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Multiplicity and energy distributions in high-energy e^+e^-, pp, and $p\bar{p}$ collisions

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An attempt is made to understand the most striking features of multiplicity distribution observed in the recent $p\bar{p}$ collider experiments. It is shown in particular that the validity of Koba-Nielsen-Olesen (KNO) scaling in the central rapidity region, the growth of the central plateau with increasing energy, and the existence and the properties of forward-backward multiplicity correlation can be readily understood in terms of a three-fireball model for nondiffractive hadron-hadron collisions. It is suggested that KNO scaling functions reflect reaction mechanisms. The qualitative difference between the KNO scaling curve in e^-e^+ annihilation and that in nondiffractive hadron-hadron collisions is discussed as an example.

INTRODUCTION

One of the most striking features of the recent $p\bar{p}$-collider data[1,2] is the observed scaling property of the multiplicity distribution of charged hadrons in the central rapidity region: When one plots $\langle n_c \rangle P(n_c)$, the probability $P(n_c)$ of finding n_c charged particles times the mean charge multiplicity $\langle n_c \rangle$ observed in a given *central rapidity region* ($|\eta| \leq 1.3$ where η is the pseudorapidity; the subscript c stand for "central") against $n_c/\langle n_c \rangle$ the data fall exactly on those obtained by plotting the same quantity at much lower bombarding energies[3] (see Fig. 1). Considering in particular the extremely wide range of energies [from center-of-mass-system (c.m.s.) energy $\sqrt{s}=24$ to 540 GeV] in which the scaling property is valid, this is indeed a remarkable fact.

Scaling behavior with respect to the *total* multiplicity n_{inel} of produced charged hadrons in high-energy inelastic hadron-hadron collisions has been proposed by Koba, Nielsen, and Olesen[4] (KNO) many years ago. A considerable number of papers on this subject have been published in the past.[5,6] In particular, it has been pointed out[6] that the original KNO scaling,[4] with respect to the variable $n_{\text{inel}}/\langle n_{\text{inel}} \rangle$, is only valid approximately.

Several questions can, and should be, asked: Why is the scaled multiplicity in the central rapidity region $n_c/\langle n_c \rangle$ a relevant variable in such a wide range of incident energies? Why is $n_c/\langle n_c \rangle$ a better scaling variable than $n_{\text{inel}}/\langle n_{\text{inel}} \rangle$? What is the reason, if any, that the "universal curve" for $\langle n_c \rangle P(n_c)$ has such a peculiar shape?

The above-mentioned observation,[6] taken together with the well-known experimental fact[7] that single-diffractive-dissociation processes contribute about 18% of the total inelastic cross sections in high-energy hadron-hadron collisions, suggests that it may be the diffractive component which causes the observed violation of scaling with respect to $n_{\text{inel}}/\langle n_{\text{inel}} \rangle$. In order to study this problem in more detail, we consider the difference between n_{inel} and n_{sd}, the multiplicity of charged hadrons in single-diffractive-dissociation processes at the same incident energies. Since at these energies the contribution of double-diffractive-dissociation processes has been estimated to be negligibly small,[8] this difference is nothing else but the multiplicity of produced charged hadrons in nondiffractive collisions. Denoting this quantity by n_{nd}, and plotting $\langle n_{\text{nd}} \rangle P(n_{\text{nd}})$ against the corresponding scaled quantity $n_{\text{nd}}/\langle n_{\text{nd}} \rangle$ for all available hadron-hadron collision data[7,9,10] above 50 GeV/c, we obtain the result

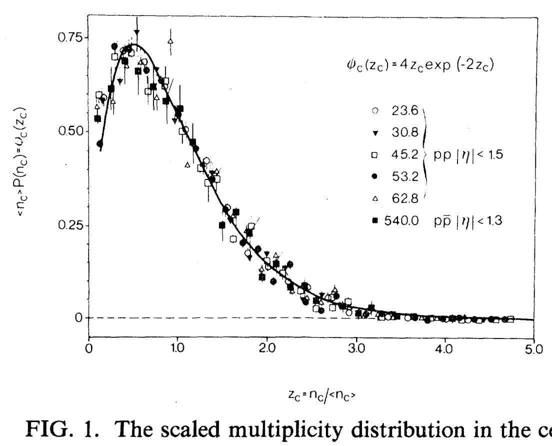

FIG. 1. The scaled multiplicity distribution in the central rapidity region. The experimental data are taken from Refs. 1 and 3. The curve is obtained from Eq. (10).

shown in Fig. 2. Here we see that the scaling property for nondiffractive multiplicity is indeed better than the scaling for total inelastic multiplicity, and that not only the *pp* data at various incident energies but also those for $\pi^- p$ and $\pi^+ p$ fall exactly on one curve. However, what we also see is that the scaling curve for $\langle n_{\rm nd}\rangle P(n_{\rm nd})$ is qualitatively different from that for $\langle n_c\rangle P(n_c)$.

