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Simultaneous fit of the spectra of light and heavy self-conjugate mesons

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It is shown that a simultaneous nonrelativistic fit of the spectra of both the light and heavy self-conjugate mesons is possible by an effective non-Coulombic power-law potential of the form $V(r)=Ar^{0.1}+V_0$. This purely phenomenological potential is found to provide a very good account of the mass spectra and the leptonic decay widths of the ρ^0, ϕ, ψ, and Υ systems in a flavor-independent manner. In spite of the smallness of the constituent quark masses involved, the nonrelativistic fit for the light mesons of ρ^0 and ϕ systems is excellent.

I. INTRODUCTION

The implications of the power-law-potential model have been investigated by many authors,[1] particularly in the context of the phenomenological logarithmic-potential model for the heavy-quark–antiquark bound systems. But in view of some recent experiments giving very accurate data on the Υ system[2] and also some fresh information about the pseudoscalar partners of ψ and ψ',[3] the power-law-potential model has again come back into the game. In fact, it has been shown[4] that the most up-to-date data on the charmonium and Υ spectra can be very well fitted by a power-law potential of the form

$$V(r)=Ar^{\nu}+V_0 \tag{1.1}$$

with ν close to 0.1 and $A>0$. The non-Coulombic short-distance behavior of this potential, which is in apparent contradiction with the predictions of quantum chromodynamics (QCD), does not pose any problem to explain the fine-hyperfine splittings, if we formally prescribe[5] the spin dependence to be generated through this static confining potential in the form of an approximately equal admixture of scalar and vector parts with no contributions from the anomalous quark magnetic moments. Now, considering flavor independence as a guiding factor, one may be tempted to extend this nonrelativistic formalism of heavy mesons to the study of light mesons. The main purpose of the present work is to modify our previous nonrelativistic fit of the ψ and Υ levels[4] by such a power-law potential to include systems containing light quarks. Work in this direction, giving a simultaneous fit of $b\bar{b}$, $c\bar{c}$, $s\bar{s}$, and $c\bar{s}$ spectra by the power-law potential, has already been done by Martin.[6] In spite of the smallness of the strange-quark mass, his nonrelativistic fit has been found to be astonishingly good. This inspires us to be more daring in order to obtain a simultaneous nonrelativistic description of the spectra of both the light and heavy mesons in the framework of our power-law-potential model in a flavor-independent manner. Here we would mainly concentrate on a group of self-conjugate light and heavy mesons ρ^0, ϕ, ψ, Υ, and the yet-to-be observed ξ which correspond to the like-flavored quark-antiquark configurations $(1/\sqrt{2}(u\bar{u}-dd)$, $s\bar{s}$, $c\bar{c}$, $b\bar{b}$, and possibly $t\bar{t}$, respectively. Our approach in this work would be to study some gross features such as mass spectra and leptonic decay widths of these mesons in a nonrelativistic Schrodinger formalism. In this connection, it must be pointed out that the usual nonrelativistic Schrödinger-type approach for heavy quarkonia has been justified on the basis of the large quark masses involved. But the same approach may be unsuitable for the ordinary light mesons due to relativistic complications, expected to be significant in these cases. On the other hand, the relativistic generalizations attempted in some limited senses by some authors,[7] are by no means simple and straightforward. Therefore nonrelativistic-potential-model studies are often extended[8] to include light hadrons, which gives, if not a quantitative, at least a qualitative understanding. With this contention in mind, we feel that a simultaneous fit of both light and heavy self-conjugate mesons is possible which would provide a comprehensive picture of the applicability of the simple phenomenological power-law potential.

II. THEORY FOR THE SCHRÖDINGER FIT

In this section, we derive the expressions for the Schrödinger bound-state masses and the leptonic decay widths of $q\bar{q}$ systems in the power-law-potential model.

First of all, let us obtain an expression for the bound-state mass of a $q\bar{q}$ system from the Schrödinger equation ($\hbar = c = 1$)

$$\frac{d^2U(r)}{dr^2} + \left[m_q[E - V(r)] - \frac{l(l+1)}{r^2} \right] U(r) = 0 , \qquad (2.1)$$

where the symbols have their usual meaning. Taking $V(r)$ as given in (1.1) and making a substitution $\rho = (r/r_0)$ with the scale factor r_0 chosen conveniently as

$$r_0 = (m_q A)^{-1/(\nu+2)} , \qquad (2.2)$$

Eq. (2.1) reduces to the form

$$\frac{d^2U(\rho)}{d\rho^2} + \left[\epsilon - \rho^\nu - \frac{l(l+1)}{\rho^2} \right] U(\rho) = 0 \qquad (2.3)$$

