Effective-potential study of symmetry breaking in scalarless SU(2)×U(1)

K. T. Mahanthappa and J. Randa
Department of Physics, University of Colorado, Boulder, Colorado 80309
(Received 24 August 1982)

We derive the effective potential as a functional of composite operators for massless, scalarless SU(2)×U(1); and we obtain conditions under which this effective potential is the vacuum energy density for the case of composite operators which are nonlocal in time. General considerations of the number of Goldstone bosons prevent the construction of a realistic model, but we are able to study dynamical symmetry breaking for various unrealistic spectra of physical particles. We first use the standard linearized approximation (LA) to solve the equations obtained for the propagators from the effective potential, and we compare these results to the most-attractive-channel hypothesis. These LA solutions reproduce the standard vector-boson mixing and in addition yield a (reasonable) relation between the vector masses and those of the fermions. However, the linearized equations are also satisfied by the symmetric (massless) solution. In order to determine which solution corresponds to the true vacuum, we use the effective potential in a variational calculation. The linearized-approximation solutions are used to determine the functional forms of the propagators, and the physical masses are treated as variational parameters in minimizing the effective potential. In an Abelian approximation, in which the effects of the vector self-couplings are absent, the mass relations of the LA survive the inclusion of nonlinear effects. On the other hand, if a Hartree-Fock approximation is used for the vector self-couplings, all the desirable features of the LA are lost: The breaking is no longer $I=\frac{1}{2}$, and the relation between vector and fermion masses requires fermions much heavier than the vectors (or very large numbers of lighter fermions). In either approximation, whether spontaneous symmetry breaking occurs depends on the number and quantum numbers of the fermions. Significance of the results and possible future directions are discussed.

I. INTRODUCTION

The presence of fundamental scalars in the standard model seems distasteful to most high-energy physicists. The scalars certainly serve a necessary purpose, but they do not seem to return much more than was invested. They allow the possibility of spontaneous symmetry breaking (SSB), but they do not require it; they provide a mechanism for generating fermion and gauge-boson masses, but these masses are not calculable. The mass(es) and number of the scalars themselves are not known. Some of the parameters accompanying the scalars require unnaturally fine tuning in grand unified theories. The entire framework has all the appearances of an effective theory for a phenomenon arising from the fundamental interactions. In that case, one would naturally prefer to get SSB from, rather than impose it upon, the dynamics.

Most of the effort in this direction over the past few years has been devoted to hypercolor-type models,[1] in which very heavy hyperquarks are postulated, interacting through a new, very strong force, e.g., an $SU(3)_{c'}$ with $\Lambda_{c'} \simeq 500$ GeV. Some of the bound states of the hyperquarks, the hyperpions, serve as the traditional scalars, giving mass to the gauge bosons. This generates masses without recourse to fundamental scalars, but it leaves the problem of fermion masses and introduces the new problem of the origin of the hyperquark masses. One can appeal to another new interaction at a still higher mass scale in order to generate the fermion masses. A suggestion which naturally incorporates this is the idea of tumbling,[2] in which some initial large gauge group breaks dynamically at some very large mass scale M_1. This occurs because as one goes down in energy the running coupling constant increases (for an asymptotically free theory), and at some point the binding of fermion-antifermion becomes sufficiently strong to alter the structure of the vacuum, forming a condensate and breaking the symmetry. In general, some subgroup of the initial group will remain unbroken by the (first) condensate, and we can track it down to lower energy where the same phenomenon can recur, at mass scale M_2. This could then continue down to hypercolor [conceivably even to $SU(2)\times U(1)$], giving mass to the hyperquarks at a scale $M_N \simeq 1$ TeV. (No phenomenologically successful models have been constructed yet.)

So far as the breaking of just $SU(2)\times U(1)$ itself is concerned, hypercolor schemes are much like the standard model, with scalars introduced from "outside" to break the symmetry. Another possibility is that $SU(2)\times U(1)$ breaks within itself, i.e., that the theory with only massless vectors and fermions undergoes symmetry breaking, in the manner envisioned by Nambu and Jona-Lasinio[3] and alluded to above in connection with tumbling. The early work on $O(2)\times U(1)$ and $U(1)\times U(1)$ by Cornwall and Norton[4] and Jackiw and Johnson[5] showed that in a linearized approximation these gauge symmetries could break dynamically. Such studies employing the linearized approximation (LA) share a fundamental weakness, however, in that they are unable to determine whether SSB does in fact occur. The LA and similar approximations yield homogeneous equations which admit trivial solutions (no symmetry breaking) as well as the symmetry-breaking solution. Subsequent work by Cornwall, Jackiw, and Tomboulis[6] (CJT) using an effective potential and including nonlinear effects showed that dynamical symmetry breaking (DSB) does indeed occur in $O(2)\times U(1)$. These studies indicated that the mass generated for the vector bosons was of the same order of magnitude as (the symmetry-breaking part of) the fermion masses, a major obstacle to model building so long as only the light ($\lesssim 5$ GeV) fermions were known. However, as the experimental lower bound on the mass of the top quark creeps upward, this problem evaporates and things begin to look more hopeful for DSB in $SU(2)\times U(1)$. Salient features of such a scheme have been deduced by analogy to the $O(2)\times U(1)$ case[7] and from the LA to the theory,[8] using one quark or lepton left-handed doublet plus right-handed singlet(s), and putting in the mixing of the neutral vectors from the beginning. The next step would be to determine whether the results of the LA survive the inclusion of nonlinear effects and to determine whether the symmetry does in fact break. We shall address this question, using a variational calculation of the effective potential.[6] A brief summary of the results has been reported elsewhere.[9]

At the outset we must recognize general considerations which will preclude the construction of a realistic model. Because the fermions in the initial Lagrangian are massless, there are in general global chiral symmetries in addition to the $SU(2)\times U(1)$ symmetry. If the fermions and appropriate vectors acquire mass, the chiral symmetry is broken as well as $SU(2)\times U(1)$. The breaking of $SU(2)\times U(1)$ into $U(1)$ entails three broken generators and therefore three Goldstone bosons, just enough to give mass to the $W^{\pm}$ and Z^0. Any additional Goldstone bosons from the chiral symmetry breaking will not be consumed, however, a serious embarassment if we claim to be describing the real world. Any such extra Goldstone bosons also constitute a problem for the calculational method we employ. It will be described more fully below, but the basic point is that the effective potential as computed for the nonsymmetric vacuum does not include the effects of any physical Goldstone bosons. Since the true vacuum minimizes the effective potential, the effective potential we compute will be an upper limit on the true vacuum energy density. Therefore, when extra Goldstone bosons are required, $V(\text{broken}) < V(\text{sym})$ is a sufficient but not a necessary condition for DSB if $V(\text{broken})$ does not take Goldstone bosons into account.

The presence of Goldstone bosons from chiral symmetry breaking prevents us from making extensive contact with reality. Nevertheless, the calculation is worth pursuing. Besides its relevance to the tumbling schemes mentioned above, DSB figures prominently in the recent surge of interest in supersymmetric theories[10] and in the breaking of chiral symmetry by the strong interactions,[11] and of course the hope of relevance to electroweak interactions still glimmers. In these circumstances, it is useful to acquire any available insight into the workings of DSB in a model which is moderately complex, even if not realistic.

We shall allow the possibility of different quantum numbers and numbers of fermions. The only case which is not afflicted by residual Goldstone bosons is one left-handed doublet plus one right-handed singlet. For the sake of illustration consider $(\nu,e)_L$, e_R. Then $(\bar{e}_L e_R + \bar{e}_R e_L)$ is the condensate, acquiring a vacuum expectation value. The aspiring Goldstone bosons $\bar{e}_R \nu_L$, $\bar{e}_L e_R - \bar{e}_R e_L$, $\bar{\nu}_L e_R$ are consumed, giving mass to the $W^{\pm}$ and Z^0. The correspondence with the conventional scalar doublet is

$$\begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} \sim \begin{bmatrix} \bar{e}_R \nu_L \\ \bar{e}_R e_L \end{bmatrix} . \tag{1.1}$$

The breaking of any additional chiral symmetries in the Lagrangian would result in additional massless states which would remain in the physical spectrum.

For choices of fermion representations which would lead to Adler-Bell-Jackiw anomalies, we shall implicitly assume additional appropriately charged fermions which remain massless. Provided they acquire no mass, they do not affect the difference between the effective potential evaluated in the symmetric vacuum and in the unsymmetric vacuum. They are therefore neglected in the effective-potential calculation.

The calculation itself proceeds as follows. We begin with the Lagrangian for the $SU(2)_L \times U(1)$ theory with only the gauge bosons and (massless) fermions. The strong interactions are ignored. Any quark masses generated will be the current masses. A generating functional is constructed in the usual manner, except that in addition to the standard sources for single fields we add to the action sources of composite fields, e.g.,

$$\int d^4x\, d^4y \bar{\psi}(x) K_\psi(x,y)\psi(y) .$$

By performing a Legendre transform with respect to these sources, we obtain the effective action, which is also the generating functional for two-particle irreducible vertices. It is a functional of the functions conjugate to the sources, and these functions can be shown to be the propagators for the fundamental fields of the theory. The effective potential can then be obtained from the effective action by removing a four-dimensional volume. The next section of the paper is devoted to construction of the effective potential for $SU(2)\times U(1)$ following the prescription of CJT. In Sec. III, we require that the effective potential be stationary against variations of the propagator functions, as required by the subsidiary conditions of the Legendre transform. This yields Dyson-Schwinger equations for the propagators, which we solve in the LA. We obtain the conditions which determine whether a fermion can acquire mass in the LA, and a relation between the vector and fermion masses also results. Diagonalization of the neutral-vector mass matrix results in the usual definitions of the photon and Z fields and in the $M_W{}^2/M_Z{}^2=\cos^2\theta$ relation. These results are then discussed in the framework of the most-attractive-channel (MAC) hypothesis.[2] Assuming that the symmetry does break, the LA results agree completely with MAC expectations.

In Sec. IV, we use the solutions to the LA as test functions in the effective potential. The LA solutions give the functional form for the masses and propagators as functions of the momentum, but do not determine the physical masses. Use of these trial functions therefore means that instead of requiring stationarity of $V[G]$ for arbitrary variations of G, we stipulate that G be of the form $G_{\rm LA}(m)$. This reduces the problem to one of minimizing $V[G_{\rm LA}(m)]$, an algebraic function of the masses. We obtain this algebraic function and minimize it. The results obtained in the Hartree-Fock approximation are very different from those of the LA. Non-Abelian effects, which are neglected in the LA, tend to restore the symmetry, and when the symmetry does break they modify mass ratios. However, the Hartree-Fock approximation for multivector vertices does not preserve gauge invariance in the presence of dynamically generated gauge boson masses; and Cornwall[12] has argued that the gauge-boson loop graphs do not contribute to the effective potential in a gauge-invariant treatment. This leads us to consider an "Abelian approximation," in which such graphs are neglected. Even then, the LA results can be misleading. The mass ratios are as given by the LA, but whether the symmetry breaks depends strongly on the number and quantum numbers of the fermions which acquire mass. A discussion of the results is contained in Sec. V. There are two appendices, one devoted to a discussion of whether the $V[G]$ we calculate is indeed an effective potential and the other containing various integrals which arise in the calculation.

II. THE EFFECTIVE POTENTIAL

A. Review of general formalism

The calculation of the effective potential as a functional of composite operators is based on the formalism of multiple Legendre transforms of the generating functional of Green's functions for composite fields.[13] If a theory is described by a classical action $S=\int d^4x\, \mathscr{L}(x)$, where $\mathscr{L}$ is the effective Lagrangian including ghosts if necessary, then one defines the generating functional for Green's functions of composite (nonlocal) fields,

$$\begin{aligned} W[J,K] &= \exp\{iZ[J,K]\} \\ &= \int [d\Phi] \exp\left\{ i \int d^4x \left[\mathscr{L}(x) + \Phi_i(x)J_i(x) + \eta_{ij} \int d^4y\, \Phi_i(x)K_{ij}(x,y)\Phi_j(y) \right]\right\} , \end{aligned} \tag{2.1}$$

where the index i represents all Lorentz and flavor indices. The factor η_{ij} is $\frac{1}{2}$ for $i=j$ bosons, 1 for $i\neq j$ bosons, and -1 for fermions, by our convention. In the rest of this subsection, we shall assume boson fields and diagonal sources, $K_{ij} \propto \delta_{ij}$. The few generalizations necessary will be made in the next subsection.

The quantity $W[J,0]$ is of course the familiar generating functional for the Green's functions of the theory, and $Z[J,0]$ generates the connected Green's functions. By Legendre transforming $Z[J,0]$, one obtains the generating functional for proper (one-particle irreducible) vertices $\Gamma[\phi]$. Γ is also the effective action and therefore leads directly to the effective potential as a function of ϕ_i, the expectation values of the fields Φ_i. By introducing into our generating functional Z a source K_{ij} for composite fields, we are able to construct an effective action which depends on the expectation value of the composite field $\Phi_i\Phi_j$. This is just what we need if we are to study symmetry breaking without fundamental scalar fields. The formal construction of the effective action is standard[6,13]; one performs a double Legendre transform on $Z[J,K]$,

$$\Gamma[\phi,G]=Z[J,K]-\int d^4x\,\phi(x)J(x) -\tfrac{1}{2}\int d^4x\,d^4y[\phi(x)K(x,y)\phi(y) +K(x,y)G(x,y)]\,, \tag{2.2}$$

where we have suppressed all indices and where ϕ and G are given by

$$\frac{\delta Z[J,K]}{\delta J(x)}=\phi(x)\,,\qquad \frac{\delta Z[J,K]}{\delta K(x,y)}=\tfrac{1}{2}[\phi(x)\phi(y)+G(x,y)]\,. \tag{2.3}$$

The conjugate relations to Eq. (2.3) are

$$\frac{\delta\Gamma[\phi,G]}{\delta\phi(x)}=-J(x)-\int d^4y\,K(x,y)\phi(y)\,,\qquad \frac{\delta\Gamma[\phi,G]}{\delta G(x,y)}=-\tfrac{1}{2}K(x,y)\,. \tag{2.4}$$

Since physical processes correspond to vanishing sources, we have

$$\frac{\delta\Gamma[\phi,G]}{\delta\phi(x)}=0\,,\qquad \frac{\delta\Gamma[\phi,G]}{\delta G(x,y)}=0 \tag{2.5}$$

for physical solutions. If we require translationally invariant solutions, then the effective action takes the form

$$\Gamma[\phi,G]\,|_{\rm TI}=-V[\phi,G]\int d^4x\,, \tag{2.6}$$

where V is to be identified as the effective potential.

The validity of this identification has been questioned by Banks and Raby,[14] who point out that the use of sources which are nonlocal in time leads to a breakdown of the usual demonstration that the standard effective potential is the vacuum energy density.[6,15] This point is considered in detail in Appendix A. We are led to agree that in general the proof fails for sources which are nonlocal in time. However, there is a simple restriction on the functional form of the sources which allows one to carry through the proof. Furthermore, we argue that the restriction to this certain class of sources is implicit in the use of the test functions we choose for the variational calculation of Sec. IV. The situation then is as follows. The Dyson-Schwinger equations used in the next section follow from Eq. (2.5) and do not require that V be the effective potential (although at the stationary point it clearly is, since the sources vanish there[16]). The variational calculation of Sec. IV obviously does require that V be the effective potential in a neighborhood of the stationary point; and it is, provided we restrict ourselves to a certain class of sources. But we have done so by our choice of test functions, and therefore no further restriction or apology is required beyond that which is already obvious when we adopt a variational approach. These will be noted at the beginning of Sec. IV.

Equations (2.5) and (2.6) provide us with a means of determining the physical Green's functions and vacuum expectation values of the fields—they are those for which V is a minimum. V itself, or Γ, can be calculated from the prescription derived by CJT. Define

$$i\mathscr{D}^{-1}[\phi;x,y]=\left[\frac{\delta^2 S[\Phi]}{\delta\Phi(y)\delta\Phi(x)}\right]_{\Phi=\phi}\,,$$
$$iD^{-1}(x,y)=\left[\frac{\delta^2}{\delta\Phi(y)\delta\Phi(x)}(S[\Phi]-S_{\rm int}[\Phi])\right]_{\Phi=\phi} =[i\mathscr{D}^{-1}(\phi;x,y)]_{\phi=0}\,, \tag{2.7}$$

where $S_{\rm int}$ contains all terms which are at least cubic in the fields. The effective action is then given (symbolically) by

$$\Gamma[\phi,G]=S[\phi]+\frac{i}{2}{\rm Tr}\{\ln DG^{-1}\} +\frac{i}{2}{\rm Tr}\{\mathscr{D}^{-1}[\phi]G\}+\Gamma_2[\phi,G] +{\rm const}\,. \tag{2.8}$$

$\Gamma_2[\phi,G]$ is computed from two-particle irreducible vacuum graphs in a shifted theory ($S[\Phi]\to S[\Phi+\phi]$) with propagators given by G. Clarifying details can be found in CJT or inferred from the treatment of SU(2)×U(1), which follows.

B. Application to SU(2)×U(1)

The Lagrangian density is that of the standard $SU(2)_L \times U(1)$ with the scalar fields omitted,

$$\mathscr{L}_0 = -\tfrac{1}{4}A^i_{\mu\nu}A^{i\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} + i\sum_a \bar{R}_a\left[\partial\!\!\!/ - i\frac{g_B}{2}Y_{aR}B\!\!\!/\right]R_a + i\sum_a \bar{L}_a\left[\partial\!\!\!/ - \frac{ig_B}{2}Y_{aL}B\!\!\!/ - \frac{ig_A}{2}\tau^i A\!\!\!/^i\right]L_a ,$$

$$A^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g_A\epsilon^{ijk}A^j_\mu A^k_\nu , \qquad (2.9)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu .$$

The U(1) hypercharge, Y, satisfies $Q = I_L{}^3 + Y/2$, so that for quarks $Y_{uR} = \frac{4}{3}$, $Y_{dR} = -\frac{2}{3}$, $Y_{qL} = \frac{1}{3}$,

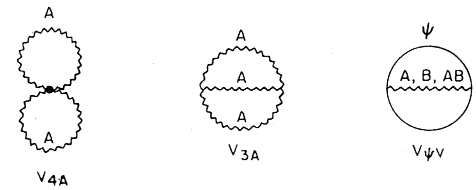

FIG. 1. Two-loop graphs contributing to V_2.

and for leptons $Y_{eR} = -2$, $Y_{\nu R} = 0$, $Y_{lL} = -1$. It will be more convenient to have $\mathscr{L}_0$ written in terms of the different fermion flavors rather than the right-handed singlets (R_a) and left-handed doublets (L_a). Defining

$$y_a = Y_{aR} + Y_{aL} ,$$
$$x_a = Y_{aR} - Y_{aL} , \qquad (2.10)$$

$\mathscr{L}_0$ takes the form

$$\mathscr{L}_0 = -\tfrac{1}{4}A^i_{\mu\nu}A^{i\mu\nu} - \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} + i\bar{\Psi}\partial\!\!\!/\Psi + \frac{g_A}{2}\bar{\Psi}\tau^i A\!\!\!/^i\left[\frac{1-\gamma_5}{2}\right]\Psi + \frac{g_B}{2}\bar{\psi}_1 B\!\!\!/\left[\frac{y_1+x_1\gamma_5}{2}\right]\psi_1 + \frac{g_B}{2}\bar{\psi}_2 B\!\!\!/\left[\frac{y_2+x_2\gamma_5}{2}\right]\psi_2 ,$$

$$\Psi = \begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix} . \qquad (2.11)$$

For simplicity, Eq. (2.11) is written for only one fermion doublet. To $\mathscr{L}_0$ we must add gauge-fixing terms for both the A^i and B fields,

$$\mathscr{L}_G = -\tfrac{1}{2}(\partial_\mu B^\mu)^2 - \tfrac{1}{2}(\partial_\mu A^{i\mu})(\partial_\nu A^{i\nu}) , \qquad (2.12)$$

and ghosts for the A^i gauge-fixing terms. This leads to the action

$$S = \int d^4x\{\mathscr{L}_0(x) + \mathscr{L}_G(x) - i\,\mathrm{Tr}[\ln(\delta_{ij}\partial^2 - \epsilon^{ijk}A^{k\mu}\partial_\mu)]\} . \qquad (2.13)$$

Once we have the action, the calculation of the effective potential proceeds as in CJT or II A, with two slight differences. We must allow for $B - A_3$ mixing and must therefore include a nondiagonal source K_{AB} and the accompanying propagator Δ_{AB}. In addition, there were no ghosts in the O(2)×U(1) case, whereas they are present here, at least at this stage.

