

Supergravity as the messenger of supersymmetry breaking

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A systematic study is made of theories in which supergravity is spontaneously broken in a "hidden" sector of superfields that interact with ordinary matter only through supergravity. General rules are given for calculating the low-energy effective potential in such theories. This potential is given as the sum of ordinary supersymmetric terms involving a low-energy effective superpotential whose mass terms arise from integrating out the heavy particles associated with grand unification, plus supersymmetry-breaking terms that depend on the details of the hidden sector and the Kähler potential only through the values of four small complex mass parameters. The result is not the same as would be obtained by ignoring grand unification and inserting small mass parameters into the superpotential from the beginning. The general results are applied to a class of models with a pair of Higgs doublets.

I. INTRODUCTION

It was widely hoped that supersymmetry would turn out to be spontaneously broken at energies no higher than a few hundred GeV, both in order to help in understanding gauge hierarchies and also to allow some chance of confirming supersymmetry experimentally. Unhappily, it has proved difficult to construct satisfactory theories along these lines.¹ We are led to the conclusion that supersymmetry if valid at all is spontaneously broken at energies very much greater than those of $SU(2) \times U(1)$ breaking. But then if any vestige of supersymmetry is to survive at ordinary energies to help establish a gauge hierarchy, the source of supersymmetry breaking must somehow be partly isolated from ordinary particles and interactions.²

Recently attention has been drawn to a class of interesting models of this sort.³⁻¹³ In these models, unextended ($N=1$) supersymmetry is broken by very large scalar-field vacuum expectation values (VEV's) of order 10^{19} GeV, but the scalars that have these large VEV's form a "hidden sector," that does not interact directly with the ordinary fields (quarks, leptons, gauge and Higgs bosons, and their superpartners) of the "observable sector." That is, the superpotential of the theory breaks up into a sum of two terms^{14,15}

$$f_{\text{TOTAL}}(S, \tilde{S}) = f(S) + \tilde{f}(\tilde{S}),$$

where S^a and \tilde{S}^h are the left-chiral superfields of the

observable and hidden sectors, respectively. With a minimal kinetic term and no other interactions, the potential of the scalar (nonauxiliary) components z^a, \tilde{z}^h of S^a, \tilde{S}^h would take the form

$$\begin{aligned} V(z, \tilde{z}) &= \sum_{\text{all } z} \left| \frac{\partial f_{\text{TOTAL}}}{\partial z} \right|^2 \\ &= \sum_a \left| \frac{\partial f(z)}{\partial z^a} \right|^2 + \sum_h \left| \frac{\partial \tilde{f}(\tilde{z})}{\partial \tilde{z}^h} \right|^2 \end{aligned}$$

and the spontaneous breakdown of supersymmetry in the hidden sector could have no effect on the observable sector. In the models of Refs. 3-12 the news that supersymmetry is broken by the \tilde{z}^h VEV's is carried over to the observable superfields by gravity and its superpartners, which interact with both sectors.

In the papers of Ref. 3, a thorough study is presented of a model with a specific linear hidden-sector superpotential \tilde{f} , and a specific grand-unified observable sector. Their results exhibit some remarkable features; in particular, the VEV's of the light Higgs scalars are of order of the gravitino mass m_g , and do not depend in any way on the grand-unified mass scale M_{GU} , but do depend on coupling parameters of heavy fields whose masses are of order M_{GU} . However, because the model studied was so specific, and the results were expressed in terms of values for scalar VEV's, it was difficult to see how the decoupling of heavy from light degrees of

freedom works in these models, and it was difficult to know what aspects of the results would apply in general.¹⁶

The papers of Refs. 4 and 6 dealt with models that were in various respects more general. Reference 4 considered the same linear superpotential for the hidden sector, but put no restrictions on the form of the superpotential for the observable sector. Reference 6 considered a general superpotential for the hidden sector, and restricted the form of the observable sector only by requiring that its superpotential be purely trilinear. However, neither of these groups considered grand-unified models, in which the observable-sector superpotential involves mass scales $M_{\text{GU}} \gg m_g$. As shown by the work of Ref. 3 (and more generally in Sec. III below), the existence of a class of superheavy particles which have to be "integrated out" to construct the low-energy effective potential changes the way that the supersymmetry-breaking corrections appear in this effective potential, in a manner that (except for purely trilinear superpotentials like that of Ref. 6) cannot be simulated by inserting mass terms in an observable-sector superpotential involving only light fields.

It seemed to us that it would be useful to present a study of this class of models, with general superpotentials for both the hidden and observable sectors, and with full attention to the complications

caused by the presence of heavy particles with masses of order $M_{\text{GU}} \gg m_g$. Our assumptions are spelled out in Sec. II, and in Sec. III we present our main result, a general formula [Eq. (3.11)] for the effective superpotential of the light scalars. In this formula the unknown properties of the hidden sector enter in the values of just two comparable mass parameters, m_g and m_g' , one of them the gravitino mass, and all aspects of the full grand-unified theory enter only in the parameters of an effective superpotential. Section IV generalizes this result to a large class of Kähler metrics, and shows that this introduces just two more unknown mass parameters, m_g'' and m_g''' . In Sec. V we show how our results can be used to derive phenomenologically interesting predictions, even without having to make any specific assumptions about the grand-unified theory or the hidden sector.

II. ASSUMPTIONS

We assume a total superpotential of the form^{14,15}

$$f_{\text{TOTAL}}(S, \tilde{S}) = f(S) + \tilde{f}(\tilde{S}). \quad (2.1)$$

Here $f(S)$ and $\tilde{f}(\tilde{S})$ are the superpotentials for the chiral superfields S^a and \tilde{S}^h of the observed and hidden sectors, respectively. The potential for the scalar field components z^a and \tilde{z}^h is then¹⁷

$$V(z, \tilde{z}) = \exp \left[8\pi G \left(\sum_a |z^a|^2 + \sum_h |\tilde{z}^h|^2 \right) \right] \left[\sum_a \left| \frac{\partial f(z)}{\partial z^a} + 8\pi G z^a [f(z) + \tilde{f}(\tilde{z})] \right|^2 + \sum_h \left| \frac{\partial \tilde{f}(\tilde{z})}{\partial \tilde{z}^h} + 8\pi G \tilde{z}^h [f(z) + \tilde{f}(\tilde{z})] \right|^2 - 24\pi G |f(z) + \tilde{f}(\tilde{z})|^2 \right] + \sum_k D_k^2. \quad (2.2)$$

This is for a quadratic d function (i.e., a flat Kähler metric); we will return to the general case later, in Sec. IV. The gauge auxiliary scalar D_k in (2.2) takes the usual form

$$D_k \equiv \sum_{a,b} (t_k)^a_b z^a z^{*b}, \quad (2.3)$$

where t_k is the Hermitian matrix representing the k th gauge generator, including coupling-constant factors. We assume that there are no Fayet-Iliopoulos terms, and that the hidden-sector fields are neutral with regard to all gauge symmetries, but it would be easy to include the effects of additional

gauge fields that interact only with the hidden sector.

Our assumptions regarding the observable and hidden superpotentials are as follows.

A. Observable sector

It is assumed that there is a set of scalar field VEV's, z_0^a , for which, in the absence of the hidden sector, supersymmetry would be unbroken and spacetime would be flat:

$$\partial f(z) / \partial z^a = 0, \quad \text{at } z = z_0, \quad (2.4)$$

$$D_k = 0, \text{ at } z = z_0, \quad (2.5)$$

$$f(z_0) = 0. \quad (2.6)$$

[Of course, we can always make $f(z)$ vanish at the z_0 defined by (2.4) and (2.5) by shifting a constant term from $f(z)$ to $\tilde{f}(\tilde{z})$.] The tree-approximation scalar spectrum in the absence of the hidden sector then consists of a complex scalar of mass M for each eigenvalue M^2 of the Hermitian matrix

$$M^2_{ab} = \sum_c f_{ac}^* f_{bc} \quad (2.7)$$

with subscripts denoting differentiation with respect to z^a, z^b , etc., at $z = z_0$:

$$f_{ab} \equiv \left[\frac{\partial^2 f(z)}{\partial z^a \partial z^b} \right]_{z=z_0},$$

$$f_{abc} \equiv \left[\frac{\partial^3 f(z)}{\partial z^a \partial z^b \partial z^c} \right]_{z=z_0}, \text{ etc. ,}$$

plus a real scalar with mass μ for each nonzero eigenvalue μ^2 of the vector-boson mass² matrix:

$$\mu^2_{kl} = (z_0^\dagger \{t_k, t_l\} z_0) = 2((t_k z_0)^\dagger (t_l z_0)). \quad (2.8)$$

