$N^*(1470)$ in the cloudy bag model

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We investigate the $N^*(1470)$ in the context of the cloudy bag model. Gluonic and pionic self-energy terms mix the two orthogonal SU(6) N^* states. After correcting the mass for spurious center-of-mass motion, we obtain a doublet of states with masses of 1418 and 1533 MeV, respectively. A calculation of $N^* \rightarrow N\pi$ and $N^* \rightarrow \Delta\pi$ partial widths is in good agreement with experiment.

I. INTRODUCTION

The cloudy bag model (CBM) (Refs. 1–6) improves the static-MIT-bag-model predictions of hadron properties and also predicts pion-baryon-interaction quantities such as the $NN\pi$ and $\Delta N\pi$ coupling constants. In the CBM, the pion is introduced as a fundamental pseudoscalar field to restore chiral invariance. In this work we apply the CBM to the radially excited nucleon states. The lowest-mass excitation, the $N^*(1470)$ or Roper resonance, is thought to be a $(1S)^2 2S$ configuration of three quarks.[7] $SU(6) \times SU(2)$ radial symmetry predicts two distinct N^* states, the radially symmetric $N^*(56)$ and the radially mixed symmetric $N^*(70)$ [where the SU(6) multiplet to which each belongs is given in parentheses]. These states are degenerate before symmetry-breaking effects such as gluon exchange remove the degeneracy and mix the two states.

Previous work has concentrated solely on the effects of gluon exchange[7,8] and bag-surface oscillations on the N^* mass matrix. Bowler and Hey,[7] using the direct gluon-exchange graphs of Figs. 1(a) and 1(b), found physical N^* states at 1543 and 1646 MeV with each possessing equal components of $N^*(56)$ and $N^*(70)$. Close and Horgan[8] included the exchange amplitude [Fig. 1(c)], as well as the negative surface-oscillation term of DeGrand and Rebbi[9] that couples only to the $N^*(56)$, to eliminate virtually all mixing and to separate widely the physical N^* states. They attempted to identify the low-mass, predominantly $\underline{56}$ state with the $P_{11}(1470)$ and the heavier $\underline{70}$ with the $P_{11}(1710)$ resonance. These are the states advocated by most previous phase-shift studies. When the CBM is applied to the N^* problem, however, we find a spectrum of two adjacent states with masses of 1418 and 1533 MeV, respectively, similar to the results of an older phase-shift analysis by Ayed.[10] (See Sec. V. Ayed also finds a state at 1730 MeV.) In addition, we calculate $N^* \rightarrow N\pi$ partial widths for the two physical N^* states and obtain good agreement with experiments. Finally, a calculation of the $N^* \rightarrow \Delta\pi$ is in reasonable accord with the meager data.

In Sec. II we introduce pionic as well as gluonic self-energy amplitudes into the N^* mass matrix. Zeroth-order diagonal elements are found in the usu-

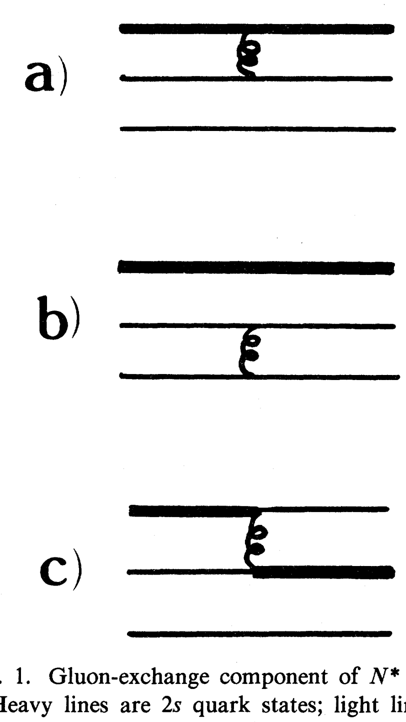

FIG. 1. Gluon-exchange component of N^* mass matrix. Heavy lines are 2s quark states; light lines are 1s states.

al way by minimizing $E_{N^*}(R)$ and applying center-of-mass corrections to this static-bag-model result. In Sec. III, pionic radiative vertex corrections renormalize the lowest-order $N^*N\pi$ and $N^*\Delta\pi$ coupling constants in the calculation of decay rates. We explore the stability of our results, in particular with regard to bag-surface oscillations, in Sec. IV. Finally, in Sec. V, we review the experimental situation and offer conclusions. In particular, we point out that the experimental situation remains uncertain regarding the question of one or more resonant structures in the $N^*(1470)$ region.