The generating functional is

$$W[K] = \exp(iZ[K]) = \int [d\Phi]\exp\left\{iS[\Phi] + i\int d^4x\,d^4y\left[\frac{1}{2}\sum_{i=1,3}A^i_\mu(x)K^{i\mu\nu}_A(x,y)A^i_\nu(y) + \tfrac{1}{2}B_\mu(x)K^{\mu\nu}_B(x,y)B_\nu(y) + B_\mu(x)K^{\mu\nu}_{AB}(x,y)A^3_\nu(y) - \sum_a \bar{\psi}_a(x)K_{\psi a}(x,y)\psi_a(y)\right]\right\} . \qquad (2.14)$$

The symbol Φ is used to represent all the fields. The source terms for single fields, e.g., $J_\mu B^\mu$ which usually appear, can be neglected in this case because none of the fundamental fields acquires a vacuum expectation value and because we shall not be using W to generate anything other than vacuum graphs, for which we expand $W[J=0,K=0]$. The effective action is obtained by Legendre transforming Z,

$$\Gamma[G_a,\Delta_{A^i},\Delta_B,\Delta_{AB}] = Z[K] - \int d^4x\,d^4y\{\mathrm{Tr}[K_{\psi a}](x,y)G_a(x,y)] + \tfrac{1}{2}\Delta^i_{A\mu\nu}(x,y)K^{i\mu\nu}_A(x,y) + \tfrac{1}{2}\Delta_{B\mu\nu}(x,y)K^{\mu\nu}_B(x,y) + \Delta_{AB\mu\nu}(x,y)K^{\mu\nu}_{AB}(x,y)\} , \qquad (2.15a)$$

$$\frac{\delta\Gamma}{\delta G_a}=-K_{\psi a},\quad \frac{\delta\Gamma}{\delta\Delta_V}=-\tfrac{1}{2}K_V,\quad \frac{\delta\Gamma}{\delta\Delta_{AB}}=-K_{AB}\;,$$
$$\frac{\delta Z}{\delta K_{\psi a}}=G_a,\quad \frac{\delta Z}{\delta K_V}=\tfrac{1}{2}\Delta_V,\quad \frac{\delta Z}{\delta K_{AB}}=\Delta_{AB}\;. \tag{2.15b}$$

From Eqs. (2.15b) and (2.14), it follows that

$$G_a(x,y)=\langle 0\,|\,T(\psi_a(y)\bar{\psi}_a(x))\,|\,0\rangle_{\text{connected}}\;, \tag{2.16}$$

and similar relations for the Δ's, which leads to the identification of the G's and Δ's as propagators.

For the present case, the prescription for calculating Γ, Eq. (2.8), becomes

$$\Gamma[G,\Delta]=-i\,\mathrm{Tr}[\mathscr{D}_{\psi a}{}^{-1}G_a+\ln G_a{}^{-1}]$$
$$+\frac{i}{2}\sum_{i=1,2}\mathrm{Tr}[\mathscr{D}_{A^i}{}^{-1}\Delta_{A^i}+\ln\Delta_{A^i}{}^{-1}]+\frac{i}{2}\mathrm{Tr}\overline{\mathscr{D}}_{AB}{}^{-1}\overline{\Delta}_{AB}+\frac{i}{2}\ln\mathrm{Det}\overline{\Delta_{AB}{}^{-1}}+\Gamma_2[G,\Delta]+\text{const}\;. \tag{2.17}$$

Γ_2 is calculated from vacuum graphs and is described below. The matrix Δ_{AB} has elements

$$\overline{\Delta}_{AB}=\begin{bmatrix}\Delta_{A^3} & \Delta_{AB}\\ \Delta_{AB} & \Delta_B\end{bmatrix},\quad \overline{\Delta_{AB}{}^{-1}}=\begin{bmatrix}\Delta_{A^3}{}^{-1} & \Delta_{AB}{}^{-1}\\ \Delta_{AB}{}^{-1} & \Delta_B{}^{-1}\end{bmatrix}, \tag{2.18}$$

and the matrix $\overline{\mathscr{D}}_{AB}$ is similar. The $\mathscr{D}$'s, calculated from Eq. (2.7), are given (in momentum space) by

$$\mathscr{D}_\Phi(p,q)=(2\pi)^4\delta^4(p-q)\mathscr{D}_\Phi(p)\;,$$
$$i\mathscr{D}_{\psi a}{}^{-1}(p)=\not{p}\equiv iS^{-1}(p)\;,$$
$$i\mathscr{D}_B{}^{-1\mu\nu}(p)=-(p^2g^{\mu\nu}-p^\mu p^\nu)\equiv iD^{-1\mu\nu}(p)\;,$$
$$i\mathscr{D}_{AB}{}^{-1\mu\nu}(p)=0\;,$$
$$i\mathscr{D}_{A^i}{}^{-1\mu\nu}(p)=iD^{-1\mu\nu}(p)+i\mathscr{G}^{-1}(p)^{\mu\nu}\;,$$
$$\mathscr{G}^{-1}(p)^{\mu\nu}=-2g_A{}^2\int\frac{d^4k}{(2\pi)^4}\frac{k^\mu(p+k)^\nu}{k^2(p+k)^2}\;, \tag{2.19}$$

where we have chosen to work in the Landau gauge. The $\mathscr{G}^{-1}$ contribution to $\mathscr{D}_{A^i}{}^{-1}$ is due to the ghost term. Substitution of these $\mathscr{D}$'s into Eq. (2.17) and use of Eq. (2.6) leads to the effective potential

$$V[G,\Delta]=-i\sum_a\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\ln G^a(k)-S^{-1}(k)G^a(k)]$$
$$+\frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\ln\Delta_B(k)-D^{-1}(k)\Delta_B(k)+\ln\Delta_{AB}(k)+\ln\Delta_{BA}(k)]$$
$$+\frac{i}{2}\sum_i\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\{\ln\Delta_A^i(k)-[D^{-1}(k)+\mathscr{G}^{-1}(k)]\Delta_A^i(k)\}+V_2[G,\Delta]+\text{const}\;, \tag{2.20}$$

where $V_2[G,\Delta]$ is computed from the two-particle irreducible vacuum graphs occurring in the expansion of the generating functional Z.

$$Z[0]=-i(\text{vac graphs})_{2\text{PI}}\int d^4x=-V_2\int d^4x\;, \tag{2.21}$$

where the propagators in the vacuum graphs are replaced by the G's and Δ's. The lowest order in which DSB could occur is two loops; the relevant graphs are therefore those of Fig. 1. After a little algebra they are given by

$$V_2=V_{3A}+V_{4A}+V_{\psi V}\,,$$

$$V_{3A}=\frac{ig_A{}^2}{4}|\epsilon^{ijk}|\int\frac{d^4k_1d^4k_2}{(2\pi)^8}k_1^\alpha k_1^\beta\Delta^{i\gamma\delta}(k_1)$$
$$\times[\Delta^j_{\alpha\beta}(k_2)\Delta^k_{\gamma\delta}(k_1+k_2)+\Delta^k_{\alpha\beta}(k_1+k_2)\Delta^j_{\gamma\delta}(k_2)-2\Delta^j_{\alpha\delta}(k_2)\Delta^k_{\beta\gamma}(k_1+k_2)]\,,$$

$$V_{4A}=\frac{g_A{}^2}{4}(1-\delta_{jk})\int\frac{d^4k_1d^4k_2}{(2\pi)^8}[\Delta^{j\mu}_{\mu}(k_1)\Delta^{k\nu}_{\nu}(k_2)-\Delta^j_{\mu\nu}(k_1)\Delta^{k\nu\mu}(k_2)]\,, \tag{2.22}$$

$$V_{\psi V}=\frac{ig_A{}^2}{8}\int\frac{d^4k_1d^4k_2}{(2\pi)^8}\mathrm{Tr}\left[\bar G(k_1)\tau^i\gamma_\mu\left[\frac{1-\gamma_5}{2}\right]\bar G(k_2)\tau^i\gamma_\nu\left[\frac{1-\gamma_5}{2}\right]\right]\Delta^{i\mu\nu}_A(k_1-k_2)$$
$$+\frac{ig_B{}^2}{8}\sum_a\int\frac{d^4k_1d^4k_2}{(2\pi)^8}\mathrm{Tr}\left[G^a(k_1)\gamma_\mu\left[\frac{y_a+x_a\gamma_5}{2}\right]G^a(k_2)\gamma_\nu\left[\frac{y_a+x_a\gamma_5}{2}\right]\right]\Delta^{\mu\nu}_B(k_1-k_2)$$
$$+\frac{ig_Ag_B}{4}\sum_a\int\frac{d^4k_1d^4k_2}{(2\pi)^8}\mathrm{Tr}\left[G^a(k_1)\tau^3_{aa}\gamma_\mu\left[\frac{1-\gamma_5}{2}\right]G^a(k_2)\gamma_\nu\left[\frac{y_a+x_a\gamma_5}{2}\right]\right]\Delta^{\mu\nu}_{AB}(k_1-k_2)\,,$$

$$\bar G=G^j\delta_{jk};\quad j,k=1,2\,.$$

In Eq. (2.22), we have used perturbative vertices only (the Hartree-Fock approximation). If the SU(2)×U(1) symmetry is broken, with the fermions and gauge bosons acquiring masses, these vertices do not satisfy the Ward-Takahashi identities. In $V_{\psi V}$ this does not matter provided we work in the Landau gauge. Then the Goldstone pole contribution to the $\bar\psi V\psi$ vertex, which is proportional to the boson momentum q^μ, vanishes when contracted into the boson propagator $\Delta_{\mu\nu}(q)$. For covariance purposes we must still retain the nonperturbative part of the vertices when Dyson-Schwinger equations are derived for the vector propagators, but this will be done in the treatment of the LA in the next section. The non-Abelian terms V_{3A} and V_{4A} are a different matter. Use of the Landau gauge does not eliminate the nonperturbative contributions to the vertices. We return to this point in Sec. IV.

III. LINEARIZED TREATMENT

Having obtained the effective potential, Eqs. (2.20) and (2.22), we can use the stationariness conditions of Eq. (2.5) to derive the Dyson-Schwinger equations for the propagators,

$$G_1{}^{-1}(p)=S^{-1}(p)+\frac{g_B{}^2}{4}\int\frac{d^4k}{(2\pi)^4}\left[\gamma_\mu\left[\frac{y_1+x_1\gamma_5}{2}\right]G_1(k)\gamma_\nu\left[\frac{y_1+x_1\gamma_5}{2}\right]\right]\Delta^{\mu\nu}_B(p-k)$$
$$+\frac{g_A{}^2}{4}\int\frac{d^4k}{(2\pi)^4}\left[2\gamma_\mu\left[\frac{1-\gamma_5}{2}\right]G_2(k)\gamma_\nu\left[\frac{1-\gamma_5}{2}\right]\Delta^{\mu\nu}_W(p-k)\right.$$
$$\left.+\gamma_\mu\left[\frac{1-\gamma_5}{2}\right]G_1(k)\gamma_\nu\left[\frac{1-\gamma_5}{2}\right]\Delta^{\mu\nu}_{A^3}(p-k)\right]$$
$$+\frac{g_Ag_B}{4}\int\frac{d^4k}{(2\pi)^4}\left[\gamma_\mu\left[\frac{1-\gamma_5}{2}\right]G_1(k)\gamma_\nu\left[\frac{y_1+x_1\gamma_5}{2}\right]+\left[\frac{1-\gamma_5}{2}\right]\gamma_\nu\left[\frac{y_1+x_1\gamma_5}{2}\right]G_1(k)\gamma_\mu\right]$$
$$\times\Delta^{\mu\nu}_{AB}(k-p)\,, \tag{3.1a}$$

$$\Delta_B^{-1\mu\nu}(p)=D^{-1\mu\nu}(p)-\frac{g_B{}^2}{4}\int\frac{d^4k}{(2\pi)^4}\sum_a\mathrm{Tr}[G_a(k)\gamma^\mu_{Ba}G_a(k-p)\Gamma^\nu_{Ba}]\,, \tag{3.1b}$$

$$\Delta_{AB}^{-1\mu\nu}(p)=-\frac{g_Ag_B}{4}\int\frac{d^4k}{(2\pi)^4}\sum_a\mathrm{Tr}[G_a(k)\gamma^\mu_{Aa}G_a(k-p)\Gamma^\nu_{Bb}]\,, \tag{3.1c}$$

$$\Delta_{A^3}^{-1\mu\nu}(p)=D^{-1\mu\nu}(p)+\mathcal{G}^{-1\mu\nu}(p)+2ig_A{}^2\int\frac{d^4k}{(2\pi)^4}[\Delta_{W\alpha}^{\alpha}(k)g^{\mu\nu}-\Delta_W^{\mu\nu}(k)]$$
$$-\frac{g_A{}^2}{4}\int\frac{d^4k}{(2\pi)^4}\sum_a \mathrm{Tr}[G_a(k)\gamma_{Aa}^{\mu}G_a(k-p)\Gamma_{Aa}^{\mu}]$$
$$-g_A{}^2\int\frac{d^4k}{(2\pi)^4}\{k_\alpha k_\beta[\Delta_W^{\alpha\beta}(k)\Delta_W^{\mu\nu}(p+k)-\Delta_W^{\alpha\nu}(k)\Delta_W^{\beta\mu}(p+k)$$
$$-\Delta_W^{\alpha\mu}(k)\Delta_W^{\beta\nu}(p+k)+2\Delta_W^{\mu\nu}(k)\Delta_W^{\alpha\beta}(p+k)]+[\Delta_{W\alpha\beta}(p+k)+\Delta_{W\alpha\beta}(p-k)]$$
$$\times[k^\mu k^\nu\Delta_W^{\alpha\beta}(k)-k^\mu k^\alpha\Delta_W^{\beta\nu}(k)-k^\nu k^\alpha\Delta_W^{\beta\mu}(k)]\}\ , \tag{3.1d}$$

$$\Delta_W^{-1\mu\nu}(p)=D^{-1\mu\nu}(p)+\mathcal{G}^{-1\mu\nu}(p)$$
$$+ig_A{}^2\int\frac{d^4k}{(2\pi)^4}\{[\Delta_{W\alpha}^{\alpha}(k)+\Delta_{A^3\alpha}^{\alpha}(k)]g^{\mu\nu}-[\Delta_W^{\mu\nu}(k)+\Delta_{A^3}^{\mu\nu}(k)]\}$$
$$-\frac{g_A{}^2}{4}\int\frac{d^4k}{(2\pi)^4}\sum_i\{\mathrm{Tr}[G_1^i(k)\gamma_W^{\mu}G_2^i(k-p)\Gamma_W^{\nu}]+(1\leftrightarrow 2)\}$$
$$-\frac{g_A{}^2}{2}\int\frac{d^4k}{(2\pi)^4}\{k_\alpha k_\beta[\Delta_W^{\alpha\beta}(k)\Delta_{A^3}^{\mu\nu}(p+k)+2\Delta_W^{\mu\nu}(k)\Delta_{A^3}^{\alpha\beta}(p+k)$$
$$-\Delta_W^{\alpha\nu}(k)\Delta_{A^3}^{\beta\mu}(k+p)-\Delta_W^{\alpha\mu}(k)\Delta_{A^3}^{\beta\nu}(p+k)]+[\Delta_{A^3\alpha\beta}(p+k)+\Delta_{A^3\alpha\beta}(p-k)]$$
$$\times[k^\alpha k^\beta\Delta_W^{\mu\nu}(k)+k^\mu k^\nu\Delta_W^{\alpha\beta}(k)-k^\mu k^\alpha\Delta_W^{\beta\nu}(k)-k^\nu k^\alpha\Delta_W^{\beta\mu}(k)]+(A^3\leftrightarrow W)\}\ . \tag{3.1e}$$

Much of the notation in Eq. (3.1) requires explanation. In (3.1e), fermions 1 and 2 are in the same weak doublet. The equation for $G_2{}^{-1}$ is obtained by interchanging all subscript 1's and 2's and changing the sign of the $g_A g_B$ term in (3.1a). The sums over a in Eqs. (3.1b)—(3.1d) run over all fermions; the sum over i in (3.1e) runs over all fermion doublets. We have of course identified $W^{\pm}=(1/\sqrt{2})(A_1\pm iA_2)$ and have used $\Delta_{W^+}=\Delta_{W^-}\equiv\Delta_W$. For the $\bar{\psi}_a V\psi_a$ vertices, where V is one of the vectors, we have introduced

$$\Gamma_{Va}^{\mu}=\gamma_{Va}^{\mu}+\Gamma_{Va}^{\mu}(\text{nonpert})\ , \tag{3.2}$$

where γ_{Va}^{μ} is the perturbative vertex and Γ_{Va}^{μ}(nonpert) is the additional piece required by the Ward-Takahashi identity. As discussed above, the nonperturbative piece is not required in the equation for G^{-1}. The perturbative vertices are

$$\gamma_{Ba}^{\mu}=\gamma^{\mu}\left[\frac{y_a+x_a\gamma_5}{2}\right],$$
$$\gamma_{Aa}^{\mu}=\gamma^{\mu}\left[\frac{1-\gamma_5}{2}\right]\tau_{aa}^3\ , \tag{3.3}$$
$$\gamma_W^{\mu}=\gamma^{\mu}\left[\frac{1-\gamma_5}{2}\right],$$

where the subscript A refers to A^3.