The existence of *two different kinds* of KNO scaling (two different functions of two different scaling variables) in a single collision process, namely, nondiffractive hadron-hadron collision process, has to be considered as a striking empirical fact. Hence, it seems useful to know the following: Why are $n_c/\langle n_c\rangle$ and $n_{\rm nd}/\langle n_{\rm nd}\rangle$ good scaling variables? What is the origin of these two types of scaling functions? What is the relationship, if any, between them?

Furthermore, scaling behavior with respect to the KNO variable $n_{ee}/\langle n_{ee}\rangle$ has also been observed[11] in e^+e^- reactions (the subscripts *ee* stand for e^+e^-). The shape of the "universal curve" $\langle n_{ee}\rangle P(n_{ee})$ is, however, again *very much different* from that for $\langle n_c\rangle P(n_c)$ and that for $\langle n_{\rm nd}\rangle P(n_{\rm nd})$. Does this imply the existence of a *third* kind of KNO scaling?

Existence of KNO scaling functions has also been reported[12] in connection with hadron-nucleus and nucleus-nucleus reactions. Are there connections between these functions and those for the hadron-hadron and/or e^+e^- collisions?

The purpose of this paper is to show that high-energy nondiffractive hadron-hadron collisions can be understood in terms of a three-fireball model. In particular, explicit expressions for the KNO scaling functions $\psi(n_c/\langle n_c\rangle)=\langle n_c\rangle P(n_c)$ and $\psi(n_{\rm nd}/\langle n_{\rm nd}\rangle)=\langle n_{\rm nd}\rangle P(n_{\rm nd})$ can be obtained in this framework.

In order to avoid confusion of language and to compare the properties of nondiffractive hadron-hadron collisions with those of other high-energy processes, we discuss in some detail the underlying physical picture of the proposed model.

PHYSICAL PICTURE

The physical picture is based on the following hypotheses. Parts of them are very well known and have already been checked experimentally.[13]

(I) While electrons and positrons are pointlike particles, hadrons are spatially extended objects of many degrees of freedom.

(II) In contrast to e^+e^- annihilation processes, the colliding objects in high-energy hadronic processes may go through each other.[14] Because of the different possibilities for the constituents of the colliding objects to interact with one another, the possible final states of a hadronic collision process can be the following.

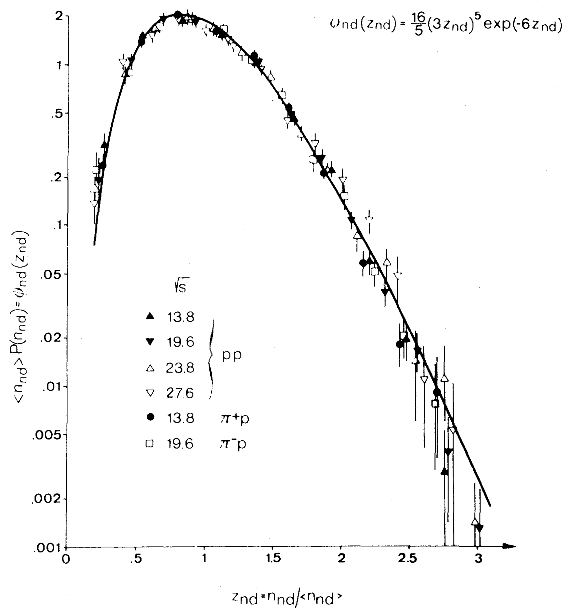

FIG. 2. The scaled multiplicity distribution for nondiffractive collisions. The experimental data are taken from Refs. 7, 9, and 10. The curve is obtained from Eq. (11).

(a) The colliding objects hit each other so violently[15,16] that they stop each other and give their *entire amount* of kinetic energy to a common system. The new system formed by the two colliding objects decays after expansion. Such processes are associated with extremely high multiplicity and/or high average transverse momentum. Geometrically, such processes correspond to those with extremely small impact parameters.

