

Spontaneous breaking of global $B-L$ symmetry and matter-antimatter oscillations in grand unified theories

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We explore the consequences of the hypothesis that $B-L$ symmetry, which is exact in the standard model based on the $SU(2)_L \times U(1) \times SU(3)_c$ gauge group and its extension to the minimal $SU(5)$ theory, is a spontaneously broken global symmetry. This results in possibly observable rates for $\Delta(B-L)=0$, $\Delta B=2$ hydrogen-antihydrogen transitions and double proton decay $pp \rightarrow e^+e^+$ ($\mu^+\mu^+$) and $\Delta(B-L) \neq 0$, $\Delta B=2$ neutron-antineutron mixing and $p+n \rightarrow \pi^+$'s; furthermore, a relation between the transition times $\tau_{H-\bar{H}}$, $\tau_{n-\bar{n}}$, Majorana neutrino mass, and the mass of a doubly charged Higgs particle Δ^{++} emerges. At the level of $SU(2)_L \times U(1) \times SU(3)_c$, for $\tau_{n-\bar{n}} \simeq 10^7$ sec and $m_{\Delta^{++}} \simeq 20$ GeV, we predict $\tau_{H-\bar{H}} \simeq 10^3$ yr and $\tau_{pp} \simeq 10^{31}$ yr, both of which are experimentally testable numbers. The embedding of our model in $SU(5)$ is discussed in detail, with the analysis of τ_p and $\sin^2\theta_W$ besides matter oscillations.

I. INTRODUCTION

It is well known that the standard electroweak model based on the gauge group $SU(2)_L \times U(1) \times SU(3)_c$ conserves baryon and lepton numbers. If one includes the nonperturbative effects of weak interactions,¹ neither the baryon number nor the lepton number are exactly conserved; however, the combination $(B-L)$ is. One may therefore look for $(B-L)$ -nonconserving effects as a way to explore new physics. The following possibilities for new interactions, then present themselves; depending on the nature of $B-L$ symmetry breaking: (i) explicit breaking, (ii) spontaneously broken local $B-L$ symmetry, or (iii) spontaneously broken global $B-L$ symmetry.

The first possibility involves a certain degree of arbitrariness associated with the new Higgs-boson couplings, new multiplets, etc., and we do not pursue this point of view. On the other hand, in cases (i) and (iii), a unique set of selection rules emerges for baryon-nonconserving processes simply from symmetry consideration and is substantially independent of the detailed Higgs structure.

As has been noted before,² case (ii), i.e., local $B-L$ symmetry, implies the extension of the standard model to the left-right symmetric theory³ based on $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ gauge

group. Parity becomes a spontaneously broken symmetry of nature and its breaking dictates $\Delta B=0$, $\Delta L=2$ and $\Delta B=2$, $\Delta L=0$ selection rules.

In this paper we focus our attention on case (iii), i.e., spontaneous breaking of global $B-L$ symmetry,⁴ which necessarily leads to the existence of a real Goldstone boson—the Majoron.⁵ The small, purely pseudoscalar, couplings of the Majoron to matter ensure its existence consistent with the existing information on long-range forces.⁵

Furthermore, since $B-L$ is a global symmetry, only the Higgs sector of the theory must be extended by including new multiplets which transform as bilinears in quark and lepton fields. This leads to the prediction⁶ of hydrogen-antihydrogen oscillation, and double proton decay $pp \rightarrow e^+e^+$ processes which conserve $B-L$, but change baryon and lepton numbers by two units. If no new mass scale beyond m_W is assumed, we predict the $H-\bar{H}$ oscillation time $\tau_{H-\bar{H}} \gtrsim 10^{13}$ yr and $\tau_{2p} \gtrsim 10^{31}$ yr, with lower limits being not far from the existing experimental bounds.⁷ Moreover, $B-L$ breaking leads to the existence of neutron-antineutron ($n-\bar{n}$) mixing with the strength proportional to the neutrino Majorana mass which is the measure of $B-L$ breaking. Assuming m_ν in the eV region, we predict $\tau_{n-\bar{n}} \gtrsim 10^7$ sec, again possibly close to the experimental limit.

We also present a grand unified version of our

model, based on $SU(5)$ gauge group with Higgs multiplets transforming as $\underline{5}$, $\underline{24}$, $\underline{15}$, and $\underline{50}$ dimensional representations. In order to obtain observable $\tau_{n-\bar{n}}$ in such models,⁸ one needs further fine tuning of the parameters, beyond the one needed to establish the hierarchy between X and W boson masses. The precise predictions for τ_p and $\sin^2\theta_W$ depend on the assumptions on Higgs-boson masses, and the simplest case suggests essentially no change compared to the minimal model.

The presentation of the rest of the material in this paper is as follows: In Sec. II a brief operator analysis of matter-antimatter oscillations is given and a possible connection between $n-\bar{n}$ and $H-\bar{H}$ oscillations for certain types of operators is discussed. In Sec. III we discuss the $SU(2)_L \times U(1) \times SU(3)_c$ model with spontaneous breakdown of global $B-L$ symmetry, and its predictions for $n-\bar{n}$ and $H-\bar{H}$ oscillations. Next, in Sec. IV the $SU(5)$ extension of the model is introduced, with the discussion of the predictions for $\sin^2\theta_W$ and τ_p . Section V offers a summary of our results.

II. OPERATOR ANALYSIS OF MATTER OSCILLATIONS

If we accept B and L nonconservation, it is possible to contemplate oscillations between neutral matter and antimatter, such as $\nu-\bar{\nu}$ (Majorana neutrino mass), $n-\bar{n}$, and $H-\bar{H}$ ($pe \rightarrow \bar{p}e^+$). We can write down the corresponding effective operators,⁹ independent of the model under consideration, by requiring them to respect $SU(2)_L \times U(1) \times SU(3)_c$ symmetry. The fermionic multiplets are taken to be the

$$O_{n-\bar{n}}^{(1)} = G^{(1)} \epsilon^{ijk} \epsilon^{lmn} (u_{Ri}^T C^{-1} d_{Rl}) (u_{Rj}^T C^{-1} d_{Rm}) (d_{Rk}^T C^{-1} d_{Rn}) + \text{permutations}, \quad (2.5)$$

where $G^{(1)} = h^3/M^5$. These operators are important in local $B-L$ symmetry models.²

(ii) Mixed-helicity operators involving weak triplets:

$$O_{n-\bar{n}}^{(2)} = G^{(2)} \epsilon^{ikm} \epsilon^{ilm} (Q_{Li}^T C^{-1} \tau_2 \vec{\tau} Q_{Lj}) (Q_{Lk}^T C^{-1} \tau_2 \vec{\tau} Q_{Ll}) (d_{Rm}^T C^{-1} d_{Rn}) + \text{permutation}. \quad (2.6)$$

Again these operators are important in left-right symmetric models, with $G^{(2)} \simeq G^{(1)}$.