We then write $G_a{}^{-1}$ and $\Delta_V{}^{-1}$ in the form

$$G_a{}^{-1}(p)=S^{-1}(p)-iA_a(p^2)\not{p}+iB_a(p^2)+iC_a(p^2)\gamma_5-iD_a(p^2)\not{p}\gamma_5\ ,$$
$$\Delta_V{}^{-1\mu\nu}(p)=D^{-1\mu\nu}(p)-iP^{\mu\nu}\pi_V(p^2),\quad P^{\mu\nu}=\left[g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right], \tag{3.4}$$
$$\Delta_{AB}^{-1\mu\nu}(p)=-iP^{\mu\nu}\pi_{AB}(p^2)\ ,$$

and substitute into Eq. (3.1). Linearized in the standard manner,[4–6] Eq. (3.1a) leads to

$$A_a(p)\not p=-\frac{i}{16}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(p-k)^2}\left[\not k+2(\not p-\not k)\frac{k\cdot(p-k)}{(p-k)^2}\right]$$
$$\times\left[\frac{g_B{}^2}{\alpha_a(k)k^2-\mu_a{}^2(k)}\{(x_a{}^2+y_a{}^2)[1+A_a(k)]-2x_ay_aD_a(k)\}\right.$$
$$+2g_A{}^2\{2[1+A_b(k)+D_b(k)]/[\alpha_b(k)k^2-\mu_b{}^2(k)]$$
$$\left.+[1+A_a(k)+D_a(k)]/[\alpha_a(k)k^2-\mu_a{}^2(k)]\}\right],$$

$$B_a(p)=-\frac{3i}{4}g_B{}^2Y_{aR}Y_{aL}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(p-k)^2}\frac{1}{\alpha_a(k)k^2-\mu_a{}^2(k)}B_a(k),\tag{3.5}$$

$$C_a(p)=-\frac{3i}{4}g_B{}^2Y_{aR}Y_{aL}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(p-k)^2}\frac{1}{\alpha_a(k)k^2-\mu_a{}^2(k)}C_a(k),$$

$$D_a(p)\not p=-\frac{i}{16}\int\frac{d^4k}{(2\pi)^4}\frac{1}{(p-k)^2}\left[\not k+2(\not p-\not k)\frac{k\cdot(p-k)}{(p-k)^2}\right]$$
$$\times\left[\frac{g_B{}^2}{\alpha_a(k)k^2-\mu_a{}^2(k)}\{2x_ay_a[1+A_a(k)]-(x_a{}^2+y_a{}^2)D_a(k)\}\right.$$
$$+2g_a{}^2\{2[1+A_b(k)+D_b(k)]/[\alpha_b(k)k^2-\mu_b{}^2(k)]$$
$$\left.+[1+A_a(k)+D_a(k)]/[\alpha_a(k)k^2-\mu_a{}^2(k)]\}\right],$$

$$\alpha_a(k)\equiv[1+A_a(k)]^2-D_a(k)^2,\quad \mu_a{}^2(k)=B_a(k)^2-C_a(k)^2.$$

The subscript a can refer to any fermion, with b then denoting its isodoublet partner.

It is consistent and convenient to take $A_a, A_b, D_a, D_b = 0 + O(g^2)$. The equations for B_a and C_a are then of the standard form

$$F_a(p)=-\frac{3i}{4}g_B{}^2Y_{aL}Y_{aR}\times\int\frac{d^4k}{(2\pi)^4}\frac{1}{(p-k)^2}\frac{1}{k^2-\mu_a{}^2(k)}F_a(k),\tag{3.6}$$

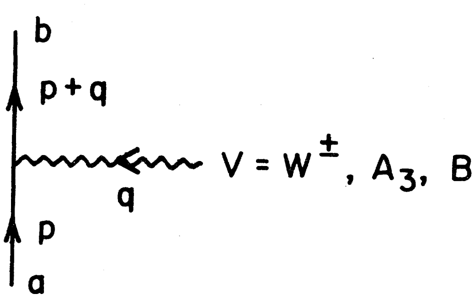

FIG. 2. Notation for fermion-vector vertex in Eq. (3.10).

where F is B or C. It is a good approximation to neglect the k dependence of $\mu_a{}^2(k)$ in the denominator,[17] letting $\mu_a{}^2(k)\simeq m_a{}^2$ there. Then Eq. (3.6) has the solution[4,5,8,17]

$$F_a(p)=f_a\Gamma(1+r_a)\Gamma(2-r_a)\times{}_2F_1\left[r_a,1-r_a,2;\frac{p^2}{m_a{}^2}\right],\quad |p^2|<m_a{}^2,$$

$$F_a(p)\simeq f_a\left[-\frac{p^2}{m_a{}^2}\right]^{-r_a},\quad |p^2|\to\infty,\tag{3.7}$$

$$r_a=\frac{1}{2}\left[1+\left[1-\frac{3g_B{}^2}{16\pi^2}Y_{aL}Y_{aR}\right]^{1/2}\right]$$
$$\simeq\frac{3g_B{}^2}{64\pi^2}Y_{aL}Y_{aR},$$

provided $Y_{aL}Y_{aR}>0$. The fact that r_a is small justifies neglecting the k dependence in the denominator and also means that integrals like that occurring in Eq. (3.6) are dominated by very large k, $|k^2| \gg m_a{}^2, p^2$. It is easy to see that it is this region which leads to the $1/r_a$ (and therefore $1/g^2$) divergence of the integral as $g_B{}^2 \to 0$. Combining the definition of $\mu_a{}^2(k)$ in Eq. (3.5) with the form of B_a and C_a from Eq. (3.7), we have

$$\mu_a{}^2(k) \simeq m_a{}^2 \left[-\frac{p^2}{m_a{}^2} \right]^{-2r_a} \quad (\text{large } |p^2|) , \qquad m_a{}^2 = b_a{}^2 - c_a{}^2 , \tag{3.8}$$

where b_a and c_a are the constant f_a in Eq. (3.7). The absence of $g_A{}^2$ from (3.6) results from its pure left-handed coupling.

The $Y_{aL} Y_{aR} > 0$ requirement has interesting consequences. For one thing, if a fermion has no right-handed component, then $Y_{aR} = 0$ and it will be prevented from acquiring a mass (in the LA). If we consider the standard quantum-number assignments for quark and lepton doublets, then

$$Y_{aL} Y_{aR} = \begin{cases} \frac{4}{9}, & u \\ -\frac{2}{9}, & d \\ 2, & e \\ 0, & \nu_e \, . \end{cases} \tag{3.9}$$

We therefore see that the neutrino does not acquire a mass; but, embarassingly, neither does the down (or strange or bottom) quark. In fact, from $Q = I_L^3 + Y/2$ we deduce that for fermions in a left-handed doublet and right-handed singlets, only one member of a doublet can acquire a mass if the charge of the $I_L^3 = +\frac{1}{2}$ fermion is in the range $Q \in [0,1]$.

We then consider the gauge-boson propagators and masses. If we were to use the perturbative vertices in Eqs. (3.1b)–(3.1e) we would find inconsistencies in the gauge on different sides of the same equation. We could extract the correct result by keeping only terms proportional to $g^{\mu\nu}$, but it is more reassuring to use vertices which satisfy the Ward-Takahashi identities in the presence of dynamically generated masses. Referring to Fig. 2 for notation, the fermion–gauge-boson vertices for $A_3(\Gamma_A)$, $W^{\pm}(\Gamma_W)$, and $B(\Gamma_B)$ are [cf. Eq. (3.3)]

$$\begin{aligned} \Gamma^{\mu}_{Aa}(q) &= \gamma^{\mu}_{Aa} + \tau^3_{aa} \frac{q^{\mu}}{q^2} \left[\left[\frac{1+\gamma_5}{2} \right] [B_a(p) + C_a(p)] - \left[\frac{1-\gamma_5}{2} \right] [B_a(p+q) - C_a(p+q)] \right] , \\ \Gamma^{\mu}_{Wab}(q) &= \gamma^{\mu}_{W} + \frac{q^{\mu}}{q^2} \left[\left[\frac{1+\gamma_5}{2} \right] [B_a(p) + C_a(p)] - \left[\frac{1-\gamma_5}{2} \right] [B_b(p+q) - C_b(p+q)] \right] , \\ \Gamma^{\mu}_{Ba}(q) &= \gamma^{\mu}_{Ba} + \frac{q^{\mu}}{q^2} \left[\left[\frac{y_a - x_a\gamma_5}{2} \right] [B_a(p) + \gamma_5 C_a(p)] - \left[\frac{y_a + x_a\gamma_5}{2} \right] [B_a(p+q) + \gamma_5 C_a(p+q)] \right] . \end{aligned} \tag{3.10}$$

When Eqs. (3.4) and (3.10) are used in Eq. (3.1d) for the (inverse) A^3 propagator, it takes the form

$$\begin{aligned} P^{\mu\nu}\pi_A(p) &= -\frac{i g_A{}^2}{4} \int \frac{d^4k}{(2\pi)^4} \sum_a \mathrm{Tr}\{ G_a(k) \gamma^{\mu}_{Aa} G_a(k-p) [\gamma^{\nu}_{Aa} + \Gamma^{\nu}_{Aa}(-p, \text{nonpert})] \} + O(g^2) \\ &\equiv I^{\mu\nu}_{pp} + I^{\mu\nu}_{pnp} \end{aligned} \tag{3.11}$$

in the LA. (All vector self-couplings are nonlinear and are therefore neglected in the LA.) The integral in Eq. (3.11) is proportional to $1/g_B{}^2$, so that $\pi_A \sim (g^2)^0$. The integrals $I^{\mu\nu}_{pp}$ and $I^{\mu\nu}_{pnp}$ are divergent and require regularization. We shall employ dimensional regularization throughout the paper. A subtraction scheme will not be required since all poles in $(n-4)$ will occur in nonleading orders. Using (3.3), (3.4), and (3.10) in (3.11) yields

$$I^{(n)\mu\nu}_{pp} = \frac{g_A{}^2}{8} \mathrm{Tr}[\gamma_\alpha \gamma^\mu \gamma_\beta \gamma^\nu (1-\gamma_5)] \sum_a \int \frac{d^nk}{(2\pi)^n} \frac{k^\alpha (k-p)^\beta}{[k^2 - \mu_a{}^2(k)][(p-k)^2 - \mu_a{}^2(k-p)]} \, . \tag{3.12}$$

The masses have disappeared from the numerator due to the chiral form of the vertex. Introducing Feynman parameters, this becomes

$$I_{pp}^{(n)\mu\nu}=\frac{g_A{}^2}{2}\sum_a\int\frac{d^nk}{(2\pi)^n}\int d\alpha[2k^\mu k^\nu-k^2g^{\mu\nu}+\alpha(1-\alpha)(p^2g^{\mu\nu}-2p^\mu p^\nu)+\mu_a{}^2(k)g^{\mu\nu}]$$
$$\times[k^2+\alpha(1-\alpha)p^2-\mu_a{}^2(k)]^{-2}$$
$$-\frac{g_A{}^2}{2}g^{\mu\nu}\sum_a\int\frac{d^nk}{(2\pi)^n}\int d\alpha\,\mu_a{}^2(k)[k^2+\alpha(1-\alpha)p^2-\mu_a{}^2(k)]^{-2}\,, \tag{3.13}$$

where we have used $\mu_a(k-p)\simeq\mu_a(k)$, a safe approximation for small r_a. The first integral would be a familiar one encountered in perturbative QED[18] if $\mu_a(k)$ were a constant. The k dependence of μ_a complicates the analysis, but it proceeds in the same manner. From Appendix B, (B15) and (B16),

$$\frac{g_A{}^2}{2}\sum_a\int\frac{d^nk}{(2\pi)^n}\int d\alpha[2k^\mu k^\nu-k^2g^{\mu\nu}+\alpha(1-\alpha)(p^2g^{\mu\nu}-2p^\mu p^\nu)+\mu_a{}^2(k)g^{\mu\nu}]\,[k^2+\alpha(1-\alpha)p^2-\mu_a{}^2(k)]^{-2}$$
$$=g_A{}^2g^{\mu\nu}\sum_a\frac{2r_a}{n}\int d\alpha\int\frac{d^nk}{(2\pi)^n}\mu_a{}^2(k)[k^2+\alpha(1-\alpha)p^2-\mu_a{}^2(k)]^{-2}$$
$$-\frac{4g_A{}^2}{n-4}(p^2g^{\mu\nu}-p^\mu p^\nu)\sum_a\int d\alpha\,\alpha(1-\alpha)\int\frac{d^nk}{(2\pi)^n}[\alpha(1-\alpha)p^2-(1+2r_a)\mu_a{}^2(k)]$$
$$\times[k^2+\alpha(1-\alpha)p^2-\mu_a{}^2(k)]^{-3}\,. \tag{3.14}$$

The first term has no $n=4$ pole. The integral is proportional to $1/r_a$ for $n=4$, and so the contribution to $I_{pp}^{(n)\mu\nu}$ is of order g^2 and hence nonleading. The second term does have a pole at $n=4$, but the integral leads to no powers of $1/r$ (for $n=4$) and therefore the second term is also nonleading. That leaves

$$I_{pp}^{(n)\mu\nu}=-\frac{g_A{}^2}{2}g^{\mu\nu}\sum_a\int d\alpha\int\frac{d^nk}{(2\pi)^n}\mu_a{}^2(k)[k^2+\alpha(1-\alpha)p^2-\mu_a{}^2(k)]^{-2}\,, \tag{3.15}$$

which is finite and proportional to $(g^2)^0$ for $n=4$. For future convenience we define

$$M_a{}^2(p)\equiv-i\int d\alpha\int\frac{d^4k}{(2\pi)^4}\mu_a{}^2(k)[k^2+\alpha(1-\alpha)p^2-\mu_a{}^2(k)]^{-2}\,. \tag{3.16}$$

Turning to the second integral in Eq. (3.11),

$$I_{pnp}^{(n)\mu\nu}=\frac{g_A{}^2}{4}\sum_a\int\frac{d^nk}{(2\pi)^n}\mathrm{Tr}\left\{[\not k+B_a(k)-C_a(k)\gamma_5]\gamma^\mu\left[\frac{1-\gamma_5}{2}\right][\not k-\not p+B_a(k-p)-C_a(k-p)\gamma_5]\left[-\frac{p^\nu}{p^2}\right]\right.$$
$$\left.\times\left[[B_a(k)+C_a(k)]\left[\frac{1+\gamma_5}{2}\right]-[B_a(k-p)-C_a(k-p)]\left[\frac{1-\gamma_5}{2}\right]\right]\right\}$$
$$\times[(k-p)^2-\mu_a{}^2(k-p)]^{-1}[k^2-\mu_a{}^2(k)]^{-1}$$
$$=-\frac{g_A{}^2}{2}\frac{p^\nu}{p^2}\sum_a\int\frac{d^nk}{(2\pi)^n}[\mu_a{}^2(k)(k-p)^\mu-\mu_a{}^2(k-p)k^\mu][k^2-\mu_a{}^2(k)]^{-1}[(k-p)^2-\mu_a{}^2(k-p)]^{-1}\,.$$

After Feynman parametrization and use of $\mu_a(k-p)\simeq\mu_a(k)$, which can be obtained by expanding about $p=0$ for large k, this becomes

$$I_{pnp}^{(n)\mu\nu}=\frac{ig_A{}^2}{2}\frac{p^\mu p^\nu}{p^2}\sum_a M_a{}^2(p)\ . \tag{3.17}$$

Using that and Eq. (3.15) in Eq. (3.11) yields

$$P^{\mu\nu}\pi_A(p)=\frac{g_A{}^2}{2}P^{\mu\nu}\sum_a M_a{}^2(p)\ . \tag{3.18}$$

The calculations for $\pi_B(p)$ and $\pi_{AB}(p)$ are very similar. The results are

$$\pi_B(p)=\frac{g_B{}^2}{2}\sum_a x_a{}^2M_a{}^2(p)\ ,$$
$$\pi_{AB}(p)=-\frac{g_Ag_B}{2}\sum_a x_a\tau_{aa}^3M_a{}^2(p)\ . \tag{3.19}$$

It is easy to see that $\pi_A(p)$ can also be written this way, and we therefore obtain the mass matrix for the neutral gauge bosons,

$$\overline{M^2}(p)=\tfrac{1}{2}\sum_a M_a{}^2(p)\begin{bmatrix} g_B{}^2x_a{}^2 & -g_Ag_Bx_a\tau_{aa}^3 \\ -g_Ag_Bx_a\tau_{aa}^3 & g_A{}^2\end{bmatrix}\ . \tag{3.20}$$

For fermions with quark or lepton quantum numbers, a convenient simplification occurs, since $x_a{}^2=x_a\tau_{aa}^3=+1$ for such fermions. In that case diagonalization of $M^2(p)$ yields the following eigenvalues and corresponding eigenvectors:

$$\begin{aligned}
&\pi_1(p)\equiv\pi_\gamma(p)=0\ ,\\
&V_1^\mu=B^\mu\cos\theta+A_3^\mu\sin\theta\equiv A_\gamma^\mu\ ,\\
&\pi_2(p)\equiv\pi_Z(p)=\tfrac{1}{2}(g_A{}^2+g_B{}^2)\sum_a M_a{}^2(p)\ ,\\
&V_2^\mu=B^\mu\sin\theta-A_3^\mu\cos\theta\equiv Z^\mu\ ,\\
&\sin\theta=g_B/(g_A{}^2+g_B{}^2)^{1/2}\ .
\end{aligned} \tag{3.21}$$

We have thus obtained a massless photon and a massive Z^0, and have reproduced the usual mixing pattern. In addition, the Z mass is given in terms of the fermion masses.

The W mass will also be determined by the dynamically generated fermion masses. Starting with Eq. (3.1e), inserting Eqs. (3.4) and (3.10), linearizing, and proceeding as we did for the neutral vectors, we obtain

$$\pi_W(p)=-ig_A^2\sum_a\int d\alpha\,\alpha\int\frac{d^4k}{(2\pi)^4}\mu_a{}^2(k)[k^2+\alpha(1-\alpha)p^2-\alpha\mu_a{}^2(k)]^{-2}\ , \tag{3.22}$$

where we have used the fact that only one member of a quark or lepton doublet acquires mass in the LA.

The W and Z masses are given by $M_V{}^2=\pi_V(M_V{}^2)$. Since the integrals in (3.21) and (3.22) are controlled by very large k, we can use $M_V{}^2\simeq\pi_V(p^2=0)$. The integrals are evaluated in Appendix B (B17) and result in

$$M_W{}^2=\frac{1}{3}\frac{g_A{}^2}{g_B{}^2}\sum_a\frac{m_a{}^2}{Y_{aL}Y_{aR}}\ ,$$
$$M_Z{}^2=\frac{1}{3}\frac{g_A{}^2+g_B{}^2}{g_B{}^2}\sum_a\frac{m_a{}^2}{Y_{aL}Y_{aR}}=M_W{}^2/\cos^2\theta\ . \tag{3.23}$$

The standard $I=\frac{1}{2}$ breaking is thus reproduced, and each quark contributes

$$m_q{}^2/(3\sin^2\theta Y_{qL}Y_{qR})=3m_q{}^2/(4\sin^2\theta)$$

($m_l{}^2/6\sin^2\theta$ for leptons) to the Z mass squared. The $I=\frac{1}{2}$ pattern of breaking is due to the fact that only fermion-antifermion intermediate states are considered in the equation for the gauge-boson self-energies in the LA.

