Negative-norm states, superselection rules, and the lepton family

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Field theories containing states of both positive and negative norm are considered. With the correct definition of the number operators for the quantum fields, all physical quantities are rendered canonically normalized. If the theory admits a global symmetry leading to a superselection rule which forbids transitions between positive- and negative-norm states, then the negative-norm states are allowed to be physical. Specifically, a spinor theory with higher-order field equations and multiple excitations is considered and applied to the charged lepton system: e,μ,τ. In this model, the negative norm of the muon state allows us to understand the nonexistence of $\mu\to e\gamma$ decay. For minimal coupling, the theory is renormalizable and equivalent to three separate fermion electrodynamics with the additional prediction of equal charge for the leptons. A further anomalous magnetic moment coupling can only allow one of the decays $\tau\to\mu\gamma$ or $\tau\to e\gamma$.

I. INTRODUCTION

Negative-norm states or Hilbert spaces with indefinite metric have a long history in quantum field theory going back to the work of Dirac,[1] Pauli,[2] Heisenberg,[3] and many others.[4] While in general the indefinite metric is introduced as a tool to overcome some of the difficulties of field theory (for example, in the covariant formulation of electrodynamics) which is eventually to be eliminated, or an indication of a nonlocal theory, we discuss in this paper the cases where the appearance of the indefinite metric becomes a virtue and extends the scope of the field theory. This happens when the indefinite metric occurs in connection with a global symmetry and associated superselection rule. The latter prohibits transitions from one sector of the Hilbert space to another of opposite norm.

After some general considerations about norms, transition probabilities, and fields with internal excitations, we exhibit the connection between indefinite metric and superselection rules explicitly in terms of a third-order spinor field theory and apply it to the lepton family. The appearance of indefinite metric in higher-order wave equations goes back to Pais and Uhlenbeck.[5]

II. NEGATIVE-NORM STATES AND SUPERSELECTION RULES

In the usual formulation of Lagrangian field theory, the classical field equations are invariant under a change of scale of the Lagrangian density

$$\Lambda'=\kappa\Lambda\ , \tag{2.1}$$

where κ is an arbitrary nonzero constant. Since the physical quantities of interest P (energy-momentum tensor, angular momentum tensor, currents, etc.) are linear functionals of the Lagrangian density, the rescaling

$$P'[\Lambda']\equiv\frac{1}{\kappa}P[\Lambda']=P[\Lambda] \tag{2.2}$$

restores the standard normalization of these quantities, as indicated.

When we pass to quantum theory this invariance is no longer naively present; indeed, the Lagrangian density is chosen such that the "kinetic" terms for the fields always have a definite sign and magnitude. Relative to such a field, another field appearing with the opposite sign is said to have negative norm and is often denoted (slandered) with the term "ghost." We will now demonstrate that, with one crucial proviso, there is no fundamental objection to having physical ghost states, and the negative norm can be a virtue in understanding the presence of certain superselection rules.

To begin, we observe that the quantum field theory will certainly be invariant under both transformations (2.1) and (2.2). However, the interesting nontrivial situation arises when the theory contains fields of both positive and negative norm. In this case, no transformation of form (2.1) can render all norms positive nor can a transformation of form (2.2) render all physical quantities canonically normalized. Therefore we take the following approach. We will assume that all physical quantities are canonically defined (except for the Lagrangian density). The only change necessary will be a redefinition of the number operators for the quantum fields.

We will consider and compare free quantum field theories defined by Lagrangian densities Λ (taken to be canonical) and Λ' [given by Eq. (2.1)], where κ in general may be negative. We take the conjugate momenta to be given by their canonical forms

$$\Pi_i \equiv \frac{\partial \Lambda}{\partial \dot{\phi}_i} , \tag{2.3}$$

$$\Pi_i' \equiv \frac{\partial \Lambda'}{\partial \dot{\phi}_i} = \kappa \Pi_i , \tag{2.3'}$$

where ϕ_i represents one component of a Fermi or Bose field.