II. N^* MASS MATRIX

A. Bag contribution

The SU(6)×SU(2)-symmetric flavor-spin-space N^* wave functions are[11,12]

$$N^*(56)=\frac{1}{\sqrt{2}}R_S(I_{MS}S_{MS}+I_{MA}S_{MA}) \ , \quad (1a)$$

$$N^*(70)=\tfrac{1}{2}[R_{MS}(I_{MA}S_{MA}-I_{MS}S_{MS}) + R_{MA}(I_{MA}S_{MS}+I_{MS}S_{MA})] \ , \quad (1b)$$

where R, I, and S are space, isospin, and spin wave functions, respectively, and the subscripts S, MS, and MA stand for the symmetric, mixed symmetric, and mixed antisymmetric three-quark combinations of Close.[12] For $I_3=+\frac{1}{2}$ states, the mixed-symmetry states are

$$I_{MS}=\frac{1}{\sqrt{6}}[(ud+du)u-2uud] \ , \quad (2a)$$

$$I_{MA}=\frac{1}{\sqrt{2}}(ud-du)u \ , \quad (2b)$$

and the symmetric state

$$I_S=\frac{1}{\sqrt{3}}(uud+udu+duu) \quad (2c)$$

for up and down quarks. The radial (spin) states are of the same form with 1s (↑) and 2s (↓) wave functions substituted. An antisymmetric color wave function multiplying Eqs. (1a) and (1b) gives overall antisymmetry.

The $J=\frac{1}{2}^+$ space states are those of the MIT static bag model[13]

$$\psi_n(\vec{r})=\frac{N_n}{\sqrt{4\pi}}\begin{bmatrix} j_0(\omega_n r) \\ i j_1(\omega_n r)\vec{\sigma}\cdot\hat{r}\end{bmatrix} \ , \quad (3)$$

where n is the radial quantum number and ω_n is the eigenenergy of the linear boundary condition:

$$\Omega_n=\omega_n R=\begin{cases} 2.04 & (n=1) \\ 5.4 & (n=2) \ , \end{cases} \quad (4)$$

where N_n is the normalization,

$$N_n{}^2R^3=\frac{\Omega_n{}^2}{1-j_0{}^2(\Omega_n)} \ , \quad (5)$$

with a bag radius R.

Before symmetry breaking the bag energy of N^* states is written[14]

$$E_{N^*}(R)=\tfrac{4}{3}\pi R^3 B+\frac{2\Omega_1+\Omega_2}{R}-\frac{Z_0}{R} \ . \quad (6)$$

The first term is the volume energy of the bag, the second the kinetic energy of the interior quarks, and the last term the zero-point energy. Using previously determined values for the parameters $B^{1/4}$ (0.15 GeV) (Refs. 6 and 14) and Z_0 (~ 1) (Ref. 15) and minimizing E gives

$$E_{N^*}=1.872 \text{ GeV} \ , \quad (7a)$$

$$R_{N^*}=6.04 \text{ GeV}^{-1} \ . \quad (7b)$$

The bag energy is large compared to the experimental mass of the $N^*\approx 1.47$ GeV if E is interpreted as the mass. But when spurious center-of-mass motion[15,16] is subtracted, the mass is given by

$$M^2=E_{N^*}{}^2-\langle P_{\text{c.m.}}{}^2\rangle \ . \quad (8)$$

If we approximate

$$\langle P_{\text{c.m.}}{}^2\rangle \approx 2\left[\frac{\Omega_1}{R_{N^*}}\right]^2+\left[\frac{\Omega_2}{R_{N^*}}\right]^2 = 1.03 \text{ GeV}^2 \ , \quad (9)$$

we find

$$M=1.573 \text{ GeV} \quad (10)$$

much closer to the measured value.

The nominal N^* mass (10) will be renormalized by gluon- and pion-induced self-interaction that will also break the N^*(56) and N^*(70) degeneracy. These effects are considered below.

B. Gluon-exchange contribution

Gluon-quark interactions of the forms shown in Fig. 1 proceed via the exchange of TE(1) bagged gluons. The formalism of Ref. 8 leads to the gluonic matrix elements

$$\langle f\,|\,H_G\,|\,i\rangle=-\frac{8\alpha_s}{3R}\sum_k \langle \omega_{f_1}\,|\,I_k\,|\,\omega_{i_1}\rangle \frac{1}{\omega^2-k^2}\langle \omega_{f_2}\,|\,I_k\,|\,\omega_{i_2}\rangle 3\langle N^*(f)\,|\,\vec{\sigma}_1\cdot\vec{\sigma}_2\,|\,N^*(i)\rangle \ , \quad (11)$$

