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Brief Reports

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Dynamical symmetry breaking in $SU(2)_L \times SU(2)_R \times U(1)$

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Results of weak-coupling approach to dynamical symmetry breaking within $SU(2)_L \times SU(2)_R \times U(1)$ are presented. The mixing of the neutral vector bosons is wrong, and a simple reason for this failure is given.

Recently, the possibility of dynamical symmetry breaking (DSB) within scalarless $SU(2)_L \times U(1)$ was studied using an effective potential approach in the weak-coupling limit.[1] Two potentially fatal problems arose in that study. One was the loss of the $M_W = M_Z \cos\theta$ relation which was due to the inclusion of vector self-couplings (in the Hartree-Fock approximation). The other difficulty was that physical Goldstone bosons were almost unavoidable. Only three gauge bosons acquire mass, and for most choices of fermion content more than three generators of the theory's chiral symmetries will be broken when the fermions acquire mass. That leads to the hope that things could be better—or at least different—in $SU(2)_L \times SU(2)_R \times U(1)$,[2] in which more mass-gaining vectors are present to consume unwanted Goldstone bosons. In $SU(2)_L \times U(1)$, for example, a massless left-handed quark doublet plus the two accompanying right-handed singlets have an initial $SU(2)_L \times SU(2)_R \times U(1)$ chiral symmetry. If the quarks acquire different masses, the residual symmetry is just $U(1)$, resulting in three massive vector bosons and three massless Goldstone bosons. If the gauge group were $SU(2)_L \times SU(2)_R \times U(1)$, on the other hand, there would be just enough gauge bosons acquiring mass to allow the chiral $SU(2)_L \times SU(2)_R \times U(1)$ to break into $U(1)$ without the appearance of any Goldstone bosons. In practice, the number of additional vectors is insignificant compared to the number of Goldstone bosons which would arise if three generations of massless fermions gained unequal masses. Nevertheless, $SU(2)_L \times SU(2)_R \times U(1)$ is a serious electroweak candidate theory and deserves DSB consideration in its own right. There is considerable aesthetic appeal to treating the left- and right-handed sectors in the same manner. In addition, it is quite natural for the neutrino to have a (Majorana) mass in such a theory,[3] and the $U(1)$ charge can be $(B-L)$.[4]

Pagels[5] briefly treated DSB in $SU(2)_L \times SU(2)_R$. Recently, Konetschny[6] pointed out problems in applying a standard hypercolor treatment to $SU(2)_L \times SU(2)_R \times U(1)$, a point to which we shall return later. This report will present the effective potential as a functional of composite operators for the $SU(2)_L \times SU(2)_R \times U(1)$ theory without fundamental scalars. The effective potential is obtained through two loops in the Hartree-Fock approximation. The Dyson-Schwinger equations resulting from minimizing the effective potential are solved in the linear approximation (LA), with special attention paid to the mixing of the neutral vector bosons. A massless vector results, but it couples *not* to the electromagnetic current, but rather to the third component of isospin. This failure is traced to the exotic quantum numbers of the Higgs representations in the standard treatment of $SU(2)_L \times SU(2)_R \times U(1)$.

Assuming all fermions are in $SU(2)_{L,R}$ doublets and there are no fundamental scalar fields, the $SU(2)_L \times SU(2)_R \times U(1)$ Lagrangian can be written

$$L_0 = -\tfrac{1}{4}A^i_{\mu\nu}A^{i\mu\nu} - \tfrac{1}{4}B^i_{\mu\nu}B^{i\mu\nu} - \tfrac{1}{4}U_{\mu\nu}U^{\mu\nu} + \sum_d \overline{\Psi}_d\left[i\not{\partial} + \frac{g_A}{2}\tau^i \not{A}^i\left(\frac{1-\gamma_5}{2}\right) + \frac{g_B}{2}\tau^i\not{B}^i\left(\frac{1-\gamma_5}{2}\right) + \frac{g_U}{2}\not{U}\left(\frac{y_d + x_d\gamma_5}{2}\right)\right]\Psi_d ,$$

$$y_d = Y_{dR} + Y_{dL},\quad x_d = Y_{dR} - Y_{dL},\quad \Psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \tag{1}$$

where the sum is over all doublets d, and Y_{dL} (Y_{dR}) is the hypercharge of the left- (right-) handed d doublet. In a true left-right-symmetric theory, $Y_R = Y_L$ and $x=0$, but for illustrative purposes we allow x to be nonzero for now. In constructing the generating functional $W[J]$ for the theory, one must add to L_0 gauge-fixing terms and the accompanying ghost terms. However, ghost terms do not affect the present treatment,[1] and so we shall just state that the Landau gauge is used and will not explicitly display the gauge-ghost terms.

