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Monte Carlo simulation of SU(2) lattice gauge theory with internal quark loops

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Dynamical effects of quark loops in lattice gauge theory with the icosahedral group are studied. We employ the standard Wilson action and calculate the fermionic part by a discrete pseudofermionic method. The masses of the π, ρ, and ω are computed and the average value of an effective fermionic action is evaluated.

Since the lattice formulation was proposed by Wilson[1] as a powerful tool to regularize gauge theories, two important advances have been made in this field. Creutz and other authors have shown that Monte Carlo simulation can work really well for investigating pure gauge theories[2,3] and, secondly, dynamical quarks have been introduced into the calculations.[4–6] Since then, interesting and challenging attempts to evaluate static properties of hadrons started.[7–13]

All Monte Carlo evaluations of physical properties of hadrons so far use a quenched approximation or a hopping-parameter expansion.[14] Though they are effective methods for reducing computer time, each has its defect. In the quenched approximation, we cannot study effects of internal quark loops which are neglected. At the present time, for the hopping-parameter-expansion calculations, it is doubtful whether the series expansion is enough to reach convergence.

In this paper, we report on the first study of Monte Carlo simulation without these approximations. We employ the standard Wilson action with gauge group SU(2), which is replaced by the 120-element icosahedral subgroup[15,16] in the real calculation. The simulation is done on a periodic 4^4 lattice.

The action has the form

$$S = S_G + \sum_{m,n} \bar{\psi}_m \Delta(m,n) \psi_n \ , \tag{1}$$

where the first term represents the kinetic term of gauge fields, U, with bare coupling g. Meson fields are written in the form $\bar{\psi}(x)\Gamma\psi(x)$, where Γ is a matrix with Dirac and flavor indices. After integrating over the fermionic degrees of freedom, we get the formula for meson Green's functions:

$$G(x,y) = \int \mathfrak{D}U \mathcal{G}(x,y) e^{-S_{\text{eff}}} \Big/ \int \mathfrak{D}U e^{-S_{\text{eff}}} \ , \tag{2}$$

where

$$\mathcal{G}(x,y) = \mathrm{Tr}[\Gamma\Delta^{-1}(x,x)]\,\mathrm{Tr}[\tilde{\Gamma}\Delta^{-1}(y,y)] - \mathrm{Tr}[\Gamma\Delta^{-1}(x,y)\tilde{\Gamma}\Delta^{-1}(y,x)] \ , \tag{3}$$

with $\tilde{\Gamma} = \gamma^4\Gamma\gamma^4$. The first term of $\mathcal{G}$ splits ρ and ω meson masses.[17] The effective action, S_{eff}, is defined by

$$S_{\text{eff}} = S_G + S_{\text{EF}} \ , \quad S_{\text{EF}} = -\ln\det\Delta \ . \tag{4}$$

For a small change of gauge fields,

$$\delta S_{\text{EF}} \simeq -\tfrac{1}{2}\,\mathrm{Tr}[(\Delta^\dagger\Delta)^{-1}\delta(\Delta^\dagger\Delta)] \ . \tag{5}$$

We employ the pseudofermionic method, which was proposed in Ref. 4, to evaluate $(\Delta^\dagger\Delta)^{-1}$.

Here is one trick to reduce the computer time; we use discrete pseudofermionic fields, ϕ. Suppressing Dirac and site indices, we can write

$$\phi = \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} = r\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \ , \tag{6}$$

with $|\alpha|^2 + |\beta|^2 = 1$. Note that α and β satisfy the same condition as a column vector of a 2×2 SU(2) matrix. We replace this by a set of column vectors of the representation of the icosahedral group. We have already employed this finite group for the gauge fields. Therefore we can use a multiplication table of the group in the computation.

