Suppression of color screening at large N

J. Greensite and M. B. Halpern
Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720

(Received 7 September 1982)

In large-N QCD, deconfinement by color screening is suppressed. The adjoint string tension is twice the fundamental string tension. Consequences for models of confinement are discussed, and a simple model of a confining large-N master field is given.

Consider an adjoint quark-antiquark pair in finite-N QCD. If there is a (double) string between the quarks (confined state), it is expected that it is energetically favorable for the string to break (vacuum polarization), resulting in a state of free screened quark plus free screened antiquark.[1] In strong-coupling Euclidean lattice gauge theory, the situation corresponds to Figs. 1(a) and 1(b). A fixed-time slice of the "sandwich" [Fig. 1(a)] is the bound state of the two quarks (connected by a double string). A fixed-time slice of the "tube" [Fig. 1(b)] is the screened state (each quark free and wrapped in glue). The statement is that the sandwich contributes an area piece to the Wilson loop, while the tube contributes a perimeter piece. When the loop is large the sandwich is small and we see a perimeter law. This is color screening, in which the adjoint quark charges can be shielded by pure glue. Of course, the quark charges can also be screened (the string can break) by production of quark-antiquark pairs in the vacuum. However, we will assume that the quarks are very heavy (nondynamical) and neglect this effect.

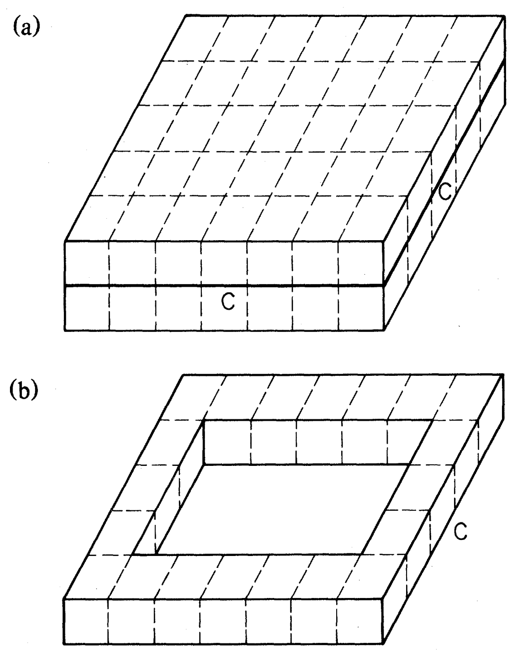

FIG. 1. Strong-coupling lattice diagrams responsible for (a) area-law and (b) perimeter-law falloffs of the Wilson loop C in the adjoint representation.

Our point in this Brief Report is that screening by pure glue is suppressed in the large-N limit, and this fact has serious implications for certain models of confinement. Computation of the diagrams in the figure gives

$$\langle \mathrm{Tr}_A U[C]\rangle \sim N^2(e^{-2\sigma_F A[C]}+N^{-2}e^{-4\sigma_F P[C]}) \ , \qquad (1)$$

where Tr_A is the trace in the adjoint representation, and σ_F is the string tension for the fundamental quark. The first term is from the sandwich, the second from the tube. In the extreme large-N limit, the tube is suppressed (color screening is suppressed), and we see an area law for the adjoint loop. In fact, comparing the terms in (1), we get an estimate for the screening length (at which the string will snap),

$$L \approx \left[\frac{\ln N^2}{2\sigma_F}\right]^{1/2} \ . \qquad (2)$$

A general argument for large-N suppression of color-screening deconfinement is available. Since (F = fundamental)

$$\mathrm{Tr}_A U[C] = \begin{cases}\mathrm{Tr}_F U[C]\mathrm{Tr}_F U^\dagger[C] & [\mathrm{U}(N)] \ , \\ \mathrm{Tr}_F U[C]\mathrm{Tr}_F U^\dagger[C]-1 & [\mathrm{SU}(N)] \ , \end{cases} \qquad (3)$$

it follows immediately from large-N factorization that

$$\langle \mathrm{Tr}_A U[C]\rangle \underset{N}{=} |\langle \mathrm{Tr}_F U[C]\rangle|^2 \ . \qquad (4)$$

If $\sigma_{F(A)}$ is the string tension in the fundamental (adjoint) respresentation, Eq. (4) says that

$$\sigma_A \underset{N}{=} 2\sigma_F \ , \qquad (5)$$

thus verifying our scenario above in generality. The relations (1) and (2) are also generic. Further, it is known that the topology of the leading contribution to the fundamental loop is planar, and order N. Equation (4) then states that the leading contribution to the adjoint loop has the topology of a sphere (with the adjoint loop at the equator), and order N^2. Note that the sandwich [Fig. 1(a)] is a sphere, while the tube [Fig. 1(b)] is a torus, and hence down in N.

In an extreme large-N universe, we have seen that transitions bound $\rightarrow$screened are suppressed. By the same token, the number of free screened adjoint quarks is absolutely conserved (they cannot be produced or annihilated). In such a universe, created with no free screened adjoint quarks, none can arise, and it is fair to say all adjoint quarks are confined.