where

$$\epsilon = m_q(E - V_0)(m_q A)^{-2/(\nu+2)} . \qquad (2.4)$$

Now for $\nu > 0$, a bound-state solution to Eq. (2.3) is always possible either by the semiclassical method[9] or by an exact numerical method which would give a positive definite $\epsilon = \epsilon_{nl}$ (say) corresponding to a $q\bar{q}$ bound state of radial and orbital quantum numbers "n" and "l", respectively, with the binding energy $E = E_{nl}$ obtained from (2.4) as

$$E_{nl} = a(a/m_q)^{\nu/(\nu+2)} \epsilon_{nl} + V_0 , \qquad (2.5)$$

where

$$a = (A)^{1/(\nu+1)} . \qquad (2.6)$$

Then the nonrelativistic Schrödinger bound-state mass of the $q\bar{q}$ system follows immediately,

$$M_{nl}(q\bar{q}) = 2m_q + V_0 + a(a/m_q)^{\nu/(\nu+2)} \epsilon_{nl} . \qquad (2.7)$$

If we take the semiclassical WKB results of the solution of (2.3), then we have ϵ_{nl} and the absolute square of the S-state wave functions at the origin $|\Psi_{ns}(0)|^2$ in the following form:

$$\epsilon_{nl} = [F(\nu)(n + \tfrac{1}{2}l - \tfrac{1}{4})]^{2\nu/(\nu+2)} , \qquad (2.8)$$

$$|\Psi_{ns}(0)|^2 = \frac{1}{2\pi^2} \left[\frac{\nu}{\nu+2} \right] \{ a m_q [a F(\nu)]^\nu \}^{3/(\nu+2)} \times (n - \tfrac{1}{4})^{2(\nu-1)/(\nu+2)} , \qquad (2.9)$$

where

$$F(\nu) = \frac{2\sqrt{\pi}\,\Gamma(\frac{3}{2} + 1/\nu)}{\Gamma(1 + 1/\nu)} . \qquad (2.10)$$

Now using these results one can obtain the gross features like the bound-state masses and the leptonic decay widths of $q\bar{q}$ systems. Equation (2.7), aided with (2.8) and (2.10), gives obviously the Schrödinger bound-state masses of $q\bar{q}$ systems corresponding to various possible radial ($n = 1,2,3,\ldots$) and orbital ($1 = 0,1,2,\ldots$) states. But since the semiclassical formulas are approximate ones, they become worse for the excited states, particularly with $L \neq 0$. For example, Eq. (2.7) would predict a degenerate $2S$ and $1D$ level of the $q\bar{q}$ bound systems, which is not true at least in case of the charmonium spectrum. Therefore for exactness, we calculate the ϵ_{nl} values by numerically solving Eq. (2.3) for different values of n and l. Now Eq. (2.7) with these computed values of ϵ_{nl} would give the accurate bound-state masses of $q\bar{q}$ systems. Again in order to be more accurate in calculating the $|\Psi_{ns}(0)|^2$ values from these values of ϵ_{nl}, Eq. (2.9) may be expressed with the help of (2.8) as

$$|\Psi_{ns}(0)|^2 = \frac{1}{2\pi^2} \left[\frac{\nu}{\nu+2} \right] (a^{\nu+1} m_q)^{3/(\nu+2)} \times F(\nu) \epsilon_{nl}^{(\nu-1)/\nu} . \qquad (2.11)$$

The leptonic decay widths can be obtained by using the Van Royen—Weisskopf formula[10] in the form

$$\Gamma(nS \to e^+e^-) = \frac{16\pi\alpha^2 e_q^2}{M_{ns}^2(q\bar{q})} |\Psi_{ns}(0)|^2 . \qquad (2.12)$$

However, this formula should not be trusted too much in its absolute value, as it is affected by large but, not quite certain corrections.[11] In that case the leptonic-decay-width ratio given by

$$R_w = \frac{\Gamma(ns \to e^+e^-)}{\Gamma(1S \to e^+e^-)} = \left[\frac{M_{1S}(q\bar{q})}{M_{ns}(q\bar{q})} \right]^2 \frac{|\Psi_{ns}(0)|^2}{|\Psi_{1S}(0)|^2} \qquad (2.13)$$

would serve as a meaningful and reliable quantity. Now using Eq. (2.11), Eq. (2.13) can be expressed in terms of ϵ_{nl} as

$$R_w = \left[\frac{M_{1S}(q\bar{q})}{M_{ns}(q\bar{q})} \right]^2 (\epsilon_{1S}/\epsilon_{ns})^{(1-\nu)/\nu} \qquad (2.14)$$

TABLE I. Schrödinger mass spectrum for ρ^0 and ϕ systems.