The effective potential is constructed using the prescription of Cornwall, Jackiw, and Tomboulis.[7] The entire calculation parallels that in Ref. 1, with only some details changed, and we follow the notation and conventions used there. The effective potential is given by

$$\begin{aligned} V[G,\Delta] = {} & \text{const} - i\sum_a \int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}[\ln G^a(k) - S^{-1}(k)G^a(k)] \\ & -\frac{i}{2}\int \frac{d^4k}{(2\pi)^4}\left\{D^{-1}(k)\Delta_U(k) + \sum_{i=1}^{3}[D^{-1}(k)+\mathcal{G}_A{}^{-1}(k)]\Delta_A^i(k) + \sum_{i=1}^{3}[D^{-1}(k)+\mathcal{G}_B{}^{-1}(k)]\Delta_B^i(k)\right\} \\ & +\frac{i}{2}\int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}[\ln\det\underline{\Delta}_0(k) + \ln\det\underline{\Delta}_1(k) + \ln\det\underline{\Delta}_2(k)] + V_{4A}+V_{4B}+V_{3A}+V_{3B}+V_{\psi V}\ . \end{aligned} \tag{2}$$

The G^a's and Δ_V's are, respectively, the fermion and vector-boson propagators, with S and D the bare propagators. The $\mathcal{G}$'s are ghost terms (cf. Ref. 1) and are irrelevant to the present development. The matrices $\underline{\Delta}_i$ are

$$\underline{\Delta}_0 = \begin{pmatrix} \Delta_A^3 & \Delta_{AB}^3 & \Delta_{AU} \\ \Delta_{AB}^3 & \Delta_B^3 & \Delta_{BU} \\ \Delta_{AU} & \Delta_{BU} & \Delta_U \end{pmatrix},\quad \underline{\Delta}_i = \begin{pmatrix} \Delta_A^i & \Delta_{AB}^i \\ \Delta_{AB}^i & \Delta_B^i \end{pmatrix},\quad i=1,2\ . \tag{3}$$

The two-loop terms are given by ($I=A,B$)

$$\begin{aligned} V_{4I} = {} & \frac{g_I{}^2}{4}(1-\delta_{jk})\int \frac{d^4k_1 d^4k_2}{(2\pi)^8}[\Delta_{I\mu}^{j\,\mu}(k_1)\Delta_{I\nu}^{k\,\nu}(k_2) - \Delta_{I\mu\nu}^{j}(k_1)\Delta_I^{k\mu\nu}(k_2)]\ , \\ V_{3I} = {} & \frac{ig_I{}^2}{4}|\epsilon^{ijk}|\int \frac{d^4k_1 d^4k_2}{(2\pi)^8} k_1^\alpha k_1^\beta \Delta_I^{i\,\gamma\delta}(k_1)[\Delta_{I\alpha\beta}^{j}(k_2)\Delta_{I\gamma\delta}^{k}(k_1+k_2) \\ & + \Delta_{I\alpha\beta}^{k}(k_1+k_2)\Delta_{I\gamma\delta}^{j}(k_2) - 2\Delta_{I\alpha\delta}^{j}(k_2)\Delta_{I\beta\gamma}^{k}(k_1+k_2)] \\ V_{\psi V} = {} & \sum_{V=A^i,B^i,U}\frac{ig_V{}^2}{8}\int \frac{d^4k_1 d^4k_2}{(2\pi)^8}\,\mathrm{Tr}[\underline{G}(k_1)\Gamma_{V\mu}\underline{G}(k_2)\Gamma_{V\nu}]\Delta_V{}^{\mu\nu}(k_1-k_2) \\ & + \frac{ig_A g_B}{4}\sum_{i=1}^{3}\int \frac{d^4k_1 d^4k_2}{(2\pi)^8}\,\mathrm{Tr}[\underline{G}(k_1)\Gamma_{A^i}^{\mu}\underline{G}(k_2)\Gamma_{B^i}^{\nu}]\Delta_{AB\mu\nu}^{i}(k_1-k_2) \\ & + \sum_{V=A^3,B^3}\frac{ig_U g_V}{4}\int \frac{d^4k_1 d^4k_2}{(2\pi)^8}\,\mathrm{Tr}[\underline{G}(k_1)\Gamma_{V\mu}\underline{G}(k_2)\Gamma_{U\nu}]\Delta_V{}^{\mu\nu}(k_1-k_2)\ , \end{aligned} \tag{4}$$

