PHYSICAL REVIEW D VOLUME 27, NUMBER 10 15 MAY 1983

Rapid Communications

The Rapid Communications section is intended for the accelerated publication of important new results. Manuscripts submitted to this section are given priority in handling in the editorial office and in production. A Rapid Communication may be no longer than 3½ printed pages and must be accompanied by an abstract. Page proofs are sent to authors, but, because of the rapid publication schedule, publication is not delayed for receipt of corrections unless requested by the author.

Probabilistic interpretation of the dipole ghost models

Ion I. Cotăescu
Department of Physics, University of Timişoara, Timişoara, Romania
(Received 1 March 1983)

It is shown that a specific symmetry of the dipole ghost models allows one to derive a good probabilistic interpretation for a particular spin-half model. The arguments given indicate that the interpretation presented holds for any model in which the interaction does not break this symmetry.

It is known that the dipole ghost models (DGM's), in which the mass condition has the quadratic Klein-Gordon form, are not unitary because of the indecomposable nonunitary representation of the translation group T(4).[1,2] The space of states is of an indefinite metric, and therefore the physical interpretation cannot be given as usual. The general opinion is that the dipole ghost states have no physical significance and thus they must be eliminated by using a reconstruction method of the physical-state space.[3,4] Another attitude is to try to find a new probabilistic interpretation for the DGM's, so that positive and conserved probabilities should be derived directly from the non-Hilbert dipole ghost space. Such an attempt has been made by Kiskis[5] who defines a particle-dipole transition probability which is conserved but not always positive, in the first order of the perturbation theory. On the other hand, in Ref. 2, another interpretation is suggested so that the probabilities are positive but their conservation is not certain. Thus there appears the problem of the conditions for which the transition probabilities should be conserved and positive in each order of the perturbation theory.

Here, we want to point out that the method of Ref. 5 leads to positive probabilities if the DGM has a specific symmetry. Our arguments will be illustrated on a simple example of the spin-half DGM, the physical content of which would be easily compared with the QED one.

Let us consider the spin-half free DGM constructed, by analogy to the Froissart[6] spinless model, starting with two coupled free Dirac equations:

$$(i\gamma^\mu\partial_\mu - m)\psi_1 = 0, \quad (i\gamma^\mu\partial_\mu - m)\psi_2 = m\psi_1 \ .$$

If we denote

$$\psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \quad \beta^\mu = \begin{pmatrix}0 & \gamma^\mu\\ \gamma^\mu & 0\end{pmatrix}, \quad \alpha = \begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix} \ ,$$

we obtain the spin-half equation of Ref. 2 in a suitable form for the Lagrangian theory:

$$(i\beta^\mu\partial_\mu - \alpha m)\psi = 0 \ . \tag{1}$$

The corresponding free Lagrangian density is

$$\mathcal{L} = \frac{i}{2}(\bar\psi\beta^\mu\partial_\mu\psi - \partial_\mu\bar\psi\beta^\mu\psi) - m\bar\psi\alpha\psi \ ,$$

where $\bar\psi = (\bar\psi_1, \bar\psi_2)$ is the Dirac adjoint of the eight-component spinor ψ. The invariance transformations of the Lagrangian are the Poincaré transformations, the Abelian gauge, and a new class of transformations of the form

$$\psi_1 \to \psi_1 \ , \psi_2 \to \psi_2 + i\lambda\psi_1, \quad \lambda \in \mathrm{R} \tag{2}$$

which will be called the dipole-gauge (D) transformation. As will be shown in the following, this is a characteristic symmetry of the free DGM's.