It is instructive to compare these results with MAC expectations and to see whether they make physical sense. For simplicity, we consider one left-handed quark doublet plus two (or one—it does not matter) right-handed singlet(s). Considering only $S=0$ states, we see that only the U(1) interaction can provide the attractive potential, since $SU(2)_L$ does not couple to right-handed fermions. The $\bar{f}f$ potential is then proportional to $-Y_{fR}Y_{fL}$ and is attractive only for $Y_{fR}Y_{fL}>0$, exactly the condition for mass generation obtained from (3.7). Pursuing the matter further, of all $\bar{f}f'$ or ff' spin-singlet states, the MAC's are $\bar{u}_Rd_L$, $\bar{d}_Lu_R$, and $(\bar{u}_Ru_L\pm\bar{u}_Lu_R)$. The condensate is $(\bar{u}_Ru_L+\bar{u}_Lu_R)$, giving mass to the u, but not to the d. The other three states are the aspiring Goldstone bosons which give three gauge bosons mass. The W's obviously get mass from $\bar{u}_Rd_L$ and $\bar{d}_Lu_R$; and $(\bar{u}_Ru_L-\bar{u}_Lu_R)$ couples to the combination $(-g_AA_3+g_BB)$, but not to $(g_BA_3+g_AB)$, producing a massive Z^0 and massless γ. Exactly analogous results are obtained for a lepton doublet plus singlet(s). The LA results are therefore consistent with the MAC hypothesis. If the symmetry does break we would expect the pattern of breaking produced by the LA.

If we inquire no further, things look very promising. Starting with massless fermions and gauge bosons, we have obtained massive fermions, properly mixed vector bosons, and a relation between the Z or W mass and those of the fermions. With three generations and assuming the top quark mass dominates the sum in (3.23), one obtains

$$m_t \simeq \frac{2\sin\theta}{\sqrt{3}} M_Z = 0.54 M_Z \ . \tag{3.24}$$

For $M_Z = 93$ GeV, this gives a top mass of about 50 GeV. The $t\bar{t}$ vector meson would have a mass of about $2m_t -$binding energy$\simeq 99$ GeV (Ref. 19), not too far from the Z mass.

Unfortunately, this attractive facade crumbles under closer scrutiny. The fact that the $I_3 = -\frac{1}{2}$ quarks have failed to acquire a mass is not viewed as particularly devastating. It is still possible that they may gain a mass when the treatment is refined or when other interactions are included. The first major problem arises when we begin counting the number of symmetries which have been broken and the consequent number of Goldstone bosons required.[20] We want no physical Goldstone bosons, both because none are observed experimentally and because the variational method used in the next section is weakened considerably if they are present. There are, however, only three gauge bosons acquiring a mass, and consequently only three Goldstone bosons will disappear. We therefore wish to break the (global) symmetry associated with only three generators. The breaking of $SU(2) \times U(1)$ to $U(1)$ results in three broken generators. Therefore any additional broken symmetries will result in physical Goldstone bosons. The only choice of fermion representations we have found which does not have the additional broken (chiral) symmetries is the case of only one left-handed doublet and one right-handed singlet. This appears to preclude construction of a realistic theory, except perhaps for the case of preons. However, we can still study the mechanics of DSB in a model which is realistically complex, even if it is not realistic because of Goldstone bosons or too few fermions.

There is an additional obvious shortcoming of the LA. The integral equations for the fermion masses (3.5) and (3.6) are homogeneous, and therefore they admit the trivial solution, $m = 0$, corresponding to the symmetry remaining unbroken. This is symptomatic of the LA and similar approaches. Even if one believes the approximation, all that can be determined is the functional form of the symmetry-breaking masses if they are nonzero. It is impossible to determine *whether* the breaking occurs. Since we have an effective potential, we will be able to determine whether the zero or nonzero mass solution corresponds to the lower energy density, and consequently whether or not symmetry breaking does occur. We will find that even ignoring non-Abelian effects—as the LA does—whether DSB occurs is critically dependent on the fermion quantum numbers Y_{aR} and Y_{aL}. The neglect of non-Abelian terms is itself another potential flaw of the LA. We shall address it in some detail in the next section.

IV. VARIATIONAL CALCULATION

A. General and $\Omega_{\rm kin}$

Thus far the effective potential has only enabled us to derive Schwinger-Dyson equations which we could have written down in the first place. To solve these equations we merely resorted to the same linearization procedure used in the past. The effective potential does enable us to go beyond this approximation. The V of Eqs. (2.20) and (2.22) is a functional of the propagators G_a, Δ_V, and its minimization yielded the intractable equations (3.1) satisfied by these functions. If we knew the functional form of $G_a(p)$, $\Delta_V(p)$, then the minimization of V would yield algebraic equations for parameters occurring in these functional forms. This approach was advocated by CJT, who implemented it for $O(2) \times U(1)$. That was one motivation for obtaining the linearized results in the preceding section: They will provide the set of parametrized test functions for G_a and Δ_V. CJT showed that use of the linearized results as test functions assured the absence of divergences in their calculation. We shall assume this feature carries over to the present case—unless compelled to conclude otherwise.

There is, of course, a drawback to the variational approach—we have no absolute assurance that our test functions constitute the optimal set. If we find that our propagators raise the vacuum energy density when the symmetry is broken, it is still possible that there is some other set of propagators giving a lower energy than the symmetric vacuum. If we find that the symmetry-broken vacuum has lower energy than the symmetric vacuum, then the symmetry does break; but it is still possible that there is some other vacuum, corresponding to different propagators and physical masses, which has still lower energy. (Recall[21] that use of the LA solutions as test functions automatically restricts the form of sources via $K_i = \delta V / \delta G_i$.) By restricting ourselves to such test functions (sources), it is in principle possible that we may overlook the true vacuum, but the LA solutions do constitute a good parametrization of the propagators. They have the general form one would expect for massive propagators, and the functional form of the masses eliminates logarithmic divergences from V. Furthermore, although they are obtained from linearized Dyson-Schwinger equations, to leading order in g^2 they do satisfy the full (sub)set of Dyson-Schwinger equations (3.1a)—(3.1e)—at least in the Abelian sector. (For vector self-coupling terms, things are still unclear due to the aforementioned gauge-invariance problems.) It therefore may not be such a serious constraint to restrict our attention to those particular functional forms for the propagators.

Referring to (3.4), (3.8), (3.21), and (3.22), the following forms are used for the propagators:

$$G_a(p)=\frac{i}{\not p-\mathcal{M}_a(p)} ,$$

$$\mathcal{M}_a(p)=\begin{cases}0, & r_a<0\\ B_a(p)+C_a(p)\gamma_5, & r_a>0 ,\end{cases} \tag{4.1a}$$

$$B_a(p)=b_a\left[-\frac{p^2}{m_a^2}\right]^{-r_a} ,$$

$$C_a(p)=c_a\left[-\frac{p^2}{m_a^2}\right]^{-r_a} ,$$

$$m_a^2=b_a^2-c_a^2 ,$$

$$\mu_a^2(p)\equiv m_a^2\left[-\frac{p^2}{m_a^2}\right]^{-2r_a} ,$$

$$r_a=\frac{3}{64\pi^2}g_B^2Y_{aL}Y_{aR} ,$$

$$\Delta_W^{\mu\nu}(p)=-iP^{\mu\nu}\frac{1}{p^2-\pi(p)} ,$$

$$\pi(p)=\sum_a \pi_a\left[-\frac{p^2}{m_a^2}\right]^{-2r_a} , \tag{4.1b}$$

$$\Delta_Z^{\mu\nu}(p)=-iP^{\mu\nu}\frac{1}{p^2-\zeta(p)} ,$$

$$\zeta(p)=\sum_a \zeta_a\left[-\frac{p^2}{m_a^2}\right]^{-2r_a} , \tag{4.1c}$$

$$\Delta_\gamma^{\mu\nu}(p)=-iP^{\mu\nu}\frac{1}{p^2} . \tag{4.1d}$$

The variational parameters are then π_a, ζ_a, b_a, and c_a, although in practice b_a and c_a only occur in the combination $m_a^2=b_a^2-c_a^2$. The p^2 dependence given is for large $|p^2|$, which is all that is needed. The p dependence of $\pi(p)$ and $\zeta(p)$ was not given explicitly in Sec. III, but was obtained in Appendix B (B17).

The effective potential for the broken symmetry vacuum, Eqs. (2.20) and (2.22), contains numerous divergences. A more manageable quantity to calculate is the difference between that and V evaluated in the symmetric (perturbation) vacuum. Defining

$$\Omega=V[G,\Delta]-V[\text{sym}] , \tag{4.2}$$

we shall see that Ω is finite, at least in the order to which we calculate. If we use Eq. (2.20) for V, we will obtain an expression for Ω in terms of propagators in the A^i-B basis, whereas the trial functions have been given in the W-Z-γ basis. Writing Ω in terms of the physical fields, and breaking Ω into smaller pieces, we get the form

$$\begin{aligned}\Omega&=[\Omega_G+\Omega_W+\Omega_{\gamma Z}]+\Omega_{\psi V}+[\Omega_{4A}+\Omega_{3A}]\\ &\equiv\Omega_{\text{kin}}+\Omega_{\psi V}+\Omega_{NA} .\end{aligned} \tag{4.3}$$

The free-field terms are given by

$$\begin{aligned}\Omega_G&=i\sum_a\int\frac{d^4k}{(2\pi)^4}\text{Tr}\{\ln[G_a^{-1}(k)S(k)]+S^{-1}(k)G_a(k)-\mathbb{1}\} ,\\ \Omega_W&=-i\int\frac{d^4k}{(2\pi)^4}\text{Tr}\{\ln[\Delta_W^{-1}(k)D(k)]+D^{-1}(k)\Delta_W(k)-\mathbb{1}+\mathcal{G}^{-1}(k)[\Delta_W(k)-D(k)]\} ,\\ \Omega_{\gamma Z}&=-\frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\text{Tr}\{\ln[\Delta_\gamma^{-1}(k)D(k)]+\ln[\Delta_Z^{-1}(k)D(k)]\\ &\qquad+D^{-1}(k)[\Delta_\gamma(k)+\Delta_Z(k)]-2+\mathcal{G}^{-1}(k)[\sin^2\theta\Delta_\gamma(k)+\cos^2\theta\delta_Z(k)-D(k)]\} .\end{aligned} \tag{4.4}$$

For the interaction contributions, the V's are given by

$$V_{4A}[G,\Delta]=\frac{g_A{}^2}{2}\int\frac{d^4k_1d^4k_2}{(2\pi)^8}\{\Delta^{\mu}_{W\mu}(k_1)[\Delta^{\nu}_{W\nu}(k_2)+2\Delta^{\nu}_{\gamma\nu}(k_2)\sin^2\theta+2\Delta^{\nu}_{Z\nu}(k_2)\cos^2\theta]$$

$$-\Delta_{W\mu\nu}(k_1)[\Delta^{\nu\mu}_{W}(k_2)+2\Delta^{\nu\mu}_{\gamma}(k_2)\sin^2\theta+2\Delta^{\nu\mu}_{Z}(k_2)\cos^2\theta]\}\ , \qquad (4.5a)$$

$$V_{3A}=\frac{ig_A{}^2}{2}\int\frac{d^4k_1d^4k_2}{(2\pi)^8}k_1^{\alpha}k_1^{\beta}\{[\Delta^{\gamma\delta}_{W}(k_1)\Delta_{W\alpha\beta}(k_2)d_{\gamma\delta}(k_1+k_2)$$

$$+\Delta^{\gamma\delta}_{W}(k_1)d_{\alpha\beta}(k_2)\Delta_{W\gamma\delta}(k_1+k_2)+d^{\gamma\delta}(k_1)\Delta_{W\alpha\beta}(k_2)\Delta_{W\gamma\delta}(k_1+k_2)]$$

$$+[\Delta^{\gamma\delta}_{W}(k_1)\Delta_{W\gamma\delta}(k_2)d_{\alpha\beta}(k_1+k_2)+\Delta^{\gamma\delta}_{W}(k_1)d_{\gamma\delta}(k_2)\Delta_{W\alpha\beta}(k_1+k_2)$$

$$+d^{\gamma\delta}(k_1)\Delta_{W\gamma\delta}(k_2)\Delta_{W\alpha\beta}(k_1+k_2)]$$

$$-2[\Delta^{\gamma\delta}_{W}(k_1)\Delta_{W\alpha\delta}(k_2)d_{\beta\gamma}(k_1+k_2)+\Delta^{\gamma\delta}_{W}(k_1)d_{\alpha\delta}(k_2)\Delta_{W\beta\gamma}(k_1+k_2)$$

$$+d^{\gamma\delta}(k_1)\Delta_{W\alpha\delta}(k_2)\Delta_{W\beta\gamma}(k_1+k_2)]\}\ ,$$

$$d^{\mu\nu}(k)\equiv\sin^2\theta\Delta^{\mu\nu}_{\gamma}(k)+\cos^2\theta\Delta^{\mu\nu}_{Z}(k)\ , \qquad (4.5b)$$

$$V_{\psi V}=\frac{ig_A{}^2}{8}\sum_i\int\frac{d^4k_1d^4k_2}{(2\pi)^8}\Delta^{\mu\nu}_{W}(k_1-k_2)\mathrm{Tr}[G_1^i(k_1)\gamma_\mu(1-\gamma_5)G_2^i(k_2)\gamma_\nu(1-\gamma_5)]$$

$$+\frac{ig_A{}^2g_B{}^2}{32(g_A{}^2+g_B{}^2)}\sum_a\int\frac{d^4k_1d^4k_2}{(2\pi)^8}\Delta^{\mu\nu}_{\gamma}(k_1-k_2)\mathrm{Tr}[(1+\tau^3_{aa}y_a)^2G_a(k_1)\gamma_\mu G_a(k_2)\gamma_\nu$$

$$-(1-\tau^3_{aa}x_a)^2G_a(k_1)\gamma_\mu G_a(-k_2)\gamma_\nu]$$

$$+\frac{i(g_A{}^2+g_B{}^2)}{32}\sum_a\int\frac{d^4k_1d^4k_2}{(2\pi)^8}\Delta^{\mu\nu}_{Z}(k_1-k_2)\mathrm{Tr}[(\cos^2\theta-\tau^3_{aa}y_a\sin^2\theta)^2G_a(k_1)\gamma_\mu G_a(k_2)\gamma_\nu$$

$$-(\cos^2\theta+\tau^3_{aa}x_a\sin^2\theta)^2G_a(k_1)\gamma_\mu G_a(-k_2)\gamma_\nu]\ , \qquad (4.5c)$$

where the sums on a are over all flavors and the sum on i is over all doublets. The corresponding Ω's are formed by subtracting V evaluated in the perturbative vacuum, i.e., with

$$G_a(k)\to S(k)\ ,$$
$$\Delta_\gamma(k),\Delta_Z(k),\Delta_W(k)\to D(k)\ . \qquad (4.6)$$

The next step is to insert the functional forms of Eq. (4.1) into (4.4) and (4.5), thereby obtaining an expression for Ω in terms of the variational parameters m_a,π_a,ξ_a. Beginning with Ω_G, the logarithmic term can be written (in n dimensions)

$$\mathrm{Tr}\{\ln[G_a{}^{-1}(k)S(k)]\}=-\mathrm{Tr}\sum_{j=1}^{\infty}\frac{1}{j}\left[\frac{1}{k^2}\mathscr{M}_a(k)\not{k}\right]^j$$
$$=\frac{n}{2}\ln\left[1-\frac{\mu_a{}^2(k)}{k^2}\right]\ , \qquad (4.7)$$

where we have used the fact that $\mathscr{M}_a(k)\gamma_\mu\mathscr{M}_a(k)=\mu_a{}^2(k)\gamma_\mu$. Therefore, in n dimensions,

$$\Omega_G^{(n)}=i\frac{n}{2}\int\frac{d^nk}{(2\pi)^n}\left[\ln\left[1-\frac{\mu_a{}^2(k)}{k^2}\right]+2\frac{\mu_a{}^2(k)}{k^2-\mu_a{}^2(k)}\right]\ . \qquad (4.8)$$

Appendix B contains the evaluation of various integrals encountered in the dimensional regularization of Ω. They are somewhat different from those normally encountered, due to the k^2 dependence of $\mu^2(k)$. Using (B4), (B8), and (B11), $\Omega_G^{(n)}$ is given by

$$\Omega_G^{(n)}=\frac{4i}{n}\sum_a (n-1+2r_a)\left[\frac{1+2r_a}{1-2r_a}\right] \times \int \frac{d^n k}{(2\pi)^n}\frac{\mu_a{}^4(k)}{[k^2-\mu_a{}^2(k)]^2} \; . \tag{4.9}$$

The expression derived for V or Ω includes contributions through two loops. The same is true for $\Omega_G^{(n)}$. We shall, however, retain only the most singular contribution to Ω as g^2 or r_a goes to zero. In doing so, we neglect terms of order $r_a \ln(m_i{}^2/m_j{}^2)$ compared to one, where m_i and m_j are two of the dynamically generated masses. Since

$$r_a=\frac{3}{64\pi^2}g_B{}^2 Y_{aL} Y_{aR} \sim 10^{-3} \; ,$$

such an approximation should be safe for quark, lepton, and weak-boson masses.

The integral in Eq. (4.9) is convergent in four dimensions for $g_B{}^2 \neq 0$. The most singular contribution to Ω diverges like $g_B{}^{-2}$ for $g_B{}^2 \to 0$ and arises from the large-k region of the integral. The large-k approximation for $\mu^2(k)$ is therefore justified and we obtain, (B10),

$$\Omega_G \simeq -\frac{1}{g_B{}^2}\sum_a \frac{m_a{}^4}{Y_{aL} Y_{aR}} \; . \tag{4.10}$$

The calculation of Ω_W proceeds in the same manner, except for the presence of the ghost contribution. That is given by

$$\begin{aligned}\Omega^{(n)}&=-i\int \frac{d^n k}{(2\pi)^n}\mathrm{Tr}\{\mathcal{G}^{-1}(k)[\Delta_W(k)-D(k)]\}\\ &=-2g_A{}^2\int \frac{d^n k_1 d^n k_2}{(2\pi)^{2n}}\pi(k_1)\left[-1+\frac{(k_1\cdot k_2)^2}{k_1{}^2 k_2{}^2}\right]\{k_1{}^2(k_1+k_2)^2[k_1{}^2-\pi(k_1)]\}^{-1} \; .\end{aligned} \tag{4.11}$$

The two terms in parentheses differ only in the angular integration and consequently lead to the same dependence of the integral on $(1/g_B{}^2)$. Considering just the first term,

$$\int \frac{d^n k_1 d^n k_2}{(2\pi)^{2n}}\pi(k_1)\{k_1{}^2(k_1+k_2)^2[k_1{}^2-\pi(k_1)]\}^{-1} = \left[\int \frac{d^n k_1}{(2\pi)^n}\pi(k_1)\{k_1{}^2[k_1{}^2-\pi(k_1)]\}^{-1}\right]\left[\int \frac{d^n K}{(2\pi)^n}K^{-2}\right] . \tag{4.12}$$

The k_1 integral is proportional to $g_B{}^{-2}$. The K integral is independent of g^2. Therefore, $\Omega \propto g_A{}^2 g_B{}^{-2} \propto (g^2)^0$, and the ghosts do not contribute to the singular part of Ω. The remainder of the calculation of Ω_W proceeds as that for Ω_G until

$$\begin{aligned}\Omega_W&=-\tfrac{3}{2}i\int \frac{d^4 k}{(2\pi)^4}\frac{\pi^2(k)}{[k^2-\pi(k)]^2}\\ &=\frac{1}{g_B{}^2}\sum_{a,b}\frac{\pi_a \pi_b}{Y_{aL} Y_{aR}+Y_{bL} Y_{bR}}\left[\frac{M_W{}^2}{m_a{}^2}\right]^{-2r_a}\left[\frac{M_W{}^2}{m_b{}^2}\right]^{-2r_a} \; .\end{aligned} \tag{4.13}$$

There are two small qualitative differences from the Ω_G result, one of them potentially interesting. Because $\pi(k)$ is a sum of contributions from different fermions, there will be cross terms in π^2 if more than one fermion acquires mass. The more interesting point is that the scale for the momentum dependence of $\pi(k)$ is set not by the W mass, but by the fermion masses. This leads to factor of $(M_W{}^2/m_a{}^2)^{-2r_a}$ in Ω_W. For $r_a \to 0$,

$$(M_W{}^2/m_a{}^2)^{-2r_a} \simeq 1-2r_a \ln(M_W{}^2/m_a{}^2) \; ,$$

so that in the present calculation such factors are ignored. However, it is tantalizing to observe that a calculation of Ω to the next order (in g^2) would include such terms, and therefore minimization of Ω *could* yield constraints on $r_a \ln(M_W{}^2/m_a{}^2)$. That would provide a mechanism for large disparities in masses. For example, if $r \ln M^2/m^2 \simeq O(1)$, then $M/m \simeq \exp(1/2r) \simeq 10^{26}$, an enormous difference in mass scales.