(iii) Mixed-helicity operators involving weak singlets:

$$O_{n-\bar{n}}^{(3)} = G^{(3)} \epsilon^{ijk} \epsilon^{lmn} (Q_{Li}^T C^{-1} \tau_2 Q_{Lj}) (Q_{Ll}^T C^{-1} \tau_2 Q_{Lm}) (d_{Rk}^T C^{-1} d_{Rn}). \quad (2.7)$$

These operators are suppressed, since the bilinear $Q_{Li}^T C^{-1} \tau_2 Q_{Lj} \epsilon^{ijk}$ transforms as a color triplet and so it can couple to $Q_{Lk}^T C^{-1} \tau_2 \psi_L$ so as to give rise to proton decay. Therefore, $G^{(2)}$ has to be orders of magnitude smaller than $G^{(1)}$ or $G^{(3)}$.

C. Hydrogen-antihydrogen oscillation

There is a purely left-handed operator that can induce $n-\bar{n}$ and $H-\bar{H}$ transitions.

(iv) Purely left-handed operator takes the form

$$O_{H-\bar{H}}^{(4)} = G^{(4)} \epsilon^{ikm} \epsilon^{ilm} (Q_{Li}^T C^{-1} \tau_2 \vec{\tau} Q_{Lj}) (Q_{Lk}^T C^{-1} \tau_2 \vec{\tau} Q_{Ll}) (Q_{Lm}^T C^{-1} \tau_2 \vec{\tau} Q_{Ln}) (\psi_L^T C^{-1} \tau_2 \vec{\tau} \psi_L) + \text{permutation}, \quad (2.8)$$

conventional ones:

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad u_R, \quad d_R, \quad (2.1)$$

$$\psi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L, \quad e_R.$$

A. $\nu-\bar{\nu}$ oscillation

The lowest-dimension operator that can give ν_L a (Majorana) mass is

$$O^{(\nu)} = \frac{1}{M^2} (\psi_L^T C^{-1} \tau_2 \vec{\tau} \psi_L) (\psi_L^T C^{-1} \tau_2 \vec{\tau} \psi_L), \quad (2.2)$$

where we imagine M not to be much bigger than M_W . If the following ν_L condensate forms

$$\langle \nu_L^T C^{-1} \nu_L \rangle = m^3 \quad (2.3)$$

we obtain from (2.2)

$$m_\nu = \frac{m^3}{M^2}. \quad (2.4)$$

If $m \simeq m_f$, a typical mass (~ 10 MeV) for the first-generation fermion, together with $M \simeq 100$ GeV would give $m_\nu \simeq 10^{-1}$ eV.

B. $n-\bar{n}$ oscillation¹⁰

This process is represented by the following six-quark operators¹¹:

(i) Purely right-handed operators:

where $G^{(4)} = h^4/M^8$, M again being the typical mass in the theory. The above operator clearly induces $\Delta B = 2$, $\Delta L = 2$, but $\Delta(B-L) = 0$ hydrogen-antihydrogen oscillation ($pe \leftrightarrow \bar{p}e^+$) and double proton decay $p+p \rightarrow e^+ + e^+$ or $p+p \rightarrow \mu^+ + \mu^+$. If we write for the effective $H-\bar{H}$ Hamiltonian

$$\mathcal{H}_{H-\bar{H}} = G_{H-\bar{H}} (p_L^T C^{-1} p_L) (e_L^T C^{-1} e_L) \quad (2.9)$$

with

$$G_{H-\bar{H}} = G^{(4)} |\psi_N(0)|^4, \quad (2.10)$$

where $\psi_N(0)$ is the value of the three-quark wave function of the nucleon at the origin, mixing of hydrogen and antihydrogen atoms, $\delta m_{H-\bar{H}}$, can be expressed in terms of $G_{H-\bar{H}}$ as follows:

$$\delta m_{H-\bar{H}} = G_{H-\bar{H}} \frac{(\alpha m_e)^3}{\pi}. \quad (2.11)$$

We will define $\tau_{H-\bar{H}} \simeq \hbar/m_{H-\bar{H}}$ and astrophysical γ rays appear to put bounds⁷ on $\tau_{H-\bar{H}} \geq 10^{12}$ yr. We will come back to the phenomenological analysis of $H-\bar{H}$ oscillation and related phenomena in a subsequent section.

Now, coming back to $n-\bar{n}$ oscillation, if we use Eq. (2.3), we obtain

$$O_{n-\bar{n}}^{(4)} = G_{n-\bar{n}} (Q_{Li}^T C^{-1} \tau_2 \bar{\tau} Q_{Lj}) (Q_{Lk}^T C^{-1} \tau_2 \bar{\tau} Q_{Ll}) \times (d_{Lm}^T C^{-1} d_{Ln}) \epsilon^{ikm} \epsilon^{jln}, \quad (2.12)$$

where $G_{n-\bar{n}} = G^{(4)} m^3$. Noting the fact that

$$\delta m_{n-\bar{n}} = G_{n-\bar{n}} |\psi_N(0)|^4 \quad (2.13)$$

and using Eq. (2.4), we obtain

$$\frac{\tau_{H-\bar{H}}}{\tau_{n-\bar{n}}} = \frac{\pi M^2 m_\nu}{(\alpha m_e)^3}. \quad (2.14)$$

We see that for the models in which the dominant contribution to $n-\bar{n}$ oscillation involves pure left-handed quark operators, $\tau_{H-\bar{H}}$ can be predicted in terms of $\tau_{n-\bar{n}}$ and m_ν . For example, if we choose $M \simeq 10^2$ GeV, we expect

$$\tau_{H-\bar{H}} \simeq 5 \times 10^{20} [m_\nu (\text{GeV})] \tau_{n-\bar{n}}. \quad (2.15)$$

For $\tau_{n-\bar{n}} \simeq 1$ yr, $m_\nu \simeq 1-10$ eV, $\tau_{H-\bar{H}} \simeq 10^{12}-10^{13}$ yr, which is close to the lower bound inferred from astrophysics. The important point to note is that acceptable and interesting baryon nonconserving am-

plitudes can be predicted in theories without any mass scale beyond $M \simeq m_W$.