The solutions of a free DGM can be written in covariant[1] or noncovariant[5,6] form, the latter being preferred for spinless models. In the spinorial case, the covariant form, which contains only Dirac spinors, is the simplest because, in the noncovariant form, there also appear the derivatives of these spinors. In the covariant form, the system of fundamental solutions of Eq. (1) contains two particlelike solutions $U^i_{\vec{p},\sigma}(x)$ $(i=1,2)$, and two antiparticlelike solutions, $V^i_{\vec{p},\sigma}(x)$, of momentum p and spin projection σ. These solutions depend on two pairs of spinors, $U^i(\vec{p},\sigma)$ and $V^i(\vec{p},\sigma)$, related to the Dirac spinors,

u_D and v_D. By using the compact notation $U^i = W^i\ (\epsilon=1)$ and $V^i = W^i\ (\epsilon=-1)$, we can write the dipole ghost spinors as

$$W^1(\vec{p},\sigma,\epsilon)=\begin{pmatrix}0\\ w_D(\vec{p},\sigma,\epsilon)\end{pmatrix},\quad W^2(\vec{p},\sigma,\epsilon)=\begin{pmatrix}w_D(\vec{p},\sigma,\epsilon)\\ 0\end{pmatrix}\ , \tag{3}$$

and the fundamental solutions

$$W^1_{\vec{p},\sigma}(x,\epsilon)=(2\pi)^{-3/2}(m/E)^{1/2}\exp(-i\epsilon px)\,W^1(\vec{p},\sigma,\epsilon)\ ,$$
$$W^2_{\vec{p},\sigma}(x,\epsilon)=(2\pi)^{-3/2}(m/E)^{1/2}\exp(-i\epsilon px)[cW^1(\vec{p},\sigma,\epsilon)+W^2(\vec{p},\sigma,\epsilon)-i\epsilon pxW^1(\vec{p},\sigma,\epsilon)]\ ,$$

where $E=(\vec{p}^{\,2}+m^2)^{1/2}$, and c is an arbitrary constant. We choose $c=\frac{3}{2}$ so that the following condition should be satisfied:

$$\int d^3x\ W^i_{\vec{p},\sigma}(x,\epsilon)\beta^0 W^j_{\vec{p}',\sigma'}(x,\epsilon')=\eta^{ij}\delta_{\epsilon,\epsilon'}\delta_{\sigma,\sigma'}\delta^3(\vec{p}-\vec{p}')\ ,$$

where the 2×2 matrix η has the elements $\eta^{11}=\eta^{22}=0$, and $\eta^{12}=\eta^{21}=1$. With this condition, the solutions of Eq. (1) can be written as

$$\psi(x)=\int d^3p\sum_{i,j,\sigma}[\,U^i_{\vec{p},\sigma}(x)\eta^{ij}a^j(\vec{p},\sigma)+V^i_{\vec{p},\sigma}(x)\eta^{ij}b^{\dagger j}(\vec{p},\sigma)]\ ,$$

depending on the particlelike annihilation dipole ghost operators a^i and antiparticlelike creation ones, $b^{\dagger i}$.

According to the naive quantization procedure, we obtain, as in the spinless case,[5,6] the anticommutators of these operators; the nonvanishing ones are

$$\{a^i(\vec{p},\sigma),a^{\dagger j}(\vec{p}',\sigma')\}=\{b^i(\vec{p},\sigma),b^{\dagger j}(\vec{p}',\sigma')\}=\eta^{ij}\delta_{\sigma,\sigma'}\delta^3(\vec{p}-\vec{p}')\ . \tag{4}$$

These relations lead to the well-known dipole ghost structure of the space of states. This is a non-Hilbert space with a sesquilinear Hermitian form, defined in the one-dipole sector $[\,|i,p\sigma\rangle=a^{\dagger i}(\vec{p},\sigma)|0\rangle$, $i=1,2]$ as follows:

$$\langle i,\vec{p}\sigma|j,\vec{p}'\sigma'\rangle=\eta^{ij}\delta_{\sigma,\sigma'}\delta^3(\vec{p}-\vec{p}')\ . \tag{5}$$

The Hermitian adjoint of the operators is considered with respect to this form.