where $\omega=\omega_{f_1}-\omega_{i_1}=\omega_{i_2}-\omega_{f_2}$ and k is the eigenenergy of the mth gluon mode

$$k_m R=2.75,6.12,9.32,\ldots,m\pi \text{ (large } m) .$$

The overlap integrals are

$$\langle \omega_{f_1} | I_k | \omega_{i_1}\rangle=\int_0^R r^2 dr\, N_f N_i N_k R^{3/2} j_1(kr)[j_0(\omega_{f_1}r)j_1(\omega_{i_1}r)+j_0(\omega_{i_1}r)j_i(\omega_{f_1}r)] \quad (12)$$

with the normalization for the gluon wave function

$$N_k R^{3/2}=\frac{1}{j_1(kR)}\left[\frac{kR}{(kR)^2-2}\right]^{1/2} \quad (13)$$

and α_s is the strong coupling constant, equal to 0.55 in Ref. 8. We set $\alpha_s=0.3$ since pionic interactions account for part of the N-Δ splitting[3] originally attributed solely to gluon exchange.[14] We roughly take asymptotic freedom into account by setting $R=R_N=5\ \text{GeV}^{-1}$ as in Ref. 8. The gluonic self-energy mass generated by Fig. 1 is

$$\delta\widetilde{M}^{\text{gluon}}=\begin{matrix}\underline{56}\\ \underline{70}\end{matrix}\begin{bmatrix}-66 & +17\\ +17 & -43\end{bmatrix}\ \text{MeV} . \quad (14)$$

C. Pion-exchange contribution

The unrenormalized quark-pion pseudoscalar surface coupling constant of the cloudy bag model is[1–6]

$$\frac{f_{q_i q_f \pi}}{m_\pi}\vec{\sigma}\cdot\vec{k}\underline{\tau}\cdot\hat{\phi}\nu(kR)=\int d^3r\frac{1}{2f_\pi}\bar{q}_f(r)\gamma_5\underline{\tau}q_i(r)\cdot\hat{\phi}e^{i\vec{k}\cdot\vec{r}}\delta(r-R) , \quad (15)$$

where $f_\pi=93$ MeV is the pion decay constant, $\hat{\phi}=\underline{\phi}/|\phi|$, and $\nu(kR)$ is a form factor. The wave functions of Eq. (3) when inserted into Eq. (15) lead to

$$f_{q_1q_1\pi}=\frac{m_\pi}{3f}N_{1S}{}^2R^3j_0{}^2(\Omega_1)=0.486 , \quad (16a)$$

$$f_{q_2q_2\pi}=\frac{m_\pi}{3f}N_{2S}{}^2R^3j_0{}^2(\Omega_2)=0.302 , \quad (16b)$$

$$f_{q_2q_1\pi}=\frac{m_\pi}{3f}N_{1S}N_{2S}R^3j_0(\Omega_1)j_0(\Omega_2)=-0.385 , \quad (16c)$$

for the $(1S)(1S)\pi$, $(2S)(2S)\pi$, and $(2S)(1S)\pi$ coupling constants, respectively. The form factor

$$\nu(kR)=\frac{3j_1(kR)}{kR} \quad (17)$$

is normalized to unity for $k=0$.

The $N^*(i)\to N^*(j)\pi$ pion-emission process can proceed via either Fig. 2(a) or (2b). The $N^*(i)N^*(j)\pi$ coupling constants are found from

$$\langle S_Z=+\tfrac{1}{2},I_3=+\tfrac{1}{2}\,|f^0_{N^*(i)N^*(j)\pi}\sigma_Z\tau_3|\,S_Z=+\tfrac{1}{2},I_3=+\tfrac{1}{2}\rangle =\langle N^*(j)\,|\,3\sigma_z^{(3)}\tau_3^{(3)}(f^0_{q_1q_1\pi}b_{1S}^{(3)\dagger}b_{1S}^{(3)}+f^0_{q_2q_2\pi}b_{2S}^{(3)\dagger}b_{2S}^{(3)})\,|\,N^*(i)\rangle , \quad (18)$$

where we have specialized to the third quark and multiplied by 3 on the right-hand side. The coefficients of the quark-pion coupling constants are projection operators. We find

$$f^0_{N^*(56)N^*(56)\pi}=\tfrac{5}{9}(2f_{q_1q_1\pi}+f_{q_2q_2\pi})=0.71 , \quad (19a)$$

$$f^0_{N^*(56)N^*(70)\pi}=\tfrac{4}{9}(f_{q_1q_1\pi}-f_{q_2q_2\pi})=0.082 , \quad (19b)$$

$$f^0_{N^*(70)N^*(70)\pi}=\tfrac{1}{9}(5f_{q_2q_2\pi}-2f_{q_1q_1\pi})=0.059 . \quad (19c)$$

