\psi_k(\vec{r})=\omega^2(k)\delta^3(\vec{k}'-\vec{k})\ . \tag{B1}$$

Using the fact that the Green's function for the Klein-Gordon operator of Eq. (A4) and the Green's function for the square root of the Klein-Gordon operator are related, namely,

$$G_k^{1/2}(\vec{r},\vec{r}\,')=2\omega(k)G_k(\vec{r},\vec{r}\,') \tag{B2}$$

it follows from Eqs. (2.33a) and (2.33b) that the scattering wave functions defined by Eq. (2.33a) also diagonalize the square root of the Klein-Gordon operator such that

$$\int d^3r'd^3r\,\psi_{k'}^*(\vec{r}\,')f(\vec{r}\,',\vec{r})\psi_k(\vec{r})=\omega(k)\delta^3(\vec{k}'-\vec{k})\ , \tag{B3}$$

where

$$f(\vec{r}\,',\vec{r})=\int\frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot(\vec{r}-\vec{r}\,')}\omega(k)\ . \tag{B4}$$

The relation of Eq. (B3) will become useful shortly. Substituting the expansion of $\phi_i'(\vec{r})$, given by Eq. (2.36) and the corresponding expansion of $\pi_i'(\vec{r})$ into H' defined by Eq. (2.31) we have

$$H'=\tfrac{1}{2}\int d^3k\,d^3k'\int d^3r\,d^3r'\psi_k^*(\vec{r})f(\vec{r},\vec{r}\,')\psi_{k'}(\vec{r}\,')[a_i(\vec{k})a_i^\dagger(\vec{k}')+a_i^\dagger(\vec{k})a_i(\vec{k}')]\ , \tag{B5}$$

where we have used Eqs. (B1) and (B3) to obtain this form of H'. The vacuum state is defined such that

$$a_i(\vec{k})\,|\,\text{vac}\rangle=0\ .$$

It then follows from Eq. (B5), the commutation relations between $a_i(\vec{k})$ and $a_i^\dagger(\vec{k}')$, and the definition of $|\,\text{vac}\rangle$ that the vacuum energy is given by

$$E'_{\text{vac}}=\tfrac{3}{2}\int d^3k\,d^3r\,d^3r'\psi_k^*(\vec{r})f_\Lambda(\vec{r},\vec{r}\,')\psi_k(\vec{r}\,')\ , \tag{B6}$$

where the subscript Λ on f indicates that we have introduced a cutoff. If we now use the completeness relation, namely,

$$\int d^3k\,\psi_k^*(\vec{r})\psi_k(\vec{r}\,')=\delta^3(\vec{r}-\vec{r}\,')-F(r)F(r')\ , \tag{B7}$$

then the vacuum energy E_{vac} can be rewritten in the form

$$E'_{\text{vac}}=\tfrac{3}{2}\int d^3r\,f(\vec{r},\vec{r})-\tfrac{3}{2}\int d^3r\,d^3r'F(r)f(\vec{r},\vec{r}\,')F(r') \tag{B8}$$

which is equivalent to Eq. (2.40). Note that in Eq. (B8) we have taken the cutoff to infinity.

APPENDIX C

In this appendix we prove the projection theorem of Eq. (2.50). Suppose we are given two vector operators $\underline{Q}$ and $\hat{q}$, where $\underline{Q}$ is an $N\times N$ Hermitian matrix of vector operators and $\hat{q}$ is a unit vector operator. Consider the eigenvalue equation of the scalar product of these operators, namely,

$$\underline{Q}\cdot\hat{q}\chi_\eta(\Omega_1)=\eta\chi_\eta(\Omega_q)\ , \quad (C1)$$

where η are the eigenvalues, Ω_q is the solid angle subtended by $\hat{q}$, and the χ implicitly depend on a set of rotational quantum numbers, say I and I_z. We want to show that