[The second version of this formula follows from the first and Eq. (2.5).] The gauge symmetries of $f(z)$ [plus (2.4)] imply that

$$\sum_b f_{bc} (t_k z_0)^b = \sum_b M^2_{ab} (t_k z_0)^b = 0. \quad (2.9)$$

These are the ‘‘Goldstone’’ eigenvectors of M^2_{ab} , for which the corresponding scalars are eliminated by the Higgs mechanism. We assume for reasons of naturalness that $f(z)$ depends on only a single grand-unified mass scale M_{GU} (presumably $M_{\text{GU}} \sim 10^{17}$ GeV), so that aside from coupling-constant factors, we have $z_0^a \sim M_{\text{GU}}$, $f_{ab} \sim M_{\text{GU}}$, $f_{abc} \sim 1$, and the eigenvalues of M^2_{ab} and μ^2_{kl} are either of order M_{GU}^2 or zero.¹⁸ We adopt a basis in which these matrices are diagonal, with the zero and nonzero eigenvalues of μ^2_{kl} labeled κ, λ, \dots and K, L, \dots , respectively (one nonzero eigenvalue for each linearly independent Goldstone vector $t_k z_0$), and the Goldstone, zero non-Goldstone, and nonzero eigenvalues of M^2_{ab} labeled K, L, \dots ; α, β, \dots ; and A, B, \dots , respectively. That is

$$\mu^2_{KL} = \delta_{KL} \mu_K^2, \quad (2.10)$$

$$\mu^2_{\kappa L} = \mu^2_{\kappa \lambda} = 0, \quad (2.11)$$

$$M^2_{AB} = \delta_{AB} M_A^2, \quad (2.12)$$

$$M^2_{\alpha B} = M^2_{\alpha \beta} = M^2_{\alpha K} = M^2_{KB} = M^2_{KL} = 0, \quad (2.13)$$

with μ_K^2 and M_A^2 nonzero and of order M_{GU}^2 .

Correspondingly, the fields z^a are classified as follows:

z^α : light complex scalars, corresponding to non-Goldstone eigenvectors of M^2_{ab} with eigenvalue zero (Higgs bosons, s-quarks, s-leptons).

z^A : superheavy complex scalars, corresponding to nonzero eigenvalues of M^2_{ab} .

z^K : superheavy real scalars, degenerate with superheavy gauge bosons, corresponding to independent Goldstone eigenvectors $(t_K z_0)^a$ of M^2_{ab} , one for each nonzero eigenvalue of μ^2_{KL} . (The z^K are real because the imaginary part of the coefficients of $t_K z_0$ are Goldstone bosons, eliminated by the Higgs mechanism.)

Because z^α and z^A are orthogonal to z^K , we have

$$(t_K z_0)^\alpha = (t_K z_0)^A = 0. \quad (2.14)$$

Also, because the z^L correspond to nonzero eigenvalues μ_L^2 of μ^2_{kl} , we have

$$(t_K z_0)^L = \mu_L \delta_{KL} \text{ nonsingular}. \quad (2.15)$$

Further, (2.9), (2.12), and (2.13) yield

$$f_{\alpha\beta} = f_{\alpha A} = f_{\alpha K} = f_{KA} = f_{KL} = 0, \quad (2.16)$$

$$f_{AB} \text{ nonsingular} \sim M_{\text{GU}}. \quad (2.17)$$

We will not need to assume that $f(z)$ is a cubic polynomial, as it would be if we started with a renormalizable theory. Finally, our results will turn out to depend critically on the assumption that the light scalars do not get nonvanishing VEV's from the breakdown of the grand gauge group

$$z_0^\alpha = 0. \quad (2.18)$$

This is an automatic consequence of symmetries like $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$ for Higgs bosons and scalar quarks and leptons, but may require fine tuning for light $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$ -neutral scalars. Also, it is an automatic consequence of the supersymmetry condition (2.5) that the scalar superpartners of the superheavy gauge bosons have zero VEV¹⁸:

$$z_0^K = 0. \quad (2.19)$$

Some of the z_0^A may also vanish, but they are generally of order M_{GU} .

B. Hidden sector

The superpotential $\tilde{f}(\tilde{z})$ is assumed to be proportional to a relatively small factor μ^3 , but otherwise to depend only on \tilde{z}^h and on a mass scale of order $M_{\text{PL}} = 1/\sqrt{G}$:

$$\tilde{f}(\tilde{z}) = \mu^3 \times \text{function of } \tilde{z}\sqrt{G}. \quad (2.20)$$

In the absence of the observable sector, the potential would take the form

$$\tilde{V}(\tilde{z}) = \exp \left[8\pi G \sum_h |\tilde{z}^h|^2 \right] \left[\sum_h \left| \frac{\partial \tilde{f}}{\partial \tilde{z}^h} + 8\pi G \tilde{z}^h \tilde{f} \right|^2 - 24\pi G |\tilde{f}|^2 \right]. \quad (2.21)$$

We assume that there is at least a local minimum of $V(\tilde{z})$ at a point \tilde{z}_0 , and that the additive constant in \tilde{f} can be adjusted so that \tilde{V} vanishes at this point

$$\tilde{V}(\tilde{z}) = \partial \tilde{V}(\tilde{z}) / \partial \tilde{z}^h = 0 \text{ at } \tilde{z} = \tilde{z}_0. \quad (2.22)$$

Since $\tilde{V}(\tilde{z})$ equals $\mu^6 G$ times a function of $\tilde{z}\sqrt{G}$, this condition yields a μ -independent value of order $1/\sqrt{G} = M_{\text{PL}}$ for \tilde{z}_0^h .

The supergravitational coupling between the hidden and observable sectors will introduce what appears as intrinsic supersymmetry-breaking terms in the effective Lagrangian of the observable sector. As we shall see below, the magnitude of these terms is characterized by a mass parameter

$$m_g = 8\pi G \tilde{f}(\tilde{z}_0) \exp \left[4\pi G \sum_A |z_0^A|^2 \right] \exp \left[4\pi G \sum_h |\tilde{z}_0^h|^2 \right]. \quad (2.23)$$

It so happens that $|m_g|$ is the gravitino mass, but for us the important thing about m_g is that it sets the mass scale of particles like the \tilde{W}^\pm and Z^0 . We therefore assume that

$$|m_g| \ll M_{\text{GU}} \text{ and } |m_g| \ll M_{\text{PL}} \equiv 1/\sqrt{G} \quad (2.24)$$

and for orientation we may think of m_g as roughly of order 100 GeV.¹⁹

From now on we will use m_g rather than μ to characterize the smallness of the hidden-sector superpotential. That is, μ in (2.20) is taken to be of order $(m_g/G)^{1/3}$ (or 10^{13} GeV for $m_g \simeq 100$ GeV) so that \tilde{f} is of order m_g/G , as required by (2.16). Of course, we do not at present know why μ should take this particular value, so for now m_g is simply a parameter put in by hand.

III. RESULTS

In order to characterize the breaking of supersymmetry in the observable sector in theories of the sort described in Sec. II, it seems to us most useful to calculate the complete effective potential of the light observable scalars z^α , from which we can obtain whatever information we want about scalar VEV's and masses. We do this by "integrating out" the heavy scalars z^A and z^K , expressing them as functions of z^α and \tilde{z}^h by imposing the condition that

$$\begin{aligned} \partial V / \partial z^A = \partial V / \partial z^K = 0 \text{ at } z^A = z^A(z^\alpha, \tilde{z}^h), \\ z^K = z^K(z^\alpha, \tilde{z}^h). \end{aligned} \quad (3.1)$$

Leaving the hidden fields for the moment as free parameters, the effective potential of the light scalars is then

$$V_{\text{eff}}(z^\alpha, \tilde{z}^h) = V(z^\alpha, z^A(z^\alpha, \tilde{z}^h), z^K(z^\alpha, \tilde{z}^h), \tilde{z}^h). \quad (3.2)$$

To render this calculation tractable, it is necessary at every point to use a power-series expansion in m_g . We take the light fields z^α to be of order m_g and the hidden fields \tilde{z}^h to be of order M_{PL} , because that is where experience teaches us to look for the minimum of V . The mass m_g also enters as the "smallness" parameter in \tilde{f} . Apart from m_g , the only masses in the problem are M_{GU} (perhaps 10^{17} GeV) and $M_{\text{PL}} = 1.2 \times 10^{19}$ GeV. These are not very different, so our expansion parameter will be taken as m_g/M , with M_{GU} and M_{PL} regarded as roughly of the same order of magnitude M . The expansion for the heavy fields then takes the form

$$\begin{aligned} z^A = z_0^A + z_1^A + z_2^A + \cdots, \\ z^K = z_0^K + z_1^K + z_2^K + \cdots, \end{aligned} \quad (3.3)$$

with z_n^A and z_n^K of order $M(m_g/M)^n$. To repeat, M now stands for the grand-unification mass and/or the Planck mass.