The $N^*(i)\to N\pi$ or $\Delta\pi$ pion-emission amplitude is represented graphically in Fig. 2(c). The coupling constants are obtained from

$$\langle S_Z=+\tfrac{1}{2},I_3=+\tfrac{1}{2}\,|f^0_{N^*(i)N\pi}\sigma_Z\tau_3|\,S_Z=+\tfrac{1}{2},I_3=+\tfrac{1}{2}\rangle=\langle N\,|\,3\sigma_z^{(3)}\tau_3^{(3)}f_{q_2q_1\pi}b_{1S}^{(3)\dagger}b_{2S}^{(3)}\,|\,N^*(i)\rangle \quad (20)$$

and

$$\langle S=\tfrac{3}{2},S_Z=+\tfrac{1}{2},I=\tfrac{3}{2},I_3=+\tfrac{1}{2}\,|\,f^0_{N^*(i)\Delta\pi}S_Z T_3\,|\,S=\tfrac{1}{2},S_Z=+\tfrac{1}{2},I=\tfrac{1}{2},I_3=+\tfrac{1}{2}\rangle$$
$$=\langle \Delta\,|\,3\sigma_Z^{(3)}\tau_3^{(3)}f_{q_2q_1\pi}b_{1S}^{(3)\dagger}b_{2S}^{(3)}\,|\,N^*(i)\rangle \ , \quad (21)$$

where $S_Z(T_3)$ are spin (isospin) transition operators for $\frac{3}{2}\leftrightarrow\frac{1}{2}$ transitions.[17] The nucleon and Δ component-quark wave functions are

$$|N\rangle=R_S\frac{1}{\sqrt{2}}(S_{MS}I_{MS}+S_{MA}I_{MA}) \ , \quad (22a)$$

$$|\Delta\rangle=R_S S_S I_S \ , \quad (22b)$$

where

$$R_S=(1s)(1s)(1s) \ . \quad (23)$$

We find

$$f^0_{N^*(56)N\pi}=\frac{5\sqrt{3}}{9}f_{q_2q_1\pi}=-0.369 \ , \quad (24a)$$

$$f^0_{N^*(70)N\pi}=-\frac{4\sqrt{3}}{9}f_{q_2q_1\pi}=+0.295 \quad (24b)$$

and

$$f^0_{N^*(56)\Delta\pi}=3\left[\frac{4\sqrt{6}}{9}\right]f_{q_2q_1\pi}=-1.25 \ , \quad (25a)$$

$$f^0_{N^*(70)\Delta\pi}=3\left[\frac{4\sqrt{6}}{9}\right]f_{q_2q_1\pi}=-1.25 \ . \quad (25b)$$

The pion-induced self-energy contributions to the N^* mass matrix are represented in Fig. 3. In the static baryon limit they are

$$\delta M_{fi}^{\text{pion}}(M^0_{N^*})=\sum_B[3,\tfrac{1}{3}]\mathrm{P}\int_{m_\pi}^{\omega_{\max}}\frac{d\omega\,k^3v^2(kR)f^0_{N_f^*B\pi}f^0_{N_1^*B\pi}}{(2\pi)^2m_\pi{}^2(E-M_B-\omega)}\Bigg|_{E=M^0_{N^*}} \ , \quad (26)$$

where $B=[N^*$ (or N), $\Delta]$ and $M^0_{N^*}=1.573$ GeV for the purpose of calculating the pionic contributions to the mass matrix elements. The pion energy ω ranges from m_π to $(k_1{}^2+m^2)^{1/2}$ where k_1R is the first zero of $v(kR)$ following DeTar.[6] Increasing this limit, established to enable a numerical integration, has little effect on results. The bag radius R is chosen to be $R_N=5$ GeV^{-1}. Although somewhat artificial, this ansatz has the virtue of defining R and rendering the bag states orthogonal. More sophisticated means of accomplishing the same ends are discussed in the work of Barnhill.[18]

Performing the sum over $B=N$, Δ, $N^*(56)$, and $N^*(70)$ in Eq. (26) yields the pionic contributions to the mass matrix:

$$\delta\widetilde{M}^{\text{pion}}=\begin{matrix}\underline{56}\\\underline{70}\end{matrix}\begin{bmatrix}-62 & -66\\ -66 & -23\end{bmatrix}\text{MeV} \ . \quad (27)$$

D. Physical N^* states

The complete N^* mass matrix includes Eq. (27) as well as the gluonic contributions of Eq. (14):

$$M=M^0_{N^*}\widetilde{I}+\delta\widetilde{M}^{\text{pion}}+\delta\widetilde{M}^{\text{gluon}}$$
$$=\begin{bmatrix}1.445 & -0.049\\ -0.049 & 1.507\end{bmatrix}\text{GeV} \ . \quad (28)$$

Notice how the off-diagonal pionic and gluonic contributions are of opposite sign, reducing the mixing in a natural fashion.