In Sec. III we will discuss a model where the last possibility is realized.

III. $SU(2)_L \times U(1) \times SU(3)_c$ MODEL WITH SPONTANEOUS BREAKDOWN OF GLOBAL $B-L$ SYMMETRY

In this section we present a discussion of matter oscillations in a single model based on $SU(2)_L \times U(1) \times SU(3)_c$ gauge group. This material is largely based on our recent work.⁶ The main idea is to incorporate the notion of spontaneously broken global $B-L$ symmetry into a study of $n-\bar{n}$ and $H-\bar{H}$ transitions; the result of which, as we shall see, is the connection between these processes and neutrino Majorana mass.

We start by describing the model. The gauge group is the standard $SU(2)_L \times U(1) \times SU(3)_c$ model of electroweak and strong interactions, with both the conventional fermionic and Higgs multiplets.

Fermions:

$$\begin{aligned} \psi_L &= \begin{bmatrix} \nu \\ e \end{bmatrix}_L (2, -1, 1_c), \quad e_R (1, -2, 1_c), \\ Q_L &= \begin{bmatrix} u \\ d \end{bmatrix}_L (2, \frac{1}{3}, 3_c), \\ u_R &(1, \frac{4}{3}, 3_c), \quad d_R (1 - \frac{2}{3}, 3_c). \end{aligned} \quad (3.1)$$

Higgs bosons:

$$\phi (2, 1, 1_c)$$

and the following additional $SU(2)_L$ Higgs triplets:

$$\Delta_q (3, -\frac{2}{3}, \bar{6}_c), \quad \Delta_l (3, 2, 1_c), \quad (3.2)$$

where the numbers in parentheses denote the representation content under $SU(2)_L$, $U(1)$, and $SU(3)_c$, respectively. The extension to more generations of fermions is straightforward and is not included only to keep the notation simple (see below, though).

We assign the usual B and L quantum numbers to the multiplets in (3.2) and assume $B_{\Delta_q} = -\frac{2}{3}$, $L_{\Delta_q} = 0$ and $B_{\Delta_l} = 0$, $L_{\Delta_l} = -2$. As was mentioned in the Introduction, we demand the full Lagrangian to possess global $U(1)_{B-L}$ symmetry. The most general Yukawa interaction and Higgs boson potential are then given by $[\Delta \equiv (1/\sqrt{2})\bar{\tau} \cdot \Delta]$

$$\mathcal{L}_Y = h_u \bar{Q}_L (i\tau_2 \phi^*) u_R + h_d \bar{Q}_L \phi d_R + h_e \bar{\psi}_L \phi e_R + f_q Q_L^T C^{-1} \tau_2 \Delta_q Q_L + f_e \psi_L^T C^{-1} \tau_2 \Delta_l \psi_L + \text{H.c.} \quad (3.3)$$

and

$$\begin{aligned}
V = & -\mu_\phi^2 \phi^\dagger \phi + \lambda_\phi (\phi^\dagger \phi)^2 - \mu_l^2 \text{Tr} \Delta_l^\dagger \Delta_l + \lambda_l (\text{Tr} \Delta_l^\dagger \Delta_l)^2 + \lambda'_l \text{Tr} \Delta_l^\dagger \Delta_l \Delta_l^\dagger \Delta_l - \mu_q^2 \text{Tr} \Delta_q^\dagger \Delta_q + \lambda_q (\text{Tr} \Delta_q^\dagger \Delta_q)^2 \\
& + \lambda'_q \text{Tr} \Delta_q^\dagger \Delta_q \Delta_q^\dagger \Delta_q + \alpha \text{Tr} \Delta_l^\dagger \Delta_l \text{Tr} \Delta_q^\dagger \Delta_q + \alpha' \text{Tr} \Delta_q^\dagger \Delta_q \Delta_l^\dagger \Delta_l + \phi^\dagger \phi (\beta_l \text{Tr} \Delta_l^\dagger \Delta_l + \beta_q \text{Tr} \Delta_q^\dagger \Delta_q) \\
& + \phi^\dagger (\gamma_l \Delta_l^\dagger \Delta_l + \gamma_q \Delta_q^\dagger \Delta_q) \phi + \lambda \epsilon_{ikm} \epsilon_{jln} (\text{Tr} \Delta_q^{ij} \Delta_q^{kl}) (\text{Tr} \Delta_q^{mn} \Delta_l) + \text{H.c.}, \quad (3.4)
\end{aligned}$$

where we have explicitly included the only nontrivial color indices in the last term of (3.4). That term is very important: in its absence the Lagrangian admits two separate $U(1)$ symmetries corresponding to baryon and lepton numbers; however, $\lambda \neq 0$ leaves only $U(1)_{B-L}$ broken.

Incidentally, the theory as a by-product possesses a discrete symmetry D_B , with all the fields having the transformation property (field) $\rightarrow e^{i\pi B}$ (field). In particular under D_B , $p \rightarrow -p, n \rightarrow -n$, but $e \rightarrow e$ and mesons \rightarrow mesons. This symmetry, as we shall see below, plays a crucial role in guaranteeing the stability of the proton.