The equal-time (anti-) commutation relations are written in canonical form as

$$[\Pi_i(\vec{x},t),\phi_j(\vec{y},t)]_{\pm} = -i\delta^3(\vec{x}-\vec{y})\delta_{ij} , \tag{2.4}$$

$$[\Pi_i'(\vec{x},t),\phi_j(\vec{y},t)]_{\pm} = -i\delta^3(\vec{x}-\vec{y})\delta_{ij} . \tag{2.4'}$$

Now we expand $\phi_i(x)$ in plane waves,

$$\phi_i(x) = \int \frac{d^3k}{(2\pi)^3} \sum_{\alpha} [X_{i\alpha}^{+}(\vec{k},x)a_{\alpha}(\vec{k}) + X_{i\alpha}^{-}(\vec{k},x)b_{\alpha}^{\dagger}(\vec{k})] , \tag{2.5}$$

where $X_{i\alpha}^{\pm}(\vec{k},x)$ form a complete orthogonal set of plane-wave solutions to the free field equations of positive and negative energy. With the usual normalization of these states, Eqs. (2.4), (2.4'), and (2.5) yield

$$[a_{\alpha}(\vec{k}),a_{\beta}^{\dagger}(\vec{p})]_{\pm} = \delta_{\alpha\beta}\delta^3(\vec{k}-\vec{p}) = [b_{\alpha}(\vec{k}),b_{\beta}^{\dagger}(\vec{p})]_{\pm} , \tag{2.6}$$

$$[a_{\alpha}(\vec{k}),a_{\beta}^{\dagger}(\vec{p})]_{\pm} = \frac{1}{\kappa}\delta_{\alpha\beta}\delta^3(\vec{k}-\vec{p}) = [b_{\beta}(\vec{k}),b_{\beta}^{\dagger}(\vec{p})]_{\pm} , \tag{2.6'}$$

all other (anti-) commutators being zero. In order to understand the significance of the factor $1/\kappa$ in Eq. (2.6') we write this equation as

$$[a_{\alpha}(\vec{k}),\kappa a_{\beta}^{\dagger}(\vec{p})]_{\pm} = \delta_{\alpha\beta}\delta^3(\vec{k}-\vec{p}) = [b_{\alpha}(\vec{k}),\kappa b_{\beta}^{\dagger}(\vec{p})]_{\pm} . \tag{2.7}$$

If we make the usual interpretation that a and b are the annihilation operators for the particles and antiparticles, then the creation operators must be $\kappa a^{\dagger}$ and $\kappa b^{\dagger}$ instead of the usual $a^{\dagger}$ and $b^{\dagger}$. (Note that there are other ways of including the factor κ, all of which yield the same conclusions below.) Therefore the number operators for particles and antiparticles are

$$N_{\alpha}(\vec{p}) = a_{\alpha}^{\dagger}(\vec{p})a_{\alpha}(\vec{p}), \quad \bar{N}_{\alpha}(\vec{p}) = b_{\alpha}^{\dagger}(\vec{p})b_{\alpha}(\vec{p}) , \tag{2.8}$$

$$N_{\alpha}'(\vec{p}) = \kappa a_{\alpha}^{\dagger}(\vec{p})a_{\alpha}(\vec{p}), \quad \bar{N}_{\alpha}'(\vec{p}) = \kappa b_{\alpha}^{\dagger}(\vec{p})b_{\alpha}(\vec{p}) . \tag{2.8'}$$

Since we are considering (for the moment) a free field theory, all canonically defined physical quantities are bilinear in the fields, and hence linear in the number operators. Because the physical quantities are also linear functionals of the Lagrangian density, the additional factor of κ appearing in the "primed" formulation is absorbed by the appropriate redefinition of the number operators, and we recover the standard normalization of the physical quantities. We stress again that it is important to treat the quantum theory in this fashion [as opposed to a simple rescaling as in Eq. (2.2)] because in the situation in which the theory contains both positive- and negative-norm states this overall rescaling does not work. The treatment given here, however, has the virtue of only changing the signs for the negative-norm states, rendering all physical quantities appropriately normalized.

The case of interacting fields presents the most interesting feature of these generalized field theories. In general, the probability that an initial state at $t=-\infty$ is to be found in a final state at $t=+\infty$ is given by

$$P_{fi} = \frac{|\langle f \mid S \mid i \rangle|^2}{||f|| \; ||i||} . \tag{2.9}$$

Here $\langle f \mid S \mid i \rangle$ is the scattering matrix element for the initial and final states in question, and $||i||$ and

$||f||$ are the norms of the initial and final states. This expression is valid for arbitrarily normalized states. Being a probability it must, of course, be positive. Observe that there is no problem if $||i||$ and $||f||$ are either both positive or both negative. However, the scattering matrix element must vanish in the case that the initial and final states have opposite norm. Therefore, in order for such a theory to be physically realizable, there must exist a superselection rule (and a corresponding global symmetry) which excludes transitions between positive- and negative-norm physical states. In other words, the Hilbert space of physical states must be split into two disjoint sectors corresponding to positive and negative norms, and no interaction may exist which induces transitions between these two subspaces.