(b) The colliding objects hit each other so gently[14] that they completely go through each other, forming two distinct systems in the total phase space. *Only a very limited amount* of kinetic energy is converted into internal (excitation) energy. Elastic scattering and diffractive-dissociation processes belong to this category.[14,17] Geometrically, these processes are predominantly collisions with very large (compared to the mean value of all collisions) impact parameters. That is, the overwhelming part of such collisions is extremely peripheral.

(c) During the process of going through each other, a considerable amount of kinetic energy of the colliding objects is converted into internal energy E^* which materializes. We shall call it hereafter "prematter."[18] It is clear that the prematter will eventually become hadrons, which are distributed not only in the two rapidity regions $R(P^*)$ and $R(T^*)$ where the excited projectile P^* and the excited target T^* are located, but also in the corresponding intermediate region $R(C^*)$. Hence, it may be envisaged that in every event the energy E^* (of the prematter) is distributed in three distinct rapidity regions $R(P^*)$, $R(T^*)$, and $R(C^*)$, and that the prematter in these three regions can approximately be considered as three independent systems which we denote by P^*, T^*, and C^*, respectively. The hadrons produced by P^* and T^* contribute mainly to the projectile and target fragmentation[14] regions, respectively, while those produced by C^* contribute to the "plateau."[13] Note that an overwhelming part of inelastic hadron-hadron collision processes are events of this category. Almost all the corresponding impact parameters have values between those mentioned in the two extreme cases (a) and (b).

(III) The hadronization of the prematter takes place in such a way that the multiplicity n of the observed charged particles is uniquely determined by the internal energy E^* of the decaying hadronic system. In particular, for nondiffractive hadron-hadron collisions we have, for each decaying system,

$$n \propto E^* . \tag{1}$$

This assumption is based on the empirical fact[13] that the overwhelming parts of the produced particles are pions with approximately the same distribution in transverse momentum [its average value is above 0.3 GeV/c up to CERN ISR energies and 0.5 GeV/c at the CERN-$p\bar{p}$-collider energy (540 GeV)]. It follows from Eq. (1) that the KNO scaling variable[4] $n/\langle n\rangle$ is nothing else but the *scaled* internal (excitation) energy $E^*/\langle E^*\rangle$ where $\langle E^*\rangle$ is the average value of E^*.[19,20]

Before proceeding with the discussion on hadron-hadron collisions, which is the main topic of this paper, let us, for the sake of comparison, briefly discuss the e^+e^- annihilation process: In terms of the present picture the energetic pointlike particles (e^+e^-) hit each other so violently that their entire amount of primary energies is deposited into a common system formed by the colliding objects. Since this system is the only one after the collision, its internal energy is nothing else but the total incident energy $(\sqrt{s})$ in the c.m.s. frame. Having the above-mentioned violent nature of the collision process in mind, one can imagine that, for a given $\sqrt{s}$, the energy per particle is relatively large (compared to the pion mass, say) and may not be very much different from event to event. Let us, for the sake of simplicity, first consider the idealized case in which the average energy per particle is the same in every event. In this case, we have

$$n_{ee} = \langle n_{ee}\rangle , \tag{2}$$

where $\langle n_{ee}\rangle$ is the mean value of n_{ee} averaged over all events with the same $\sqrt{s}$. That is, the multiplicity distribution can be written in the form of a δ function $\delta(n_{ee}/\langle n_{ee}\rangle - 1)$. The experimental fact is that the multiplicity data[11] indeed shows a sharp maximum at $n_{ee}/\langle n_{ee}\rangle = 1$ in the KNO plot. Taken together with the observation that the average kinetic energy of each produced particle is indeed large ($\sim$1 GeV) the sharp maximum at $n_{ee} = \langle n_{ee}\rangle$ strongly suggests that detailed information about production and decay of particles at the intermediate stage does not play a significant role in the determination of the multiplicity distribution. It is interesting to see that the distribution around the maximum $n_{ee}/\langle n_{ee}\rangle = 1$ can approximately be expressed as a Gaussian distribution for the relative fluctuation $n_{ee}/\langle n_{ee}\rangle - 1$:

$$P(n_{ee}) = A \exp[-4(n_{ee}/\langle n_{ee}\rangle - 1)^2] , \tag{3}$$

where the constant A is determined by the normalization condition for $P(n_{ee})$. The result is

$$\langle n_{ee}\rangle P(n_{ee}) = 2.3 \exp[-4(n_{ee}/\langle n_{ee}\rangle - 1)^2] . \tag{4}$$

It is also interesting to see the role played by the normalization condition. It is this condition which leads Eq. (3) to the result that $\langle n_{ee}\rangle P(n_{ee})$ is a function of the single variable $n_{ee}/\langle n_{ee}\rangle$.