The potential in Eq. (3.4) is minimized for the following values of the fields, which break the global $B-L$ symmetry¹¹:

$$\langle \phi \rangle = \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix}, \quad \Delta_l = \begin{bmatrix} 0 & 0 \\ \kappa/\sqrt{2} & 0 \end{bmatrix}, \quad (3.5)$$

leading to the usual expressions for quark and electron masses and a Majorana mass for the neutrino

$$m_\nu = \sqrt{2} f_e \kappa. \quad (3.6)$$

The leptonic aspect of this model has been discussed at length before.¹²⁻¹⁴ The spontaneous breakdown of global $B-L$ symmetry leads to an existence of a zero mass Goldstone boson, the Majorana,^{5,12} whose presence requires redoing some of the usual weak interaction phenomenology. The only constraint of importance for us is $f_e \leq 7 \times 10^{-3}$ from π and K meson decays¹⁴ and $\kappa \leq 100$ keV from the astrophysical analysis of energy loss from red giants.^{13,16} This results in $m_{\nu_i} < 700$ eV; obviously not a very useful bound.

A. Matter-antimatter transitions

We now turn our attention to $\Delta B=2, \Delta L=0$ $n-\bar{n}$ transitions and $\Delta B=\Delta L=2$ $H-\bar{H}$ transitions. Notice that, since a discrete symmetry D_B remains unbroken, proton is absolutely stable. That means that we have no reason to assume the existence of new large mass scales in the theory; in what follows we shall make the most natural assumption of a single mass scale, i.e., that of weak interactions ~ 100 GeV.

1. $H-\bar{H}$ oscillations

This process proceeds in the manner shown in Fig. 1. Its strength is easily estimated to be

$$G_{H-\bar{H}} \simeq \frac{\lambda f_q^3 f_e}{m_{\Delta_q}^6 m_{\Delta^{++}}^2}. \quad (3.7)$$

This in turn leads to an effective hydrogen-antihydrogen mixing of the strength

$$\begin{aligned}
H_{\text{eff}} \simeq & G_{H-\bar{H}} |\psi_N(0)|^4 (p_L^T C^{-1} p_L) (e_L^T C^{-1} e_L) \\
& + \text{H.c.}, \quad (3.8)
\end{aligned}$$

where $|\psi_N(0)|^4$ is the quark wave function inside the nucleus, and e and p are electron and proton fields. From Ref. 7 and Sec. II we can then estimate the mixing time for $H-\bar{H}$ oscillations as

$$\tau_{H-\bar{H}}^{-1} \simeq G_{H-\bar{H}} |\psi_N(0)|^4 \frac{(m_e \alpha)^3}{\pi}. \quad (3.9)$$

In what follows we will take $m_{\Delta_q} \sim 100$ GeV and $\lambda \sim 1$ as natural values. Since $f_e \leq 7 \times 10^{-3}$, $m_{\Delta^{++}} \geq 15$ GeV,¹⁵ by assuming $f_q \leq 10^{-3}$ and taking¹¹ $|\psi(0)|^4 \simeq 10^{-3}$ GeV⁶ we get from (3.7) and (3.9)

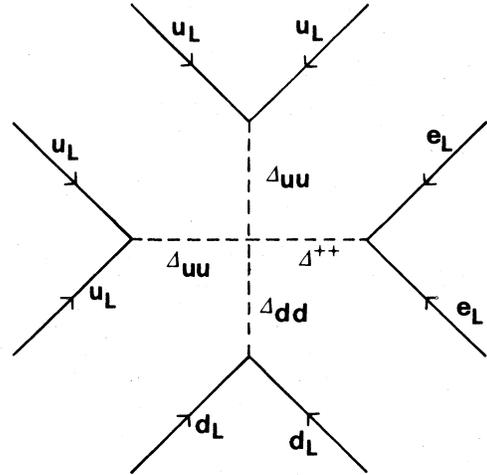


FIG. 1. Example of a quark diagram involving $H \rightarrow \bar{H}$ process, Δ_{uu} and Δ_{dd} are charge $-\frac{4}{3}$ and $+\frac{2}{3}$ members of the Δ_q triplet, and Δ^{++} is the doubly charged Higgs scalar from Δ_l .

$$\tau_{H-\bar{H}} \gtrsim 3 \times 10^{13} \text{ yr.} \quad (3.10)$$

We wish to add, that as is usual with $\Delta B \neq 0$ processes, our prediction depends sensitively on the choice of Higgs particle masses; however, our estimate is fairly reasonable. We eliminate a large degree of uncertainty, when we make a comparison of $\tau_{H-\bar{H}}$ with (possibly) observable $\tau_{n-\bar{n}}$.

2. $n-\bar{n}$ oscillations

It is easy to estimate $\tau_{n-\bar{n}}$ from Fig. 2, in exactly the same manner as for $H-\bar{H}$ transitions

$$\tau_{n-\bar{n}}^{-1} \simeq \frac{\lambda f_q^3}{m_{\Delta_q}^6} \frac{m_\nu}{\sqrt{2} f_e} |\psi_N(0)|^4, \quad (3.11)$$

where we have used (3.6). For the same values of parameters as before, we obtain

$$\tau_{n-\bar{n}} \simeq 10^{22} m_\nu^{-1}. \quad (3.12)$$

The experimental limit $\tau_{n-\bar{n}} \gtrsim 10^7$ sec requires then $m_\nu \leq 10^{-1}$ eV (of course, the smaller couplings and/or larger value of m_{Δ_q} would suppress the above process and allow for heavier neutrinos).

Using Eqs. (3.7), (3.9), and (3.11) we obtain our main result, shown in (2.14), in which all dependence on quark couplings f_q and diquark Higgs (Δ_q) masses has disappeared. Expression (2.14) provides a definite prediction that can serve as a test of the idea of spontaneous breaking of global $B-L$ symmetry.

As far as laboratory experimental searches for $H-\bar{H}$ mixing is concerned, long mixing times would appear to make it inaccessible to observation in existing setups. However, the processes such as

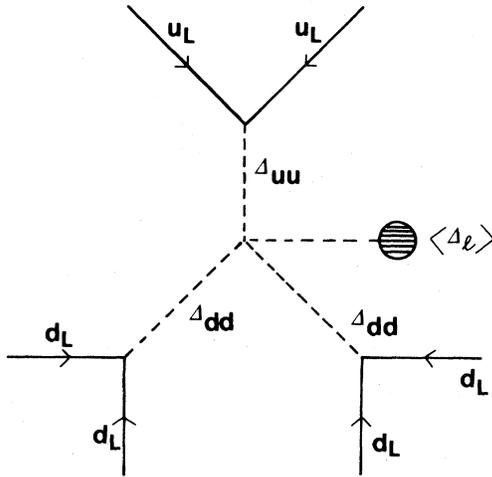


FIG. 2. A basic quark diagram showing $n-\bar{n}$ transitions.