where in $V_{\psi V}$,

$$\Gamma_{A^i}^{\mu} = \tau^i\gamma^\mu\left(\frac{1-\gamma_5}{2}\right),\quad \Gamma_{B^i}^{\mu} = \tau^i\gamma^\mu\left(\frac{1+\gamma_5}{2}\right)\ , \qquad \Gamma_U^{\mu} = \gamma^\mu\left(\frac{y+x\gamma_5}{2}\right),\quad \underline{G} = \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix}\ . \tag{5}$$

The requirement that the effective potential be stationary under variations of the G's and Δ's leads to Dyson-Schwinger equations. In the linearized approximation (LA)[8] the equations for the fermion masses become

$$\mu_a(p) = -\frac{3i}{4}g_U{}^2 Y_{aL}Y_{aR} \times \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-\mu_a{}^2(k)}\frac{1}{(p-k)^2}\mu_a(k)\ . \tag{6}$$

The asymptotic ($|p|\to\infty$) solution is

$$\mu_a(p) \simeq m_a\left(-\frac{p^2}{m_a{}^2}\right)^{-r_a},\qquad r_a = \frac{3}{64\pi^2}g_U{}^2 Y_{aL}Y_{aR}\ , \tag{7}$$

for small r_a, if $Y_{aL}Y_{aR}>0$. If $Y_{aL}Y_{aR}\leqslant 0$, no nontrivial solution exists. The first real difference from the $SU(2)_L\times U(1)$ case occurs at this point. Since both members of a doublet have the same Y_L and Y_R, if one fermion can acquire mass ($Y_LY_R>0$) so can its doublet partner. In $SU(2)_L\times U(1)$, on the other hand, there is no such requirement on Y_R; and whereas the electron and up quark have $Y_LY_R>0$ and can acquire mass, the neutrino and down quark have $Y_LY_R\leqslant 0$ and are doomed to remain massless (in the LA).[1,9]

Continuing in the LA, the mass matrix for the neutral vector bosons takes the form

$$\mathfrak{M}=\frac{2}{3g_U{}^2}\sum_i\frac{1}{Y_{iR}Y_{iL}}\begin{pmatrix} g_A{}^2\sigma_i & -g_Ag_B\sigma_i & -g_Ag_Ux_i\delta_i\\ -g_Ag_B\sigma_i & g_B{}^2\sigma_i & g_Bg_Ux_i\delta_i\\ -g_Ag_Ux_i\delta_i & g_Bg_Ux_i\delta_i & g_U{}^2x_i{}^2\sigma_i\end{pmatrix}\ , \tag{8}$$

where the sum is over all doublets i and where

$$\sigma_i\equiv m_{i1}{}^2+m_{i2}{}^2,\quad \delta_i\equiv m_{i1}{}^2-m_{i2}{}^2 \tag{9}$$

are, respectively, the sum and the difference of the masses squared of the two members of the doublet.

Considering for simplicity the case of one left- and right-handed doublet, the eigenvalues of $\mathfrak{M}$ are

$$M_1{}^2=0,\ M_2{}^2=\frac{2}{3g_U{}^2}\frac{1}{Y_LY_R}\sigma\lambda_-,\ M_3{}^2=\frac{2}{3g_U{}^2}\frac{1}{Y_LY_R}\sigma\lambda_+,$$
$$\lambda_\pm=\tfrac{1}{2}\{(g_A{}^2+g_B{}^2+x^2g_U{}^2)\pm[(g_A{}^2+g_B{}^2-x^2g_U{}^2)^2+x^2g_U{}^2(g_A{}^2+g_B{}^2)\hat{\delta}]^{1/2}\}\ ,$$
$$\hat{\delta}\equiv\delta/\sigma=(m_1-m_2)/(m_1+m_2)\ . \tag{10}$$