Our observations make it clear that conjectured confinement mechanisms having to do with the center of the group (Z_N fluxons,[2] "spaghetti" vacuum[3]) cannot survive at large N. Assume (fundamental) confinement is describable at all N in terms of a sum over fluxon configurations alone. Since the fluxons have no effect on the adjoint loop, the hypothesis is inconsistent with large-N factorization, Eq. (4). A further set of *ad hoc* configurations X might be assumed to confine the adjoint quarks at large N, but X must be mysteriously correlated with the fluxons to produce (5). We find this unnatural. It is simpler to believe that the center of the group plays no role in large-N confinement. The center of the group, therefore, stands to confinement roughly as instantons stand to the U(1) problems. It would be preferable to find a unified (all-N) confinement mechanism. From a different direction, Lovelace has recently drawn similar conclusions about monopoles.[4]

Indeed the very idea of a *group* of configurations being necessary for large-N confinement is presumably in contradiction with factorization. The large-N limit, as discussed in Refs. 5–7, is analogous to a thermodynamic limit, in which the statistical fluctuations of gauge-invariant quantities vanish. This means that a Wilson loop $\mathrm{Tr}\,U(C)$ evaluated over any set of equilibrium configurations (generated, e.g., by the Monte Carlo procedure or the Langevin equation) will take on the *same* value [up to $O(1/N)$ corrections] for each configuration. In other words, the dispersion

$$\frac{1}{(\mathrm{Tr}1)^2}\langle[\mathrm{Tr}\,U(C)-\langle\mathrm{Tr}\,U(C)\rangle]^2\rangle=0+O(1/N^2) \tag{6}$$

vanishes in the $N\rightarrow\infty$ limit. This behavior has actually been verified in a recent Monte Carlo calculation.[7]

Now factorization is a simple consequence of the vanishing of fluctuations in gauge-invariant quantities in the large-N limit. However, Eq. (6) is at odds with any explanation of confinement, such as the fluxon or monopole picture, in which typical vacuum configurations can give rise to wildly different values for $\mathrm{Tr}\,U(C)$. In such models, the Wilson loop attains an exponentially small value by averaging over large (positive and negative) fluctuations in the value of the loop, induced by the confining configurations. But that kind of behavior conflicts with Eq. (6), so it seems that these models are ruled out as an explanation of confinement in the large-N limit. An alternative framework for studying confinement at large N is the master-field approach.[8] We now mention a simple toy master field that confines correctly and incorporates asymptotic freedom.

In Ref. 5, we have found an exact matrix equation (the quenched Langevin equation) for the QCD master field. In our approach, the master field is a translationally covariant function of $5N$ uniform random (quenched) momenta $\bar{p}^*=(p^*_{\mu a},p^*_{5a})$ and a four-vector $N\times N$ Hermitian Gaussian random noise matrix n_μ^{*ab}. To zeroth order, the master field is

$$A_\mu^{*ab}(x)\approx e^{i(p_a^*-p_b^*)\cdot x}n_\mu^{*ab}\;[i(p^*_{5a}-p^*_{5b})+(p_a^*-p_b^*)^2]^{-1}+\text{gauge terms}\;. \tag{7}$$

The prescription for the toy master field is to modify $(p_a^*-p_b^*)^2$ in Eq. (7) by an expression involving a logarithm,

$$gA_\mu^*(x)=e^{i(p_a^*-p_b^*)\cdot x}\left[\delta_{\mu\nu}-\frac{(p_a^*-p_b^*)_\mu(p_a^*-p_b^*)_\nu}{(p_a^*-p_b^*)^2}\right]\eta_\nu^{ab}$$
$$\times\left[i(p^*_{5_a}-p^*_{5_b})+\frac{11}{48\pi^2}N(p_a^*-p_b^*)^2\ln\left[1+\frac{(p_a^*-p_b^*)^2}{\mu^2}\right]\right]^{-1}+\text{gauge terms}\;. \tag{8}$$

This gives rise to a two-point function which corresponds to Richardson's potential.[9] The infrared k^{-4} behavior of the two-point function is consistent with a truncated set of Schwinger-Dyson equations.[10] It should be emphasized that a possible objection to k^{-4} propagators, namely, that they confine Wilson loops in any nontrivial representation, has been converted to a virtue. This insensitivity of confining forces to the group center should indeed be a property of the planar sector of QCD.

Note added. We have learned that T. Banks and A. Casher [Nucl. Phys. B167, 215 (1980)] have also pointed out that Eq. (4) implies the suppression of color screening at large N, although their article does not discuss the implications of this fact for models of confinement, nor deal with the other points raised above. We thank Poul Olesen for bringing this article to our attention.

We would like to thank Orlando Alvarez and Neal Snyderman for useful discussions. This work was supported in part by Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098 and in part by the National Science Foundation under Grant No. PHY-81-18547.

---

[1]J. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975).
[2]G. 't Hooft, Nucl. Phys. B138, 1 (1978); G. Mack, in *Recent Developments in Gauge Theories,* Proceedings of the Cargese Summer Institute, 1979, edited by G. 't Hooft *et al.* (Plenum, New York, 1980), p. 217.
[3]See J. Ambjørn and P. Olesen, Nucl. Phys. B170[FS1], 265 (1980), and references therein.
[4]C. Lovelace, Nucl. Phys. B201, 333 (1982).
[5]J. Greensite and M. B. Halpern, Nucl. Phys. B211, 343 (1983).
[6]O. Haan, Phys. Lett. 106B, 207 (1981).
[7]J. Carlson, J. Greensite, M. B. Halpern, and T. Sterling, Berkeley Report No. UCB-PTH-82/24 (unpublished).
[8]E. Witten, in *Recent Developments in Gauge Theories,* Ref. 2, p. 403.
[9]J. Richardson, Phys. Lett. 82B, 272 (1979).
[10]U. Bar Gadda, Nucl. Phys. B163, 312 (1980); R. Anishetty *et al.,* Phys. Lett. 86B, 52 (1979); S. Mandelstam, Phys. Rev. D 20, 3223 (1979).