$p+n \rightarrow e^+ + \nu$ and $n \rightarrow p + e^+ + \nu$, and especially double proton decay described below, are free from the suppression due to atomic wave functions and so could be searched for in the same set-ups that look for proton decays.

B. Double proton decay $p+p \rightarrow e^+ + e^+$

This process proceeds through the same diagram in Fig. 1 that describes $H-\bar{H}$ transition and can be characterized by the same effective strength $G_{H-\bar{H}}$ defined in (3.7). The absence of spontaneous fission of ^{232}Th led the authors of Ref. 7 to put the limit on the lifetime of this decay: $\tau_{pp} \gtrsim 10^{23}$ yr; or equivalently the bound on the coupling: $G_{H-\bar{H}} |\psi(0)|^4 \lesssim 10^{-25} \text{ GeV}^{-2}$. On the other hand, for the same values of the parameters as before, we estimate in our model $G_{H-\bar{H}} |\psi(0)|^4 \lesssim 10^{-29} \text{ GeV}^{-2}$, or in other words $\tau_{pp} \gtrsim 10^{31}$ yr.

For values of $\tau_{H-\bar{H}}$ that we predict, there also exists the possibility of observing $H-\bar{H}$ mixing effects in astrophysics by looking for energetic interstellar γ rays.²

As for other distinguishable features of this model, we note the following.

(i) Doubly charged Higgs bosons (with $m_{\Delta_{++}} = 15$ GeV or so) should be visible in existing high-energy machines in the not too distant future.

(ii) Electron neutrinos should weigh about 10^{-1} eV, if both $n-\bar{n}$ and $H-\bar{H}$ processes are to have detectable rates.

(iii) There exist direct $\Delta S=2$ tree-level amplitude through the exchange of a diquark Δ_{dd} , with the strength $f_d f_s / m^2 \Delta_q$. For $f_d < 10^{-3}$ and $m_{\Delta_q} \simeq 100$ GeV, we need $f_s / f_d \leq 10^{-3}$ in order to guarantee the smallness of K_L-K_S mass difference. This would predict the large suppression of possible $\Lambda-\bar{\Lambda}$ transition, compared to $n-\bar{n}$ process.

In conclusion, we have presented an $SU(2)_L \times U(1)$ model where global $B-L$ symmetry is spontaneously broken, leading to both $n-\bar{n}$ and $H-\bar{H}$ oscillations. Using the present limits on $\tau_{n-\bar{n}}$, we obtain $\tau_{H-\bar{H}} \gtrsim 10^{13}$ yr. In the next section we shall see how this model can be embedded.

In closing we should mention that our model provides the simplest framework which realizes the general possibility of mutually related $\tau_{n-\bar{n}}$ and $\tau_{H-\bar{H}}$ when only left-handed operators contribute. The connection is necessary since we work in the context of purely left-handed, standard model.

IV. $SU(5)$ MODEL WITH SPONTANEOUSLY BROKEN GLOBAL $B-L$ SYMMETRY

We have shown in the previous section how a fairly simple extension of the Higgs sector in the stand-

ard model can lead to mutually connected and possibly observable $n-\bar{n}$ and $H-\bar{H}$ oscillations. Here, we shall discuss its grand unified version, with special emphasis paid to the predictions for proton lifetime and $\sin^2\theta_W$, besides matter oscillations.

Gauge group:

$$\underline{5}=F_R = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ e^+ \\ \nu^c \end{pmatrix}_R, \quad \underline{10}=T_L = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & e^+ \\ d_1 & d_2 & d_3 & -e^+ & 0 \end{pmatrix}_L. \quad (4.1)$$

Higgs sector. Apart from 5-dimensional representation H and 24-dimensional representation Σ of the minimal model, we include the following additional Higgs multiplets to implement our idea of spontaneous $B-L$ symmetry breakdown:

$$\underline{15}: \Delta_{pq}, \quad \underline{50}: \chi_{rst}^{pq}, \quad (4.2)$$

where Δ_{pq} is symmetric, $\Delta_{pq} = \Delta_{qp}$, and χ_{rst}^{pq} is totally antisymmetric in upper and lower indices, respectively, and is traceless, $\chi_{pst}^{pq} = 0$.

The most general Yukawa interaction consistent with $SU(5)$ gauge symmetry is

$$\mathcal{L}_Y = h_d F_R^p H^q T_{Lpq} + h_u \epsilon^{pqrst} T_{Lpq}^T H_r C^{-1} T_{Lst} + h_\nu F_{Rp}^T C^{-1} \Delta^{pq} F_{Rq} + f \epsilon^{pqrst} T_{Lpp_1}^T C^{-1} \chi_{rst}^{p_1 q_1} T_{Lqq_1} + \text{H.c.} \quad (4.3)$$

It is easy to see that the above interaction is invariant under a $U(1)_G$ global symmetry $U = e^{i\theta G}$, where

$$GT_L = T_L, \quad GF_R = 3F_R, \quad (4.4)$$

$$GH = -2H, \quad G\Delta = 6\Delta, \quad G\chi = -2\chi,$$

all other fields of the theory having no G charge. The invariance of the rest of the Lagrangian under this symmetry is not automatic. It requires omitting various renormalizable couplings such as $H\Delta^+H$, $\Sigma\Delta^+\Sigma$, etc. Therefore, we demand $U(1)_G$ to be a symmetry of the full Lagrangian.