The details of this calculation are presented in Appendix A. A crucial result is that the potential V_{eff} turns out to be independent of the light scalars z^α not only in orders M^4 and $m_g M^3$, but also in orders $m_g^2 M^2$ and $m_g^3 M$. It is therefore possible to choose a z^α -independent value of the hidden fields \tilde{z}^h where the potential V_{eff} to this order is stationary in \tilde{z} , and adjust an additive constant in the superpotential \tilde{f} to make the potential vanish to this order. The values of the hidden scalars and the additive constant in \tilde{f} turn out to be just those that we would calculate according to Eq. (2.22) in the absence of the observable sector, plus small corrections of order m_g in \tilde{z}^h and of order $m_g^2 M$ in \tilde{f} . With \tilde{z}^h and the constant term in \tilde{f} fixed in this way, the leading terms in V_{eff} are of order m_g^4 . As long as we are not interested in higher terms (of order m_g^5/M , etc.) it is an adequate approximation then to neglect the

corrections to \tilde{z}^h and to the constant term in \tilde{f} , and simply take them to have the values given by (2.22), which will be indicated with a subscript 0.

Our result for the $O(m_g^4)$ terms in V_{eff} can most conveniently be expressed in terms of a low-energy effective superpotential

$$f_{\text{eff}} = f^{(3)} + f^{(2)} + f^{(1)} + f^{(0)}. \tag{3.4}$$

Here $f^{(3)}$ is proportional to the part of the original superpotential that is trilinear in *light* scalars

$$f^{(3)} \equiv \frac{1}{6} E_0^{1/2} \sum_{\alpha\beta\gamma} f_{\alpha\beta\gamma} z^\alpha z^\beta z^\gamma \tag{3.5}$$

while $f^{(2)}$, $f^{(1)}$, and $f^{(0)}$ result from the first-order shift in the heavy scalars

$$f^{(2)} \equiv \frac{1}{2} E_0^{1/2} \sum_{\alpha\beta A} f_{\alpha\beta A} z^\alpha z^\beta z_1^A, \tag{3.6}$$

$$f^{(1)} \equiv \frac{1}{2} E_0^{1/2} \sum_{\alpha AB} f_{\alpha AB} z^\alpha z_1^A z_1^B, \tag{3.7}$$

$$f^{(0)} \equiv \frac{1}{6} E_0^{1/2} \sum_{ABC} f_{ABC} z_1^A z_1^B z_1^C. \tag{3.8}$$

Also E_0 is the constant factor

$$E_0 = \exp \left[8\pi G \sum_A |z_0^A|^2 \right] \exp \left[8\pi G \sum_h |\tilde{z}_0^h|^2 \right] \tag{3.9}$$

and z_1^A is the first-order shift in the heavy scalars

$$z_1^A = - \sum_B f_{AB}^{-1} z_0^B m_g^*. \tag{3.10}$$

Note that f_{AB} and z_0^B are both of order M_{GU} and independent of G , so z_1^A is of order m_g , and otherwise independent of both M_{GU} and M_{PL} , as well as of z^α . It turns out that $z_1^K = 0$, so only z_1^A appears in (3.6)–(3.8).

Our main result is the formula for the $O(m_g^4)$ terms in the effective potential of the light scalars:

$$\begin{aligned} V_{\text{eff}} = & \sum_\alpha \left| \frac{\partial f_{\text{eff}}}{\partial z^\alpha} \right|^2 + 2 \text{Re}(m_g^* f^{(3)}) \\ & + 4 \text{Re}(m_g^* f^{(2)}) + 2 \text{Re}[(4m_g - m_g^*) f^{(1)}] \\ & + |m_g|^2 \sum_\alpha |z^\alpha|^2 + \frac{1}{2} \sum_\kappa (z_0^\dagger t_\kappa z_0)^2 + V_0. \end{aligned} \tag{3.11}$$

Here m_g is the gravitino mass (2.23), which we can write as

$$m_g = 8\pi G E_0^{1/2} \tilde{f}_0 \tag{3.12}$$

and m_g' is a comparable mass parameter

$$m_g' = 8\pi G E_0^{1/2} \left[\sum_h \tilde{z}_0^h \left(\frac{\partial \tilde{f}}{\partial \tilde{z}^h} \right)_0 + 8\pi G \tilde{f}_0 \sum_h |\tilde{z}_0^h|^2 \right] \tag{3.13}$$

while t_κ are the $SU(3) \times SU(2) \times U(1)$ gauge generators, and V_0 is a constant of order m_g^4 . (It is important to note that $f^{(2)}$ and $f^{(1)}$ are, respectively, proportional to m_g and m_g^2 , and do not involve m_g' .) We can arrange to cancel the vacuum expectation value of V_{eff} , including V_0 and all radiative corrections, by a shift in the hidden-sector superpotential \tilde{f} by a constant term of order m_g^3 . The first term in (3.11) is just what we would expect in a globally supersymmetric theory with superpotential f_{eff} , while the other terms explicitly break supersymmetry.

Several features of our result are worth special mention:

(a) It is amazing how little we need to know in order to calculate the effective potential. All the unknown features of the hidden sector are embodied in just two complex mass parameters m_g and m_g' , of comparable magnitude. Also, all aspects of the grand unified theory have been boiled down to the parameters in the effective superpotential. In particular, and somewhat surprisingly, there are no terms in V_{eff} of order $m_g^4 (GM_{\text{GU}}^2)^N$, so to order m_g^4 the effective potential does not even depend on the grand-unification mass scale M_{GU} .

(b) Despite the fact that (3.11) does not depend on M_{GU} , the supernormalizable terms in the effective potential that arise here from the shifts in the heavy scalars z^A are very different from those that would arise directly from linear and quadratic terms in the original superpotential in a theory without heavy scalars. In the latter case, the potential of the light scalars to order m_g^4 would be (as in Ref. 4)

$$\begin{aligned} V = & \sum_\alpha \left| \frac{\partial f_{\text{eff}}}{\partial z^\alpha} \right|^2 + 2 \text{Re}(m_g^* f^{(3)}) + 2 \text{Re}[(m_g^* - m_g^*) f^{(2)}] + 2 \text{Re}[(m_g^* - 2m_g^*) f^{(1)}] \\ & + |m_g|^2 \sum_\alpha |z^\alpha|^2 + \frac{1}{2} \sum_\kappa (z_0^\dagger t_\kappa z_0) + V_0. \end{aligned} \tag{3.14}$$

[See Appendix C. Here f_{eff} is simply the constant $E_0^{1/2}$ times the original superpotential, with any terms of fourth or higher orders in the light fields deleted, and $f^{(n)}$ is defined as in Eqs. (3.5)–(3.8).] Comparison of (3.14) with (3.11) shows that these results are not the same, and cannot be brought into the same form by any redefinition of the constants m_g and m'_g .

(c) The bilinear and linear terms $f^{(2)}$ and $f^{(1)}$ in the potential (3.11), which distinguish our result from (3.14), may be of importance in developing realistic models.²⁰ The appearance of such terms in order m_g^4 in the effective superpotential is governed in part by the mechanism that is responsible for their nonappearance in order $m_g^3 M_{\text{GU}}$ or $m_g^2 M_{\text{GU}}^2$ in theories with a superheavy mass scale M_{GU} . As indicated in Ref. 18, there are several possibilities for this mechanism. If the breakdown of the grand-unified gauge group leaves some scalars massless and with zero VEV because of an unbroken symmetry of the whole theory, then $f^{(1)}$ and $f^{(2)}$ will not appear even in order m_g^4 , and there will be no difference between (3.11) and (3.14). If these scalar masses and VEV's vanish because of a fine tuning of the theory then $f^{(1)}$ and $f^{(2)}$ terms will in general appear in order m_g^4 , but they will be unstable to tiny changes in the fine tuning. We wish to stress that it is also possible for scalar masses and VEV's to vanish in the limit $m_g \rightarrow 0$ automatically, but not because of an unbroken symmetry of the whole theory, and in this case we generally expect $f^{(1)}$ and $f^{(2)}$ terms to arise naturally in order m_g^4 . One way that this can happen is for masses and VEV's of the light scalars to be kept zero in the limit $m_g \rightarrow 0$ by an R symmetry of the whole theory,¹⁴ which is spontaneously broken in the hidden sector. The news of R -invariance breaking would then be carried to observable fields by supergravity. For instance, suppose that the chiral superfields of the observable sector comprise a set Y^n with $R=0$ plus one X with $R=2$. The observable-sector superpotential then must take the form