From the Noether theorem, we can calculate the momentum operators P_μ and the generator D of the D transformations (2). These are Hermitian operators of the form

$$P_\mu=\int d^3p\sum_\sigma p_\mu\left\{a^{\dagger 1}(\vec{p},\sigma)a^1(\vec{p},\sigma)+\sum_{i,j}a^{\dagger i}(\vec{p},\sigma)\eta^{ij}a^j(\vec{p},\sigma)+\text{antiparticle terms}\right\}$$

and

$$D=\int d^3p\sum_\sigma[a^{\dagger 1}(\vec{p},\sigma)a^1(\vec{p},\sigma)-b^{\dagger 1}(\vec{p},\sigma)b^1(\vec{p},\sigma)]\ .$$

According to Eqs. (4), we obtain the following relations:

$$[P_\mu,a^{\dagger 1}(\vec{p},\sigma)]=p_\mu a^{\dagger 1}(\vec{p},\sigma)\ ,$$
$$[P_\mu,a^{\dagger 2}(\vec{p},\sigma)]=p_\mu[a^{\dagger 2}(\vec{p},\sigma)+a^{\dagger 1}(\vec{p},\sigma)]\ ,$$

which show that the operators P_μ are the generators of a nonunitary indecomposable representation of the T(4) group.[1,2] On the other hand, we must observe that the generator D, which satisfies

$$[D,a^{\dagger 1}(\vec{p},\sigma)]=0\ ,\qquad [D,a^{\dagger 2}(\vec{p},\sigma)]=a^{\dagger 1}(\vec{p},\sigma)\ , \tag{6}$$

is related to the nilpotent part[2] of the translation generators. This means that the D symmetry is characteristic of all DGM's with indecomposable translation representations.

Now we shall discuss the probabilistic interpretation of the model, starting with the method of Ref. 5, which generalizes the definition of the probability of unitary case, so that the conservation of the probability should be certain. In Ref. 5 the transition probability of a particle-dipole process is defined as the expectation value in a particle state, of the projector of the dipole ghost subspace, associated to a single physical state. Following this idea, we shall also associate the one-dipole ghost subspace ($|i,\alpha\rangle$) with the physical state $|\alpha\rangle$. In the free DGM, the probability may be calculated, only between the two above-defined physical states, as the expectation value in the state of the projector of the subspace ($|i,\beta\rangle$), corresponding to the physical state $|\beta\rangle$. The form of the projector is known[1,5]:

$$\sum_{i,j}|i,\beta\rangle\eta^{ij}\langle j,\beta|\ ,$$

but the expectation value in a dipole state has not yet been defined. We suggest the following definition for the expectation value of an operator A, in the $|\alpha\rangle$ state, corresponding to the one-dipole subspace ($|i,\alpha\rangle$):

$$\langle\alpha|A|\alpha\rangle=\frac{1}{2}\sum_{i,j}\langle i,\alpha|A|j,\alpha\rangle\eta^{ij} \ . \qquad (7)$$

As can be easily seen this is invariant under translations and D transformations. According to the above definitions, the probability in the one-dipole sector of the free model can be written as

$$P_{\alpha,\beta}=\frac{1}{2}\sum_{i,j,k,l}\langle i,\alpha|k,\beta\rangle\langle l,\beta|j,\alpha\rangle\eta^{ij}\eta^{kl} \ . \qquad (8)$$

This probability is conserved by definition[5] and, according to Eq. (5), it is equal to $|\langle 1,\alpha|2,\beta\rangle|^2$, which is the positive probability of Ref. 2. It can prove that these properties stand for any dipole (or antidipole) sector of the free model. Thus, the two discussed interpretations are equivalent in the free case, and lead to well-defined probabilities.

In the presence of the interaction, the conserved transition probability will be defined as the expectation value in the "out" state of a projector of the "in" subspace. The expectation value, in the states containing many dipole ghosts, can be obtained by generalizing the definition (7), when, for the pure particle states, they will be calculated as usual. The transition probabilities which result will depend on the transition matrix elements derived from the perturbation theory. Particularly, the dipole-dipole transition probability will be obtained by replacing the corresponding transition matrix elements in Eq. (8). These definitions are consistent only if they lead to positive transition probabilities in each order of the perturbation theory.