Before presenting numerical results, we comment briefly upon the reliability of the program. A pure gauge part of the program is essentially the same as that of Ref. 18. Weingarten has reported the critical value of the hopping parameter κ, at which the pion mass becomes zero, to be 0.220 on a 4^4 lattice after averaging 8 gauge configurations.[9] Under the same conditions, we switch off internal quark loops and calculate the time slice propagator, $G^\pi(\tau)$, over 16 gauge configurations, which is shown in Fig. 1(a). Within error bars, the behavior of $G(\tau)$ is consistent with zero pion mass. Next, we compare $\langle\bar{\psi}(x)\psi(x)\rangle$ of our calculation with that of the hopping-parameter expansion[19] at $\kappa = 0.1$ where convergence of the expansion may be very good. In Fig. 2, $8 - \langle\bar{\psi}(x)\psi(x)\rangle$ is plotted as a function of κ. The agreement is quite good.

In Fig. 1(b), we present the value of $G(\tau)$ when we switch on quark loops. Values of the parameters $\kappa = 0.218$ and $\beta = 1.0515$ are the same as in Ref. 9. The behavior is distinctly different from the result of

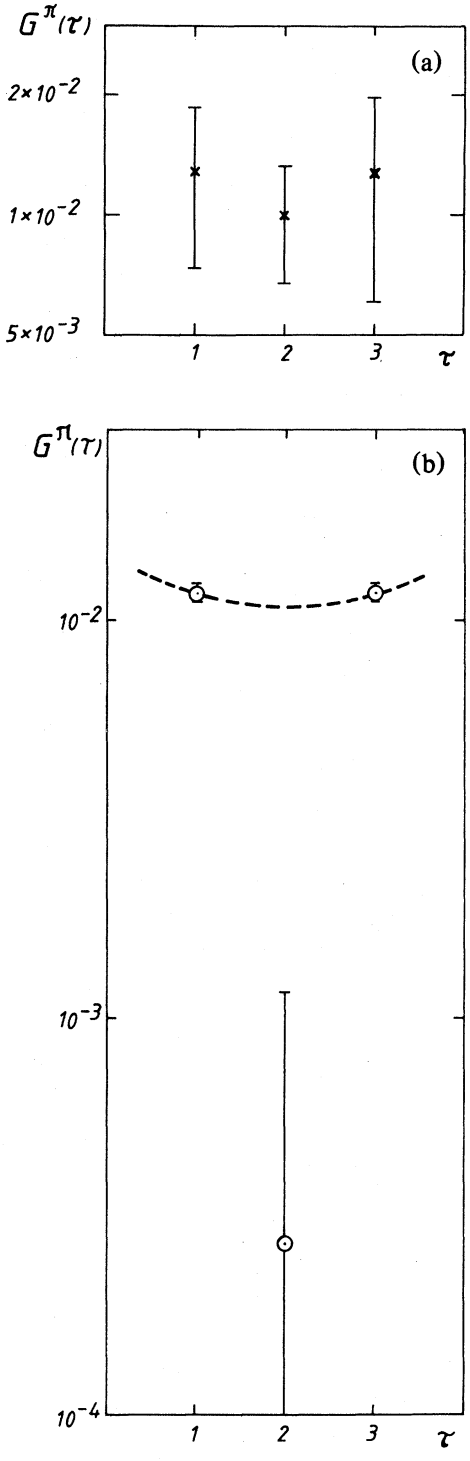

FIG. 1. (a) Numerical results for the pion time-slice propagator at $\kappa=0.220$ and $\beta=1.0515$ when quark loops are switched off. Normalization is arbitrary. (b) Numerical results for the pion time-slice propagator at $\kappa=0.218$ and $\beta=1.0515$. The dashed line is a fit of the results of Ref. 9 (quenched approximation) at the same values of the parameters. Normalization is arbitrary.