			$M_{n1}(\rho^0)$ (GeV)		$M_{n1}(\phi)$ (GeV)	
n	l	ϵ_{n1}	Theory	Experiment	Theory	Experiment
1	S	1.2364	0.770	0.770 ± 0.005	1.0196	1.0196 ± 0.0001
2	S	1.3347	1.402	$\sim1.25(\rho'?)$	1.640	1.65
3	S	1.3923	1.772	~1.6 $(\rho''?)$	2.004	~1.90
4	S	1.4335	2.037		2.264	
5	S	1.4657	2.224		2.467	
1	P	1.3071	1.225	$1.31(A_2?)$	1.466	1.44 ±0.005
2	P	1.3731	1.649		1.883	
1	D	1.3544	1.529		1.765	

Equation (2.14) with the calculated values of ϵ_{nl} and M_{ns} enables us to obtain the leptonic-decay-width ratios of $q\bar{q}$ systems.

III. FIXATION OF PARAMETERS AND THE RESULTS

Now for phenomenological study we first of all fix the parameter $\nu=0.1$ in close agreement with our previous finding[4] and solve numerically Eq. (2.3) to obtain ϵ_{nl} values for different quark-antiquark bound states. Then, taking the experimental masses of ψ, ψ', Υ, Υ', and ϕ as inputs in (2.7), we determine the potential parameters a, V_0 and the constituent quark masses m_s, m_c, and m_b as

$$(a,V_0)=(5.6811,-8.0248)\text{ GeV},$$
$$(m_s,m_c,m_b)=(0.6192,1.8567,5.2024)\text{ GeV}. \tag{3.1}$$

However for the light meson $\rho^0(770)$ considered to be a $q\bar{q}$ configuration such as $(u\bar{u}-d\bar{d})/\sqrt{2}$, we take its experimental ground-state mass as the input and obtain the effective quark mass $m_q=0.4228$ GeV.

Then, using the potential parameters and the constituent quark masses thus obtained in Eq. (2.7), we can calculate the bound-state mass spectrum for ρ^0, ϕ, ψ, and Υ systems. The results in comparison with the corresponding experimental masses are presented in Tables I and II. We observe that particularly for ψ and Υ systems, for which numerous and convincing experimental data are available, the agreement with our computed values is excellent. For the light meson ϕ, some new experimental material has come out. The first radial excitation of ϕ called ϕ' has been discovered by the Orsay group at DCI with a mass around 1.65 GeV.[12] The second radial excitation ϕ'' is experimentally obscure, with a possible candidate at ~1.9 GeV.[13] A new evidence for the old E meson[14] has been obtained both in hadron collisions[15] and in radiative decay of the J/ψ,[16] with a mass 1.44 GeV. Similarly for the radially excited states of the light meson ρ^0, the experimental situation is not clear yet. There have been several experiments[17] in favor of a $\rho'(1600)$, whereas some other experiments[18] at the same time claim good evidence for a $\rho'(1250)$ as the 2S excited states of ρ^0. But from our calculation we find the 2S and 3S levels for ρ^0 at mass values 1402 MeV and 1772 MeV, respectively. However, we would not like to attach too much quantitative significance to these results, since for the excited states of the lighter mesons, the nonrelativistic approach used here may

TABLE II. Schrödinger mass spectrum for ψ and Υ systems.

			$M_{n1}(\psi)$ (GeV)		$M_{n1}(\Upsilon)$ (GeV)	
n	l	ϵ_{n1}	Theory	Experiment	Theory	Experiment
1	S	1.2364	3.097	3.097 ± 0.002	9.4336	9.4336 ± 0.0002
2	S	1.3347	3.686	3.686 ± 0.003	9.9944	9.9944 ± 0.00004
3	S	1.3923	4.031	4.030 ± 0.005	10.3230	10.3231 ± 0.00004
4	S	1.4335	4.278		10.5581	10.5476 ± 0.00011
5	S	1.4657	4.471	4.417 ± 0.01	10.7418	
1	P	1.3071	3.521	3.521	9.8369	
2	P	1.3731	3.916		10.2135	
1	D	1.3544	3.804	3.772	10.1068	

TABLE III. Leptonic-decay-width ratios, $R_w=\Gamma(ns\to e^+e^-)/\Gamma(1S\to e^+e^-)$ for ρ^0, ϕ, ψ, and Υ systems. Within parentheses are the available experimental values which are given for comparison.