The corresponding eigenvectors are

$$V_1^\mu=N_1(g_BA_3^\mu+g_AB_3^\mu)\ ,$$
$$V_2^\mu=N_2[-g_AA_3^\mu+g_BB_3^\mu-(g_A{}^2+g_B{}^2-\lambda_-)/(xg_U\hat{\delta})U^\mu]\ , \tag{11}$$
$$V_3^\mu=N_3[-g_AA_3^\mu+g_BB_3^\mu+(g_A{}^2+g_B{}^2-\lambda_+)/(xg_U\hat{\delta})U^\mu]\ ,$$

provided $x\hat{\delta}\neq 0$, where the N_i are normalization factors. For $x=0$, as it is in a true left-right-symmetric theory, $M_1=M_2=0$, $M_3{}^2=2\sigma(g_A{}^2+g_B{}^2)/(3Y_LY_Rg_U{}^2)$, and there are two massless vector bosons.[10] For $x\neq 0$, $|\hat{\delta}|=|m_1-m_2|/(m_1+m_2)\ll 1$, the masses are given approximately by

$$M_1{}^2=0,\quad M_2{}^2\simeq\frac{2}{3}\frac{1}{Y_LY_R}x^2\sigma,$$
$$M_3{}^2\simeq\frac{2}{3}\frac{1}{Y_LY_R}\left[\frac{g_A{}^2+g_B{}^2}{g_U{}^2}\right]\sigma\ . \tag{12}$$

In this limit there is one massless vector, and the ratio of masses squared for the other two vectors is $x^2g_U{}^2/(g_A{}^2+g_B{}^2)$, which can be safely small. For $\hat{\delta}\simeq 1$, the masses are

$$M_1{}^2=0,$$
$$M_2{}^2\simeq\frac{2}{3}\frac{1}{Y_LY_R}(1-\hat{\delta}^2)\sigma\frac{g_A{}^2+g_B{}^2}{g_A{}^2+g_B{}^2+x^2g_U{}^2}\ , \tag{13}$$
$$M_3{}^2\simeq\frac{2}{3}\frac{1}{Y_LY_R}\sigma\frac{g_A{}^2+g_B{}^2+x^2g_U{}^2}{g_U{}^2}\ .$$

Again $M_3{}^2$ can be much larger than $M_2{}^2$, although now $M_2{}^2$ is small compared to fermion masses squared. There is therefore no trouble in achieving the desired spectrum for the neutral vector bosons.

The problem arises when we investigate the coupling of the massless vector to fermions. The lack of a U(1) component in V_1 in Eq. (10) should make us apprehensive, and that fear is borne out when the Lagrangian is rewritten in terms of the V_i. The current to which V_1 couples is

$$J_1^\mu=N_1\frac{g_Ag_B}{2}\sum_i\overline{\Psi}_i\tau^3\gamma^\mu\Psi_i\ , \tag{14}$$

which is not the desired electromagnetic current. This failure can be understood if we recall the Higgs content of the standard $SU(2)_L\times SU(2)_R\times U(1)$ treatments. In Ref. 3, for example, there are scalars in $SU(2)_L\times SU(2)_R\times U(1)$ representations $(\frac{1}{2},\frac{1}{2},0)$, (1,0,2), and (0,1,2). In a DSB scheme the role of the Higgs bosons is played by fermion-antifermion bound states. In the present treatment the only fermions are the fundamental quarks or leptons of the theory, and it is impossible to construct a (1,0,2) state from $\bar{q}q$ or $\bar{l}l$.[11] This same cause underlies (some of) the problems noted in Ref. 6. It would require an unusual hypercolor scheme to provide composite scalars with the quantum numbers required, in particular, with $(B-L)=2$.

We have not addressed the question of *whether* DSB occurs in $SU(2)_L\times SU(2)_R\times U(1)$. The LA has nothing to say about that matter. To investigate that question, one needs to determine whether the vacuum energy density is lower for the broken-symmetry solution than it is for the symmetric vacuum. This could be done by using the LA solutions as test functions in the effective potential of Eq. (2) and minimizing V with respect to the fermion and boson masses.[1,7] In view of the undesirable vector mixing we do not pursue this any further. $SU(2)_L \times SU(2)_R \times U(1)$ without scalars may or may not undergo spontaneous symmetry breaking, but, if it does, the mixing of the neutral gauge bosons is wrong (in the LA); and if it breaks because of techniscalars, they would need to have unusual quantum numbers.

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[10]We have also investigated $SU(2)_L \times SU(2)_R \times U(1) \times U(1)$ in the same approximation and find there are always at least two massless vectors in it.

[11]Even if the initial U(1) charge is not identified as $(B-L)$, the same problems persist since the U(1) charge is additive and therefore $\bar{q}_a q_b$ and $\bar{l}_a l_b$ will be U(1) singlets.