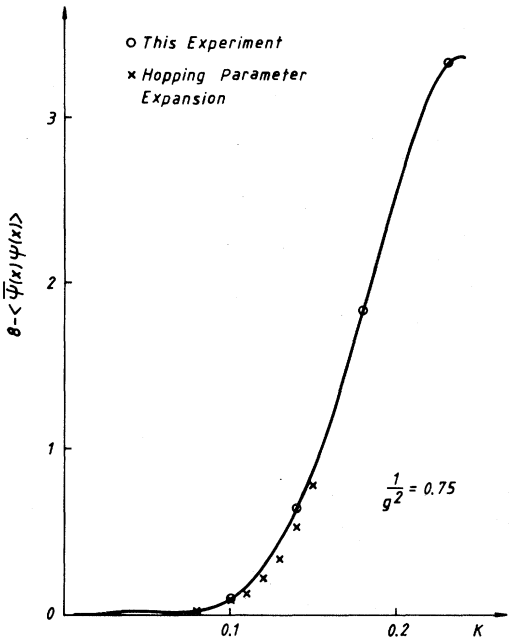

FIG. 2. Observed value of $8-\langle\bar{\psi}\psi\rangle$ (circle). The solid line is a polynomial fit. The crosses ($\times$) are the results of Ref. 19 (hopping-parameter expansion). The coupling is fixed at $1/g^2=0.75$.

the quenched approximation. This may come partly from higher continuum states, i.e., multimeson states.

Our numerical evaluation of the π, ρ, and ω masses and $\langle\bar{\psi}(x)\psi(x)\rangle$ in the lattice distance unit for $\kappa=0.24$ and $1/g^2=0.75$ are shown in Table I. The number of the gauge configurations is $128+8$; we have not used the first 8 configurations for measurements. For each gauge configuration we generate $175+25$ pseudofermionic Monte Carlo iterations; we have not used the first 25 configurations. In order to reduce statistical errors, we calculate $G(\tau)$ for four directions of the lattice and sum them up.

For the same purpose, we average the propagators over all possible charge and spin states. All error

TABLE I. Numerical results on the π, ρ, and ω masses. Also the result for $\langle\bar{\psi}\psi\rangle$ for one flavor is shown. $\kappa=0.24$ and $1/g^2=0.75$. The number of the gauge configurations is 128.

$m_\pi a$	$m_\rho a$	$m_\omega a$	$\langle\bar{\psi}\psi\rangle$
2.5 ±0.1	2.9 ±0.2	3.0 ±0.6	4.42 ±0.01

bars are given in a very naive manner: we divide the data into two subgroups and recalculate quantities in each group.

The mass ratio between π and ρ is far from its experimental value. This is mainly because our lattice is too small. In a 4^4 lattice, we cannot subtract the effects of higher masses.

From Table I, we find the ω/ρ mass ratio to be 1.03 which is near the experimental value, 1.02. We calculate this ratio also for the last 96 gauge configurations and get the value 1.1. For $\kappa=0.23$ and 0.245, the value is 1.04 and 1.03, respectively.

It is interesting to estimate the magnitude of $\det\Delta$ or $S_{\rm EF}$. This is related to $\langle\bar{\psi}(x)\psi(x)\rangle$ in the following form:

$$\left\langle \kappa\frac{dS_{\rm EF}}{d\kappa}\right\rangle = N^4[\,\langle\bar{\psi}(x)\psi(x)\rangle - 8\,]\ , \qquad (7)$$

for color SU(2) and for one flavor, where N^4 is lattice size. In Fig. 2, we show observed values of $8-\langle\bar{\psi}\psi\rangle$ at several κ for $1/g^2=0.75$. The solid line is a polynomial fit for these data. From this fit, assuming there is no singular behavior in this region, we can get $\langle S_{\rm EF}\rangle/N^4$ by the formula (7). The averaged effective fermion action per one flavor is shown in Fig. 3(a). In Fig. 3(b), we show calculated values of $\langle S_G\rangle/N^4$ as a function of κ. These figures tell us that we may discard $S_{\rm EF}$ in comparison with S_G below $\kappa\sim 0.1$. The loopless approximation is justified for heavy-quark phenomenology. For $\kappa\gtrsim 0.2$, however, $S_{\rm EF}$ makes a sizable contribution to the total action.

In this experiment, we cannot determine the critical κ and, therefore, we cannot get the quark mass. Though the values of our parameters $1/g^2$ and κ are consistent with the SU(3) hopping-parameter-expansion results,[10] there is a possibility that we have made observation above κ_c. We think, however, the sign and the order of the ρ-ω splitting is correct because the ρ/ω mass ratio is very stable when we change κ.