Returning to the calculation at hand, $\Omega_{\gamma Z}$ is almost identical to Ω_W. Inserting (4.1c) and (4.1d) into (4.4) yields

$$\Omega_{\gamma Z}=\frac{1}{2g_B{}^2}\sum_{a,b}\frac{\zeta_a\zeta_b}{Y_{aL}Y_{aR}+Y_{bL}Y_{bR}} \; . \tag{4.14}$$

Combining (4.10), (4.13), and (4.14), we obtain what we call the kinetic contribution to Ω,

$$\Omega_{\text{kin}}=-\frac{1}{g_B{}^2}\sum_a\frac{m_a{}^4}{Y_{aL}Y_{aR}}+\frac{1}{2g_B{}^2}\sum_{a,b}\frac{2\pi_a\pi_b+\zeta_a\zeta_b}{Y_{aL}Y_{aR}+Y_{bL}Y_{bR}} \; . \tag{4.15}$$

For the case of only one fermion doublet, this reduces to

$$\Omega_{\text{kin}}(1\text{ doublet})=\frac{1}{4g_B{}^2Y_{fL}Y_{fR}}(M_Z{}^4+2M_W{}^4-4m_f{}^4) \; , \tag{4.16}$$

where f denotes the fermion which acquires mass.

B. $\Omega_{\psi V}$ and linearization revisited

$\Omega_{\psi V}$ arises from the fermion loop graphs of Fig. 1(c),

$$\begin{aligned}
&\Omega_{\psi V}=\Omega_{\psi W}+\Omega_{\psi Z}+\Omega_{\psi\gamma} \; ,\\
&\Omega_{\psi W}^{(n)}=\frac{ig_A{}^2}{8}\sum_i\int\frac{d^nk_1d^nk_2}{(2\pi)^{2n}}\{\Delta_W^{\mu\nu}(k_1-k_2)\text{Tr}[G_1^i(k_1)\gamma_\mu(1-\gamma_5)G_2^i(k_2)\gamma_\nu(1-\gamma_5)]\\
&\qquad -D^{\mu\nu}(k_1-k_2)\text{Tr}[S(k_1)\gamma_\mu(1-\gamma_5)S(k_2)\gamma_\nu(1-\gamma_5)]\} \; ,\\
&\Omega_{\psi Z}^{(n)}=\frac{i(g_A{}^2+g_B{}^2)}{32}\sum_a\int\frac{d^nk_1d^nk_2}{(2\pi)^{2n}}\{\Delta_Z^{\mu\nu}(k_1-k_2)\text{Tr}[(\cos^2\theta-\tau_{aa}^3y_a\sin^2\theta)^2G_a(k_1)\gamma_\mu G_a(k_2)\gamma_\nu\\
&\qquad -(\cos^2\theta+\tau_{aa}^3x_a\sin^2\theta)^2G_a(k_1)\gamma_\mu G_a(-k_2)\gamma_\nu]\\
&\qquad -D^{\mu\nu}(k_1-k_2)\text{Tr}[(\cos^2\theta-\tau_{aa}^3y_a\sin^2\theta)^2S(k_1)\gamma_\mu S(k_2)\gamma_\nu\\
&\qquad -(\cos^2\theta+\tau_{aa}^3x_a\sin^2\theta)^2S(k_1)\gamma_\mu S(-k_2)\gamma_\nu]\} \; ,\\
&\Omega_{\psi\gamma}^{(n)}=\frac{ig_A{}^2g_B{}^2}{32(g_A{}^2+g_B{}^2)}\sum_a\int\frac{d^nk_1d^nk_2}{(2\pi)^{2n}}D^{\mu\nu}(k_1-k_2)\\
&\qquad\times\{\text{Tr}[(1+\tau_{aa}^3y_a)^2G_a(k_1)\gamma_\mu G_a(k_2)\gamma_\nu-(1-\tau_{aa}^3x_a)^2G_a(k_1)\gamma_\mu G_a(-k_2)\gamma_\nu]\\
&\qquad -\text{Tr}[(1+\tau_{aa}^3y_a)^2S(k_1)\gamma_\mu S(k_2)\gamma_\nu-(1-\tau_{aa}^3x_a)^2S(k_1)\gamma_\mu S(-k_2)\gamma_\nu]\} \; .
\end{aligned} \tag{4.17}$$

The W, Z, and γ calculations are all virtually the same, and so we do only $\Omega_{\psi W}$ in any detail. Using the explicit variational forms for the propagators,

$$\Omega_{\psi W}^{(n)}=-\frac{g_A{}^2}{4}\sum_i\int\frac{d^nk_1d^nk_2d^nk_3}{(2\pi)^{2n}}\delta^n(k_1-k_2-k_3)\left[g^{\mu\nu}-\frac{k_3^\mu k_3^\nu}{k_3{}^2}\right]\mathrm{Tr}[\not k_1\gamma_\mu\not k_2\gamma_\nu(1-\gamma_5)]$$
$$\times(\{[k_1{}^2-\mu_{i1}{}^2(k_1)][k_2{}^2-\mu_{i2}^2(k_2)][k_3{}^2-\pi(k_3)]\}^{-1}-[k_1{}^2k_2{}^2k_3{}^2]^{-1})$$
$$=-g_A{}^2\sum_i\int\frac{d^nk_1d^nk_2d^nk_3}{(2\pi)^{2n}}\delta^n(k_1-k_2-k_3)\left[k_1\cdot k_2+2\frac{k_1\cdot k_3k_2\cdot k_3}{k_3{}^2}\right]$$
$$\times\{[k_1{}^2-\mu_{i1}{}^2(1)][k_2{}^2-\mu_{i2}{}^2(2)][k_3{}^2-\pi(3)]\}^{-1}$$
$$\times[\hat\mu_{i1}{}^2(1)+\hat\mu_{i2}{}^2(2)+\hat\pi(3)-\hat\mu_{i1}{}^2(1)\hat\mu_{i2}{}^2(2)-\hat\mu_{i1}{}^2(1)\hat\pi(3)-\hat\mu_{i2}{}^2(2)\hat\pi(3)]\ , \tag{4.18a}$$

plus terms which are regular at $g^2=0$, and where

$$\mu(j)=\mu(k_j)\ ,\ \ \hat\mu^2(j)=\mu^2(j)/k_j{}^2\ ,\ \ \hat\pi(j)=\pi(j)/k_j{}^2\ . \tag{4.18b}$$

The integral of Eq. (B14) is then used to regulate the integrals arising from the first three terms in the final brackets. That results in

$$\Omega_{\psi W}^{(n)}=-g_A{}^2\sum_i\int\frac{d^nk_1d^nk_2d^nk_3}{(2\pi)^{2n}}\delta^n(k_1-k_2-k_3)\left[k_1\cdot k_2+2\frac{k_1\cdot k_3k_2\cdot k_3}{k_3{}^2}\right]$$
$$\times\{[k_1{}^2-\mu_{i1}{}^2(1)][k_2{}^2-\mu_{i2}{}^2(2)][k_3{}^2-\pi(3)]\}^{-1}$$
$$\times[\hat\mu_{i1}{}^2(1)\hat\mu_{i2}{}^2(2)+\hat\mu_{i1}{}^2(1)\hat\pi(3)+\hat\mu_{i2}{}^2(2)\hat\pi(3)]\ , \tag{4.19}$$

which is finite for $n=4$. Using (B19)—(B21) and the fact that only one fermion in the ith doublet acquires mass, $\Omega_{\psi W}$ is then

$$\Omega_{\psi W}=-\frac{2}{3}\frac{g_A{}^2}{g_B{}^4}\sum_{a,b}\frac{m_a{}^2}{Y_{aL}Y_{aR}}\frac{\pi_b}{(Y_{aL}Y_{aR}+Y_{bL}Y_{bR})}\ . \tag{4.20}$$

Similarly,

$$\Omega_{\psi Z}=-\frac{1}{3}\frac{g_A{}^2+g_B{}^2}{g_B{}^4}\sum_{a,b}\frac{m_a{}^2}{Y_{aL}Y_{aR}}\frac{\zeta_b}{Y_{aL}Y_{aR}+Y_{bL}Y_{bR}}+\frac{16}{3g_B{}^2}\sum_a Q_a(Q_a\sin^2\theta-I_a{}^3)\frac{m_a{}^4}{(Y_{aL}Y_{aR})^2}\ ,$$
$$\Omega_{\psi\gamma}=\frac{16}{3}\frac{1}{g_B{}^2}\cos^2\theta\sum_a Q_a{}^2\frac{m_a{}^4}{(Y_{aL}Y_{aR})^2}\ , \tag{4.21}$$

where we have used $Q=I_L{}^3+Y/2$ and have done some algebra. The $m_a{}^4$ contributions to $\Omega_{\psi\gamma}$ and $\Omega_{\psi Z}$ were absent from $\Omega_{\psi W}$ due to the pure left-handed coupling of the W [cf. Eq. (3.6)]. The fact that $\Omega_{\psi\gamma}$ is proportional to the charge squared is reassuring, but its sign warrants further comment. The fact that $\Omega_{\psi\gamma}>0$, which indicates that the photon-fermion interactions tend to restore the symmetry, seems to clash with earlier results on U(1),[14] and with one's intuition that an attractive force favors condensation and symmetry breaking. The apparent conflict can be resolved by comparing $\Omega_{\psi\gamma}$ to $\Omega_{\psi Z}$. The term in $\Omega_{\psi Z}$ which favors DSB goes like $M_Z{}^2\times m_f{}^2$. It is absent from $\Omega_{\psi\gamma}$ because the LA gives us a propagator for a massless photon. The fact that $\Omega_{\psi\gamma}>0$ then indicates not that interactions tend to prevent DSB in U(1), but that they tend to prevent it *if* we require that the photon remain massless. In the present case that choice is made for us by the LA (or MAC), in which the photon does not couple to the condensate.

We are now in a position to learn a lesson about the linearized approximation and similar approaches leading to the same homogeneous integral equations for the symmetry-breaking masses. Since non-Abelian effects are inherently nonlinear, they are neglected in the linearized approximation. Consequently, the linearized equations of the preceding section would arise from an effective potential containing only the kinetic and ψV parts of Ω. We shall call this the Abelian approximation to Ω; for the case of one fermion doublet it is

$$\Omega_{AA} \equiv \Omega_{\text{kin}} + \Omega_{\psi V} , \tag{4.22}$$

$$\Omega_{AA}(1\text{ doublet}) = \frac{1}{4g_B{}^2 Y_{fL} Y_{fR}}(M_Z{}^4 + 2M_W{}^4 - 4m_f{}^4) - \frac{1}{3g_B{}^2}\frac{m_f{}^2}{(Y_{fL}Y_{fR})^2}\left[\frac{g_A{}^2}{g_B{}^2}M_W{}^2 + \frac{1}{2}\left[1 + \frac{g_A{}^2}{g_B{}^2}\right]M_Z{}^2 - 16Q_f(Q_f - I_f{}^3)m_f{}^2\right] .$$

Minimizing Ω_{AA},

$$\frac{\partial}{\partial M_W{}^2}\Omega_{AA} = \frac{\partial}{\partial M_Z{}^2}\Omega_{AA} = 0 ,$$

yields

$$M_W{}^2 = \frac{1}{3}\frac{g_A{}^2}{g_B{}^2}\frac{1}{Y_{fL}Y_{fR}}m_f{}^2 ,$$
$$M_Z{}^2 = \frac{1}{3}\frac{g_A{}^2 + g_B{}^2}{g_B{}^2}\frac{1}{Y_{fL}Y_{fR}}m_f{}^2 = M_W{}^2/\cos^2\theta , \tag{4.23}$$

as in (3.23). Thus, the mass ratios of the linearized approximation do indeed correspond to an extremum of Ω_{AA}. However, they do not necessarily correspond to a minimum. Using the expressions for $M_W{}^2$ and $M_Z{}^2$ in terms of $m_f{}^2$, Ω_{AA} becomes

$$g_B{}^2\Omega_{aa} = -\frac{m_f{}^4}{Y_{fL}Y_{fR}}\left[1 - \frac{16}{3}\frac{Q_f(Q_f - I_f{}^3)}{Y_{fL}Y_{fR}} + \frac{1}{36(Y_{fL}Y_{fR})^2}\frac{g_A{}^4 + (g_A{}^2 + g_B{}^2)^2}{g_B{}^4}\right] . \tag{4.24}$$

In the case of leptons and quarks,

$$Q_f(Q_f - I_f{}^3)/(Y_{fL}Y_{fR}) = \tfrac{1}{4}$$

for the fermion acquiring mass. If we use $\sin^2\theta = 0.22$, then

$$g_B{}^2\Omega_{AA} = \begin{cases} +0.050m_f{}^4 \text{ (leptons)} \\ -9.8m_f{}^4 \text{ (quarks)} \end{cases} . \tag{4.25}$$

Therefore, whether a nonzero fermion mass raises or lowers the effective potential depends on the fermion quantum numbers. The symmetry will break (in this approximation) for a quark doublet; it will not break for a lepton doublet. This demonstrates the utility, even necessity, of the effective potential in such a calculation. The LA (or MAC) is virtually the same for the lepton and quark cases. The effective potential, however, reveals that for the LA solution the symmetric (massless) vacuum is a minimum of the vacuum energy density in the lepton case and a maximum in the quark case.

The form $\Omega = \omega m_f{}^4$ requires some explanation. The original Lagrangian has only dimensionless parameters g_A, g_B; and we are calculating Ω, which has dimensions $(E)^4$. In a higher-order calculation renormalization would be required, and the renormalization point would set the scale, replacing one of the initial dimensionless parameters through dimensional transmutation. The present calculation of Ω goes through the two-loop level and leading order in g^2, which is the lowest order at which DSB can occur and which does not require a subtraction procedure. Consequently, an energy scale is not provided by renormalization, and one of the dynamically generated masses must serve this purpose and cannot be determined in the present calculation.

It is worth noting that the Abelian approximation is not just of academic interest. In the context of a renormalizable nonlocal nonpolynomial effective Lagrangian, Cornwall[12] has argued that the gauge-boson loop graphs which contribute to $\Omega_{NA} = \Omega_{3A} + \Omega_{4A}$ must vanish to leading order. In that case the full effective potential reduces to the Abelian approximation, Eq. (4.22).

C. Non-Abelian terms and the full Ω

Finally, we confront the non-Abelian terms arising from the graphs of Figs. 1(a) and (1b). The equation(s) for the effective potential, (4.4) and (4.5), used the perturbative vertices only (the Hartree-Fock approximation). This is justified in $V_{\psi V}$, where the Goldstone pole part of the vertex does not contribute in the Landau gauge. The same is not true for $V_{3A}+V_{4A}$. The Goldstone pole parts of the vertices can contribute to these non-Abelian graphs even in the Landau gauge. To obtain the nonperturbative vertices would require derivation of the appropriate Ward-Takahashi identities and construction and use of triple and quartic vector vertices satisfying these identities, for all q, not just $q\to 0$. We have not done this and, consequently, we cannot verify the argument of Ref. 12, that the non-Abelian terms $\Omega_{NA}=\Omega_{3A}+\Omega_{4A}$ do not contribute to Ω to leading order. Since this point has yet to be verified by a full gauge-invariant calculation, it may be useful to determine the effect of such terms if they do *not* vanish. We shall use the Hartree-Fock approximation to calculate Ω_{NA}. The perturbative vertices should be sufficient for extracting qualitative features arising from the non-Abelian terms (if they do contribute). The quartic coupling graph is given by

$$\begin{aligned}\Omega_{4A}^{(n)}=\frac{g_A{}^2}{2}\int\frac{d^nk_1d^nk_2}{(2\pi)^{2n}}\{&\Delta^{\mu}_{W\mu}(k_1)[\Delta^{\nu}_{W\nu}(k_2)+2\sin^2\theta\Delta^{\nu}_{\gamma\nu}(k_2)+2\cos^2\theta\Delta^{\nu}_{Z\nu}(k_2)]\\&-3D^{\mu}_{\mu}(k_1)D^{\nu}_{\nu}(k_2)-\Delta_{W\mu\nu}(k_1)[\Delta^{\mu\nu}_{W}(k_2)+2\sin^2\theta\Delta^{\mu\nu}_{\gamma}(k_2)+2\cos^2\theta\Delta^{\mu\nu}_{Z}(k_2)]\\&+3D_{\mu\nu}(k_1)D^{\mu\nu}(k_2)\}\ ,\end{aligned}\tag{4.26}$$

which upon substitution of (4.1) becomes

$$\begin{aligned}\Omega_{4A}^{(n)}=&-\frac{g_A{}^2}{2}\int\frac{d^nk_1d^nk_2}{(2\pi)^{2n}}\left[-7+\frac{(k_1\cdot k_2)^2}{k_1{}^2k_2{}^2}\right]\frac{\hat\pi(1)}{[k_1{}^2-\pi(1)]}\left[\frac{\hat\pi(2)}{k_2{}^2-\pi(2)}+\frac{2\cos^2\theta\hat\zeta(2)}{k_2{}^2-\zeta(2)}\right]\\&+g_A{}^2\int\frac{d^nk_1d^nk_2}{(2\pi)^{2n}}\left[-7+\frac{(k_1\cdot k_2)^2}{k_1{}^2k_2{}^2}\right][k_1{}^2-\pi(1)]^{-1}\\&\times\left[\frac{\hat\pi(1)}{k_2{}^2-\pi(2)}+\sin^2\theta\frac{\hat\pi(1)}{k_2{}^2}+\frac{\cos^2\theta}{k_2{}^2-\zeta(2)}[\hat\pi(1)+\hat\zeta(2)]\right]\ .\end{aligned}\tag{4.27}$$