$$f(X, Y) = Xg(Y).$$

The conditions for a supersymmetric vacuum solution are then

$$g(y) = 0, \quad x \partial g(y) / \partial y^n = 0 \quad (\text{all } n)$$

with lower-case letters denoting scalar components of superfields. It is natural to expect that there should be a nonzero scalar field value y_0^n at which $g(y)$ vanishes, but with $\partial g(y) / \partial y^n \neq 0$ for at least some y^n , and in this case there is a supersymmetric vacuum solution with

$$y = y_0; \quad x = 0.$$

By a linear transformation we can choose the Y^n fields so that $\partial g(y) / \partial y^n$ is nonzero at y_0 for only one of the scalars, say y^1 . That is

$$[\partial g(y) / \partial y^1]_0 = M,$$

$$[\partial g(y) / \partial y^\alpha]_0 = 0,$$

with α running over values of $n > 1$, and M nonzero and of order M_{GU} . The y^α are the light scalars whose masses vanish for $m_g \rightarrow 0$. With suitable additional symmetries, it can also be natural for their VEV's y_0^α to vanish, while y_0^1 is nonzero and of order M_{GU} . The matrix f_{AB} of second derivatives of the superpotential with respect to heavy scalars then has elements

$$(\partial^2 f / \partial x \partial x)_0 = (\partial^2 f / \partial y^1 \partial y^1)_0 = 0,$$

$$(\partial^2 f / \partial y^1 \partial x)_0 = M.$$

Equation (3.10) then gives the shift in the heavy scalars x and y^1 as

$$x_1 = -M^{-1} y_0^1 m_g = O(m_g),$$

$$y_1^1 = 0,$$

and (3.4)–(3.8) give the effective low-energy superpotential as

$$f_{\text{eff}} = \frac{1}{2} E_0^{1/2} x_1 \sum_{\alpha\beta} \left[\frac{\partial^2 g(y)}{\partial y^\alpha \partial y^\beta} \right]_0 y^\alpha y^\beta.$$

This means that we can encounter bilinear mass terms in the low-energy effective superpotential without fine tuning. In a realistic model the y^α would be the Higgs doublets; we would also have to add quark and lepton superfields with $R = +1$ and perhaps additional light singlets with $R = 2$.

(d) The only mass scale appearing in (3.11) is m_g (recall that z_1^4 and m'_g are of order m_g) so apart from coupling-constant factors, all light scalar masses and VEV's will be of order m_g . It is for this reason that we have considered the potential for z^α values of order m_g , and have taken m_g to be of order m_W .

(e) It was crucial in the calculation of V_{eff} that the terms of order $m_g^2 M^2$ and $m_g^3 M$ turned out to be independent of light scalars, and could therefore be eliminated by an adjustment of the additive constant in the hidden-sector superpotential. The third-order terms are larger than those of order m_g^4 by a factor $M_{\text{GU}}/m_g \approx 10^{15}$, so even very tiny z^α -dependent corrections to these terms could completely invalidate our results for V_{eff} . Our calculation here shows that there are no z^α -dependent corrections of higher order in GM_{GU}^2 to the m_g^2 and m_g^3 terms in the tree approximation, but it is necessary also to check

both ordinary and gravitational radiative corrections to at least fifth order in α and GM_{GU}^2 . We have not done this, but in Appendix B we analyze what properties of a general potential are needed in order that the leading terms that depend on light scalars should be of fourth order in a perturbation. We anticipate that the no-renormalization theorems of supersymmetry can be used to show that the terms in the potential due to radiative corrections actually have these properties in theories with a natural hierarchy.²¹

Even accepting that there are no z^α -dependent radiative corrections to V_{eff} of order m_g^2 and m_g^3 , there certainly are such corrections in order m_g^4 . If we were to use V_{eff} to carry out calculations of quantities measured at energies of order m_g , these

radiative corrections would be of order $\alpha \ln(M_{\text{GU}}/m_g)$, and so could not be considered small. Instead we must interpret our results as giving the effective potential for energies of order M_{GU} , and use (3.11) as the input to a renormalization-group calculation that would integrate the equations for the parameters in V_{eff} down to energies of order m_g , and only then use the results as our low-energy effective potential.^{5,7,10}

IV. GENERAL KÄHLER POTENTIALS

Up to now, our results have been based on Eq. (2.2) for the potential, corresponding to a flat Kähler metric. In general, the potential would be given by a formula¹⁷

$$V = \exp(8\pi G d) \left[\sum_{NM} (g^{-1})_M^N \left(\frac{\partial f_{\text{TOT}}}{\partial Z^N} + 8\pi G \frac{\partial d}{\partial Z^N} f_{\text{TOT}} \right) \right] \left[\frac{\partial f_{\text{TOT}}}{\partial Z^M} + 8\pi G \frac{\partial d}{\partial Z^M} f_{\text{TOT}} \right]^* - 24\pi G |f_{\text{TOT}}|^2 + \text{gauge terms}, \quad (4.1)$$

where Z^N here runs over all chiral scalars $z^a, \bar{z}^{\dot{a}}$, and

$$g_N^M \equiv \frac{\partial^2 d}{\partial Z^N \partial Z^{M*}}, \quad (4.2)$$

where d , the Kähler potential, is a function of both Z^N and Z^{N*} , while $f_{\text{TOT}} = f + \bar{f}$ is still a function of Z^N alone. Equation (4.1) reduces to (2.2) in the special case

$$d = \sum_N |Z^N|^2. \quad (4.3)$$

It does not seem reasonable to expect that the Kähler potential will oblige us by taking a form as simple as (4.3). For one thing, this is not what we find if we start with a renormalizable theory of chiral superfields and then turn on supergravity; the Weyl rescaling that is necessary in this case yields

$$d = -\frac{3}{8\pi G} \ln \left[1 - \frac{8\pi G}{3} \sum_N |Z^N|^2 \right]. \quad (4.4)$$

(This is the case $\phi = -3 + 8\pi G \sum |Z|^2$ in the notation of Cremmer *et al.*¹⁷) Another argument against (4.3) arises from the presence of gravitational radiative corrections, which could not be expected to preserve a simple formula like (4.3).

On the other hand, if we do not limit the form of the Kähler potential in any way, we can derive hardly any conclusions from (4.1). We may in the end be driven to such a pessimistic conclusion, but for the present it seems reasonable at least to explore the

possibility that d belongs to a class of functions that is wide enough to be plausible and yet narrow enough to allow us to draw interesting conclusions.

We shall assume here that the Kähler potential takes the form

$$8\pi G d(Z, Z^*) = P \left[8\pi G \sum_N |Z^N|^2 \right], \quad (4.5)$$

where $P(u)$ is a power series with coefficients of order unity. This includes (4.3) and (4.4) as special cases. Also, it is reasonable to expect that gravitational radiation corrections will at least approximately respect the form of (4.5), because in the absence of a superpotential these corrections possess a $U(n)$ symmetry among the n chiral superfields²² that would require the Kähler potential to take the form (4.5). We would expect any violations of this $U(n)$ symmetry in $d(u)$ due to gravitational radiative corrections to be suppressed by whatever small factors (m_g/M_{PL} or Yukawa couplings) appear in the superpotential.

From (4.5), we obtain the Kähler metric

$$g_N^M = P'(u) \delta_N^M + 8\pi G P''(u) Z^{N*} Z^M, \quad (4.6)$$

where

$$u \equiv 8\pi G \sum_N |Z^N|^2. \quad (4.7)$$

This has inverse

$$(g^{-1})_N^M = P'^{-1}(u)\delta_N^M + 8\pi G Q(u)Z^N Z^M, \quad (4.8) \quad Q(u) = -P''(u)/[P'^2(u) + uP'(u)P''(u)]. \quad (4.9)$$

where

Using (4.8) in (4.1) yields

$$V = e^{P(u)} \left[P'^{-1}(u) \sum_N \left| \frac{\partial f_{\text{TOT}}}{\partial Z^N} + 8\pi G P'(u) Z^N f_{\text{TOT}} \right|^2 + Q(u) \left| \sum_N Z^N \frac{\partial f_{\text{TOT}}}{\partial Z^N} + u P'(u) f_{\text{TOT}} \right|^2 - 24\pi G |f_{\text{TOT}}|^2 \right] + \text{gauge terms}. \quad (4.10)$$