We conclude that as long as such a superselection rule can be derived from the theory in question, there is no fundamental objection to a theory possessing physical ghost states.

This rather simple observation has many interesting ramifications. Possible applications include gauge field theories of graded Lie algebras and theories where negative-norm collective variables arise. Here we will discuss a theory in which one field has many excitations.

III. FIELDS WITH MULTIPLE INTERNAL EXCITATIONS

This result that positive- and negative-norm states cannot be dynamically connected has important consequences in any theory in which one field exhibits many excitations. In particular, these excitations (dynamically generated or otherwise) will manifest themselves as additional poles in the propagator of that field, and the residue of each pole will determine the norm of that particular excited state. If the theory in question is unitary, we may conclude that all states of negative norm will be stable against decay into states of positive norm and vice versa. One specific example is the hydrogen atom. If one treats the hydrogen atom within the framework of infinite-component fields, the propagator for the hydrogen-atom field has a pole for each bound state and a branch cut for the continuous spectrum of scattering states. One can show that the bound states all have positive norm. Of course, this is as expected since all transitions between excited states are allowed.

Consider, however, the system of charged leptons: e,μ,τ. It is very tempting to conjecture that the μ and τ particles are excited states of the electron in the sense that the electron propagator may develop additional poles through some dynamical mechanism (of electromagnetic origin or otherwise). Note that we are not necessarily making any compositeness assumptions about the leptons. There now exists the possibility of understanding why the process $\mu\to e\gamma$ seems to be absolutely forbidden. If we assume that the underlying theory, whatever it may be, is unitary, and that the residue of the pole at the muon mass is negative, then the decay is dynamically forbidden. If this is indeed the case, then only one of the possible electromagnetic τ decays, $\tau\to\mu\gamma$ or $\tau\to e\gamma$, will be allowed, depending on whether the residue of the pole at the τ mass is negative or positive.

General statements can also be made concerning the neutral leptons: ν_e,ν_μ,ν_τ. We will assume that these form the usual SU(2) doublets with the charged leptons: $(e,\nu_e),(\mu,\nu_\mu),(\tau,\nu_\tau)$. Since the global SU(2) transformations mix components within each doublet, these components must have the same norm. (We are assuming a similar dynamical mechanism for the occurrence of the μ and τ neutrinos, and therefore these neutrinos must have mass.) We see that the decay $\mu^-\to e^-\bar{\nu}_e\nu_\mu$ is allowed since the norms of both the initial and final states are negative.[6] Note, however, that the processes $\mu^-\to e^-\nu_e\bar{\nu}_\mu$ and $\mu^+e^-\to\mu^-e^+$, for example, are also allowed (they may, of course, be suppressed for other reasons). Since they are not absolutely forbidden, we would expect to see these processes. In essence, a negative norm for the μ doublet acts like a multiplicative quantum number. Also observe that ν_e and ν_μ are not allowed to mix, but one of the cases ν_e and ν_τ or ν_μ and ν_τ can mix, depending on the norm of the τ doublet.

The question of what types of interactions might dynamically generate additional poles in the electron propagator is one of great interest, but it will not be discussed in detail here. It is important to note, however, that the basic electromagnetic interaction (minimal coupling) is probably not sufficient. This is apparent if one examines the Källén-Lehman spectral decomposition of the electron propagator.[7] In particular, the spectral functions must satisfy positivity constraints, and it therefore seems impossible to generate negative-norm poles. From this point of view, it is not at all surprising that the poles in the hydrogen-atom propagator all have the same sign. It appears that there must be a fundamental negative-norm state in the underlying theory in order to realize this possibility. Such a theory, involving a negative-norm boson, is currently under investigation.