$$\langle I'I_z'\eta\,|\,Q_i\,|\,II_z\eta\rangle=\eta\langle I'I_z'\eta\,|\,\hat{q}_i\,|\,II_z\eta\rangle \quad (C2)$$

for all I, I_z, I', I_z', and η. From the Hermiticity of the operator $\underline{Q}\cdot\hat{q}$ and Eq. (C1) it follows that

$$\chi^\dagger_{\eta'}(\Omega_q)\chi_\eta(\Omega_q)=0 \quad (C3)$$

for $\eta\neq\eta'$ and all angles, Ω_q. Taking the inner product of Eq. (C1) and inserting a complete set of states we have

$$\sum_{I'I_z'\kappa}(-1)^\kappa\langle II_z\eta\,|\,Q_\kappa^{(1)}\,|\,I'I_z'\eta\rangle\langle I'I_z'\eta\,|\,\hat{q}_\kappa^{(1)}\,|\,II_z\eta\rangle=\eta\ , \quad (C4)$$

where we have used the orthogonality condition in Eq. (C3) and we have introduced the usual irreducible tensors. Similarly, from the fact that $\hat{q}$ is a unit vector operator we have

$$\sum_{I'I_z'\kappa}(-1)^\kappa\langle II_z\eta\,|\,\hat{q}_\kappa^{(1)}\,|\,I'I_z'\eta\rangle\langle I'I_z'\eta\,|\,\hat{q}_{-\kappa}^{(1)}\,|\,II_z\eta\rangle=1\ , \quad (C5)$$

where again we have used Eq. (C3). We can perform the sums over I_z' and κ with the aid of the Wigner-Eckhardt theorem so that Eqs. (C4) and (C5) become

$$\sum_{I'}\langle I\eta||Q^{(1)}||I'\eta\rangle\langle I'\eta||\hat{q}^{(1)}||I\eta\rangle C(I,I')=\eta \quad (C6)$$

and

$$\sum_{I'}\langle I\eta||\hat{q}^{(1)}||I'\eta\rangle\langle I'\eta||\hat{q}^{(1)}||I\eta\rangle C(I,I')=1\ , \quad (C7)$$

where

$$C(I,I')=\frac{1}{(2I+1)^{1/2}(2I'+1)^{1/2}}\sum_{I_z'\kappa}(-1)^\kappa\langle I'I_z'1\kappa\,|\,II_z\rangle\langle II_z1-\kappa\,|\,I'I_z'\rangle\ .$$

From the vector nature of $\vec{Q}$ and $\hat{q}$ Eqs. (C6) and (C7) become

$$\frac{\langle I\eta||Q^{(1)}||I+1\eta\rangle}{\eta}\langle I+1\eta||\hat{q}^{(1)}||I\eta\rangle C(I,I+1)+\frac{\langle I\eta||Q^{(1)}||I\eta\rangle}{\eta}\langle I\eta||\hat{q}^{(1)}||I\eta\rangle C(I,I)$$
$$+\frac{\langle I\eta||Q^{(1)}||I-1\eta\rangle}{\eta}\langle I+1\eta||\hat{q}^{(1)}||I\eta\rangle C(I,I-1)=1 \quad (C8)$$

and

$$\langle I\eta||\hat{q}^{(1)}||I+1\eta\rangle\langle I+1\eta||\hat{q}^{(1)}||I\eta\rangle C(I,I+1)+\langle I\eta||\hat{q}^{(1)}||I\eta\rangle\langle I\eta||\hat{q}^{(1)}||I\eta\rangle C(I,I)$$
$$+\langle I\eta||\hat{q}^{(1)}||I-1\eta\rangle\langle I-1\eta||\hat{q}^{(1)}||I\eta\rangle C(I,I-1)=1\ . \quad (C9)$$