Now, $\langle \Sigma \rangle \simeq M_X$ which is responsible for breaking of $SU(5)$ down to $SU(2)_L \times U(1) \times SU(3)_c$ will not break $U(1)_G$. On the other hand, $\langle H \rangle \simeq M_W$ which is responsible for the second stage of the symmetry breaking, breaks $U(1)_G$, since $G\langle H \rangle = -2\langle H \rangle$, but preserves $(Y+G/2)\langle H \rangle = \langle H \rangle$. A simple computation shows¹⁸ that this is nothing else but $B-L$ symmetry when it acts on fermions

$$B-L = \frac{2}{5} \left[Y + \frac{G}{2} \right]. \quad (4.5)$$

The potential of the theory can be chosen in the conventional manner to produce a minimum at

$$G = SU(5) \times U(1)_G$$

with $U(1)_G$ a global symmetry specified below; we shall call its generator G .

Fermions. As in the minimal $SU(5)$ model,¹⁷ we have

$\langle \Delta_{55} \rangle \neq 0$, which leads to the spontaneous breakdown of $B-L$ symmetry manifest through a Majorana mass for the neutrino

$$m_\nu = h_\nu \langle \Delta \rangle. \quad (4.6)$$

The associated Goldstone boson, the Majoron, is now given by

$$M = \frac{\langle H \rangle \text{Im} \Delta_{55} - \langle \Delta \rangle \text{Im} H_5}{(\langle H \rangle^2 + \langle \Delta \rangle^2)^{1/2}} \quad (4.7)$$

and its properties are essentially the same as in the $SU(2)_L \times U(1)$ model.¹⁹

We now turn our attention to matter oscillations. These processes, as we show below, are mediated by particles in Δ and χ multiplets.²⁰ Let us then first display the $SU(2)_L \times U(1) \times SU(3)_c$ representation contents of Δ and χ :

$$\Delta = (3, 2, 1_c) + (1, -\frac{4}{3}, 6_c) + (2, \frac{1}{3}, 3_c)$$

$$\Delta_{ab} = \Delta_1 \quad \Delta_{ij} = \Delta_2 \quad \Delta_{ia} = \Delta_3, \quad (4.8)$$

where a, b, \dots are $SU(2)_L$ indices and i, j, k, \dots are $SU(3)_c$ indices. Similarly, for χ we get

$$\begin{aligned}
\chi &= \underbrace{(1, -4, 1_c)}_{\chi_{ijk}^{ab} = \chi_1} + \underbrace{(1, -\frac{2}{3}, 3_c)}_{\chi_{cdi}^{ab} = \chi_2} + \underbrace{(2, -\frac{7}{3}, 3_c)}_{\chi_{cij}^{ab} = \chi_3} \\
&+ \underbrace{(1, \frac{8}{3}, 6_c)}_{\chi_{kab}^{ij} = \chi_4} + \underbrace{(3, -\frac{2}{3}, 6_c)}_{\chi_{bjk}^{ai} = \chi_5} + \underbrace{(2, 1, 8_c)}_{\chi_{jbc}^{ia} = \chi_6}.
\end{aligned}
\tag{4.9}$$

Before we discuss any physical consequences due to these Higgs scalars, we would have to know their mass scales. If one assumes minimal fine tuning, i.e., one makes no more fine tuning of the parameters beyond the one needed for the gauge hierarchy $M_W/M_X \simeq 10^{-12}$, then the survival hypothesis is operative²¹: only those particles which *have* to be light will remain light. In this case, it is the $SU(2)_L$ doublet (H_4, H_5) from H and the $SU(2)_L$ triplet Δ_1 from Δ . Namely, (H_4, H_5) contains Goldstone bosons (longitudinal components of W^\pm and Z) and the physical scalar η , whereas Δ_1 contains the Majoron to prevent the rest of the multiplet from becoming superheavy. All other Higgs scalars would however become heavy. This, of course, would imply the suppression of all other processes²² but proton decay and we somewhat uneasily give it up. We could imagine that some symmetry (maybe supersymmetry) could eventually keep other particles light, but for the moment we just assume it (it is always technically possible to do it by adjusting the parameters). Still, however, χ_2 has to be superheavy, because it mediates proton decay and so $m_{\chi_2} \simeq M_X$. As far as other particles are concerned, we will keep them light and in particular we assume that χ_1, χ_4, χ_5 , and Δ_1 submultiplets have masses of order 100 GeV. It is worth noting that χ_1 and Δ_1 are doubly charged Higgs bosons, where χ_1 couples to right-handed leptons and Δ_1 couples to the left-handed ones and is the analog of the Δ_l of Sec. III.

We are now fully prepared to study matter oscillations in this model.

A. Hydrogen-antihydrogen oscillation

There will be two classes of diagrams arising from Higgs self-couplings of the type $\text{Tr}(\chi^\dagger \chi \chi^\dagger \chi)$ (type I) and $\text{Tr}(\chi \chi \chi \Delta)$ (type II). Since $0 \neq \langle \Delta \rangle$, the second coupling will also lead to $n-\bar{n}$ oscillations, thus enabling us to relate $H-\bar{H}$ and $n-\bar{n}$ transition rates. A typical coupling of type I is

$$V_1 = \lambda_1 \chi_{rr'}^{pq} \chi_{pp'q'}^{rr'} (\chi^+)^{q's't'} (\chi^+)^{s't'u'} \tag{4.10}$$

and will lead to $H-\bar{H}$ oscillation via the graph in Fig. 3. Similarly, a type II coupling does it through Fig. 4.

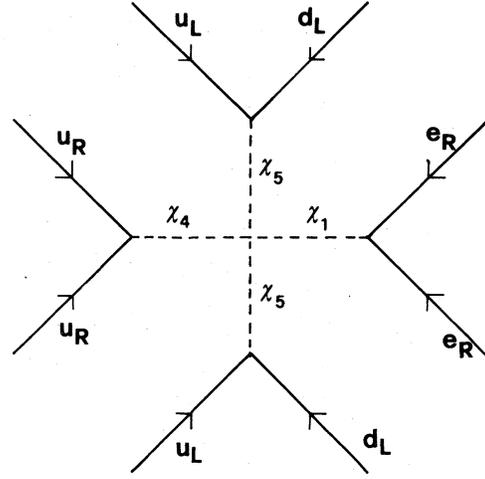


FIG. 3. A graph inducing $H \rightarrow \bar{H}$ process in $SU(5)$ model. In our notation χ_1, χ_4 , and χ_5 are the components of 50-dimensional multiplet χ .