For the purposes of this article we will content ourselves by considering a model theory which exhibits multiple excitations at the bare level. This theory has been considered previously by ourselves[8,9] (we will refer to this paper as I) and also by Fried and Plebanski.[10]

IV. THIRD-ORDER SPINOR FIELD THEORY

Our starting point is a spinor field theory exhibiting three excitations:

$$\Lambda=\frac{3}{m^2}\bar{\psi}(i\partial\!\!\!/-m_1)(i\partial\!\!\!/-m_2)(i\partial\!\!\!/-m_3)\psi . \quad (4.1)$$

The factor 3 is for convenience and the factor m^2 is to ensure that the field ψ has the canonical dimension. These factors can be absorbed into the definition of ψ without changing the physical content of the theory. The propagator $S(k)$ for the spinor field may be written as

$$S(k)=\frac{im^2}{3}[(k\!\!\!/-m_1)(k\!\!\!/-m_2)(k\!\!\!/-m_3)]^{-1}$$
$$=\frac{i}{3}\sum_{j=1}^{3}Z_j(k\!\!\!/-m_j)^{-1} , \quad (4.2)$$

where we have defined Z_j as the residue of the poles:

$$Z_j=m^2(m_j-m_i)^{-1}(m_j-m_k)^{-1},\quad i\neq j\neq k . \quad (4.3)$$

We will assume that these three excitations represent the charged leptons. However, before we introduce interactions, it is important to examine the norms of these three states. Note that the mass of a fermion may be positive or negative,[9,11] and we may assume that the electron mass m_1 is positive without loss of generality. We will take m_2 as the μ mass and m_3 as the τ mass so we have $|m_1|<|m_2|<|m_3|$. There are now four possible cases:

(A) $m_2>0,m_3>0\to Z_1>0,Z_2<0,Z_3>0$,

(B) $m_2<0,m_3>0\to Z_1<0,Z_2>0,Z_3>0$,

(C) $m_2>0,m_3<0\to Z_1<0,Z_2>0,Z_3>0$,

(D) $m_2<0,m_3<0\to Z_1>0,Z_2<0,Z_3>0$.

Therefore, the only transition which may be dynamically allowed in each case is

(A) $\tau\to e\gamma$,

(B) $\tau\to\mu\gamma$,

(C) $\tau\to\mu\gamma$,

(D) $\tau\to e\gamma$.

So we have the interesting results that, in this model at least, no matter what the choices are for the signs of the masses, the decay $\mu\to e\gamma$ is forbidden.

To include the possible electromagnetic interactions we begin by making the minimal substitution in the Lagrangian density (4.1):

$$\Lambda=-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$+\frac{3}{m^2}\bar{\psi}(iD\!\!\!\!/-m_1)(iD\!\!\!\!/-m_2)(iD\!\!\!\!/-m_3)\psi , \quad (4.4)$$

where we have defined

$$D_\mu=\partial_\mu+ieA_\mu . \quad (4.5)$$

In paper I we have demonstrated that this theory forbids all transitions between the three excitations, and therefore the minimal substitution is not the most general interaction allowed in this case. However, it is instructive to see how the absence of transitions comes about in this case. This is most easily seen by transforming the theory to first-order form. Although the physical interpretation of this theory becomes manifest in first-order form, finding the appropriate transformation is a highly nontrivial mathematical problem. Since these details are not of special interest for the purposes of this paper, we only outline the steps, give the result, and indicate a remarkable feature of this transformation.

The generating functional for this theory, in the absence of external sources, may be written as

$$Z=\int[dA][d\bar{\psi}][d\psi]\exp\left[i\int d^4x\,\Lambda\right] , \quad (4.6)$$

where Λ is given in Eq. (4.4). We now introduce two auxiliary fields X_1 and X_2 in such a way that the effective Lagrangian density written in terms of ψ, X_1, and X_2, satisfies the following three conditions.

(i) The spinor part of the effective Lagrangian density is only first order in the derivatives.

(ii) The effective Lagrangian density is Hermitian.