The physical N^* masses are found by diagonalizing the mass matrix (28). We obtain

$$\begin{bmatrix}M_a & 0\\ 0 & M_B\end{bmatrix}=\begin{bmatrix}1.418 & 0\\ 0 & 1.533\end{bmatrix} \quad (29)$$

for physical states

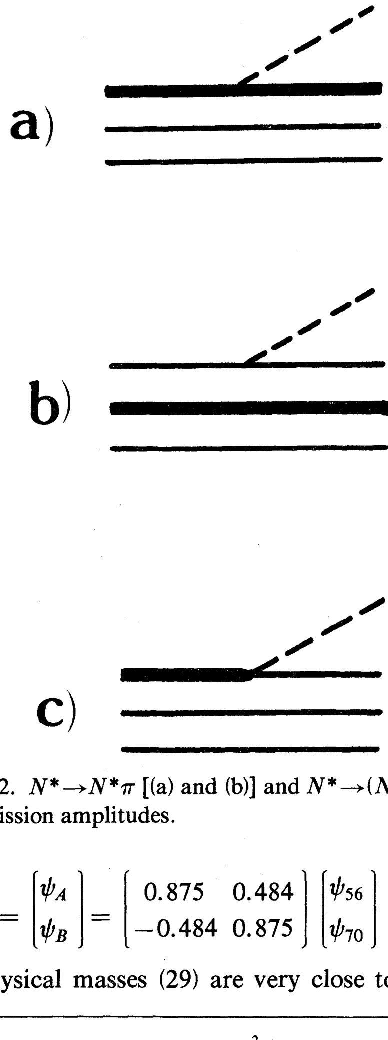

FIG. 2. $N^* \to N^* \pi$ [(a) and (b)] and $N^* \to (N \text{ or } \Delta)\pi$ (c) pion-emission amplitudes.

$$\tilde{\psi}_H = \begin{bmatrix} \psi_A \\ \psi_B \end{bmatrix} = \begin{bmatrix} 0.875 & 0.484 \\ -0.484 & 0.875 \end{bmatrix} \begin{bmatrix} \psi_{56} \\ \psi_{70} \end{bmatrix} . \quad (30)$$

The physical masses (29) are very close to those of Ayed[10] ($M_A = 1.413$ GeV and $M_B = 1.532$ GeV) which is certainly somewhat fortuitous considering both the approximations involved in the calculations and the quality of the data used in the phase-shift analysis. However, our results do support a splitting of the Roper resonance similar to that first observed by Ayed[10] and supported more recently by the results of the group in Leningrad.[19] The controversial experimental situation arising from the fact that most other phase-shift studies report only one P_{11} resonance in this energy region is reviewed in Sec. V.

III. N^* PARTIAL DECAY WIDTHS

The $N^* N\pi$ and $N^* \Delta\pi$ coupling constants are also renormalized by pionic radiative vertex corrections and by a renormalization of the baryon wave functions. The latter (for N^* states) is given by

$$\begin{bmatrix} Z_A & 0 \\ 0 & Z_B \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{d}{dE}(U\tilde{M}\,U^{-1}) \Big|_{E = M^0_{N^*}} , \quad (31)$$

where U is the transformation matrix in (30). The external N^* line in a zeroth-order pion-emission amplitude is multiplied by $\sqrt{Z_I}$ $(I = A, B)$. The Δ and nucleon wave-function-renormalization constants were calculated elsewhere[6] and are given in Table I along with the results of Eq. (31).

The pionic radiative vertex corrections for the N^* pion-emission vertex are (in the notation of Fig. 4)

$$\delta f_{N_i^* B\pi} = \sum_{B_1 B_2} C^{B_1 B_2}_{N_i^* B\pi} P \int \frac{q^3 d\omega}{(2\pi)^2} f^0_{N_i^* B\pi} f_{B_1 B_2 \pi} f_{B_2 B \pi} v^2(qR) \frac{1}{(M^0_{N^*} - M_{B_1} - \omega)(M^0_{N^*} - M_{B_2} - \omega)} , \quad (32)$$

where the static-baryon approximation is used. The coefficient $C^{B_1 B_2}_{N_i^* B\pi}$ depends only on the spin and isospin of the participating baryons and is therefore invariant under interchange of N, N^*_{56}, and N^*_{70}. The eight independent C's are given in Table II. A principal-value integration is performed where necessary and we use $f_{NN\pi} = 1$, $f_{\Delta\Delta\pi} = \frac{4}{5}$, and $f_{\Delta N\pi} = (\frac{72}{25})^{1/2}$.