We would like to point out that the simple expression given by Eq. (4) needs to be improved for the following reasons. (A) Although it gives a good description for most of the data[11] (see Fig. 3) it does not have the right behavior near $n_{ee}/\langle n_{ee}\rangle = 0$. (B) Although the Gaussian distribution does in some sense reflect the violent nature of the e^-e^+ annihilation processes, in contrast to the overwhelming part of high-energy inelastic hadron-hadron collisions (see below), it cannot describe the details of the reaction mechanism of the above-mentioned processes. Further studies (to be discussed elsewhere) show that Eq. (4) is a good approximation of a more complicated formula which can be derived from a "flux-tube" model. But, in order to reach the goal set at the beginning of this paper, it is sufficient to know that not only the KNO scaling curves of e^-e^+ annihilation processes are qualitatively different from those in nondiffractive hadron-hadron collisions, but also the corresponding reaction mechanisms are totally different from each other.

From the viewpoint of the proposed picture, the basic difference between the hadronization in e^+e^- annihilation and that in inelastic hadron-hadron collision is the following: In the latter case, not only the amount of primary energy, which is converted from kinetic energy into internal (excitation) energy, is in general different in every collision event, but also its distribution in different regions of the allowed phase space is in general different from event to event. To be more precise, in hadron-hadron collisions, the total internal energy is constant *only* in violent collisions [that is, the collision events of category (a) discussed in hypothesis II]. But, since such events are obviously very rare,[15,16] one is forced to deal with the following questions. What is the relationship between the average internal (excitation) energy of each system in events of category (b) or (c) mentioned in hypothesis II? How is the internal energy E_i^* distributed within the system i ($i = C^*$, P^*, and T^*)? In the framework of the three-fireball model these questions are answered by the following ansatz, which can be (and partly has already been) tested experimentally.

(IV) As a working ansatz, we assume the following.

(α) In inelastic hadron-hadron collisions at sufficiently high bombarding energies, the overwhelming part of events are nondiffractive [category (c) in hypothesis (II)]. In fact it consists of *all inelastic events except* the (few percent) *violent collisions* [category (a)] and the [approximately 18% (Refs. 7 and 8)] diffractive-dissociation processes [category (b)].

(β) The average internal (excitation) energies $\langle E_c^* \rangle$, $\langle E_P^* \rangle$, and $\langle E_T^* \rangle$ in the systems C^*, P^*, and T^* are approximately the same. For the sake of simplicity and definiteness, we assume[21]

$$\langle E_c^* \rangle = \langle E_P^* \rangle = \langle E_T^* \rangle \ . \tag{5}$$

This ansatz is consistent with the experimental observations [recall that the fragmentation regions $R(P^*)$ and $R(T^*)$ only *appear* to be small when the rapidity or pseudorapidity variable is used]. As we shall see later on [see Eq. (11) and Ref. 21] although the final result is very insensitive to the equality of the three average values given in Eq. (5), the corresponding expression is drastically simplified when they are set to be equal.

(γ) Based on the empirical fact[12,13] that the time for the formation of multiparticle final states in high-energy hadron-hadron collisions is *long*—much longer than that for a high-energy hadron to travel a few fermis, we assume that the hadronization of prematter in each system takes place in *two steps.* In the first stage—that is, immediately after the collision—the prematter can be considered as an incompressible fluid (may be superfluid). The internal energy E_i ($i = C^*, P^*, T^*$) manifests itself as surface (i.e., a two-dimensional) vibration. Eventually, more and more degrees of freedom in the system i will be excited such that the constituents of each system are set into random motion. This is the second stage before the decay of these systems.

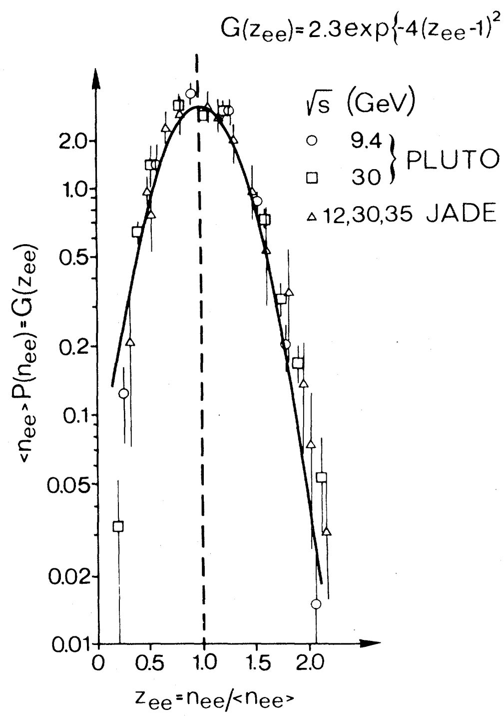

FIG. 3. The scaled hadronic multiplicity distribution in e^+e^- annihilation. The experimental data are taken from Ref. 11. The curve is obtained from Eq. (4).