$$V_2 = \lambda_2 \epsilon^{pqrst} \Delta_{p'q'} \chi_{r's'}^{p'r'} \chi_{pr'u}^{q's'} \chi_{qst}^{u't'} \tag{4.11}$$

The amplitude for $H-\bar{H}$ process can then be written as (if one ignores helicity differences)

$$G_{H-\bar{H}} \simeq |\psi_N(0)|^4 \left[\frac{f^4 \lambda_1}{m_{\chi_5}^4 m_{\chi_1}^2 m_{\chi_4}^2} + \frac{f^3 h_\nu \lambda_2}{m_{\chi_5}^6 m_{\Delta_1}} \right] \tag{4.12}$$

If it turns out that χ_4 is heavier than the rest of the fields. (say superheavy for some reason), then the second term would dominate, leading to the kind of

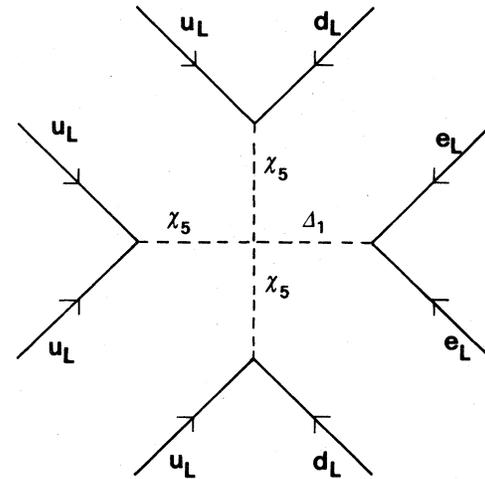


FIG. 4. Similar graph inducing again $H \rightarrow \bar{H}$ process in $SU(5)$. Δ_1 is an $SU(2)_L$ triplet (Δ_l) from 15-dimensional multiplet Δ .

situation described in Sec. III. On the other hand, if the opposite is true, then both contributions compete against each other and due to different helicity structures, the situation is somewhat complicated.

B. Neutron-antineutron oscillations

They arise from type II couplings, when we set $\langle \Delta_{55} \rangle \neq 0$ (see Fig. 2) and so

$$G_{n-\bar{n}} \simeq |\psi_N(0)|^4 \frac{f^3 \langle \Delta \rangle}{m_{\chi_5^6}}. \quad (4.13)$$

As we mentioned already, if for some reason $m_{\chi_4} \gg m_{\chi_5}$, the connection between $\tau_{n-\bar{n}}$ and $\tau_{H-\bar{H}}$ discussed in Secs. II and III is obtained

$$\frac{\tau_{H-\bar{H}}}{\tau_{n-\bar{n}}} \simeq \frac{\pi m_{\Delta_1}^2 m_\nu}{h_\nu^2 (m_e \alpha)^3}, \quad (4.14)$$

where m_{Δ_1} is, strictly speaking, the mass of the doubly charged Higgs field. We therefore reproduce in this case the predictions of the $SU(2)_L \times U(1) \times SU(3)_c$ model of Sec. III. In other words, we have a simple grand unified theory that evades the picture of a complete desert when M_W and M_X , through an oasis at low energies manifested in matter oscillations and the double proton decay. Let us just recall from Sec. III that under reasonable assumptions we obtain $\tau_{H-\bar{H}} \gtrsim 10^{13}$ yr and $\tau_{2p \rightarrow 2e} \gtrsim 10^{31}$ yr.

The discussion of our model cannot be complete if we do not include the analysis of the predictions for ordinary $\Delta B = 1$ proton decay and $\sin^2 \theta_W$.

C. $\sin^2 \theta_W$ and τ_p

Recall that in the minimal $SU(5)$ model,²³ $\sin^2 \theta_W \simeq 0.22$ and $\tau_p \simeq 10^{28} - 10^{31}$ yr. The presence of extra light Higgs bosons of our model needed to mediate the matter oscillations will affect these predictions.

For this purpose let us start with the general formula for arbitrary Higgs multiplets. Write down the familiar evolution equations for α_3 , α_2 , and α_1 , $SU(3)_c$, $SU(2)_L$, and $U(1)$ gauge couplings, respectively,

$$\frac{d\alpha_i}{dt} = -\frac{1}{2\pi} b_i \alpha_i^2 \quad (i=1,2,3), \quad (4.15)$$

where

$$b_3 = 11 - \frac{4}{3} n_g - \frac{1}{6} \sum_a T_a^{(3)}(R) \theta(t-t_a),$$

$$b_2 = \frac{22}{3} - \frac{4}{3} n_g - \frac{1}{6} \sum_a T_a^{(2)}(R) \theta(t-t_a), \quad (4.16)$$

$$b_1 = -\frac{4}{3} n_q - \frac{1}{10} \sum_a \left[\frac{Y_a}{2} \right]^2 \theta(t-t_a),$$

where $t_a = \ln m_a$; m_a being the common mass of a given $SU(2)_L \times U(1) \times SU(3)_c$ Higgs multiplet and n_g is the number of fermionic families. From (4.15) and (4.16), we get

$$\frac{1}{\alpha(M_W)} - \frac{8}{3} \frac{1}{\alpha_3(M_W)} = \frac{1}{2\pi} \ln \frac{M_X^{\mathcal{B}}}{M_W^{B_0} \prod_a m_a^{B_a}}, \quad (4.17)$$

$$\sin^2 \theta_W(M_W) = \frac{3}{8} - \frac{5\alpha(M_W)}{16\pi} \ln \frac{M_X^{\mathcal{A}}}{M_W^{A_0} \prod_a m_a^{A_a}},$$

where M_X is the unification mass

$$A = (b_2 - b_1), \quad B = (b_3 - \frac{3}{8} b_2 - \frac{5}{8} b_1),$$

$$\mathcal{A} = A_0 + \sum_a A_a, \quad \mathcal{B} = B_0 + \sum_b B_b. \quad (4.18)$$