For a superpotential of the form (2.1), Eq. (4.10) takes the form

$$V = e^{P(u)} \left[P'^{-1}(u) \sum_a \left| \frac{\partial f}{\partial z^a} + 8\pi G P'(u) z^a (f + \tilde{f}) \right|^2 + P'^{-1}(u) \sum_h \left| \frac{\partial \tilde{f}}{\partial \tilde{z}^h} + 8\pi G P'(u) \tilde{z}^h (f + \tilde{f}) \right|^2 + Q(u) \left| \sum_a z^a \frac{\partial f}{\partial z^a} + \sum_h \tilde{z}^h \frac{\partial \tilde{f}}{\partial \tilde{z}^h} + u P'(u) (f + \tilde{f}) \right|^2 - 24\pi G |f + \tilde{f}|^2 \right] + \text{gauge terms}, \quad (4.11)$$

where now

$$u = 8\pi G \left[\sum_a |z^a|^2 + \sum_h |\tilde{z}^h|^2 \right]. \quad (4.12)$$

We follow the same procedure as in Sec. III, integrating out the heavy scalars by setting them at values where V is stationary with respect to them. Again, we find that terms of order $m_g^2 M^2$ and $m_g^3 M$ are independent of light scalars, and can be made to be stationary with respect to the hidden-sector scalars and vanish by adjusting the value of the hidden-sector scalars and the additive constant in \tilde{f} . By a lengthy calculation just like that of Appendix A, the terms of order m_g^4 are found to take the form

$$V_{\text{eff}} = \sum_\alpha \left| \frac{\partial f_{\text{eff}}}{\partial z^\alpha} \right|^2 + 2 \text{Re}(m_g' f^{(3)}) + 4 \text{Re}(m_g^* f^{(2)}) + 2 \text{Re}[(4m_g - m_g^*) f^{(1)}] + |m_g''|^2 \sum_\alpha |z^\alpha|^2 + V_0 + \text{gauge terms}. \quad (4.13)$$

Here f_{eff} is an effective superpotential and $f^{(3)}$, $f^{(2)}$, and $f^{(1)}$ are its trilinear, bilinear, and linear parts, given by Eqs. (A20) or (3.4)–(3.8), but with E_0 replaced with e^P at $z^a=0$, $\tilde{z}^h=\tilde{z}_0^h$. (An additional factor $1/P'$ would appear here, but we absorb it into the normalization of z^a in order to avoid P' factors in the kinematic and gauge parts of the Lagrangian.) The first-order shift z_1^A in the heavy scalars is given here by

$$z_1^A = -m_g''' \sum_B f^{-1}{}_{AB} z_0^{B*} \quad (4.14)$$

so that $f^{(2)}$ and $f^{(1)}$ are proportional to m_g''' , and $(m_g''')^2$, respectively. The constants m_g , m_g' , m_g'' , and m_g''' are given by complicated formulas in terms of the hidden-sector superpotential and P and its derivatives at $z^a=0$, $\tilde{z}^h=\tilde{z}_0^h$, but they are all of

the same order of magnitude, roughly that of the gravitino mass. The only substantial difference between these results and those of Sec. III is that the properties of the hidden sector are now represented by four independent mass parameters m_g , m_g' , m_g'' , and m_g''' , rather than just m_g and m_g' .

V. APPLICATIONS

We now take up some examples. Much of this analysis is already present in the articles of Refs. 3–11; we go into it here in order to illustrate the use of our results when the mass scale in the low-energy effective superpotential arises from a more fundamental theory involving superheavy particles about which nothing is explicitly known.

Consider an $SU(2) \times U(1)$ low-energy effective

gauge theory with a pair of doublet Higgs left-chiral superfields:

$$H = (H^0, H^-), \quad H' = (H'^+, H'^0). \quad (5.1)$$

Additional chiral superfields will be added later. The most general effective superpotential is

$$f_{\text{eff}} = \tilde{m}_g (H^T \epsilon H'), \quad (5.2)$$

where ϵ is the usual antisymmetric 2×2 matrix, and \tilde{m}_g is a coefficient of the order of the gravitino mass, given by (3.6) and (4.14) as

$$\tilde{m}_g = -\frac{1}{2} m_g''' e^{P_0/2} \sum_{AB} f^{-1}{}_{AB} z_0^{A*} f_{BHH'}. \quad (5.3)$$

We know almost nothing about m_g''' , which depends on the hidden-sector superpotential and the Kähler potential, or about the quantities appearing in the sums over heavy scalars, which depend on the underlying grand-unified model. Never mind—all these uncertainties appear here only in the value of a single unknown complex constant \tilde{m}_g , which will have to be taken from experiment.

From (5.2) and (4.13), we obtain the effective potential

$$\begin{aligned} V_{\text{eff}} = & (|\tilde{m}_g|^2 + |m_g''|^2) (\mathcal{H}^\dagger \mathcal{H} + \mathcal{H}'^\dagger \mathcal{H}') \\ & + 4 \text{Re}(m_g^* \tilde{m}_g \mathcal{H}^T \epsilon \mathcal{H}') \\ & + \frac{1}{2} g^2 (\mathcal{H}^\dagger \vec{t} \mathcal{H} + \mathcal{H}'^\dagger \vec{t} \mathcal{H}')^2 \\ & + \frac{1}{2} g'^2 (\frac{1}{2} \mathcal{H}^\dagger \mathcal{H} - \frac{1}{2} \mathcal{H}'^\dagger \mathcal{H}')^2. \end{aligned} \quad (5.4)$$

We here use script letters for the (first) scalar components of left-chiral superfields; \vec{t} denotes the electroweak isospin generator, and g and g' are the usual gauge-coupling constants.

This cannot yield a satisfactory picture of $SU(2) \times U(1)$ breaking. For charge-conserving scalar VEV's, both the gauge and $\mathcal{H}^0 \mathcal{H}'^0$ terms in (5.4) are minimized on the surface of constant $|\langle \mathcal{H}^0 \rangle|^2 + |\langle \mathcal{H}'^0 \rangle|^2$ along the direction

$$\langle \mathcal{H}^0 \rangle = -e^{i\alpha} \langle \mathcal{H}'^0 \rangle^* \quad (5.5)$$

with phase α chosen to minimize the $\mathcal{H}^0 \mathcal{H}'^0$ term

$$\alpha = \text{Arg}(m_g \tilde{m}_g^*). \quad (5.6)$$

Along this direction, the effective potential is a quadratic

$$V_{\text{eff}} = 2(|\tilde{m}_g|^2 + |m_g''|^2 - 2|m_g||\tilde{m}_g|) |\mathcal{H}^0|^2. \quad (5.7)$$

We see that $SU(2) \times U(1)$ is unbroken if

$$|\tilde{m}_g|^2 + |m_g''|^2 > 2|m_g||\tilde{m}_g| \quad (5.8)$$

and otherwise it can be broken only at a scale very much greater than m_g , where nonperturbative effects may halt the decrease of V_{eff} . It is easy to show that this undesired conclusion obtains also when we add quark and lepton superfluids, or include arbitrary numbers of Higgs doublet superfields.

In Refs. 5, 7, and 10 it is noted that the symmetry between \mathcal{H}^0 and \mathcal{H}'^0 that is responsible for the unsatisfactory features of this model is actually broken by the different Yukawa couplings of \mathcal{H} and \mathcal{H}' to quarks and leptons, which enter in the renormalization-group equations used to integrate the parameters in V_{eff} down from grand-unification energies to ordinary energies. However, as they point out, this solves the problem only if there exist some extraordinarily heavy quarks or leptons, with masses above about 100 GeV.

An alternative possibility that has been explored by most of the authors of Refs. 3–11 is to include in the low-energy theory an $SU(3) \times SU(2) \times U(1)$ -neutral left-chiral superfield J which allows trilinear terms $JH^T \epsilon H'$ in the superpotential. Usually J is identified as the “sliding singlet,” needed to keep the Higgs doublets from getting very large masses like its $SU(5)$ partners. This runs into severe difficulties, either through J mixing with the hidden sector or through its scalar component picking up a large VEV, either of which would wreck the hierarchy of mass scales. However, for us J is simply one more chiral superfield that happens like H and H' to remain massless (and with zero VEV) in the breakdown of some grand-unified symmetry, for reasons into which we do not here inquire.²³

With J included, the most general effective superpotential is

$$\begin{aligned} f_{\text{eff}} = & \tilde{m}_g^{(1)} (H^T \epsilon H') + (\tilde{m}_g^{(2)})^2 J + \tilde{m}_g^{(3)} J^2 \\ & + \lambda (H^T \epsilon H') J + \lambda' J^3. \end{aligned} \quad (5.9)$$

We would here have to regard $\tilde{m}_g^{(n)}$ as three mass parameters of the order of the gravitino mass, which are given by formulas like (5.3), but which for practical purposes must be regarded as unknown. The constants λ and λ' are to be taken directly from the trilinear terms in the superpotential of the grand-unified theory, but for our present purposes are also just unknown dimensionless coupling constants.