The energy spectrum of E_i ($i = C^*, P^*, T^*$) at the first stage can be taken to be the same as that of a system of two uncoupled harmonic oscillators. That is, the probability of having the excitation energy E_i^* in the system is given by

$$P(E_i^*) = A_i E_i^* \exp(-B_i E_i^*) \tag{6}$$

and the constants A_i and B_i are determined by the usual normalization conditions

$$\int P(E_i^*) dE_i^* = 1 \ , \tag{7}$$

$$\int E_i^* P(E_i^*) dE_i^* = \langle E_i^* \rangle \ . \tag{8}$$

Before explicitly calculating the multiplicity distributions, let us first point out the following experimental fact which directly supports the assumption (γ): Once we accept the proposed relationship between n_c and E_c^* [Eq. (1)], a comparison between the above-mentioned experimental plot[1,3] and the well-known[22] formula for the energy distribution for a system of N (a positive integer) uncoupled oscillators shows that there can be no more than a few excited degrees of freedoms. This strongly suggests the existence of a collective mode of excitation in such gentle collision processes.

SCALING FUNCTIONS

We now calculate the multiplicity distribution in hadron-hadron collisions. First, we consider the central rapidity region $R(C^*)$. From Eqs. (6)—(8) we obtain

$$\langle E_c^* \rangle P(E_c^*) = 4E_c^*/\langle E_c^* \rangle \exp[-2E_c^*/\langle E_c^* \rangle] \ . \tag{9}$$

Hence, by using Eq. (1), the distribution of the central-region multiplicity n_c in terms of the corresponding KNO scaling variable $Z_c = n_c/\langle n_c \rangle$ is

$$\langle n_c \rangle P(n_c) = \psi_c(n_c/\langle n_c \rangle) \ , \tag{10a}$$

$$\psi_c(Z_c) = 4Z_c \exp(-2Z_c) \ . \tag{10b}$$

Comparison of Eq. (10) with the CERN collider[1] and the ISR (Ref. 3) experiments is shown in Fig. 1. Note that not only the constants A_c and B_c in Eq. (6) but also the fact that $\langle n_c \rangle P(n_c)$ is a function of the variable $n_c/\langle n_c \rangle$ alone (that is, KNO scaling) is determined by the normalization conditions (7) and (8).

Next, we consider the distribution of the nondiffractive multiplicity $n_{\rm nd}$ by including also those (n_P and n_T) obtained in the rapidity regions $R(P^*)$ and $R(T^*)$. (That is, $n_{\rm nd} = n_c + n_P + n_T$). According to hypothesis (IV), we have to deal with three independent two-dimensional vibrating systems. It follows from a trivial calculation that[21]

$$\langle n_{\rm nd} \rangle P(n_{\rm nd}) = \psi_{\rm nd}(n_{\rm nd}/\langle n_{\rm nd} \rangle) \ , \tag{11a}$$

$$\psi(Z_{\rm nd}) = \tfrac{16}{5}(3Z_{\rm nd})^5 \exp(-6Z_{\rm nd}) \ , \tag{11b}$$

which is nothing else but the energy distribution of six uncoupled harmonic oscillators. Comparison between experiments and Eq. (11) is shown in Fig. 2.[23]

Furthermore, since the average multiplicity $\langle n_{\rm sd} \rangle$ in single-diffractive events is relatively low and the occurrence of these events is comparatively seldom, the contribution from such processes to the total inelastic multiplicity $n_{\rm inel} = n_{\rm nd} + n_{\rm sd}$ should not play an important role in the $n_{\rm inel}$ distributions. Hence, we expect to see a qualitative agreement also between the $\langle n_{\rm inel} \rangle P(n_{\rm inel})$ data[3,9,24] and the calculated curve for $\langle n_{\rm nd} \rangle P(n_{\rm nd})$ from Eq. (11). A comparison between experimental and calculated values is shown in Fig. 4.