(iii) Functional integration over the auxiliary field variables X_1 and X_2 reproduces Eq. (4.6). The essentially unique result for the spinor part of the effective Lagrangian density is

$$\Lambda_{\text{eff}}=(\bar{\psi}\ \ \bar{X}_1\ \ \bar{X}_2)\begin{bmatrix}\alpha_1&\beta_3&\beta_2\\ \beta_3&\alpha_2&\beta_1\\ \beta_2&\beta_1&\alpha_3\end{bmatrix}\begin{bmatrix}\psi\\ X_1\\ X_2\end{bmatrix} , \quad (4.7)$$

where we have defined

$$\alpha_1=\tfrac{1}{3}(\lambda_1+\lambda_2+\lambda_3),\ \ \alpha_2=\tfrac{1}{6}(4\lambda_1+\lambda_2+\lambda_3),\ \ \alpha_3=\tfrac{1}{2}(\lambda_2+\lambda_3)\,,$$
$$\beta_1=\frac{1}{2\sqrt{3}}(-\lambda_2+\lambda_3),\ \ \beta_2=\frac{1}{\sqrt{6}}(\lambda_2-\lambda_3),\ \ \beta_3=\frac{1}{3\sqrt{2}}(2\lambda_1-\lambda_2-\lambda_3)\,, \tag{4.8}$$
$$\lambda_1=Z_1{}^{-1}(i\not{D}-m_1),\ \ \lambda_2=Z_2{}^{-1}(i\not{D}-m_2),\ \ \lambda_3=Z_3{}^{-1}(i\not{D}-m_3)\,,$$

and the Z_i are given in Eq. (4.3).

We now perform a unitary transformation on the spinor fields to diagonalize the effective Lagrangian density. This transformation is given by

$$\begin{bmatrix}\psi\\ X_1\\ X_2\end{bmatrix}=\frac{1}{\sqrt{6}}\begin{bmatrix}\sqrt{2}&\sqrt{2}&\sqrt{2}\\ 2&-1&-1\\ 0&\sqrt{3}&-\sqrt{3}\end{bmatrix}\begin{bmatrix}\psi_1\\ \psi_2\\ \psi_3\end{bmatrix}, \tag{4.9}$$

and the spinor part of the effective Lagrangian density now becomes

$$\Lambda_{\rm eff}=(\bar\psi_1\ \ \bar\psi_2\ \ \bar\psi_3)\begin{bmatrix}\lambda_1&0&0\\ 0&\lambda_2&0\\ 0&0&\lambda_3\end{bmatrix}\begin{bmatrix}\psi_1\\ \psi_2\\ \psi_3\end{bmatrix}, \tag{4.10}$$

where the λ_i are given in Eq. (4.8). From this result we may immediately draw the following conclusions.

(i) The norms of the physical states are Z_i, as expected from Eq. (4.2).

(ii) There are no transitions between the different fermion excitations.

(iii) The theory is essentially equivalent to QED with the separate fields (necessarily with the same charge), the only difference being the different norms of the physical states.

(iv) Since the theory is equivalent to QED, it is renormalizable, though the renormalizability of the theory in form (4.4) is not immediately evident.[9]

A remarkable property of transformation (4.9) is the following: If we consider possible permutations of the physical fields ψ_1,ψ_2,ψ_3, we see that the fields ψ and (X_1,X_2) form, respectively, one- and two-dimensional irreducible representations of the permutation group S_3. In fact, the original field ψ must form the one-dimensional symmetric representation, and the auxiliary fields must form the two-dimensional representation of S_3 in order to allow the transformation from Eq. (4.6) to Eq. (4.7). This is why we stated that the transformation is essentially unique [obviously a rotation in the (X_1,X_2) plane does not effect the final outcome]. This result generalizes to field theories of order N: The original field ψ forms the symmetric representation of S_N, while the auxiliary fields $(X_1,X_2,\dots,X_{N-1})$ form an $(N-1)$-dimensional representation (the one below the symmetric representation on the Young tableau). The precise nature and reason for this connection between special representations of the permutation group and properties of functional integral is currently under investigation. However, also note that there seems to be a connection between the appearance of the S_3 symmetry here and the work on permutation symmetry for the leptons of Derman.[12] These issues will be discussed elsewhere. We will exploit the presence of this symmetry to find interactions which will yield the allowed transition.