The renormalized coupling constants for the physical states are given by

$$\begin{bmatrix} f_{N_A^* B\pi} \\ f_{N_B^* B\pi} \end{bmatrix} = \begin{bmatrix} (Z_{N_A^*})^{1/2} & 0 \\ 0 & (Z_{N_B^*})^{1/2} \end{bmatrix} U \begin{bmatrix} f^0_{N^*_{56} B\pi} \\ f^0_{N^*_{70} B\pi} \end{bmatrix} \sqrt{Z_B} + U \begin{bmatrix} \delta f_{N^*_{56} B\pi} \\ \delta f_{N^*_{70} B_\pi} \end{bmatrix} \quad (33)$$

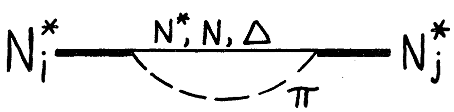

FIG. 3. Pionic-exchange component of N^* mass matrix.

TABLE I. Wave-function-renormalization constants.

$Z_{N_A} = 1.01$	$Z_N = 0.77$
$Z_{N_B} = 1.11$	$Z_\Delta = 0.89$

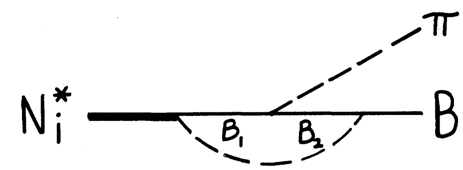

FIG. 4. Pion-vertex corrections to N^* decay amplitude. B_1 and B_2 represent N, Δ, and N^* intermediate states while B is an N or Δ final state.

with the results as shown in Table III. These are to be inserted into the formulas for two-body decay:

$$\Gamma(N_I^* \to N\pi) = \frac{3 f_{N_I^* N\pi} v^2(k_N R) k_N{}^3 M_N}{2\pi m_\pi{}^2 M_{N^*}} , \quad (34a)$$

$$\Gamma(N_I^* \to \Delta\pi) = \frac{1}{3} \frac{f_{N_I^* \Delta\pi} v^2(k_\Delta R) k_\Delta^3 M_\Delta}{2\pi m_\pi{}^2 M_{N^*}} , \quad (34b)$$

where the on-shell decay momentum is given by

$$k_B{}^2 = \frac{[M_{N^*}{}^2 - (M_B + m_\pi)^2][M_{N^*}{}^2 - (M_B - m_\pi)^2]}{4 M_{N^*}{}^2}$$

with $M_{N^*} = 1.47$ GeV, $m_\pi = 0.138$ GeV, and $M_B = M_N = 0.938$ GeV or $M_\Delta = 1.232$ GeV. We follow the policy of using the theory to generate values for dynamical quantities like matrix elements while using experimental results to give kinematic quantities like the recoil $B\pi$ momentum.

The predicted decay widths are compared to experiments in Table IV. The $N^*N\pi$ partial widths are from Ayed[10] while the poorly known $N^*\Delta\pi$ rate represents a compilation of data assuming a single N^* state. The agreement is good. We have not included center-of-mass corrections to $f^0_{N^*B\pi}$ as has been done in the past for the π-N system[6] because of uncertainties in handling $2s$-$1s$ transitions. One might expect c.m. effects to be small because of the larger-mass baryons involved. We believe an upper limit on $\langle P_{\text{c.m.}}{}^2 \rangle$ to be

TABLE II. Pionic-vertex-correction coefficients.

$C^{NN}_{N^*N\pi} = \frac{1}{3}$	$C^{NN}_{N^*\Delta\pi} = \frac{4}{3}$
$C^{N\Delta}_{N^*N\pi} = \frac{8}{27}$	$C^{N\Delta}_{N^*\Delta\pi} = \frac{25}{24}$
$C^{\Delta N}_{N^*N\pi} = \frac{8}{27}$	$C^{\Delta N}_{N^*\Delta\pi} = \frac{5}{27}$
$C^{\Delta\Delta}_{N^*N\pi} = \frac{25}{108}$	$C^{\Delta\Delta}_{N^*\Delta\pi} = \frac{25}{24}$

TABLE III. Physical N^* pion decay constants.