MULTIPLICITY MOMENTS

The scaling law proposed by Koba, Nielsen, and Olesen[4] is sometimes expressed in terms of the following relationship between the moments of the multiplicities n:

$$\langle n^q \rangle = C_q \langle n \rangle^q \ , \tag{12}$$

where q is a positive integer, and C_q can be determined empirically.

In the present model, a simple analytical expression for C_q can be obtained for the central region as well as for nondiffractive multiplicities. In fact, it follows from Eqs. (10) and (11)

$$C_q^c = \frac{(q+1)!}{2^q} \tag{13}$$

and

$$C_q^{\rm nd} = \frac{(q+5)!}{5!6^q} \ , \tag{14}$$

where the superscripts c and nd indicate that they are for central-region and for nondiffractive multiplicities, respectively. Comparisons between experimental and calculated values for C_q^c and $C_q^{\rm nd}$ as well as for $\langle n \rangle/D$ where $D \equiv (\langle n^2 \rangle - \langle n \rangle^2)^{1/2}$ are shown in Tables I and II.[23] It is very interesting to see that the observed[3,7–9,25,26] values for C_q^c, $C_q^{\rm nd}$, and $\langle n \rangle/D$, in particular the relative magnitude of C_2^c and $C_2^{\rm nd}$ are in surprisingly good agreement with the expressions given in Eqs. (13) and (14).

CONCLUSIONS

The present study is initiated by the following striking experimental facts.

(1) While KNO scaling[4] with respect to the *total* scaled multiplicity of charged hadrons in high-energy inelastic hadron-hadron processes is only approximately valid,[6] it is observed[1,3] that the scaled

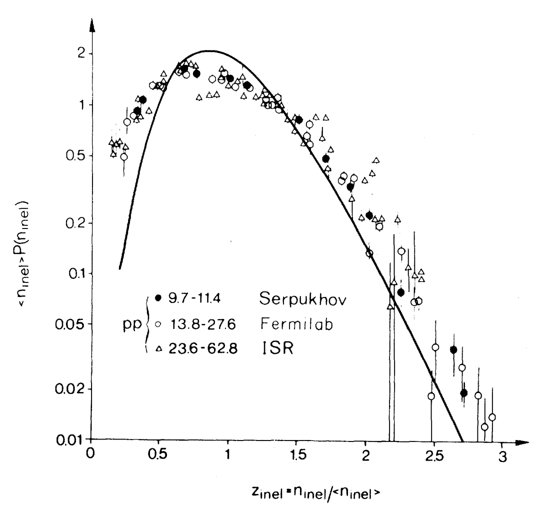

FIG. 4. Comparison of the total inelastic scattering data with the calculated nondiffractive distributions. The data are taken from Refs. 3, 9, and 24. The curve is the same as that in Fig. 2.

TABLE I. Ratio of the qth moment $\langle n_c{}^q \rangle$ to the qth power of the average multiplicity $\langle n_c \rangle^q$ and the ratio of the average multiplicity $\langle n_c \rangle$ to the multiplicity dispersion D_c in the central region. Data are taken (Ref. 25) from Refs. 3 and 26.

	Our results	Experimental results obtained at different incident energies $\sqrt{s}$ (GeV) 23.6	30.8	45.2	53.2	62.8	540
C_2	1.5	1.43±0.06	1.44±0.06	1.46±0.06	1.45±0.06	1.45±0.06	1.46±0.02
C_3	3	2.60±0.14	2.64±0.15	2.76±0.15	2.71±0.14	2.68±0.14	2.68±0.09
$\frac{\langle n \rangle}{D}$	1.414	1.53±0.05	1.51±0.05	1.47±0.05	1.49±0.06	1.49±0.06	1.43±0.05

multiplicity *in the central rapidity region* $(n_c/\langle n_c \rangle)$ is a good KNO scaling variable (see Fig. 1). That is, the product of the average multiplicity $\langle n_c \rangle$ and the probability of finding n_c, $\langle n_c \rangle P(n_c)$, depends only on $n_c/\langle n_c \rangle$, and not on the incident energy.

(2) KNO scaling is observed also in e^-e^+ annihilation processes[11] (see Fig. 3). The form of the corresponding scaling function $\psi(n_{ee}/\langle n_{ee} \rangle)$ is very much different from those in hadron-hadron collisions mentioned in point (1).

An attempt is made to understand the origin of these empirical facts and a number of other related phenomena. We show the following in this paper.