The first integral is convergent for $n=4$. In the second integral the $\sin^2\theta$ term does not contribute to $O(1/g^2)$. The first and last terms do contribute and require regularization, (B14). Upon performing the integrations, the second integral is found to be -2 times the first. The result for Ω_{4A} is

$$\Omega_{4A}=\frac{3}{2}\frac{g_A{}^2}{g_B{}^4}\left[\sum_a\frac{\pi_a}{Y_{aL}Y_{aR}}\right]\left[\sum_b\frac{\pi_b+2\cos^2\theta\zeta_b}{Y_{bL}Y_{bR}}\right]\ .\tag{4.28}$$

For the cubic coupling graph

$$\begin{aligned}&\Omega_{3A}^{(n)}=\omega_1^{(n)}+\omega_2^{(n)}+\omega_3^{(n)}\ ,\\&\omega_1^{(n)}=\frac{ig_A{}^2}{2}\int\frac{d^nk_1d^nk_2d^nk_3}{(2\pi)^{2n}}\delta^n(k_3-k_1-k_2)k_1^{\alpha}k_1^{\beta}I_{\alpha\beta}(1,2,3)\ ,\\&\omega_2^{(n)}=\frac{ig_A{}^2}{2}\int\frac{d^nk_1d^nk_2d^nk_3}{(2\pi)^{2n}}\delta^n(k_3-k_1-k_2)k_1^{\alpha}k_1^{\beta}I_{\alpha\beta}(1,3,2)\ ,\\&I_{\alpha\beta}(1,2,3)=[\Delta^{\gamma\delta}_W(1)\Delta_{W\alpha\beta}(2)d_{\gamma\delta}(3)+\Delta^{\gamma\delta}_W(1)d_{\alpha\beta}(2)\Delta_{W\gamma\delta}(3)+d^{\gamma\delta}(1)\Delta_{W\alpha\beta}(2)\Delta_{W\gamma\delta}(3)-3D^{\gamma\delta}(1)D_{\alpha\beta}(2)D_{\gamma\delta}(3)]\ ,\\&d^{\mu\nu}(i)=\sin^2\theta\Delta^{\mu\nu}_{\gamma}(i)+\cos^2\theta\Delta^{\mu\nu}_{Z}(i)\ ,\\&\omega_3^{(n)}=-ig_A{}^2\int\frac{d^nk_1d^nk_2d^nk_3}{(2\pi)^{2n}}\delta^n(k_3-k_2-k_1)\\&\qquad\times k_1^{\alpha}k_1^{\beta}[\Delta^{\gamma\delta}_W(1)\Delta_{W\alpha\delta}(2)d_{\beta\gamma}(3)\\&\qquad+\Delta^{\gamma\delta}_W(1)d_{\alpha\delta}(2)\Delta_{W\beta\gamma}(3)+d^{\gamma\delta}(1)\Delta_{W\alpha\delta}(2)\Delta_{W\beta\gamma}(3)-3D^{\gamma\delta}(1)D_{\alpha\delta}(2)D_{\beta\gamma}(3)]\ .\end{aligned}\tag{4.29}$$

From symmetry, $\omega_1^{(n)}$ and $\omega_2^{(n)}$ are equal. Consider only $\omega_2^{(n)}$. Substituting explicit forms for the propagators, it becomes

$$\omega_2^{(n)}=-\frac{g_A{}^2}{2}(2\pi)^{-2n}\int d^nk_1d^nk_2d^nk_3\delta^n(k_3-k_1-k_2)\left[k_1{}^2-\frac{(k_1\cdot k_3)^2}{k_3{}^2}\right]\left[2+\frac{(k_1\cdot k_2)^2}{k_1{}^2k_2{}^2}\right]$$
$$\times\{\sin^2\theta[S(W,W,\gamma)+S(W,\gamma,W)+S(\gamma,W,W)]$$
$$+\cos^2\theta[C(W,W,Z)+C(W,Z,W)+C(Z,W,W)]\}\ ,$$

$$S(W,W,\gamma)=\{[k_1{}^2-\pi(1)][k_2{}^2-\pi(2)]k_3{}^2\}^{-1}[\hat\pi(1)+\hat\pi(2)-\hat\pi(1)\hat\pi(2)]\ , \tag{4.30}$$
$$C(W,W,Z)=\{[k_1{}^2-\pi(1)][k_2{}^2-\pi(2)][k_3{}^2-\zeta(3)]\}^{-1}$$
$$\times[\hat\pi(1)+\hat\pi(2)+\hat\zeta(3)-\hat\pi(1)\hat\pi(2)-\hat\pi(1)\hat\zeta(3)-\hat\pi(2)\hat\zeta(3)]\ .$$

All the C's and S's are proportional to an expression of the form

$$\left[\sum_i\hat M^2(i)-\sum_{i\neq j}\hat M^2(i)\hat M^2(j)\right]\ .$$

The integral of the $\sum_{i\neq j}$ parts of the integrand is convergent for $n=4$, whereas the $\sum_i$ part must be regularized. Using (B14), we find that

$$\left[\sum_i\hat M^2(i)-\sum_{i\neq j}\hat M^2(i)\hat M^2(j)\right]\to+\sum_{i\neq j}\hat M^2(i)\hat M^2(j)\ . \tag{4.31}$$

The $i\neq j$ terms which arise in the regularization of the $\sum_i\hat M^2(i)$ integrals require further regularization, but can be shown to be nonleading for $g^2\to0$, allowing us to write (4.31).

For $n=4$, therefore,

$$\omega_2=-\frac{g_A{}^2}{2}(2\pi)^{-8}\int d^4k_1d^4k_2\left[k_1{}^2-\frac{(k_1\cdot k_3)^2}{k_3{}^2}\right]\left[2+\frac{(k_1\cdot k_2)^2}{k_1{}^2k_2{}^2}\right]$$
$$\times\{\sin^2\theta[S'(W,W,\gamma)+S'(W,\gamma,W)+S'(\gamma,W,W)]$$
$$+\cos^2\theta[C'(W,W,Z)+C'(W,Z,W)+C'(Z,W,W)]\}\ ,$$

$$S'(W,W,\gamma)=\{[k_1{}^2-\pi(1)][k_2{}^2-\pi(2)]k_3{}^2\}^{-1}\hat\pi(1)\hat\pi(2)\ , \tag{4.32}$$
$$C'(W,W,Z)=\{[k_1{}^2-\pi(1)][k_2{}^2-\pi(2)][k_3{}^2-\zeta(3)]\}^{-1}[\hat\pi(1)\hat\pi(2)+\hat\pi(1)\hat\zeta(3)+\hat\pi(2)\hat\zeta(3)]\ .$$

That can be evaluated using Appendix B, (B28)—(B39),

$$\omega_2=9g_A{}^2\{\mathscr{S}(\pi,\pi)+[\mathscr{S}(\pi,\zeta)+\mathscr{S}(\zeta,\pi)]\cos^2\theta\}$$
$$=\frac{1}{2}\frac{g_A{}^2}{g_B{}^4}\left[\sum_a\frac{\pi_a}{Y_{aL}Y_{aR}}\right]\left[\sum_b\frac{\pi_b+2\cos^2\theta\zeta_b}{Y_{bL}Y_{bR}}\right]\ . \tag{4.33}$$

For ω_3, regularization yields

$$\omega_3=g_A{}^2[\sin^2\theta(\mathscr{I}_1+\mathscr{I}_2+\mathscr{I}_3)+\cos^2\theta(\mathscr{I}_4+\mathscr{I}_5+\mathscr{I}_6)]\ ,$$
$$\mathscr{I}_i=(2\pi)^{-8}\int d^4k_1d^4k_2d^4k_3\delta^4(k_1+k_2-k_3)\frac{k_1\cdot k_2k_1\cdot k_3}{k_3{}^2}\left[1-\frac{(k_1\cdot k_2)^2}{k_1{}^2k_2{}^2}\right]A_i\ ,$$
$$A_1=S'(W,W,\gamma)\ ,\quad A_2=S'(W,\gamma,W)\ ,\quad A_3=S'(\gamma,W,W)\ , \tag{4.34}$$
$$A_4=C'(W,W,Z)\ ,\quad A_5=C'(W,Z,W)\ ,\quad A_6=C'(Z,W,W)\ .$$

The $\mathscr{I}_i$ can all be written in terms of integrals in Appendix B, (B40)—(B45), and all vanish to leading order. Consequently, ω_3 vanishes, and

$$\Omega_{3A}=2\omega_2=\frac{g_A{}^2}{g_B{}^4}\sum_a \frac{\pi_a}{Y_{aL}Y_{aR}}\sum_b \frac{\pi_b+2\cos^2\theta\zeta_b}{Y_{bL}Y_{bR}} \ . \tag{4.35}$$

Combining this result with that for Ω_{4A}, (4.28), yields

$$\Omega_{NA}=\frac{5}{2}\frac{g_A{}^2}{g_B{}^4}\sum_a \frac{\pi_a}{Y_{aL}Y_{aR}}\sum_b \frac{\pi_b+2\cos^2\theta\zeta_b}{Y_{bL}Y_{bR}} \ . \tag{4.36}$$

The full effective potential would then be

$$\begin{aligned} g_B{}^2\Omega &= g_B{}^2(\Omega_{\text{kin}}+\Omega_{\psi V}+\Omega_{NA}) \\ &= -\sum_a \frac{m_a{}^4}{Y_{aL}Y_{aR}}+\tfrac{1}{2}\sum_{a,b}\frac{2\pi_a\pi_b+\zeta_a\zeta_b}{Y_{aL}Y_{aR}+Y_{bL}Y_{bR}} \\ &\quad -\tfrac{1}{3}\cot^2\theta\sum_{a,b}\frac{m_a{}^2}{Y_{aL}Y_{aR}}\frac{1}{(Y_{aL}Y_{aR}+Y_{bL}Y_{bR})}(2\pi_b+\sec^2\theta\zeta_b) \\ &\quad +\tfrac{16}{3}\sum_a Q_a(Q_a-I_a{}^3)\frac{m_a{}^4}{(Y_{aL}Y_{aR})^2}+\tfrac{5}{2}\cot^2\theta\sum_a\frac{\pi_a}{Y_{aL}Y_{aR}}\sum_b\frac{\pi_b+2\cos^2\theta\zeta_b}{Y_{bL}Y_{bR}} \ . \end{aligned} \tag{4.37}$$

With $\Omega(m_a{}^2,\pi_a,\zeta_a)$ in hand, we can consider two types of questions. For the one-doublet case, we can see how much of the LA survives in the presence of Ω_{NA}. For other numbers of fermions, we can consider whether the minimum of $\Omega(m_a{}^2,\pi_a,\zeta_a)$ corresponds to broken (nonzero masses) or unbroken symmetry (zero masses). If there are two or more fermion doublets, there will be residual Goldstone bosons if the symmetry breaks, and these are not included in our calculation of Ω. Consequently, $\Omega>0$ for nonzero masses does not necessarily mean the symmetry is unbroken. On the other hand, $\Omega<0$ for nonzero masses is sufficient for DSB, since $\Omega(\text{true})<\Omega(\text{test})$. (There has been recent work on inclusion of bound-state effects in the effective potential,[16,22] but the methods are still in the development stage.)

We first address the one-doublet plus one-singlet case. Ω reduces to

$$\begin{aligned} g_B{}^2\Omega(1\text{ doublet}) &= \frac{m_f{}^4}{Y_{fL}Y_{fR}}\left[\frac{16}{3}\frac{Q_f(Q_f-I_f{}^3)}{Y_{fL}Y_{fR}}-1\right]+\frac{1}{4Y_{fL}Y_{fR}}(2M_W{}^4+M_Z{}^4) \\ &\quad -\tfrac{1}{6}\cot^2\theta\frac{m_f{}^2}{(Y_{fL}Y_{fR})^2}(2M_W{}^2+\sec^2\theta M_Z{}^2)+\tfrac{5}{2}\cot^2\theta\frac{M_W{}^2(M_W{}^2+2\cos^2\theta M_Z{}^2)}{(Y_{fL}Y_{fR})^2} \ . \end{aligned} \tag{4.38}$$

Minimizing Ω with respect to $M_Z{}^2$ and $M_W{}^2$ yields

$$\begin{aligned} (5+Y_{fL}Y_{fR}\tan^2\theta)M_W{}^2+5\cos^2\theta M_Z{}^2 &= \tfrac{1}{3}m_f{}^2 \ , \\ 10\cos^2\theta M_W{}^2+Y_{fL}Y_{fR}\tan^2\theta M_Z{}^2 &= \tfrac{1}{3}\sec^2\theta m_f{}^2 \ . \end{aligned} \tag{4.39}$$

Comparison with (3.23) or (4.23) shows that the non-Abelian terms have destroyed all the attractive features of the LA results. Noting that each term in (4.39) is positive, we see that it requires vector masses which are much smaller than the fermion mass and, furthermore, $M_W{}^2/M_Z{}^2\neq\cos^2\theta$. In addition, when the solutions to (4.39) are substituted into Ω, one finds $\Omega=\omega m_f{}^4$, $\omega>0$ for $\sin^2\theta=0.22$. Since for one doublet plus one singlet there are no physical Goldstone bosons, we can conclude that the symmetry does not break, for either quark or lepton quantum numbers. These results depend on use of the Hartree-Fock approximation for the three-vector and four-vector vertices. Inclusion of nonperturbative effects in these vertices could change details like coefficients in (4.39) or even whether Ω is positive or negative for a given value of $\sin^2\theta$. However, in order to regain the $I=\frac{1}{2}$ breaking and other LA results, $\Omega_{3A}+\Omega_{4A}$ would have to vanish. In that case, one is back to the Abelian approximation of the preceding subsection.

Turning to the question of whether we can find cases for which DSB does occur in the presence of Ω_{NA}, the answer is yes. For example, for a color triplet of quark doublets plus accompanying right-handed singlets, $\Omega(\text{test})<\Omega(\text{sym})=0$ for

$\sin^2\theta < 0.17$. Such a theory has nothing to do with reality (it would have massless Goldstone bosons), and furthermore our test functions may not be the true propagators of the theory. Nevertheless, since $\Omega(\text{true}) \leq \Omega(\text{test})$ and $\Omega(\text{test}) < \Omega(\text{sym})$, we have found a case in which $SU(2)\times U(1)$ does break dynamically. We have also found other cases for which DSB occurs, but none is any closer to reality and so we shall not pursue the matter.

V. DISCUSSION

A. Results

As stated in the Introduction, there are fundamental problems which prevent us from constructing a realistic model. Nevertheless, from the models we have considered we are able to distill features of DSB which we believe will be relevant to realistic applications and which are not generally appreciated.

The LA is consistent with the standard model in most respects. The A_3 and B fields mix in the usual way to form the γ and Z^0, and the ratio of W to Z masses is "correct." The vector masses are related to (and larger than) the fermion masses. One peculiar feature is that fermions with $Y_L Y_R < 0$ do not acquire mass, where $Y_R(Y_L)$ is the hypercharge of the right- (left-) handed fermion. Thus not only neutrinos but also $I^3 = -\frac{1}{2}$ quarks remain massless. All these features can be understood in the framework of the most-attractive-channel hypothesis.

When the effective potential is calculated as a function of the physical masses and then minimized, we find that the LA results can be drastically altered by the inclusion of nonlinear effects, in particular by the contribution to the effective potential of the vacuum graphs involving only the SU(2) gauge bosons. Provided these graphs do not vanish when the full nonperturbative vertices are used, we can extract qualitative features of these non-Abelian effects from the Hartree-Fock approximation. As in the calculation of the β function for the running coupling constant, the vector loops enter with the opposite sign of the fermion loops and consequently tend to preserve or restore the symmetry. For a given choice of fermions the vector loops greatly reduce (or eliminate entirely) the range of $\sin^2\theta$ for which the symmetry breaks and lead to entirely different mass ratios than those obtained in the LA. For cases in which the symmetry breaks, we no longer have $I=\frac{1}{2}$ breaking ($M_W/M_Z \neq \cos\theta$), and the vector masses are much smaller than the fermion masses.

It is, however, likely that the vector loops do vanish when the full nonperturbative vertices are used. Besides the arguments already mentioned,[12,23] it would be consistent with the fact that in a Bethe-Salpeter approach to DSB for a simple non-Abelian gauge group, the vector self-couplings can be shown not to contribute to mass generation.[24] (They could still contribute to the effective potential, and so the Bethe-Salpeter results are suggestive at most.) If the vector loops do vanish to leading order, the effective potential reduces to the Abelian approximation. The mass ratios of the LA would then be reproduced (at least in simple cases) by finding the extremum of the effective potential, but whether the symmetry broke would depend on the fermions—e.g., for a given $\sin\theta$ and one fermion left-handed doublet, DSB could occur if the fermions were quarks, and not occur if they were leptons.

Therefore, regardless of the fate of the vector loops, whether DSB occurs at all depends crucially on how many fermions the model contains and what are their quantum numbers. The obvious moral is that the LA (or similar approximations) gives no indication *whether* the symmetry breaks—beyond noting that it does not break if there is no attractive force; one needs to look at the effective potential to tell whether symmetry breaking does occur.

B. Critique

Having asserted the inadequacy of the LA, how reliable do we consider the present results? The effective potential was calculated to the two-loop level and then in the variational calculation only the leading terms in $g^2(\sim 1/g^2)$ were retained. The approximation of keeping only the most singular term as $g^2 \to 0$ is justified. Neglected terms are suppressed by a factor of $(3g_B{}^2 Y_L Y_R/32\pi^2) \sim 10^{-3}$ times possible logarithms of ratios of masses squared. It is true that we have not shown that the next order in g^2 is free of divergences. To that extent an act of faith is required, that nothing too pathological occurs.

Use of the loop expansion is also quite safe. There is a potential problem when deriving nonperturbative results as we have done, that the usual perturbative ordering of the loop expansion could be destroyed. An N-loop graph will have an explicit $g^{2(N-1)}$ (for cubic interactions), but each loop integral could lead to an $r_a^{-1} \propto 1/g^2$, cf. $\Omega_{\psi V}$ and Ω_{NA}. However, Cornwall and Shellard[23] have shown (for Abelian theories and argued it for non-Abelian theories as well) that in fact the N-loop graphs which would lead to r^{-N} are not two-particle-irreducible for $N > 2$, and therefore graphs with three or more loops cannot contribute to the singular part of Ω. The proof was done for nonvanishing bare mass, and therefore it is not directly applicable. However, since a nondynamical fermion mass will not change the ultraviolet behavior of the loop integrals, we expect the result to hold in the presence of massless fermions also. They may make a hopeless morass of the infrared behavior, but the singular contribution to Ω originates in the region where *all* loop momenta are very large, and so it should escape unscathed. If that is the case, we have calculated all $1/g^2$ contributions to Ω.