The effective potential for (5.9) is

$$\begin{aligned}
V_{\text{eff}} = & |\tilde{m}_g^{(1)} + \lambda \mathcal{F}|^2 (\mathcal{H}^\dagger \mathcal{H} + \mathcal{H}'^\dagger \mathcal{H}') + |(\tilde{m}_g^{(2)})^2 + 2\tilde{m}_g^{(3)} \mathcal{F} + 3\lambda' \mathcal{F}^2 + \lambda (\mathcal{H}^T \epsilon \mathcal{H}')|^2 \\
& + 2 \operatorname{Re}[m_g'^* (\lambda \mathcal{H}^T \epsilon \mathcal{H}' \mathcal{F} + \lambda' \mathcal{F}^3)] \\
& + 4 \operatorname{Re}[m_g^* (\tilde{m}_g^{(1)} \mathcal{H}^T \epsilon \mathcal{H}' + \tilde{m}_g^{(3)} \mathcal{F}^2)] + 2 \operatorname{Re}[(4m_g - m_g')^* (\tilde{m}_g^{(2)})^2 \mathcal{F}] \\
& + |m_g''|^2 (|\mathcal{F}|^2 + \mathcal{H}^\dagger \mathcal{H} + \mathcal{H}'^\dagger \mathcal{H}') \\
& + \frac{1}{2} g^2 (\mathcal{H}^\dagger \vec{t} \mathcal{H} + \mathcal{H}'^\dagger \vec{t} \mathcal{H}')^2 + \frac{1}{2} g'^2 (\frac{1}{2} \mathcal{H}^\dagger \mathcal{H} - \frac{1}{2} \mathcal{H}'^\dagger \mathcal{H}')^2. \tag{5.10}
\end{aligned}$$

For charge-conserving scalar VEV's, the minimum of (5.10) is again in the direction (5.5) (but with different phase α). It is well known that (5.10) has an $SU(2) \times U(1)$ -breaking minimum along this direction with $\langle \mathcal{H}^0 \rangle \neq 0$ for a variety of special cases. For instance, if all terms in (5.9) are absent except $\lambda (\mathcal{H}^T \epsilon \mathcal{H}') \mathcal{F}$, then (5.10) has an absolute minimum along the direction (5.5) with $\langle \mathcal{H}^0 \rangle \neq 0$, provided that

$$|m_g'| > 3 |m_g''|.$$

We will not specialize by choosing any specific values for the parameters in (5.9), but will just assume that they fall in the range where $SU(2) \times U(1)$ is broken, and consider those consequences of (5.10) that do not depend on the values of the parameters in this range.

Charged scalars: Inspection of (5.10) shows that the mass matrix of the charge-1 scalar boson has diagonal elements

$$\begin{aligned}
\langle \mathcal{H}^- | M^2 | \mathcal{H}^- \rangle &= \langle \mathcal{H}'^{*+} | M^2 | \mathcal{H}'^{*+} \rangle \\
&= \frac{1}{2} (m_W^2 + \Delta^2) \tag{5.11}
\end{aligned}$$

with

$$\Delta^2/2 = |\tilde{m}_g^{(1)} + \lambda \langle \mathcal{F} \rangle|^2 + |m_g''|^2 > 0. \tag{5.12}$$

By the Goldstone theorem or direct calculation, we then also have

$$\begin{aligned}
\langle \mathcal{H}^- | M^2 | \mathcal{H}'^{*+} \rangle &= \langle \mathcal{H}'^{*+} | M^2 | \mathcal{H}^- \rangle \\
&= \frac{1}{2} (m_W^2 + \Delta^2) e^{i\alpha}. \tag{5.13}
\end{aligned}$$

The eigenvalues are then 0, corresponding to the Goldstone boson eliminated by the Higgs mechanism, together with

$$m_+^2 = m_W^2 + \Delta^2. \tag{5.14}$$

Thus there is a physical charged Higgs boson heavier than the W .

Neutral scalars: There are six real scalar fields here, of which one real field is eliminated by the Higgs mechanism, leaving five real physical neutral scalars. The complete mass spectrum is quite complicated, but one of the masses is easily calculated by using the symmetry of (5.10) with $\mathcal{H}^- = \mathcal{H}'^+ = 0$ under the interchange of \mathcal{H}^0 and \mathcal{H}'^0 . By a $U(1)$ gauge transformation we can always choose the phase of $\langle \mathcal{H}^0 \rangle$ to be $(\pi + \alpha)/2$, so that (5.5) gives \mathcal{H}^0 and \mathcal{H}'^0 equal VEV's, thus preserving this symmetry. The scalars of definite mass can therefore be classified as even or odd under the symmetry $\mathcal{H}^0 \leftrightarrow \mathcal{H}'^0$: four real scalars are even, and two are odd. The Goldstone boson eliminated by the Higgs mechanism is odd (because \mathcal{H}^0 and \mathcal{H}'^0 have opposite t_3 and weak hypercharge) so there is just one physical odd neutral scalar, which does not mix with any of the other neutral scalars. Its mass is easily calculated to be

$$m_{0,\text{odd}}^2 = m_Z^2 + \Delta^2. \tag{5.15}$$

This scalar is heavier than the Z^0 , and by the same amount (counting squared masses) as the charged Higgs boson is heavier than the W .

S-quarks and s-leptons: In order to account for the quark and lepton masses, we must add terms in the superpotential of the form

$$(m_u / \langle \mathcal{H}^0 \rangle) (Q_L^T \epsilon H_L) U_R^* + (m_d / \langle \mathcal{H}^0 \rangle) (Q_L^T \epsilon H_L) D_R^* + (m_e / \langle \mathcal{H}^0 \rangle) (L_L^T \epsilon H_L) E_R^*, \tag{5.16}$$

where $Q_L \equiv \{U_L, D_L\}$ and $L_L \equiv \{N_L, E_L\}$ are left-chiral quark and lepton doublets; U_R^* , D_R^* , and E_R^* are left-chiral antiquark and antilepton singlets; and we assume one generation for notational simplicity. With vanishing VEV's for the scalar counterparts of the quarks and leptons²⁰ (s-quarks and s-leptons) there is no change in our previous discussion of Higgs and singlet masses and VEV's. Setting the neutral scalars equal to their VEV's, the terms in the effective potential that are quadratic in the s-quarks and s-leptons are

$$\begin{aligned}
m_u^2 (|\mathcal{U}_L|^2 + |\mathcal{U}_R|^2) &+ m_d^2 (|\mathcal{D}_L|^2 + |\mathcal{D}_R|^2) + m_e^2 (|\mathcal{E}_L|^2 + |\mathcal{E}_R|^2) \\
&+ 2 \operatorname{Re}\{[m_g' - e^{i\alpha} (\tilde{m}_g^{(1)} + \lambda \langle \mathcal{F} \rangle)]^* [m_u \mathcal{U}_L \mathcal{U}_R^* + m_d \mathcal{D}_L \mathcal{D}_R^* + m_e \mathcal{E}_L \mathcal{E}_R^*]\} \\
&+ |m_g''|^2 (|\mathcal{U}_L|^2 + |\mathcal{U}_R|^2 + |\mathcal{D}_L|^2 + |\mathcal{D}_R|^2 + |\mathcal{E}_L|^2 + |\mathcal{E}_R|^2 + |\mathcal{N}_L|^2 + |\mathcal{N}_R|^2). \tag{5.17}
\end{aligned}$$

The up-s-quark masses are then

$$m^2_{\mathcal{Q}\pm} = |m_g''|^2 + m_u^2 \pm m_u |m_g' - e^{i\alpha}(\tilde{m}_g^{(1)} + \lambda \langle \mathcal{F} \rangle)| \quad (5.18)$$

and likewise for the down s-quarks and s-electrons, while the s-neutrinos have mass

$$m_{\mathcal{N}}^2 = |m_g''|^2. \quad (5.19)$$

We note that for small quark and lepton masses, the s-quarks and s-leptons are nearly degenerate, and in any case the average mass² of each s-quark or s-lepton pair exceeds the corresponding quark or lepton mass² by the same amount, an amount *less* than the difference Δ^2 of \mathcal{N}^\pm and W^\pm masses².

Most of these results [except perhaps for Eq. (5.18)] have been obtained before in more specific models.³⁻¹¹ Our derivation here serves to emphasize that these results apply independently of the parameters of the low-energy superpotential or the details of the grand unified theory or even the details of the Kähler potential.