If we adjust our calculated value of the ρ mass to the experimental value, we get for lattice distance $a\simeq 0.74$ fm. In order to see whether the observed values are reasonable or not, we try to calculate the π decay constant using the PCAC (partial conservation of axial-vector current) relation. We put the quark mass m_q to be 5 MeV, which we take from the "world data."[7,8,10,11] We get $f_\pi=180$ MeV, while the experimental value is 93 MeV. There are several possible reasons for this discrepancy: (i) The f_π is a sensitive quantity because it depends on wave functions and quark mass. (ii) The PCAC formula is *not* trivial in this calculation because chiral symmetry is explicitly broken in the Wilson action due to the Wilson term. However, we pick up only the pole term and not the continuum term in the formula. (iii) Each quantity $\langle\bar{\psi}\psi\rangle$, a^{-1}, and, of course, m_q includes error. Therefore the values of κ, $\langle\bar{\psi}\psi\rangle$, and a^{-1} which we have obtained here are not inconsistent.

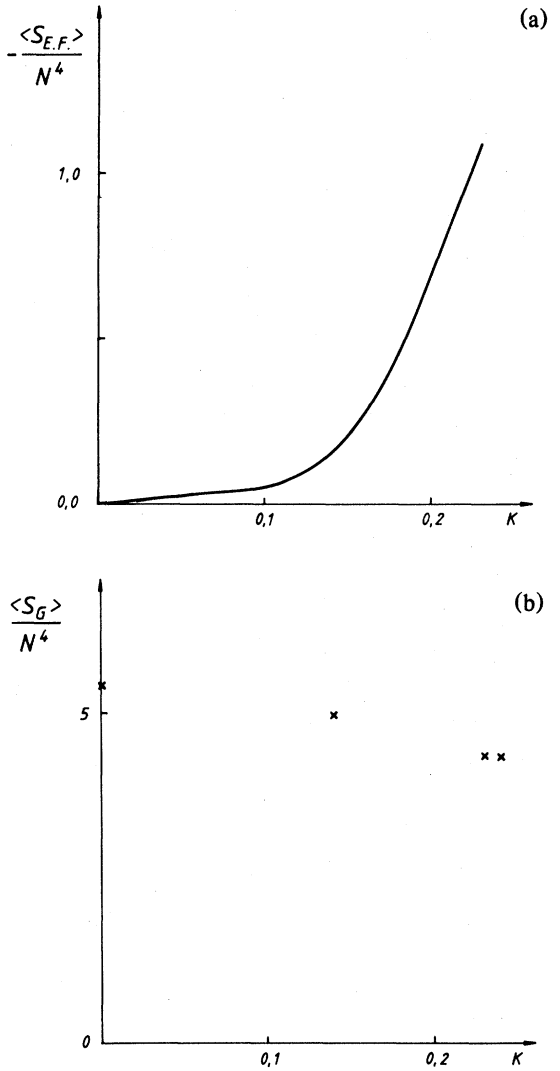

FIG. 3. (a) Plot of $-\langle S_{\rm EF}\rangle/N^4$ per one flavor as a function of the hopping parameter κ at $1/g^2=0.75$. (b) Calculated value of $\langle S_G\rangle/N^4$ as a function of the hopping parameter κ at $1/g^2=0.75$.

Though we suffer from low statistics and small lattice size, it is very encouraging that we have obtained correct ρ-ω splitting because this is the first touchstone in the study of the lattice gauge theory including dynamical quark loops. The term which splits ω from ρ consists of an initial $q\bar{q}$ annihilation and a final $q\bar{q}$ creation. Fermion loops make a significant contribution to this term and, probably, the problem of the chiral-symmetry breaking does not matter here.

We have made an estimate of $\det\Delta$. As far as we know, there has been no evaluation of this quantity in physical regions. Therefore, arguments and con-

troversies about contributions of quark loops were not well founded.

The final and, maybe, most important conclusion of this paper is that it is now feasible to include quark loops in Monte Carlo simulation of lattice gauge theories. There is a chance to investigate low-energy properties of QCD including the η problem.

All calculations have been done with a VAX11/780 at the Frascati Laboratory. Typical computer time is 1 h for 1 gauge configuration with $200+200$ pseudo-fermionic iterations for two flavors.

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