The proof proceeds by induction. Take $I=\frac{1}{2}$, then the last term in Eq. (C8) and Eq. (C9) is zero and therefore the solution to the simultaneous equations is given by

$$\langle\tfrac{1}{2}\eta||Q^{(1)}||\tfrac{3}{2}\eta\rangle=\eta\langle\tfrac{1}{2}\eta||\hat{q}^{(1)}||\tfrac{3}{2}\eta\rangle \quad (C10)$$

and

$$\langle\tfrac{1}{2}\eta||Q^{(1)}||\tfrac{1}{2}\eta\rangle=\eta\langle\tfrac{1}{2}\eta||\hat{q}^{(1)}||\tfrac{1}{2}\eta\rangle\ . \quad (C11)$$

Now consider $I=\frac{3}{2}$ and using Eqs. (C10) and (C11) we can repeat the procedure and so on, hence

$$\langle I'\eta||Q^{(1)}||I\eta\rangle=\eta\langle I'\eta||\hat{q}^{(1)}||I\eta\rangle \quad (C12)$$

which is equivalent to Eq. (C2).

APPENDIX D

In this appendix we determine the meson-nucleon T matrix for the model Hamiltonian H defined by Eq. (2.1). The total in-scattering wave function is given by

$$|\Psi_k^{(\pm)}\rangle = |\vec{k},\text{in}\rangle + \frac{1}{E_k^{(\pm)}-H}(H-E_k)|\vec{k},\text{in}\rangle , \tag{D1}$$

where the state $|\vec{k},\text{in}\rangle$ can be any of the meson-nucleon in-states defined by Eq. (2.58) and E_k is given by

$$E_k = M_N + \omega(k) ,$$

where M_N is the renormalized nucleon mass which is obtained from Eq. (2.59) by an appropriate choice of the bare mass M_0. The corresponding out-scattering wave function is given by

$$|\Psi_{k'}^{(-)}\rangle = |\vec{k},\text{out}\rangle + \frac{1}{E_{k'}^{(-)}-H}(H-E_{k'})|\vec{k}',\text{out}\rangle , \tag{D2}$$

where $|\vec{k}',\text{out}\rangle$ is the corresponding meson-nucleon out-state defined by Eq. (2.58). We note that $|\vec{k},\text{in}\rangle$ and $|\vec{k}',\text{out}\rangle$ are related via Eq. (2.38) and therefore from Eq. (D2) we obtain the relation

$$\int d^3k' S(\vec{k}',\vec{k})\,|\Psi_{k'}^{(-)}\rangle = |\vec{k},\text{in}\rangle + \frac{1}{E_k^{(-)}-H}(H-E_k)|\vec{k},\text{in}\rangle , \tag{D3}$$

where we have used the fact that the background-scattering S matrix conserves the magnitude of the momentum. Substituting Eq. (D3) into Eq. (D1) we have

$$|\Psi_k^{(+)}\rangle = \int d^3k' S(\vec{k}',\vec{k})\,|\Psi_{k'}^{(-)}\rangle + \frac{1}{E_k^{(+)}-H}(H-E_k)|\vec{k},\text{in}\rangle - \frac{1}{E_k^{(-)}-H}(H-E_k)|\vec{k},\text{in}\rangle . \tag{D4}$$

It follows from Eq. (D4) and the definition of the scattering matrix, namely, $\langle\Psi_{k'}^{(-)}|\Psi_k^{(+)}\rangle$, that the total S matrix is given by

$$S_{\text{total}}(\vec{k}',\beta;\vec{k},\alpha) = \delta_{\beta\alpha} S(\vec{k}',\vec{k}) - 2\pi i\,\delta(E_{k'}-E_k)\langle\Psi_{k'}^{(-)}|(H-E_k)|\vec{k},\text{in}\rangle , \tag{D5}$$

where α and β are the quantum numbers which specify the states $|\vec{k},\text{in}\rangle$ and $|\vec{k}',\text{out}\rangle$, and $S(\vec{k}',\vec{k})$ is given by Eq. (2.34). It is straightforward to show that on the energy shell we have

$$\langle\text{out},\vec{k}'|(H-E_{k'})|\Psi_k^{(+)}\rangle = \langle\Psi_{k'}^{(-)}|(H-E_k)|\vec{k},\text{in}\rangle . \tag{D6}$$