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APPENDIX A: CALCULATION OF THE EFFECTIVE POTENTIAL

Under the assumptions and in the notation of Sec. II, the potential is

$$V(z, \tilde{z}) = \exp \left[8\pi G \left(\sum_a |z^a|^2 + \sum_h |\tilde{z}^h|^2 \right) \right] \times \left[\sum_a |F_a|^2 + \sum_h |\tilde{F}_h|^2 - 24\pi G |f + \tilde{f}|^2 \right] + \frac{1}{2} \sum_k D_k^2, \quad (A1)$$

with

$$F_a \equiv \frac{\partial f}{\partial z^a} + 8\pi G (f + \tilde{f}) z^{a*}, \quad (A2)$$

$$\tilde{F}_h \equiv \frac{\partial \tilde{f}}{\partial \tilde{z}^h} + 8\pi G (f + \tilde{f}) \tilde{z}^{h*}, \quad (A3)$$

$$D_k \equiv \sum_{a,b} z^{a*} z^b (t_k)_b^a, \quad (A4)$$

where f and \tilde{f} are the superpotentials of the observable and hidden sectors; z^a and \tilde{z}^h are the complex scalar fields on which they, respectively, depend; and t_k are the gauge generator matrices. The scalar indices a, b, \dots run over values A, B, \dots labeling complex superheavy chiral scalars; α, β, \dots labeling complex light scalars; and K, L, \dots labeling real scalars that would be degenerate with the superheavy gauge bosons in the limit $\tilde{f} \rightarrow 0$. Also, the gauge indices k, l, \dots run over values K, L, \dots labeling superheavy gauge bosons, and values κ, λ, \dots labeling gauge bosons [of $SU(3) \times SU(2) \times U(1)$] that do not get masses from the breakdown of the grand-unified gauge group. These different index values are distinguished by the conditions

$$f_{\alpha\beta} = f_{\alpha A} = f_{\alpha K} = f_{KA} = f_{KL} = 0, \quad (A5)$$

$$f_{AB} \text{ nonsingular}, \quad (A6)$$

$$(t_K z_0)^\alpha = (t_K z_0)^A = 0, \quad (A7)$$

$$(t_K z_0)^L = \mu_{KL} \text{ nonsingular}, \quad (A8)$$

$$z_0^\alpha = z_0^K = 0, \quad (A9)$$

where z_0 is the stationary point of $f(z)$. Recall also that $f_{abc} \dots$ denotes the partial derivative of $f(z)$ with respect to z^a, z^b, z^c, \dots at $z = z_0$.

We will write the observable scalar fields as

$$z^a = z_0^a + \phi^a \quad (A10)$$

and take ϕ^a to be like \tilde{f} of order m_g . We are interested in calculating the potential to fourth order in m_g .

For heavy scalars, the leading term in F_A is of order m_g (recall that f as well as $\partial f / \partial z^a$ vanishes at $z = z_0$), so we need terms in F_A up to order m_g^3 . Grouping terms by order in m_g , we have to third order

$$F_A \simeq \left[\sum_B f_{AB} \phi^B + 8\pi G z_0^{A*} \tilde{f} \right] + \left[\frac{1}{2} \sum_{ab} f_{Aab} \phi^a \phi^b + 4\pi G z_0^{A*} \sum_{BC} f_{BC} \phi^B \phi^C + 8\pi G \phi^A \tilde{f} \right] + \left[\frac{1}{6} \sum_{abc} f_{Aabc} \phi^a \phi^b \phi^c + \frac{1}{6} 8\pi G z_0^{A*} \sum_{abc} f_{abc} \phi^a \phi^b \phi^c + 4\pi G \phi^A \sum_{BC} f_{BC} \phi^B \phi^C \right]. \quad (A11)$$

On the other hand, for light scalars and scalars degenerate with superheavy gauge bosons the leading terms in F_α and F_K are of order m_g^2 , so we need only keep these terms:

$$F_\alpha \simeq \frac{1}{2} \sum_{ab} f_{aab} \phi^a \phi^b + 8\pi G \phi^{\alpha*} \tilde{f}, \quad (\text{A12})$$

$$F_K \simeq \frac{1}{2} \sum_{ab} f_{Kab} \phi^a \phi^b + 8\pi G \phi^K \tilde{f}. \quad (\text{A13})$$

Similarly, for the hidden sector the leading terms in \tilde{F}_h are of first order in m_g , so here we need to keep terms up to third order

$$\tilde{F}_h \simeq \left[\frac{\partial \tilde{f}}{\partial \tilde{z}^h} + 8\pi G \tilde{z}^{h*} \tilde{f} \right] + \left[4\pi G \tilde{z}^{h*} \sum_{AB} f_{AB} \phi^A \phi^B \right] + \left[\frac{1}{6} \times 8\pi G \tilde{z}^{h*} \sum_{abc} f_{abc} \phi^a \phi^b \phi^c \right]. \quad (\text{A14})$$

Also, for superheavy gauge bosons there are terms in D_K of first and second order

$$D_K \simeq 2 \sum_L \mu_{KL} \phi^L + (\phi^\dagger t_K \phi) \quad (\text{A15})$$

while for light gauge bosons D_κ is entirely of second order in m_g

$$D_\kappa = (\phi^\dagger t_\kappa \phi). \quad (\text{A16})$$

Finally, the leading term in $f + \tilde{f}$ is of first order in m_g , so we need to keep terms up to third order

$$f + \tilde{f} = [\tilde{f}] + \left[\frac{1}{2} \sum_{AB} \phi^A \phi^B \right] + \left[\frac{1}{6} \sum_{abc} f_{abc} \phi^a \phi^b \phi^c \right]. \quad (\text{A17})$$

Using these approximations in (A1), and discarding terms of fifth or sixth order in m_g , we find for the terms in V of order $m_g^2 M^2$, $m_g^3 M$, and m_g^4 the following expressions:

$$V_2 = N \tilde{V} + E \sum_A \left| \sum_B f_{AB} \phi^B + 8\pi G z_0^{A*} \tilde{f} \right|^2 + 2 \sum_K \left[\sum_L \mu_{KL} \phi^L \right]^2, \quad (\text{A18})$$

$$\begin{aligned} V_3 = 16\pi G \operatorname{Re} \left[\sum_A z_0^{A*} \phi^A \right] V_2 + 2E \operatorname{Re} \left[\sum_A \left[\sum_B f_{AB} \phi^B + 8\pi G z_0^{A*} \tilde{f} \right] \right. \\ \left. \times \left[\frac{1}{2} \sum_{ab} f_{Aab} \phi^a \phi^b + 4\pi G z_0^{A*} \sum_{BC} f_{BC} \phi^B \phi^C + 8\pi G \phi^{A*} \tilde{f} \right] \right] \\ + 8\pi G E \operatorname{Re} \left[\sum_h \left[\tilde{z}^h \frac{\partial \tilde{f}}{\partial \tilde{z}^h} + 8\pi G |\tilde{z}^h|^2 \tilde{f} \right]^* \sum_{AB} f_{AB} \phi^A \phi^B \right] \\ - 24\pi G E \operatorname{Re} \left[\tilde{f}^* \sum_{AB} f_{AB} \phi^A \phi^B \right] + 2 \sum_{KL} (\phi^\dagger t_K \phi) \mu_{KL} \phi^L, \quad (\text{A19}) \end{aligned}$$

$$\begin{aligned} V_4 = \left[-128\pi^2 G^2 \left[\operatorname{Re} \sum_A z_0^{A*} \phi^A \right]^2 + 8\pi G \sum_a |\phi^a|^2 \right] V_2 + 16\pi G \operatorname{Re} \left[\sum_A z_0^{A*} \phi^A \right] V_3 \\ + E \sum_A \left| \frac{1}{2} \sum_{ab} f_{Aab} \phi^a \phi^b + 4\pi G z_0^{A*} \sum_{BC} f_{BC} \phi^B \phi^C + 8\pi G \tilde{f} \phi^{A*} \right|^2 \\ + 2E \operatorname{Re} \left[\sum_A \left[\sum_B f_{AB} \phi^B + 8\pi G z_0^{A*} \tilde{f} \right] \left[8\pi G z_0^{A*} \frac{1}{6} \sum_{abc} f_{abc} \phi^a \phi^b \phi^c + 4\pi G \phi^{A*} \sum_{AB} f_{AB} \phi^A \phi^B + \frac{1}{6} \sum_{abc} f_{Aabc} \phi^a \phi^b \phi^c \right]^* \right] \end{aligned}$$

$$\begin{aligned}
& + E \sum_{\alpha} \left| \frac{1}{2} \sum_{ab} f_{\alpha ab} \phi^a \phi^b + 8\pi G \phi^{\alpha*} \tilde{f} \right|^2 + 64\pi^2 G^2 E \sum_h |\tilde{z}^h|^2 \left| \frac{1}{2} \sum_{AB} f_{AB} \phi^A \phi^B \right|^2 \\
& + 16\pi G E \operatorname{Re} \sum_h \left[\tilde{z}^h \frac{\partial \tilde{f}}{\partial \tilde{z}^h} + 8\pi G |\tilde{z}^h|^2 \tilde{f} \right] \left[\frac{1}{6} \sum_{abc} f_{abc} \phi^a \phi^b \phi^c \right]^* - 24\pi G E \left| \frac{1}{2} \sum_{AB} f_{AB} \phi^A \phi^B \right|^2 \\
& - 48\pi G E \operatorname{Re} \left[\sum_{abc} \frac{1}{6} f_{abc} \phi^a \phi^b \phi^c \tilde{f}^* \right] + \sum_K \left| \frac{1}{2} \sum_{ab} f_{Kab} \phi^a \phi^b + 8\pi G \phi^{K*} \tilde{f} \right|^2 + \frac{1}{2} \sum_K |\phi^{\dagger t_K} \phi|^2 + \frac{1}{2} \sum_K |\phi^{\dagger t_{\kappa}} \phi|^2.
\end{aligned} \tag{A20}$$