A simple model, in which this condition is fulfilled, is that of the spin-half dipole ghost, interacting with the minimally coupled electromagnetic field. This DGM is D symmetric, since the interaction Lagrange function

$$\mathcal{L}_{\rm int}=\bar{\psi}(x)\beta^\mu\psi(x)A_\mu(x)$$

is D invariant. The behavior of the model can be illustrated by analyzing the dipole-dipole transition, in an external field, in the first order of the perturbation theory. The resulting matrix elements, which are

$$\langle i,\,\text{out}\,\vec{p}\sigma|j,\,\text{in}\,\vec{p}'\sigma'\rangle=-iem(2\pi)^{-3}(EE')^{-1/2}\bar{U}^i(\vec{p},\sigma)\beta^\mu U^j(\vec{p}',\sigma')\hat{A}_\mu(\vec{p}-\vec{p}') \ ,$$

have a remarkable property which comes from the form of the spinors (3). This is

$$\langle i,\,\text{out}\,\vec{p}\sigma|j,\,\text{in}\,\vec{p}'\sigma'\rangle=\eta^{ij}\langle\text{out},\,\vec{p}\sigma|\,\text{in}\,\vec{p}'\sigma'\rangle_{\rm QED} \ .$$

The probability (8), calculated with these matrix elements, is positive and equals that of the corresponding processes of QED. Moreover, preliminary calculations show that the renormalization has the same content as in QED. Thus we can conclude that the discussed probabilistic interpretation is valid in this case. As a matter of fact, our model is equivalent to QED and brings nothing new.

The above results are a direct consequence of the D symmetry of the considered model. The connection between this symmetry and the probabilistic interpretation seems to be general. If the interaction does not break the D symmetry, then we have $D_{\rm in}=D_{\rm out}=D$ and, according to Eqs. (6), the dipole-dipole matrix elements satisfy, in each order, the conditions

$$\langle 1,\,\text{out}\,|1,\,\text{in}\rangle=0, \quad \langle 1,\,\text{out}\,|2,\,\text{in}\rangle=\langle 2,\,\text{out}\,|1,\,\text{in}\rangle \ ,$$

which lead to the positive probability $|\langle 1,\,\text{out}\,|2,\,\text{in}\rangle|^2$. For the process involving many dipole ghosts or dipole ghosts and particles, the conserved transition probabilities have complicated expressions but, after a few D manipulations, one can verify that these are also positive. In the same way, it results that there is involved only one self-energy matrix element ($\langle 1|2\rangle=\langle 2|1\rangle$). This indicates that the renormalization has the same content as in the unitary case. In our opinion, the conclusion can be that any D-symmetric DGM, with the presented probabilistic interpretation, is equivalent with a usual unitary field theory.

In the case in which the interaction breaks the original D symmetry of the free DGM, the dipole-dipole transition probability (8), calculated in the first order, is not positively defined. This confirms the results of Ref. 5 where the model was also non-D-symmetric. Another property of the DGM is the appearance of four self-energy matrix elements ($\langle i,|j\rangle$), the divergences of which cannot be eliminated by using the standard renormalization method; here, a single normalization constant Z_2 will not suffice to assure the one-dipole stability condition. Thus it seems that there is an incompatibility between the structure of the non-D-symmetric DGM and the usual perturbation method. Therefore this must be adequately reformulated before analyzing the possibilities of physical interpretation.

[1]E. Iacopini, Nuovo Cimento A18, 205 (1973).
[2]A. Adamczyk and R. Rączka, Rep. Math. Phys. 17, 145 (1980).
[3]K. L. Nagy, *State Vector Spaces with Indefinite Metric in Quantum Field Theory* (Noordhoof, Groningen, 1966).
[4]D. Zwanziger, Phys. Rev. D 17, 457 (1978).
[5]J. Kiskis, Phys. Rev. D 11, 2178 (1975).
[6]M. Froissart, Nuovo Cimento Suppl. 14, 197 (1959).