The momentum dependence of the coupling constants has been ignored. Because only g_B enters into the generation of the fermion mass in the LA, the momentum dependence of $g_A{}^2$ should not be important.[23] Use of a constant Abelian coupling constant is standard (and necessary). It has been shown that renormalization-group effects prevent DSB in a simple Abelian theory.[25] A justification for such an approximation can be constructed on the basis of the very slow variation of the coupling constants in $SU(2)\times U(1)$ up to around 10^{16} GeV, at which point other interactions could come to the rescue in the tradition of *deus ex machina.* This is hardly satisfying, but appears to be the best one can do.

The major loophole in the calculation is the question of gauge-invariant vertices for the triple and quartic vector couplings. General arguments indicate that the relevant graphs will not contribute to the effective potential when the full vertices including nonperturbative corrections are used. If confirmed, that would reduce the full effective potential to the Abelian approximation. The LA mass relations would be unchanged, and Ω_{AA} would determine whether the symmetry breaks, DSB would occur for a number of simple (interesting) cases. If Ω_{3A} and Ω_{4A} do contribute to Ω, then one loses $I=\frac{1}{2}$ breaking and the LA mass ratios, and the range of parameters for which DSB occurs is greatly reduced.

C. Outlook

We have encountered significant problems which have prevented construction of a realistic model with DSB within $SU(2)\times U(1)$. The fundamental problem, which indicates that such a model may not even be feasible, is the presence of Goldstone bosons from chiral symmetry breaking if we require a realistic fermion spectrum. There is also the technical problem of deriving and using multivector vertices which satisfy the Ward-Takahashi identities in the presence of DSB. Lurking in the background are questions of renormalization-group effects, behavior of higher orders, ...

Despite these difficulties, prospects for practical applications of this DSB formalism are not entirely bleak. In theories other than $SU(2)\times U(1)$, it may be that the interesting fermion representations do not require "extra" Goldstone bosons (and are amenable to the weak-coupling approach we have used). In that case, it would be worthwhile to try to overcome the technical problems and attempt a realistic calculation. Even for $SU(2)\times U(1)$ the issue is not entirely closed. For one thing, it is far from certain that quarks and leptons are fundamental. If they are not, then it is the preon representations and quantum numbers that are relevant and not those of quarks and leptons, as considered here. Another possibility is that the inclusion of other interactions could ameliorate some of the difficulties present when $SU(2)\times U(1)$ is treated in isolation.

To summarize, then, Goldstone bosons pose a basic problem. Even in their absence, there are grave potential difficulties. It is very important to confirm that the vector loop graphs can be neglected. If present, they ruin the "good" mass ratios and tend to prevent DSB. The choice of fermion representations is crucial in determining whether DSB occurs. On balance, a realistic model for DSB in $SU(2)\times U(1)$ looks improbable, but perhaps not yet entirely hopeless.

ACKNOWLEDGMENTS

During the course of this work we have benefited from conversations with J. Cornwall, T. DeGrand, and R. Haymaker. We are especially indebted to Henry Tye for helpful conversations and comments. This work was supported by the U. S. Department of Energy under Grant No. DE-AC02-81ER40025.

APPENDIX A: V AS THE EFFECTIVE POTENTIAL

That the usual effective potential $V(\phi)$ is the vacuum energy density was proved by Symanzik.[26] CJT adapted the more accessible version of Coleman[15] to show that $V[\phi,G]$ is the energy density for static sources (though in their DSB calculation they did use nonstatic sources). We wish to extend that treatment to nonstatic sources which are invariant under time translations.

The outline of the problem is as follows. Given a Hamiltonian H, we wish to find the state $|\psi\rangle$ which minimizes the expectation value $\langle\psi|H|\psi\rangle$, subject to the constraints $\langle\psi|\psi\rangle=1$, and $G(x,x+z)\equiv\langle\psi|\Phi(x)\Phi(x+z)|\psi\rangle$ is independent of time x_0. In order not to clutter things up with irrelevant details, we shall ignore noncomposite operators, assume spinless particles, and impose the first constraint by hand. Using Lagrange multipliers to impose the second constraint, we have

$$\min_{\text{constr}}\{\langle\psi\,|\,H\,|\,\psi\rangle\}=\min\left[\left\langle\psi\left|H+\tfrac{1}{2}\int d^3x\,d^4z[\Phi(\vec{x}+\vec{z},x_0+z_0)\mathscr{K}(z)\Phi(x)\right.\right.\right.$$
$$\left.\left.\left.-\Phi(\vec{x}+\vec{z},t_0+z_0)\mathscr{K}(z)\Phi(\vec{x},t_0)]\right|\psi\right\rangle\right]. \quad \text{(A1)}$$

On the other hand, for space-time-translationally invariant sources, we have

$$\Gamma[G]=-V[G]\int d^4x=Z[\mathscr{K}]-\tfrac{1}{2}\int d^4x\,d^4z\,\mathscr{K}(z)G(x+z,x)\,. \quad \text{(A2)}$$

Therefore, if we can show that

$$Z[\mathscr{K}]=-\int dx_0\left\langle\psi\left|H-\tfrac{1}{2}\int d^3x\,d^4z\,\Phi(\vec{x}+\vec{z},t_0+x_0)\mathscr{K}(z)\Phi(\vec{x},t_0)\right|\psi\right\rangle$$
$$\equiv-\int dx_0\langle\psi\,|\,H'\,|\,\psi\rangle\,, \quad \text{(A3)}$$

then we will have shown that minimizing $V[G]$ is equivalent to minimizing $\langle\psi\,|\,H\,|\,\psi\rangle$ subject to the desired constraint. The standard way to show this is to note that the vacuum-to-vacuum amplitude in the presence of sources is given by

$$_{\mathscr{K}}\langle 0,\tau/2\,|\,0,-\tau/2\rangle_{\mathscr{K}}=e^{iZ[\mathscr{K}]}\,, \quad \text{(A4)}$$

where τ is the time over which the sources act ($\tau\to\infty$). In addition, of course,

$$_{\mathscr{K}}\langle 0,\tau/2\,|\,0,-\tau/2\rangle_{\mathscr{K}}={}_{\mathscr{K}}\langle 0,\tau/2\,|\,e^{-iH'\tau}\,|\,0,\tau/2\rangle_{\mathscr{K}}$$
$$=e^{-i\tau E(\mathscr{K})}\,, \quad \text{(A5)}$$

for sources which are local in time. Banks and Raby[14] made the point that for nonlocal sources $\mathscr{K}$, H' will have explicit time dependence in the Schrödinger representation, and there will not be stationary states of H', preventing us from identifying $Z[\mathscr{K}]$ with $\int dx_0\langle -H'\rangle$.

To appreciate this better, and to seek a way to circumvent it, let us consider a simple quantum-mechanical system consisting of a single harmonic oscillator at one point in space, in the presence of a source which is nonlocal in time. The role of the field operator Φ is played by the oscillator displacement x, and so we consider the Hamiltonian $H'=H_0-\mathscr{V}$, where H_0 is the simple harmonic-oscillator Hamiltonian, and $\mathscr{V}$ is defined by

$$\langle\Phi(t_0)\,|\,\mathscr{V}\,|\,\Psi(t_0)\rangle$$
$$\equiv 2\lambda\int dx\int dt'\text{Re}[K(t')\phi^*(t_0,x)$$
$$\times x^2\psi(t_0+t',x)]\,. \quad \text{(A6)}$$

Hermiticity is most conveniently ensured by defining $\mathscr{V}$ by its matrix elements; certain things are just more cumbersome in quantum mechanics than in relativistic field theory. The parameter λ is displayed explicitly because our interest lies in the neighborhood of the extremum ($\mathscr{V}\sim K\sim\delta V/\delta G=0$) and we therefore need only treat low orders in λ. As promised, $\mathscr{V}$ and consequently H' have explicit time dependence. We then search for states $|\,\Psi\rangle$ such that

$$\left\langle\Phi\left|i\frac{d}{dt}\right|\Psi\right\rangle=\langle\Phi\,|\,H'\,|\,\Psi\rangle \quad \text{(A7)}$$

for all $|\,\Phi\rangle$. Letting

$$\phi(x,t)=f(t)h_n(x)\,,$$
$$\psi(x,t)=\sum_j\psi_j(t)h_j(x)\,, \quad \text{(A8)}$$
$$H_0h_n(x)=E_nh_n(x)\,,$$

we obtain

$$if^*(t)\frac{d}{dt}\psi_n(t)=f^*(t)E_n\psi_n(t)$$
$$-2\lambda\,\text{Re}\left[f^*(t)\int dt'K(t')\{(n+\tfrac{1}{2})\psi_n(t'+t)+\tfrac{1}{2}[(n+1)(n+2)]^{1/2}\psi_{n+2}(t'+t)\right.$$
$$\left.+\tfrac{1}{2}[n(n-1)\psi_{n-2}(t+t')]^{1/2}\}\right]. \quad \text{(A9)}$$

We then expand $\psi_n(t)$ about $\lambda=0$,

$$\psi_n(t)=\sum_i\psi_n^{(i)}(t)\lambda^i\,, \quad \text{(A10)}$$

and consider the case where the oscillator was in the unperturbed ground state at some time t^*, $\psi(x,t^*)=\psi_0(t^*)h_0(x)$. From Eq. (A10), we then obtain

$$\psi^{(0)}(t)=\psi_0^{(0)}(t)=e^{-iE_0(t-t^*)},$$

but

$$if^*(t)\frac{d}{dt}\psi_0^{(1)}(t)=f^*(t)E_0\psi_0^{(1)}(t)-\mathrm{Re}[f^*(t)e^{-iE_0(t-t^*)}\bar{K}(E_0)],$$
$$\bar{K}(E_0)\equiv\int_{-\infty}^{\infty}dt'K(t')e^{-iE_0t'}. \tag{A11}$$

We therefore need $\bar{K}(E_0)=0$ if we are to have a stationary state. If $\bar{K}(E_0)=0$, then

$$\psi_0^{(1)}(t)=C_{01}e^{-iE_0(t-t^*)}$$
$$=0, \tag{A12}$$

and we can obtain a similar result for $\psi_0^{(2)}(t)$. Therefore

$$\psi(x,t)=h_0(x)\exp[-iE_0(t-t^*)]$$

satisfies Eq. (A7) up to terms of order λ^3 for all Φ, if $\bar{K}(E_0)=0$. It then follows that Eq. (A5) does hold in this case,

$$\langle\Psi(\tau/2)\,|\,\Psi(-\tau/2)\rangle=\langle\Psi(\tau/2)\,|\,e^{-iH'\tau}\,|\,\Psi(\tau/2)\rangle$$
$$=e^{-i\tau E(K)}, \tag{A13}$$
$$E(K)=\langle\Psi(t)\,|\,H'\,|\,\Psi(t)\rangle=E_0.$$

Consequently, the identification of $V[G]$ as the effective potential does go through if we restrict ourselves to sources which satisfy $\bar{K}(E_0)=0$.

What does this result mean? For the quantum-mechanical case it means that if the source has no Fourier component of the oscillator's fundamental frequency, then the ground state remains a stationary state with energy unchanged.[27] In the quantum field-theoretic generalization of this, it is not immediately obvious what should correspond to E_0, but we shall see that it does not matter.

To understand the effect of restricting ourselves to only a certain class of sources $[\bar{K}(E_0)=0]$, we recall that, Eq. (2.15b),

$$K(x,y)=\frac{\delta\Gamma[G]}{\delta G(x,y)}\sim-\frac{\delta V[G]}{\delta G(x,y)}. \tag{A14}$$

The condition on K, therefore, means that we minimize V by varying G, but consider only those G such that

$$\int_{-\infty}^{\infty}dz_0\frac{\delta V[G]}{\delta G(z)}e^{-iE_0z_0}=0. \tag{A15}$$

We obviously risk overlooking the true minimum in this manner, but the risk may not be too great—it is, in fact, related to the risk taken in choosing test functions for the variational calculation. If we set out to minimize $V[G]$ we wish to choose functional forms for the propagators which satisfy the DS equations

$$\left.\frac{\delta V[G]}{\delta G_i}\right|_{G_i=\mathscr{G}_i(\alpha_k)}=0, \tag{A16}$$

where the $\mathscr{G}_i$ are the test functions and the α_k are the parameters upon which the $\mathscr{G}_i$ depend. But (A16) may be satisfied for a range of α_i, say $\alpha_i\epsilon(a_i,a_i+\delta_i)$, and the preferred values for the α_i $(\equiv\alpha_i^*)$ must then be determined by minimizing $V[\mathscr{G}(\alpha_i)]\equiv V(\alpha_i)$. Since $\alpha_i^*\epsilon(a_i,a_i+\delta_i)$, $\mathscr{G}(\alpha_i)$ must satisfy Eq. (A16) in a neighborhood of α_i^*, and therefore (A15) is obviously satisfied and the function $V(\alpha_i)$ which we are minimizing is the vacuum energy density. We then note that our test functions, $G_{\mathrm{LA}}(m_i)$, do satisfy Eq. (A16)—at least in the Abelian sector. In particular, if we use Eq. (4.1) in Eq. (3.1), we find

$$\left.\frac{\delta V[G,\Delta]}{\delta G(p)}\right|_{G,\Delta=G(\mathrm{LA}),\Delta(\mathrm{LA})}=\left.\frac{\delta V[G,\Delta]}{\delta\Delta_B(p)}\right|_{G(\mathrm{LA}),\Delta(\mathrm{LA})}$$
$$=\left.\frac{\delta V[G,\Delta]}{\delta\Delta_{AB}(p)}\right|_{G(\mathrm{LA}),\Delta(\mathrm{LA})}=0, \tag{A17}$$

to leading order in g^2, for all values of the parameters m_a,π_a,ζ_a. Therefore, for the sources K_ψ,K_B,K_{AB}, we ensured that $\bar{K}(E_0)=0$ when we chose to use the LA results as test functions.

The question of K_A and the non-Abelian sector is not so clean, unfortunately. Equations (4.1a)—(4.1d) do not satisfy Eqs. (3.1d) and (3.1e). The failure looks suspiciously like a consequence of the gauge noninvariance of the vertices in the presence of vector masses, and this suspicion is strengthened when we note that if gauge invariance does lead to the vector loop graphs being absent from Ω, then they also do not contribute to the DS equations to leading order. In that case the LA solutions do satisfy $\delta V[G,\Delta]/\delta\Delta_A(p)=0$, and the $\bar{K}_A(E_0)=0$ constraint has also already been imposed by our choice of variational test functions.

APPENDIX B: INTEGRALS

This appendix is devoted to results for various integrals encountered in the calculation. Because of the momentum dependence in the mass, there are a few departures from standard results, particularly in regularizing integrals which would be divergent in four dimensions. We shall evaluate a few representative integrals in detail as illustrations. For the remaining integrals we merely tabulate the results.

The one convergent (in four dimensions) integral which recurs throughout the calculation, appearing first in the LA, is

$$I_0(l)=\int \frac{d^4k}{(2\pi)^4}\{k^2[k^2-\mu_a{}^2(k)]\}^{-1}\mu_a^l(k)\ , \quad l>0\ . \tag{B1}$$

Using $\mu_a(k)=m_a(-k^2/m_a{}^2)^{-r_a}$ and going over to Euclidean space,

$$\begin{aligned} I_0(l)&=\frac{i}{8\pi^2}m_a^l\int_0^\infty dz\frac{z}{z^2+1}(z^2)^{-lr_a} \\ &=\frac{i}{16\pi^2}m_a^lB(2-lr_a,lr_a)\ , \end{aligned} \tag{B2}$$

where $B(x,y)$ is the Euler β function. For small $r_a=3g_B{}^2Y_{aL}Y_{aR}/(64\pi^2)$,

$$I_0(l)\simeq\frac{4i}{3l}\frac{m_a^l}{g_B{}^2Y_{aL}Y_{aR}}\ . \tag{B3}$$

This result differs from the $l=4$ result of Ref. 8(a), but they have used the incorrect approximation

$$\int d^4k\mu^4(k)f(k)\simeq m^3\int d^4k\mu(k)f(k)\ .$$

The first integral encountered which requires regularization is

$$\begin{aligned} I_1(n,a)&=\int\frac{d^nk}{(2\pi)^n}\ln\left[1-\frac{\mu_a{}^2(k)}{k^2}\right] \\ &\equiv\frac{1}{(2\pi)^n}\left[\int d^{n-2}\Omega\right]\hat{I}_1(n,a)\ . \end{aligned} \tag{B4}$$

Integrating by parts in the usual manner yields

$$\begin{aligned} \hat{I}_1(n,a)=&-\tfrac{1}{2}\int dk_0k_0\int d|\vec{k}|\ |\vec{k}|^{n-2}\frac{d}{dk_0}\left[\ln\left[1-\frac{\mu_a{}^2(k)}{k^2}\right]\right] \\ &-\tfrac{1}{2}\int dk_0\int d|\vec{k}|\ |\vec{k}|\left[(n-2)|\vec{k}|^{n-3}\ln\left[1-\frac{\mu_a{}^2(k)}{k^2}\right]+|\vec{k}|^{n-2}\frac{d}{d|\vec{k}|}\ln\left[1-\frac{\mu_a{}^2(k)}{k^2}\right]\right]\ . \end{aligned} \tag{B5}$$

Using the explicit form for $\mu_a{}^2(k)$ and regrouping,

$$\hat{I}_1(n,a)=-\frac{n-2}{2}\hat{I}_1(n,a)-(1+2r_a)\int dk_0d|\vec{k}|\ |\vec{k}|^{n-2}\frac{\mu_a{}^2(k)}{k^2-\mu_a{}^2(k)}\ , \tag{B6}$$

and therefore

$$I_1(n,a)=-\frac{2}{n}(1+2r_a)\int\frac{d^nk}{(2\pi)^n}\frac{\mu_a{}^2(k)}{k^2-\mu_a{}^2(k)}\ . \tag{B7}$$

The integral on the right-hand side of (B7) is still not convergent for $n=4$. We proceed as above and obtain

$$I_2(n,a)\equiv\int\frac{d^nk}{(2\pi)^n}\frac{\mu_a{}^2(k)}{k^2-\mu_a{}^2(k)}=\frac{2(1+2r_a)}{n-2-4r_a}\int\frac{d^nk}{(2\pi)^n}\frac{\mu_a{}^4(k)}{[k^2-\mu_a{}^2(k)]^2}\ . \tag{B8}$$

The right-hand side of (B8) is well behaved for $n=4$; and so, defining $I_i(a)\equiv I_i(n=4,a)$,

$$\begin{aligned} I_2(a)&=\frac{1+2r_a}{1-2r_a}\int\frac{d^4k}{(2\pi)^4}\frac{\mu_a{}^4(k)}{[k^2-\mu_a{}^2(k)]^2}\ , \\ I_1(a)&=-\frac{1+2r_a}{2}I_2(a)\ . \end{aligned} \tag{B9}$$