(i) The scaled multiplicity of charged hadrons produced in *nondiffractive* hadron-hadron processes $n_{\rm nd}/\langle n_{\rm nd} \rangle$ is also a good scaling variable (see Fig. 2). The KNO scaling function $\psi(n_{\rm nd}/\langle n_{\rm nd} \rangle)$, is however, qualitatively different from $\psi(n_c/\langle n_c \rangle)$. It is also different from $\psi(n_{\rm inel}/\langle n_{\rm inel} \rangle)$, which scales approximately.

(ii) There are strong experimental indications[13,18] for the hypothesis that the multiplicity n_i of charged hadrons produced by a system i is uniquely determined by the excitation energy E_i^* of that system i. In particular, for nondiffractive hadron-hadron collisions n_i is directly proportional to E_i^*.[20] This implies in particular that formation and decay of particles at the intermediate stage do not influence the KNO distribution.

(iii) In the framework of the proposed picture, high-energy nondiffractive hadron-hadron collisions can be described by a three-fireball model. In particular, explicit expressions for the KNO scaling functions $\psi(n_c/\langle n_c \rangle)$ and $\psi(n_{\rm nd}/\langle n_{\rm nd} \rangle)$, as well as a simple relationship between the two functions, can be obtained. Comparison with experiments (see Figs. 1 and 2) shows that the agreement between the data and model is good.

(iv) Other recent experimental results obtained from $p\bar{p}$-collider experiments[26,27] are also in good agreement with the three-fireball model. For example, the observed[26] increase in the height of the central-rapidity plateau with increasing incident energy is due to the fact that internal energy (and the size) of the central fireball increases with increasing bombarding energies. The observed[27] long-range forward-backward multiplicity correlation is due to the fact that part of the observed particles in the rapidity ranges considered are products of the same (central) fireball. Hence the average value of the "backward multiplicity" $\langle n_B \rangle$ (measured in the pseudorapidity range $-4 < \eta < -1$) depends on the "forward multiplicity" n_F (measured in the pseudorapidity range $1 < \eta < 4$).

We recall that in order to study the above-mentioned long-range forward-backward multiplici-

TABLE II. Ratio of the qth moment $\langle n_{\rm nd}{}^q \rangle$ to the qth power of the average multiplicity $\langle n_{\rm nd} \rangle^q$ and the ratio of the average multiplicity $\langle n_{\rm nd} \rangle$ to the multiplicity dispersion $D_{\rm nd}$ for nondiffractive collisions. Data are taken from Refs. 7 and 9.

	Our results	Experimental results obtained at different incident energies $\sqrt{s}$ (GeV) 13.8	19.6	23.8	27.6
C_2	1.167	1.174±0.014	1.174±0.023	1.176±0.023	1.202±0.026
C_3	1.556	1.567±0.023	1.571±0.038	1.581±0.038	1.682±0.045
C_4	2.333	2.322±0.039	2.433±0.066	2.380±0.067	2.678±0.084
C_5	3.889	3.749±0.070	3.831±0.120	3.946±0.124	4.888±0.170
$\frac{\langle n \rangle}{D}$	2.449	2.398±0.028	2.396±0.047	2.381±0.047	2.225±0.049

ty correlations, the parameter b in

$$\langle n_B \rangle = a + b n_F \tag{15}$$

has been measured[27] at ISR and $p\bar{p}$-collider energies. The result, in particular the comparison of b values measured in the following three pseudorapidity intervals, $|\eta|<4$, $|\eta|<1$, and $1<|\eta|<4$ at various incident energies (see figures on pp. 14, 18, and 20 of Ref. 27), shows that the observed properties are exactly those expected by the three-fireball model. In fact, according to this model b is significantly different from zero, when and only when both of the measured quantities $\langle n_B \rangle$ and n_F have non-negligible contributions from the central fireball. Hence, one expects to see, for example, the following: (α) At a given incident energy, b becomes smaller when a part of the central rapidity region is excluded. That is, for a given $\sqrt{s}$, b is smaller for $1<|\eta|<4$ than that for $|\eta|<4$. (β) The effect mentioned in (α) is more significant for relatively small $\sqrt{s}$, because the excitation energy and hence the "length" of the central fireball in rapidity space decreases with decreasing incident energy. This is in fact the reason why b vanishes for relatively low energies ($\sqrt{s}<30$ GeV) in the rapidity region $1<|\eta|<4$). (γ) Consider a given rapidity interval, $\eta_0<|\eta|<4$ say, where b is different from zero. There will be a reduction in b if the multiplicity (n_0) in $|\eta|<\eta_0$ is fixed. This reduction is due to the fact that when n_0 is fixed, the size of the central fireball is almost fixed and hence has little dependence on the value of n_F. This is especially the case when n_0 is far away from its mean value. (δ) The parameter b will become negligibly small, when $\langle n_B \rangle$ and n_F are measured in rapidity regions separated sufficiently wide from one another ($\langle n_B \rangle$ in $-4<\eta<-3$, n_F in $3<\eta<4$, say) because the central fireball has no (or very small) contribution in these regions.