$f_{N_A^* N\pi} = -0.446$	$f_{N_B^* N\pi} = 0.288$
$f_{N_A^* \Delta\pi} = -2.18$	$f_{N_B^* \Delta\pi} = -1.55$

$$\frac{\langle P_{\text{c.m.}}{}^2 \rangle}{M_N M_{N^*}} \sim \left[\frac{2(\Omega_1/R)^2 + (\Omega_2/R)^2}{M_{N^*}{}^2} \right]^{1/2} \times \left[\frac{3(\Omega_1/R)^2}{M_N{}^2} \right]^{1/2} \quad (35)$$

leading to about a 20% increase in $f^0_{N^*B\pi}$.

IV. STABILITY OF RESULTS

One might attempt to improve upon the approximation $R = R_N = 5\ \text{GeV}^{-1}$ by using the average N^*-N bag radius $R = 5.52\ \text{GeV}^{-1}$. One obtains

$$M_A = 1.451\ \text{GeV} , \quad (36a)$$

$$M_B = 1.535\ \text{GeV} \quad (36b)$$

and

$$\Gamma_{\text{av}}(N_A^* \to N\pi) = 71\ \text{MeV} , \quad (37a)$$

$$\Gamma_{\text{av}}(N_B^* \to N\pi) = 27\ \text{MeV} , \quad (37b)$$

$$\Gamma_{\text{av}}(N^* \to \Delta\pi) = 68\ \text{MeV} \quad (37c)$$

with little significant change from the previous results.

Another question of interest concerning the N^* system is that of bag-surface oscillations[8,9] which couple only to the $N^*(56)$ and are claimed to be on the order of

$$\langle \underline{56} | H_{\text{osc}} | \underline{56} \rangle \sim \begin{cases} -150\ \text{MeV (Ref. 8)} \\ -200\ \text{MeV (Ref. 9)} \end{cases} . \quad (38)$$

Such a large negative diagonal term widely separates the two physical states, which remain almost pure 56 and 70. If we arbitrarily insert a term

TABLE IV. Pion decay rates of N^* states.

	This work	Experiment (Ref. 10)
$\Gamma_{N_A^* N\pi}$	91 MeV	98 MeV
$\Gamma_{N_B^* N\pi}$	38 MeV	12 MeV
$\Gamma_{N_A^* \Delta\pi}$	55 MeV	
$\Gamma_{N_B^* \Delta\pi}$	+28 MeV	
	83 MeV	~50 MeV

$$\langle 56 | H_{osc} | 56 \rangle = -100 \text{ MeV} \quad (39)$$

into $\tilde{M}$ [Eq. (28)], we find

$$M_A = 1.361 \text{ GeV} , \quad (40a)$$

$$M_B = 1.525 \text{ GeV} \quad (40b)$$

and

$$\Gamma(N_A^* \to N\pi) = 87 \text{ MeV} , \quad (41a)$$

$$\Gamma(N_B^* \to N\pi) = 10 \text{ MeV} , \quad (41b)$$

$$\Gamma(N^* \to \Delta\pi) = 67 \text{ MeV} \quad (41c)$$

with the lower-mass state remaining 96% $\underline{56}$. The spectrum results are only somewhat worse than before. It is clear, however, that surface oscillations of the magnitude used in Refs. 8 and 9 will seriously affect our results. The lower-mass state is driven down to well below experiment, and the lack of mixing serves to decouple the heavier state almost completely from the N channel. Our results indicate that the N_A^* and N_B^* are separated by only about 100 MeV in the vicinity of the $N^*(1470)$ rather than two distinct nucleon resonances [$P_{11}(1470)$ and $P_{11}(1710)$] as suggested in Ref. 8. The crucial difference is our subtraction of spurious center-of-mass motion from the kinetic energy of quarks in the bag and the inclusion of pionic self-energy terms that lower the masses and decrease the mixing.

V. REVIEW OF THE EXPERIMENTAL SITUATION AND CONCLUSIONS

While the low-energy region of pion-nucleon scattering up to the Δ resonance has been investigated quite extensively,[20] the amount and quality of the data available in the region of the Roper resonance is much less satisfying in general, and even more so with regard to the question of a possible splitting of the N^*. Since the splitting and the predicted widths are of the order of or less than 100 MeV, any experiment trying to look for this effect should cover the whole resonance region in small energy steps with high accuracy. To date, however, only two experiments have been reported which come anywhere near these requirements: Gordeev *et al.*[19] have measured differential cross sections for $\pi^{\pm}p$ elastic scattering at 11 momenta between 404 and 767 MeV/c and for c.m. angles between $\theta_\pi^* = 45°$ and $\theta_\pi^* = 176°$, and Sadler *et al.*[21] have performed similar measurements at 10 momenta between 378 and 687 MeV/c at 9 angles in the range $50° \le \theta_\pi^* \le 150°$. Unfortunately, the results of these two experiments disagree by as much as 20% in the backward scattering region which is the most sensitive region for observing resonance structure in πp scattering