Introducing the open- and closed-channel projection operators defined by Eqs. (5.3) and (5.4) and substituting the projection operators into the left-hand side of Eq. (D6) we have

$$\langle\text{out},\vec{k}'|(H-E_{k'})|\Psi_k^{(+)}\rangle = \langle\text{out},\vec{k}'|(H_{PP}-E_{k'})P|\Psi_k^{(+)}\rangle + \langle\text{out},\vec{k}'|H_{PQ}Q|\Psi_k^{(+)}\rangle . \tag{D7}$$

The first term on the right-hand side of Eq. (D7) is zero since the state $|\text{out},\vec{k}'\rangle$ is an eigenstate of H_{PP} with eigenvalue $E_{k'}$. By definition the closed-channel scattering wave function, $Q|\Psi_k^{(+)}\rangle$ is given by

$$Q|\Psi_k^{(\pm 0)}\rangle = \frac{1}{E_k^{(+)}-H_{QQ}}H_{QP}P|\Psi_k^{(+)}\rangle \tag{D8}$$

which in fact was used to obtain the form of the Schrödinger equation of Eq. (5.1). Substituting Eq. (D8) into Eq. (D7) we have

$$\langle\text{out},\vec{k}'|(H-E_{k'})|\Psi_k^{(+)}\rangle = \langle\text{out},\vec{k}'|H_{PQ}\frac{1}{E_k^{(+)}-H_{QQ}}H_{QP}P|\Psi_k^{(+)}\rangle . \tag{D9}$$

If we now restrict the closed-channel space Q to the nucleon and Δ states of Eq. (2.58), we obtain an approximate relation, namely,

$$\langle\text{out},\vec{k}'|(H-E_{k'})|\Psi_k^{(-)}\rangle \cong \langle\text{out},\vec{k}'|H_{PQ}^N\frac{1}{E_k-M_N}H_{QP}^N P|\Psi_k^{(+)}\rangle + \langle\text{out},\vec{k}'|H_{PQ}^{\Delta}\frac{1}{E_k-M_{\Delta}}H_{QP}^{\Delta}P|\Psi_k^{(+)}\rangle , \tag{D10}$$

where the Δ mass M_{Δ} is fixed by Eq. (2.59) for given values of the bare coupling constant g, and bag-source size R.

The complete determination of the right-hand side of Eq. (D10) requires the open-channel wave function $P\,|\,\Psi_k^{(+)}\rangle$. To this end, we consider the Schrödinger equation for the open-channel wave function defined by Eq. (5.1) which is given by

$$\left[H_{PP}+H_{PQ}\frac{1}{E_k^{(+)}-H_{QQ}}H_{QP}-E_k\right]P\,|\,\Psi_k^{(+)}\rangle=0\;. \tag{D11}$$

Again, restricting the closed-channel space Q to the nucleon and Δ states and solving for the open-channel wave function we have

$$P\,|\,\Psi_k^{(+)}\rangle=|\,\Phi,\vec{k},\mathrm{in}\rangle+\frac{1}{E_k^{(+)}-H_{PP}-U_{PP}^{N}}H_{PQ}^{\Delta}\frac{1}{E_k-M_\Delta}H_{QP}^{\Delta}P\,|\,\Psi_k^{(+)}\rangle\;, \tag{D12}$$

where U_{PP}^N is given by Eq. (5.7) and the scattering state $|\,\Phi,\vec{k},\mathrm{in}\rangle$, satisfies the Schrödinger equation given by

$$(H_{PP}+U_{PP}^N-E_k)\,|\,\Phi,\vec{k},\mathrm{in}\rangle=0\;. \tag{D13}$$

An explicit solution to Eq. (D12) can be obtained upon multiplying Eq. (D12) by $(1/E_k-M_\Delta)H_{QP}^{\Delta}$ and defining a quantity Λ_k which satisfies