It is clear from Eq. (4.10) that there are three global symmetries of the Lagrangian density:

$$\psi_i\to e^{i\alpha_i}\psi_i,\ \ i=1,2,3 \tag{4.11}$$

where the α_i are arbitrary constants. Therefore there are three superselection rules (i.e., the Hilbert space splits into separate e, μ, and τ subspaces). For this theory to have positive probabilities we require only one superselection rule. Consider the case in which all masses are positive [case (A)], so that the transition $\tau\to e\gamma$ may occur. To preserve the electromagnetic gauge invariance, the interaction which allows this transition must be of magnetic dipole form. In representation (4.7) the matrix corresponding to the permutation $(1\leftrightarrow 3)$ is given by

$$(3\ \ 2\ \ 1)=\begin{bmatrix}1&0&0\\ 0&-1/2&-\sqrt{3}/2\\ 0&-\sqrt{3}/2&1/2\end{bmatrix}. \tag{4.12}$$

So we assume that the form of the transition Lagrangian density is

$$\Lambda=(\bar\psi\ \ \bar X_1\ \ \bar X_2)(T)\begin{bmatrix}\psi\\ X_1\\ X_2\end{bmatrix}, \tag{4.13}$$

where we have defined

$$(T)=\left[s\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+t\begin{bmatrix}1&0&0\\0&-1/2&-\sqrt{3}/2\\0&-\sqrt{3}/2&1/2\end{bmatrix}\right]\sigma_{\mu\nu}F^{\mu\nu}\,, \tag{4.14}$$

where s and t are constants. We see that the term t will induce transitions between τ and e. This term will also generate anomalous magnetic moments, hence the term s is also included for generality. If we now transform to "diagonal" form using transformation (4.9), the transition Lagrangian density is

$$\Lambda_T=(\bar{\psi}_1\;\;\bar{\psi}_2\;\;\bar{\psi}_3)\begin{bmatrix}s&0&t\\0&s+t&0\\t&0&s\end{bmatrix}(\sigma_{\mu\nu}F^{\mu\nu})\begin{bmatrix}\psi_1\\\psi_2\\\psi_3\end{bmatrix}. \tag{4.15}$$

It is interesting to note that the transition term for $\tau\rightarrow e\gamma$ induces an anomalous magnetic moment for μ. Finally, it is possible to return to the higher-order field theory by integrating the auxiliary field variables, and a condition can be placed on s and t to ensure that the resulting Lagrangian density is local. However, this form of the theory is not especially enlightening and somewhat complicated so we do not give the result here.

If Eq. (4.15) is added to Eq. (4.10), the resulting theory now possesses only two global symmetries:

$$\psi_1\rightarrow e^{i\alpha}\psi_1 \text{ and } \psi_3\rightarrow e^{i\alpha}\psi_3\,,\qquad \psi_2\rightarrow e^{i\beta}\psi_2\,. \tag{4.16}$$

The simultaneous change in phase for all three fields yields conservation of charge, and the additional superselection rule is the one required for positive probability. This theory (with transitions) is not renormalizable in perturbation theory, neither in "diagonalized" forms nor in higher-order form. Our purpose in introducing the additional couplings of Eq. (4.15) was only to find a theory with negative norms preserving positive probabilities in the presence of transitions.

A final comment on the original form of this model, without transitions, is in order. We have demonstrated that the quantum field theory defined by Lagrangian density (4.4) is equivalent to usual QED with three independent fields [Eq. (4.10)]. It is quite clear, however, that the perturbation expansion of the higher-order theory will have a highly different structure than that of the usual theory.[9] There exists the possibility, therefore, that a perturbative calculation in the higher-order theory will be nonperturbative with respect to the first-order form. In fact, this seems likely since in the higher-order theory we have treated two of the three spinor fields exactly. This problem is currently under investigation.

V. SUMMARY AND CONCLUSIONS

The major result in this article can be stated as follows: In any theory containing both positive- and negative-norm states, the negative-norm states can be physical as long as the theory possesses a global symmetry and corresponding superselection rule which forbids transitions between positive- and negative-norm states.

There are other places in field theory where negative-norm states arise, but there are no superselection rules to allow them to be physical states. In gauge field theories, the presence of ghost states is necessary to ensure the gauge invariance and unitarity of the perturbation expansion. In this case, the ghost states are a mathematical construction introduced to subtract the gauge-noninvariant and nonunitary parts from the naive perturbation expansion. Another example is in the covariant formulation of quantum electrodynamics where the negative-norm scalar photon and the positive-norm longitudinal photon are projected out of the complete Hilbert space to obtain the physical states. In this case this is necessary since a transition between a physical photon and a scalar photon is not forbidden by a global symmetry (this unphysical process could occur in Compton scattering, for example).

These facts should not make us develop a "ghost prejudice." Indeed, physical ghosts may actually occur in nature, and our acceptance of them may lead to a deeper understanding of the world around us.

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