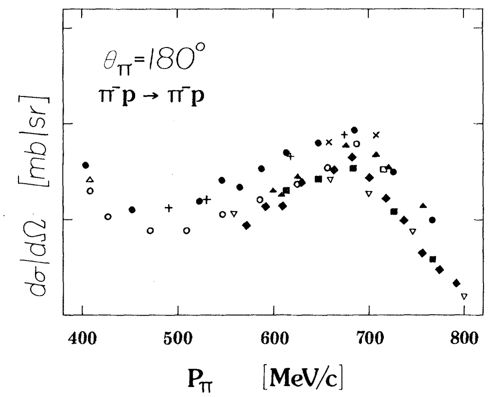

FIG. 5. Differential cross sections for $\pi^- p$ elastic scattering at $\theta_\pi = 180°$. The closed symbols are from experiments in which the cross section has been measured very close to 180° whereas the open symbols denote extrapolations from fits to angular distributions measured not further back than 170°. ●, Gordeev *et al.* (Ref. 19); ■, Debenham *et al.* (Ref. 30); ◆, Rothschild *et al.* (Ref. 31); ▲, Crabb *et al.* (Ref. 32); ○, Sadler *et al.* (Ref. 21); □, Dolbeau *et al.* (Ref. 33); △, Bussey *et al.* (Ref. 34); ▲, Brody *et al.* (Ref. 35); ×, Helland *et al.* (Ref. 36); +, Ogden *et al.* (Ref. 37).

and for determining the parameters of pion-nucleon resonances. As can be seen from Fig. 5, the various earlier measurements which are available in this energy region are of no great help in deciding which of the two results might be right.

Even less data are available on the proton polarization in the region of interest here. There are only four experiments in the literature[22–25] in which the proton spin polarization P has been measured in π^--p scattering with reasonable accuracy, while a fifth experiment is in progress.[26] Moreover, of the data yielded by these experiments, by far the most are located at energies above the peak of the N^* resonance, with only five data points on the low-energy slope of the peak. There are no data on the spin-rotation parameters A and R at all, although two groups have indicated their intent to measure these quantities in the near future.[19,21]

This experimental situation is reflected in the results of the latest phase-shift analyses which have been performed for pion-nucleon scattering.[10,27–29] Since the analysis of Ayed[10] first suggested a splitting of the $P_{11}(1470)$ resonance, neither the more recent analysis of Cutkosky *et al.*[28] nor the analysis of the Helsinki-Karlsruhe group[27] has been able to corroborate this solution. On the other hand, a new analysis of the group in Leningrad[29] which includes the data of Ref. 19 but not those of Ref. 21 results in a "more convincing confirmation of the hypothesis that the $P_{11}(1470)$ resonance splits into two resonances with a mass difference of less than 100 MeV."[19] Thus, the situation is perhaps best described by the words of Cutkosky *et al.*, who found resonances at 1450 and 1710 MeV but say, regarding their fit to the P_{11} partial wave, that "since the fit is quite poor...we do not know whether other meaningful structure might be present."[28]

We conclude with a summary of results. The qualitative features of all three of our solutions are the same. The two physical N^* states have predicted masses of about 1418 and 1533 MeV, respectively, in good agreement with the results of two of the more recent phase-shift analyses,[10,19,29] but in disagreement with others.[27,28] The lower-mass state is predominantly $\underline{56}$ and couples more strongly to the πN channel than the higher mass, primarily $\underline{70}$, state. Both πN and $\pi\Delta$ partial widths are in reasonable agreement with experiment. All phase-shift analyses agree on the existence of a P_{11} state in the vicinity of 1710 MeV. Calculations to be reported elsewhere[38] lead us to suggest that this state may be a $(1s)^3$ + TE gluon bound state.

Certainly our calculations are subject to improvement. For example, a realistic calculation of c.m. and recoil effects that is general enough to handle $2s$-$1s$ transitions and bags of different radii will soon be available.[39] Such a formalism would make possible an improved calculation of $f_{N^*N\pi}$. Moreover, it may be that $Q^4\bar{Q}$ and Q^3G components of the N^* wave function are important. Nonetheless, πN scattering experiments and phase-shift analyses with the express purpose of resolving two possible N^* states in the 1350–1550-MeV region would be of great interest.

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