A_0 and B_0 are the values of A and B in the minimal $SU(5)$ model and A_a and B_a denote the contribution of additional Higgs multiplets to A and B . From (4.17) and (4.18) one can write down the ratio of the modified proton lifetime τ_p to the corresponding value τ_p^0 in the minimal model

$$\frac{\tau_p}{\tau_p^0} = \frac{\left\{ M_W^{B_0} \prod_a m_a^{B_a} \exp \left[2\pi \left[\frac{3}{8} - \frac{1}{\alpha_3(M_W)} \right] \right] \right\}^{41\mathcal{B}}}{M_W^4 \exp \left[\frac{8\pi}{B_0} \left[\frac{3}{8} - \frac{1}{\alpha_3(M_W)} \right] \right]} \quad (4.19)$$

Similarly, we can derive the change in $\sin^2 \theta_W$, again compared to the minimal model

$$\Delta \sin^2 \theta_W(M_W) = -\frac{5}{16\pi} \alpha(M_W) \ln \frac{M_X^{\mathcal{A}-A_0}}{\prod_a m_a^{A_a}}. \quad (4.20)$$

When we apply the above formulas to our model, where the additional Higgs multiplets are 15 dimensional representation Δ and 50 dimensional representation χ , we get²⁴

$$\frac{\tau_p}{\tau_p^0} \simeq \left[\frac{m_{\Delta_1}^5 m_{\chi_1}^4 m_{\chi_3}^7 m_{\chi_4}^4}{m_{\Delta_2}^4 m_{\Delta_3} m_{\chi_2} m_{\chi_5}^6 m_{\chi_6}^8} \right]^{4/67}, \quad (4.21)$$

$$\Delta \sin^2 \theta_W(M_W) \simeq -\frac{\alpha(M_W)}{804\pi} \left[\ln \frac{m_{\Delta_1}^{153} m_{\Delta_3}^{90}}{m_{\Delta_2}^{243}} + \ln \frac{m_{\chi_5}^{741}}{m_{\chi_1}^{92} m_{\chi_2}^{94} m_{\chi_3}^{94} m_{\chi_4}^{427} m_{\chi_6}^{84}} \right].$$

Clearly, without precise knowledge of particle masses anything is possible. We just mention interesting possibilities from the point of view of matter oscillation discussed in our model.

(i) $m_{\chi_2} \simeq M_X$ (to suppress proton decay) and $m_{\text{rest}} \simeq M_W$. In this case

$$\tau_p \simeq \frac{1}{5} \tau_p^0, \quad \Delta \sin^2 \theta_W \simeq 4 \times 10^{-3} \quad (4.22)$$

essentially no change from the minimal model.

(ii) If further $m_{\chi_4} \gg m_W$, τ_p increases somewhat, whereas $\sin^2 \theta_W$ decreases. For example, for $m_{\chi_4} \simeq M_X$, we would obtain $\tau_p \simeq 100 \tau_p^0$, $\Delta \sin^2 \theta_W \simeq 0.02$ which is acceptable and may be even preferable from the point of view of a slighter shorter lifetime τ_p^0 in minimal SU(5).

In any case, although we fail to predict τ_p and $\sin^2 \theta_W$ precisely, the natural expectation is that the values of both are not substantially different from those in the minimal SU(5). We have an amusing situation, in which besides matter oscillations as new phenomena, we have the possibility of $\Delta B = 2$ double proton decay $pp \rightarrow e^+ e^+$ or $\mu^+ \mu^+$ competing against the ‘‘ordinary’’ $\Delta B = 1$ proton decay $p \rightarrow \pi^0 e^+$. We can even imagine the situation where double proton decay becomes the dominant mode, if say, χ_6 is substantially heavy to suppress τ_p [see (4.20)].

V. SUMMARY

Grand unified theories, such as the minimal SU(5) model, characterized by a single mass scale beyond M_W leave us with a picture of a desert. Proton decays, but only in $\Delta(B-L) = 0$ allowed channels such as $p \rightarrow \pi^0 + e^+$. Actually, $B-L$ symmetry remains exact to all orders in perturbation theory even in minimal SU(5) theory, as much as it does in the standard $SU(2)_L \times U(1) \times SU(3)_c$ model. Interesting consequences emerge if $B-L$ is a spontaneously broken global symmetry, as was discussed in this paper. Besides neutrino becoming a massive Majorana spinor, one in general expects matter-antimatter oscillations, such as $H-\bar{H}$ and $n-\bar{n}$. In Sec. II we have

shown how for purely left-handed effective operators oscillation periods $\tau_{n-\bar{n}}$ and $\tau_{H-\bar{H}}$ may be related to each other, simply through neutrino mass and a mass of a hypothetical doubly charged Higgs scalar Δ^{++} . If a physical mass scale that sets the strength of the processes is of order of 100 GeV, then one gets an interesting possibility of observable both $n-\bar{n}$ and $H-\bar{H}$ oscillations, with $\tau_{n-\bar{n}} \simeq 10^7$ sec and $\tau_{H-\bar{H}} \simeq 10^{13}$ yr, for $m_\nu \simeq 10^{-1}$ eV and $m_{\Delta^{++}} \simeq 20$ GeV. In Sec. III we have discussed in detail an $SU(2)_L \times U(1) \times SU(3)$ -based model, with an extended Higgs sector, which exemplifies the above situation. It is noteworthy, that besides $H-\bar{H}$ transitions, one also expects double proton decay $pp \rightarrow e^+ e^+$ with $\tau_{2p} \gtrsim 10^{31}$ yr, with the conventional $\Delta B = 1$ proton decay forbidden to all orders in perturbation theory; an amusing possibility. It turns out that this model can easily be embedded in SU(5), with spontaneous breakdown of global $B-L$ symmetry being preserved. Under a certain condition on Higgs-boson masses, the predictions of the $SU(2)_L \times U(1) \times SU(3)_c$ model can be preserved. Of course, the whole program in this case requires giving up the survival hypothesis, which turns out to be a general necessity²⁰ in grand unified theories. Could supersymmetry help here? In any case, although we fail in exactly predicting $\sin^2 \theta_W$ and τ_p , they seem to be little changed compared to the minimal SU(5) model, if light-Higgs-boson masses are not varied too much. We have an interesting possibility of double proton decay competing with the usual $\Delta B = 1$ process, if not even dominating. Even more exciting is the possibility that matter oscillations may be observable.

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