Note that the properties (α), (β), and (γ) have already been observed experimentally.[27] Point (δ) may be used as a further test of the present model.

It should be mentioned that, in the present analysis, the contribution of violent collision events [i.e., those of category (a) of hypothesis (II) mentioned above] have not been taken into account. The characteristics of such events are particles with extremely high transverse momenta and/or extremely large multiplicities. Since the occurrence of such processes is very rare (about 1%),[15,16] they will have no (or very little) influence on the value of the mean multiplicity. Hence, there will practically be no change in the KNO plots except for very high $z \equiv n/\langle n \rangle$ regions ($z>5$, say).

It should be emphasized that, according to our picture, the forms of KNO scaling functions reflect the reaction mechanism of the collision processes. That is, in contrast to other models, in which comparisons between such processes have also been made,[28] we think the KNO scaling functions can be used to differentiate different reaction mechanisms. Studies along this line are now in progress. *Note added.* After this paper was submitted for publication, the work of Cool *et al.*[29] and that of Goulianos, Sticker, and White[30] were brought to our attention. A measurement of charged multiplicity distributions of high-mass diffractive $\pi^{\pm}$, $K^{\pm}$, and $p^{\pm}$ states in 100- and 200-GeV/c hadron-proton collisions $h+p \to X+p$ has been reported by Cool *et al.* They observed that the distributions are the same when compared at the same available mass for particle production $M=M_X-M_h$, independent of the incident hadron $h=\pi^{\pm}$, $K^{\pm}$, and $p^{\pm}$, and independent of the bombarding energy. Furthermore, the distributions are described well by a Gaussian function that has a maximum at $n_0 \approx 2M^{1/2}$ and a peak-to-width ratio $n_0/D \approx 2$. It is pointed out by Goulianos, Sticker, and White that the above-mentioned properties are also valid for a number of other reactions. Their results provide us not only with further evidence for the validity of the proposed picture, but also with valuable information on the properties of the fireballs formed in high-energy collisions. In particular, the latter is very useful in studying the decay stage of the fireballs in various reactions. This, as well as other related questions, will be discussed elsewhere.

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[20]After the preliminary version of this paper [contributed paper No. 631 to the 21st International Conference on High Energy Physics, Paris, 1982 (unpublished)] was written, scaling behavior with respect to the variable $E_\perp/\langle E_\perp\rangle$ in the central rapidity region ($E_\perp$ is the "transversal energy" and $\langle E_\perp\rangle$ its mean value) was brought to our attention. [Talk given by A. Astbury at the same conference (unpublished).] Since $E_\perp$ is approximately proportional to E^*, this piece of experimental fact is also in good agreement with hypothesis (III).

[21]Equation (5) is not a necessary condition for obtaining the curves given in Figs. 2 and 4. One could, in general, introduce a parameter α, such that $\langle E_C^*\rangle=\alpha\langle E^*\rangle$, $\langle E_P^*\rangle=\langle E_T^*\rangle=(1-\alpha)\langle E^*\rangle/2$, where $\langle E^*\rangle$ is the total internal energy. A careful comparison of calculated results obtained by using different values of α shows that the result is very insensitive to the choice of α. In fact, the obtained curves are not very much different from one another as long as α is not too close to 0 or 1. Note, however, that the right-hand side of Eq. (11b) is only so simple for $\alpha=\frac{1}{3}$, i.e., when Eq. (5) is valid.

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[23]The following remarks should be made concerning the CERN-collider experiments in connection with Eq. (11). First, contributions from double diffractive dissociation have been neglected in the model, but such contributions may be appreciable at the CERN-collider energy. Second, the ratios between the average internal energies of the three fireballs may depend on incident energy; that is, the assumption given in Eq. (5) may not be valid for *all* incident energies. This and other related questions will be discussed elsewhere.

[24]See, e.g., P. Slattery, in Ref. 5 and the papers cited therein.

[25]The experimental values for C_q^c are those observed in the rapidity $|\eta|<1.5$ and not $|\eta|<3.5$. This is because the contributions for the fireballs in the fragmentation regions P^* and T^* are appreciable in $|\eta|<3.5$.

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