		$R_w=\Gamma(ns\to e^+e^-)/\Gamma(1S\to e^+e^-)$			
n	l	$\rho^0\ \left[\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})\right]$	$\phi\ (s\bar{s})$	$\psi\ (c\bar{c})$	$\Upsilon\ (b\bar{b})$
1	S	1	1	1	1
2	S	0.152	0.194	0.355 (0.46±0.06)	0.448 (0.45±0.05)
3	S	0.065	0.089	0.203 (0.16±0.02)	0.287 (0.32±0.05)
4	S	0.038	0.054	0.139 (0.09±0.06)	0.211 (0.24±0.05)
5	S	0.026	0.037	0.104	0.167

prove completely unsuitable.

We also predict the mass spectrum for the yet unobserved $t\bar{t}$ system by fixing the value of the quark mass m_t arbitrarily at 20 GeV.[19] The predictions for the different bound-state masses of the $t\bar{t}$ system are listed below:

$$\begin{aligned}
&M_{1S}(t\bar{t})=38.589\ \text{GeV}\ ,\\
&M_{3S}(t\bar{t})=39.423\ \text{GeV}\ ,\\
&M_{1P}(t\bar{t})=38.968\ \text{GeV}\ ,\\
&M_{2S}(t\bar{t})=39.115\ \text{GeV}\ ,\\
&M_{4S}(t\bar{t})=39.644\ \text{GeV}\ ,\\
&M_{1D}(t\bar{t})=39.221\ \text{GeV}\ .
\end{aligned}\tag{3.2}$$

Although recent experiments at PETRA[20] have found no evidence for such heavy mesons up to an energy scale of 36.72 GeV, they can be expected to be seen in future experiments.

Finally, using Eq. (2.14), we calculate the leptonic decay-width ratios for ρ^0, ϕ, ψ and Υ systems. These results are displayed in Table III. In the case of the heavy vector mesons of $c\bar{c}$ and $b\bar{b}$ systems, these values are found to be in excellent agreement with the recently available experimental data which are given within the brackets. As in the case of the light mesons of ρ^0 and ϕ systems these quantities are not experimentally known, we cannot make any quantitative comparison of these values with the experiment. However, for the light mesons ρ^0 and ϕ we can estimate the absolute leptonic width using the somewhat corrected Van Royen-Weisskopf formula[11]:

$$\Gamma(1S\to e^+e^-)=\frac{16\pi\alpha^2 e_q^{\,2}}{M_{1S}^{\,2}(q\bar{q})}\,|\Psi_{1S}(0)|^2(1-\tfrac{16}{3}\alpha_s)\ .\tag{3.3}$$

For this calculation, we at first compute the values of $|\Psi_{1S}(0)|^2$ for ρ^0 and ϕ from the formula (2.11) and obtain

$$(|\Psi_{1S}(0)|_{\rho^0}^{\,2},|\Psi_{1S}(0)|_{\phi}^{\,2})=(0.0198,0.0342)\ \text{GeV}^3\ .\tag{3.4}$$

Now, using these values of $|\Psi_{1S}(0)|^2$ and taking the value of the quark-gluon coupling constant for both ρ^0 and ϕ meson as $\alpha_s=0.5$ which is chosen around the value obtained in conformity with the deep-inelastic scattering data and the requirements of asymptotic freedom, we obtain from (3.3) the absolute leptonic widths for ρ^0 and ϕ mesons which can be compared with their corresponding experimental values:

$$\begin{aligned}
&\Gamma_{\text{cal}}(\rho^0\to e^+e^-)=6.76\ \text{keV}\ ,\\
&\Gamma_{\text{exp}}(\rho^0\to e^+e^-)=6.5\pm0.18\ \text{keV}\ ,\\
&\Gamma_{\text{cal}}(\phi\to e^+e^-)=1.47\ \text{keV}\ ,\\
&\Gamma_{\text{exp}}(\phi\to e^+e^-)=1.43\pm0.12\ \text{keV}\ .
\end{aligned}\tag{3.5}$$

Even though the leptonic width is much more sensitive to details of the wave function and relativistic effects than the bound-state masses, we find an excellent agreement between these calculated values and the experiment.

IV. CONCLUSION

Thus in this work we find that the effective non-Coulombic power-law potential can describe the gross features of the self-conjugate light and heavy mesons in a flavor-independent manner. The overall systematics of our phenomenological predictions such as mass spectra and the leptonic-decay-width ratios for these mesons do not reflect any inadequacy in the short- or long-distance behavior of the simple power-law potential. Therefore we can argue that even if this phenomenological potential does not possess certain behavior expected from the theoretical approaches, it is found to be quite adequate for providing a simultaneous nonrelativistic fit of the spectra of ρ^0, ϕ, ψ, and Υ systems in a unified way. The relativistic effects which are considered to be significant in the case of light mesons do not spoil the results of ρ^0 and ϕ systems. Even the absolute leptonic widths of ρ^0 and ϕ mesons agree very well with the experimental values.

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