The remaining integral is equal to $I_0(l=4)$ for small r_a,

$$\int \frac{d^4k}{(2\pi)^4}\frac{\mu_a{}^4(k)}{[k^2-\mu_a{}^2(k)]^2}=\int \frac{d^4k}{(2\pi)^4}\frac{\mu_a{}^4(k)}{k^2[k^2-\mu_a{}^2(k)]}\left[1+\frac{\mu_a{}^2(k)}{k^2-\mu_a{}^2(k)}\right]\simeq I_0(l=4)\ , \tag{B10}$$

since the second term in brackets leads to a convergent integral even for $r_a=0$. Therefore, for small r_a,

$$I_1(a)\simeq-\frac{i}{6}\frac{m_a{}^4}{g_B{}^2Y_{aL}Y_{aR}}\ ,I_2(a)\simeq\frac{i}{3}\frac{m_a{}^4}{g_B{}^2Y_{aL}Y_{aR}}\ . \tag{B11}$$

A large number of the two-loop integrals encountered are of the form

$$\begin{aligned}&I_3^{(n)}(i,j,k;a,b,c;\alpha,\beta,\gamma)=\int d^nk_1d^nk_2F^{ijk}_{abc;\alpha\beta\gamma}\ ,\\ &F^{ijk}_{abc;\alpha\beta\gamma}=\pi_1(k_1)(k_1\cdot k_2)^i(k_1\cdot k_3)^j(k_2\cdot k_3)^k[(k_1{}^2)^a(k_2{}^2)^b(k_3{}^2)^c(k_1{}^2-\pi_1)^\alpha(k_2{}^2-\pi_2)^\beta(k_3{}^2-\pi_3)^\gamma]^{-1}\ ,\\ &k_3=k_1+k_2\ ,\end{aligned} \tag{B12}$$

where the subscript on the π's in the denominator represents both the flavor and momentum subscripts, $\pi_1=\pi_1(k_1)$. Integrating by parts and assuming r_a is small,

$$\begin{aligned}I_3^{(n)}(i,j,k;a,b,c;\alpha,\beta,\gamma)&=-\frac{1}{2n}\int d^nk_1d^nk_2\left[k_{1_\mu}\frac{\partial}{\partial k_{1\mu}}+k_{2_\mu}\frac{\partial}{\partial k_{2\mu}}\right]F^{ijk}_{abc;\alpha\beta\gamma}\\ &=-\frac{1}{2n}\int d^nk_1d^nk_2F^{ijk}_{abc;\alpha\beta\gamma}\left[2i+2j+2k-2a-2b-2c-2\alpha\frac{k_1{}^2}{k_1{}^2-\pi_1}\right.\\ &\qquad\left.-2\beta\frac{k_2{}^2}{k_2{}^2-\pi_2}-2\gamma\frac{k_3{}^2}{k_3{}^2-\pi_3}\right]\ .\end{aligned} \tag{B13}$$

Writing $k^2/(k^2-\pi)=1+\pi/(k^2-\pi)$ and regrouping, we obtain

$$\begin{aligned}I_3^{(n)}(i,j,k;a,b,c;\alpha,\beta,\gamma)&=(n+i+j+k-a-b-c-\alpha-\beta-\gamma)^{-1}\\ &\quad\times\int d^nk_1d^nk_2F^{ijk}_{abc;\alpha\beta\gamma}\left[\alpha\frac{\pi_1}{k_1{}^2-\pi_1}+\beta\frac{\pi_2}{k_2{}^2-\pi_2}+\gamma\frac{\pi_3}{k_3{}^2-\pi_3}\right]\ .\end{aligned} \tag{B14}$$

In the calculation of vector masses in the LA, we must evaluate

$$M_a{}^2(p)=-i\int d\alpha\int\frac{d^4k}{(2\pi)^4}\mu_a{}^2(k)[k^2+\alpha(1-\alpha)p^2-\mu_a{}^2(k)]^{-2}\ . \tag{B15}$$

Going to Euclidean space and letting $x=k/m_a$,

$$\begin{aligned}M_a{}^2(p)&=m_a{}^2\int d\alpha\int\frac{d^4x}{(2\pi)^4}(x^2)^{-2r_a}\left[x^2+\alpha(1-\alpha)\frac{p^2}{m_a{}^2}+1\right]^{-2}\\ &=\frac{m_a{}^2}{16\pi^2}B(2-2r_a,2r_a)\int d\alpha\left[\alpha(1-\alpha)\frac{p^2}{m_a{}^2}+1\right]^{-2r_a}\ .\end{aligned} \tag{B16}$$

For large p and small r_a,

$$M_a{}^2(p)\simeq\frac{2}{3g_B{}^2Y_{aL}Y_{aR}}m_a{}^2\left[\frac{p^2}{m_a{}^2}\right]^{-2r_a}\ , \tag{B17}$$

where we recall that p^2 is a Euclidean momentum here. For $p^2=0$, $M_a{}^2(p)$ is of the form $I_0(l=2)$ of (B1) plus nonsingular terms.

Other integrals needed in the LA treatment of vector masses are

$$I_4^{(n)}(a)=\int \frac{d^nk}{(2\pi)^n}[k^2+\alpha(1-\alpha)p^2-\mu_a{}^2(k)]^{-2}$$
$$=-\frac{4}{n-4}\int \frac{d^nk}{(2\pi)^n}[\alpha(1-\alpha)p^2-(1+2r_a)\mu_a{}^2(k)][k^2+\alpha(1-\alpha)p^2-\mu_a{}^2(k)]^{-3}\,,$$
$$\int \frac{d^nk}{(2\pi)^n}(2k^\mu k^\nu-k^2g^{\mu\nu})[k^2+\alpha(1-\alpha)p^2-\mu_a{}^2(k)]^{-2}$$
$$=g^{\mu\nu}\alpha(1-\alpha)p^2I_4^{(n)}(a)-g^{\mu\nu}\left[1+\frac{4r_a}{n}\right]\int \frac{d^nk}{(2\pi)^n}\mu_a{}^2(k)[k^2+\alpha(1-\alpha)p^2-\mu_a{}^2(k)]^{-2}\,. \tag{B18}$$

Integrals arising in the evaluation of $\Omega_{\psi V}$ include

$$\int \frac{d^4k_1d^4k_2d^4k_3}{(2\pi)^8}\delta^4(k_1-k_2-k_3)k_1\cdot k_2\hat{\mu}_a^2(1)\hat{\pi}(3)\{[k_1{}^2-\mu_a{}^2(1)]k_2{}^2[k_3{}^2-\pi(3)]\}^{-1}$$
$$=\frac{4}{9g_B{}^4}\frac{m_a{}^2}{Y_{aL}Y_{aR}}\sum_b\frac{\pi_b}{Y_{aL}Y_{aR}+Y_{bL}Y_{bR}}\,, \tag{B19}$$

$$\int \frac{d^4k_1d^4k_2d^4k_3}{(2\pi)^8}\delta^4(k_1-k_2-k_3)k_1\cdot k_3k_2\cdot k_3\hat{\mu}_a{}^2(1)\hat{\pi}(3)\{[k_1{}^2-\mu_a{}^2(1)]k_2{}^2[k_3{}^2-\pi(3)]k_3{}^2\}^{-1}$$
$$=\frac{1}{9g_B{}^4}\frac{m_a{}^2}{Y_{aL}Y_{aR}}\sum_b\frac{\pi_b}{Y_{aL}Y_{aR}+Y_{bL}Y_{bR}}\,, \tag{B20}$$

$$\int \frac{d^4k_1d^4k_2d^4k_3}{(2\pi)^8}\delta^4(k_1-k_2-k_3)\hat{\mu}_a{}^2(1)\hat{\mu}_a{}^2(2)\left[k_1\cdot k_2+2\frac{k_1\cdot k_3k_2\cdot k_3}{k_3{}^2}\right]\{[k_1{}^2-\mu_a{}^2(1)][k_2{}^2-\mu_a{}^2(2)]k_3{}^2\}^{-1}$$
$$=\frac{1}{g_B{}^4}\times 0\,, \tag{B21}$$

$$\int \frac{d^4k_1d^4k_2}{(2\pi)^8}\mu_a{}^3(1)\mu_a(2)\{[k_1{}^2-\mu_a{}^2(1)]^2[k_2{}^2-\mu_a{}^2(2)](k_1+k_2)^2\}^{-1}=-\frac{4}{9g_B{}^4}\left[\frac{m_a{}^2}{Y_{aL}Y_{aR}}\right]^2\,, \tag{B22}$$

$$\int \frac{d^nk_1d^nk_2}{(2\pi)^{2n}}\mu_a(1)\mu_a(2)\{[k_1{}^2-\mu_a{}^2(1)][k_2{}^2-\mu_a{}^2(2)][(k_1+k_2)^2-\zeta(1+2)]\}^{-1}$$
$$=-\frac{8}{9g_B{}^4}\left[\frac{m_a{}^2}{Y_{aL}Y_{aR}}\right]^2-\frac{4}{9g_B{}^4}\frac{m_a{}^2}{Y_{aL}Y_{aR}}\sum_b\frac{\zeta_b}{Y_{aL}Y_{aR}+Y_{bL}Y_{bR}}\,, \tag{B23}$$

$$\int \frac{d^4k_1d^4k_2}{(2\pi)^8}\mu_a(1)\mu_a(2)\zeta(1+2)\{[k_1{}^2-\mu_a{}^2(1)][k_2{}^2-\mu_a{}^2(2)][(k_1+k_2)^2-\zeta(1+2)]^2\}^{-1}$$
$$=-\frac{4}{9g_B{}^4}\frac{m_a{}^2}{Y_{aL}Y_{aR}}\sum_b\frac{\zeta_b}{Y_{aL}Y_{aR}+Y_{bL}Y_{bR}}\,, \tag{B24}$$

where (B23) required regularization.

For Ω_{4A} the finite integral is

$$\int \frac{d^4k_1d^4k_2}{(2\pi)^8}\left[-7+\frac{(k_1\cdot k_2)^2}{k_1{}^2k_2{}^2}\right]\hat{\pi}(1)\hat{\zeta}(2)\{[k_1{}^2-\pi(1)][k_2{}^2-\pi(2)]\}^{-1}=\frac{3}{g_B{}^4}\sum_a\frac{\pi_a}{Y_{aL}Y_{aR}}\sum_b\frac{\zeta_b}{Y_{bL}Y_{bR}}\,. \tag{B25}$$

For the integrals in the evaluation of Ω_{3A}, we introduce some notation. Let

$$A_{ij}(X_1,X_2,X_3)=[(k^2-X_1)(k_2{}^2-X_2)(k_3{}^2-X_3)]^{-1}\hat{X}_i\hat{X}_j \,,$$
$$\int \frac{d^4k_1 d^4k_2 d^4k_3}{(2\pi)^8}\delta^4(k_1+k_2-k_3)f(k_1,k_2,k_3)\equiv I[f(k_1,k_2,k_3)] \,, \tag{B26}$$

where the X's can be π, ζ, or 0, (W,Z,γ), where $\pi_i=\pi(k_i)$, and where $\hat{X}_i=X_i/k_i{}^2$. Thus, for example,

$$I[k_1{}^2A_{13}(W,\gamma,Z)]=\int \frac{d^4k_1 d^4k_2 d^4k_3}{(2\pi)^8}\delta^4(k_1+k_2-k_3)k_1{}^2\hat{\pi}(k_1)\hat{\zeta}(k_3)\,[(k_1{}^2-\pi(k_1))k_2{}^2(k_3{}^2-\zeta(k_3))]^{-1} \,. \tag{B27}$$

Then the integrals of Ω_{3A} can all be written in terms of

$$I[k_1{}^2A_{12}(X_1,X_2,X_3)]=-4\mathcal{S}(X_1,X_2) \,, \tag{B28}$$

$$I\left[\frac{(k_1\cdot k_2)^2}{k_2{}^2}A_{12}(X_1,X_2,X_3)\right]=-\mathcal{S}(X_1,X_2) \,, \tag{B29}$$

$$I\left[\frac{(k_1\cdot k_3)^2}{k_3{}^2}A_{12}(X_1,X_2,X_3)\right]=-4\mathcal{S}(X_1,X_2) \,, \tag{B30}$$

$$I\left[\frac{(k_1\cdot k_2 k_1\cdot k_3)^2}{k_1{}^2k_2{}^2k_3{}^2}A_{12}(X_1,X_2,Z_3)\right]=-\mathcal{S}(X_1,X_2) \,, \tag{B31}$$

$$I[k_1{}^2A_{13}(X_1,X_2,X_3)]=-4\mathcal{S}(X_1,X_3) \,, \tag{B32}$$

$$I\left[\frac{(k_1\cdot k_2)^2}{k_2{}^2}A_{13}(X_1,X_2,X_3)\right]=-4\mathcal{S}(X_1,X_3) \,, \tag{B33}$$

$$I\left[\frac{(k_1\cdot k_3)^2}{k_3{}^2}A_{13}(X_1,X_2,X_3)\right]=-\mathcal{S}(X_1,X_3) \,, \tag{B34}$$

$$I\left[\frac{(k_1\cdot k_2 k_1\cdot k_3)^2}{k_1{}^2k_2{}^2k_3{}^2}A_{13}(X_1,X_2,X_3)\right]=-\mathcal{S}(X_1,X_3) \,, \tag{B35}$$

$$I[k_1{}^2A_{23}(X_1,X_2,X_3)]=-4[\mathcal{S}(X_2,X_3)+\mathcal{S}(X_3,X_2] \,, \tag{B36}$$

$$I\left[\frac{(k_1\cdot k_2)^2}{k_2{}^2}A_{23}(X_1,X_2,X_3)\right]=-4\mathcal{S}(X_2,X_3)-\mathcal{S}(X_3,X_2) \,, \tag{B37}$$

$$I\left[\frac{(k_1\cdot k_3)^2}{k_3{}^2}A_{23}(X_1,X_2,X_3)\right]=-\mathcal{S}(X_2,X_3)-4\mathcal{S}(X_3,X_2) \,, \tag{B38}$$

$$I\left[\frac{(k_1\cdot k_2 k_1\cdot k_3)^2}{k_1{}^2k_2{}^2k_3{}^2}A_{23}(X_1,X_2,X_3)\right]=-\mathcal{S}(X_2,X_3)-\mathcal{S}(X_3,X_2) \,, \tag{B39}$$

$$I\left[\frac{k_1\cdot k_2 k_1\cdot k_3}{k_3{}^2}A_{12}(X_1,X_2,X_3)\right]=0 \,, \tag{B40}$$

$$I\left[\frac{(k_1\cdot k_2)^3 k_1\cdot k_3}{k_1{}^2k_2{}^2k_3{}^2}A_{12}(X_1,X_2,X_3)\right]=0 \,, \tag{B41}$$

$$I\left[\frac{k_1\cdot k_2 k_1\cdot k_3}{k_3{}^2}A_{13}(X_1,X_2,X_3)\right]=-\mathcal{S}(X_1,X_3) \,, \tag{B42}$$

$$I\left[\frac{(k_1\cdot k_2)^3 k_1\cdot k_3}{k_1{}^2k_2{}^2k_3{}^2}A_{13}(X_1,X_2,X_3)\right]=-\mathcal{S}(X_1,X_3) \,, \tag{B43}$$

$$I\left[\frac{k_1\cdot k_2 k_1\cdot k_3}{k_3{}^2}A_{23}(X_1,X_2,X_3)\right]=+3\mathscr{S}(X_2,X_3)\ , \tag{B44}$$

$$I\left[\frac{(k_1\cdot k_2)^3 k_1\cdot k_3}{k_1{}^2 k_2{}^2 k_3{}^2}A_{23}(X_1,X_2,X_3)\right]=+3\mathscr{S}(X_2,X_3)\ , \tag{B45}$$

where

$$\mathscr{S}(X_i,X_j)=\frac{1}{9g_B{}^4}\sum_{a,b}X_{ia}X_{jb}\frac{1}{Y_{aL}Y_{aR}}\frac{1}{Y_{aL}Y_{aR}+Y_{bL}Y_{bR}}\ ,\quad X_i=\pi,\ \zeta,\ \text{or } 0\ , \tag{B46}$$

and only the most singular term $(1/g_B{}^4)$ is given.

[1]S. Weinberg, Phys. Rev. D 13, 974 (1976); 19, 1277 (1979); L. Susskind, *ibid.* 20, 2619 (1979). Reviews include K. D. Lane and M. Peskin, in *Electroweak Interactions and Unified Theories,* edited by J. Trân Thanh Vân (Editions Frontieres, Dreux, France, 1980), Vol. II, p. 469; E. Farhi and L. Susskind, Phys. Rep. 74, 277 (1981).

[2]S. Raby, S. Dimopoulos, and L. Susskind, Nucl. Phys. B169, 373 (1980).

[3]Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).

[4]J. M. Cornwall and R. E. Norton, Phys. Rev. D 8, 3338 (1973).

[5]R. Jackiw and K. Johnson, Phys. Rev. D 8, 2386 (1973).

[6]J. M. Cornwall, R. Jackiw, and E. Tomboulis, Phys. Rev. D 10, 2428 (1974).

[7]A. Carter and H. Pagels, Phys. Rev. Lett. 43, 1845 (1979); see also F. Englert and R. Brout, Phys. Lett. 49B, 77 (1974).

[8](a) R. Acharya, P. Narayanaswamy, and B. P. Nigam, Nuovo Cimento 60A, 265 (198); (b) K. Huang and R. Mendel, Report No. CTP 876, 1980 (unpublished).

[9]K. T. Mahanthappa and J. Randa, Phys. Lett. 121B, 156 (1983).

[10]E. Witten, Nucl. Phys. B188, 513 (1981).

[11]H. Pagels and S. Stokar, Phys. Rev. D 20, 2947 (1979); H. Pagels, *ibid.* 21, 2336 (1980).

[12]J. M. Cornwall, Phys. Rev. D 10, 500 (1974).

[13]H. D. Dahmen and G. Jona-Lasinio, Nuovo Cimento 52A, 807 (1967); A. N. Vasil'ev and A. K. Kazanskii, Teor. Mat. Fiz. 12, 353 (1972) [Theor. Math. Phys. 12, 875 (1972)].

[14]T. Banks and S. Raby, Phys. Rev. D 14, 2182 (1976).

S. Coleman, in *Laws of Hadronic Matter,* proceedings of the Summer 1973 International School "Ettore Majorana," edited by A. Zichichi (Academic, New York, 1975), p. 139.

[16]R. Haymaker and J. Perez-Mercader, Louisiana State University report, 1982 (unpublished).

[17]Th. A. J. Maris, V. E. Herscovitz, and G. Jacob, Phys. Rev. Lett. 12, 313 (1964).

[18]See, e.g., C. Nash, *Relativistic Quantum Fields* (Academic, New York, 1978), p. 105.

[19]W. Buchmüller and S.-H.H. Tye, Phys. Rev. D 24, 132 (1981).

[20]We thank Henry Tye for emphasizing this point to us.

[21]See Appendix A.

[22]R. W. Haymaker and J. Perez-Mercader, Phys. Lett. 106B, 201 (1981).

[23]J. M. Cornwall and R. C. Shellard, Phys. Rev. D 18, 1216 (1978).

[24]E. J. Eichten and F. L. Feinberg, Phys. Rev. D 10, 3254 (1974); S.-H.H. Tye, E. Tomboulis, and E. C. Poggio, *ibid.* 10, 2839 (1975).

[25]R. Stern, Phys. Rev. D 14, 2081 (1976).

[26]K. Symanzik, Commun. Math. Phys. 16, 48 (1970).

[27]See the discussion of Eqs. (3.5) and (3.6) in Ref. 16.