$$\Lambda_k=\frac{1}{E_k-M_\Delta}H_{QP}^{\Delta}P\,|\,\Psi_k^{(+)}\rangle=\frac{1}{E_k-M_\Delta}H_{QP}^{\Delta}\,|\,\Phi,\vec{k},\mathrm{in}\rangle+\frac{1}{E_k-M_\Delta}H_{QP}^{\Delta}\frac{1}{E_k^{(+)}-H_{PP}-U_{PP}^N}H_{PQ}^{\Delta}\Lambda_k\;. \tag{D14}$$

Solving for Λ_k we have

$$\Lambda_k=\frac{1}{E_k-M_\Delta-H_{QP}^{\Delta}\dfrac{1}{E_k^{(+)}-H_{PP}-U_{PP}^N}H_{PQ}^{\Delta}}H_{PQ}^{\Delta}\,|\,\Phi,\vec{k},\mathrm{in}\rangle\;. \tag{D15}$$

Therefore, Eq. (D2) for the open-channel wave function is given by

$$P\,|\,\Psi_k^{(+)}\rangle=|\,\Phi,k,\ \mathrm{in}\rangle+\frac{1}{E_k^{(+)}-H_{PP}-U_{PP}^N}H_{PQ}^{\Delta}\frac{1}{E_k-M_\Delta-H_{QP}^{\Delta}\dfrac{1}{E_k^{(+)}-H_{PP}-U_{PP}^N}H_{PQ}^{\Delta}}H_{QP}^{\Delta}\,|\,\Phi,\vec{k},\mathrm{in}\rangle\;. \tag{D16}$$

Following an analogous procedure for $|\,\Phi,\vec{k},\mathrm{in}\rangle$ we have

$$|\,\Phi,\vec{k},\mathrm{in}\rangle=|\,\vec{k},\mathrm{in}\rangle+\frac{1}{E_k^{(+)}-H_{PP}}H_{PQ}^{N}\frac{1}{E_k-M_N-H_{QP}^{N}\dfrac{1}{E_k^{(+)}-H_{PP}}H_{PQ}^{N}}H_{QP}^{N}\,|\,\vec{k},\mathrm{in}\rangle\;. \tag{D17}$$

Substituting Eqs. (D16) and (D17) into Eq. (D10) and using Eq. (D15) for Λ_k and the definition of $|\,\Phi,\vec{k},\mathrm{out}\rangle$ we obtain

$$\langle\mathrm{out},\vec{k}'\,|\,(H-E_{k'})\,|\,\Psi_k^{(+)}\rangle\cong T_d(\beta,\vec{k}';\alpha\vec{k})+T_R(\beta,\vec{k}';\alpha\vec{k})\;, \tag{D18}$$

where T_d and T_R are given by Eqs. (5.11) and (5.12), respectively. Therefore the total T matrix follows from Eq. (D5), namely,

$$T(\beta,\vec{k}';\alpha\vec{k})=\delta_{\beta\alpha}t(\vec{k}',\vec{k})+T_d(\beta,\vec{k}';\alpha\vec{k})+T_R(\beta,\vec{k}';\alpha,\vec{k})\;, \tag{D19}$$

where $t(\vec{k},\vec{k})$ given by Eq. (2.35).

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[11]It can be shown that the leading term in the second-order correction to the wave-function normalization goes like $W^{(2)} \sim (\mu R)^{1/2} \ln \mu R$ in the limit $g \rightarrow \infty$ and $R \rightarrow 0$ according to the inequality defined by Eq. (2.54). This is not true of the energy. In the same limit, the lowest-order approximation to the nucleon energy goes like $E^{(0)}_{1/2,0} \sim -g^2/8\pi R$ while the leading term in the second-order correction to the energy goes like $E^{(2)} \sim -\mu/(\mu R)^{1/2}$.

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