Here \tilde{V} is the potential of the hidden sector alone

$$\tilde{V} = \exp \left[8\pi G \sum_h |\tilde{z}^h|^2 \right] \left[\sum_h \left| \frac{\partial \tilde{f}}{\partial \tilde{z}^h} + 8\pi G \tilde{z}^h \tilde{f} \right|^2 - 24\pi G |\tilde{f}|^2 \right] \tag{A21}$$

and N and E are the exponential factors

$$N = \exp \left[8\pi G \sum_A |z_0^A|^2 \right], \tag{A22}$$

$$E = N \exp \left[8\pi G \sum |\tilde{z}^h|^2 \right]. \tag{A23}$$

Also, we remind the reader that sums over a, b, \dots run over the values A, B, \dots and α, β, \dots and K, L, \dots .

We now express the heavy scalars ϕ^A and ϕ^K as functions of $\phi^{\alpha} \equiv z^{\alpha}$ by imposing the conditions that V be stationary in heavy scalars. Expressing the heavy scalars in power series

$$\phi^A = z_1^A + z_2^A + z_3^A + \dots, \tag{A24}$$

$$\phi^K = z_1^K + z_2^K + z_3^K + \dots, \tag{A25}$$

with

$$z_n^A \text{ and } z_n^K \propto (m_g)^n \tag{A26}$$

the stationarity conditions become

$$0 = \left[\frac{\partial V_2}{\partial \phi^A} \right]_1 = \left[\frac{\partial V_2}{\partial \phi^K} \right]_1, \tag{A27}$$

$$0 = \left[\frac{\partial V_3}{\partial \phi^A} \right]_1 + \sum_B \left[\frac{\partial^2 V_2}{\partial \phi^A \partial \phi^{B*}} \right]_1 z_2^{B*} = \left[\frac{\partial V_3}{\partial \phi^K} \right]_1 + \frac{1}{2} \sum_{KL} \left[\frac{\partial^2 V_2}{\partial \phi^K \partial \phi^L} \right]_1 z_2^L, \tag{A28}$$

and so on, the subscript indicating that ϕ^A and ϕ^K are set equal to z_1^A and z_1^K , while $\phi^{\alpha} \equiv z^{\alpha}$ is a free variable. [In writing (A28), we use the fact that the only nonvanishing second derivatives of V_2 are those shown here.] From (A27) and (A28), we find

$$z_1^A = -8\pi G \tilde{f} \sum_B z_0^{B*}, \tag{A29}$$

$$\begin{aligned}
z_2^A = & - \sum_B f^{-1}_{AB} \left\{ 8\pi G \sum_C f^{-1*}_{BC} z_0^C E^{-1} V_2 + \frac{1}{2} \sum_{ab} f_{Bab} z_1^a z_1^b + 4\pi G z_0^{B*} \sum_{CD} f_{CD} z_1^C z_1^D \right. \\
& \left. + 8\pi G z_1^{B*} \left[\tilde{f} + \sum_h \left[\tilde{z}^h \frac{\partial \tilde{f}}{\partial \tilde{z}^h} + 8\pi G |\tilde{z}^h|^2 \tilde{f} \right] \right] - 24\pi G \tilde{f} z_1^{B*} \right\}, \tag{A30}
\end{aligned}$$

and

$$z_1^K = 0, \quad (\text{A31})$$

$$z_2^K = -\frac{1}{2} \sum_L \mu^{-1}_{KL} (z_1^\dagger t_L z_1), \quad (\text{A32})$$

with $z_1^\alpha \equiv z^\alpha$. We will not need the formulas for the higher-order terms in z^A and z^K .

Now we can calculate the effective potential of the light scalars by inserting our results for z^A and z^K in V . To order m_g^4 , this gives

$$\begin{aligned} V_{\text{eff}} = & (V_2)_1 + \sum_A \left[\frac{\partial V_2}{\partial \phi^A} \right]_1 z_2^A + cc + \sum_K \left[\frac{\partial V_2}{\partial \phi^K} \right]_1 z_2^K + (V_3)_1 + \sum_A \left[\frac{\partial V_3}{\partial \phi^A} \right]_1 z_2^A + cc + \sum_K \left[\frac{\partial V_3}{\partial \phi^K} \right]_1 z_2^K \\ & + \sum_{AB} \left[\frac{\partial^2 V_2}{\partial \phi^A \partial \phi^{B*}} \right]_1 z_2^A z_2^{B*} + \frac{1}{2} \sum_{KL} \left[\frac{\partial^2 V_2}{\partial \phi^K \partial \phi^L} \right]_1 z_2^K z_2^L + \sum_A \left[\frac{\partial V_2}{\partial \phi^A} \right]_1 z_3^A + cc + \sum_K \left[\frac{\partial V_2}{\partial \phi^K} \right]_1 z_3^K + (V_4)_1. \end{aligned} \quad (\text{A33})$$

Using (A27) and (A28), this simplifies to

$$V_{\text{eff}} = (V_2)_1 + (V_3)_1 + (V_4)_1 - \sum_{AB} \left[\frac{\partial^2 V_2}{\partial \phi^A \partial \phi^{B*}} \right]_1 z_2^A z_2^{B*} - \frac{1}{2} \sum_{KL} \left[\frac{\partial^2 V_2}{\partial \phi^K \partial \phi^L} \right]_1 z_2^K z_2^L. \quad (\text{A34})$$

First consider the effective potential to third order in m_g . Using (A29) and (A31) in (A18) and (A19), this is

$$(V_2)_1 + (V_3)_1 = \left[1 + 16\pi G \operatorname{Re} \sum_h z_0^A z_1^{A*} \right] N\tilde{V} + 8\pi G E \operatorname{Re} \left\{ \left[\sum_h \bar{z}^h \frac{\partial \tilde{f}}{\partial \bar{z}^h} + \left[8\pi G \sum_h |\bar{z}^h|^2 - 3 \right] \tilde{f} \right]^* \sum_{AB} f_{AB} z_1^A z_1^B \right\}. \quad (\text{A35})$$

We note that (A35) is completely independent of light fields, so we can find a z^α -independent value of the hidden fields \bar{z}^h where (A35) is stationary in \bar{z}^h , and we can adjust an additive constant in \tilde{f} to order m_g^2 so that (A35) vanishes at this point. However, the remaining terms in (A34) are already of order m_g^4 , and to this order it is an adequate approximation to calculate these terms using the lowest-order values for \bar{z}^h and $f(\bar{z}^h)$ at the stationary point. These are determined by the conditions that the second-order term $N\tilde{V}$ in (A35) vanish and be stationary, i.e., that

$$\frac{\partial \tilde{V}}{\partial \bar{z}^h} = \tilde{V} = 0. \quad (\text{A36})$$

These conditions fix \bar{z}^h to have the value denoted \bar{z}_0^h in Sec. II. From now on, we suppose that \bar{z}^h and the additive constant in \tilde{f} have been determined in this way.

With $(V_2 + V_3)_1$ absent, the effective potential is given by (A35) as

$$V_{\text{eff}} = (V_4)_1 - \sum_{AB} \left[\frac{\partial^2 V_2}{\partial \phi^A \partial \phi^{B*}} \right]_1 z_2^A z_2^{B*} - \frac{1}{2} \sum_{KL} \left[\frac{\partial^2 V_2}{\partial \phi^K \partial \phi^L} \right]_1 z_2^K z_2^L. \quad (\text{A37})$$

Setting $\phi^A = z_1^A$ and $\phi^K = z_1^K = 0$ in (A20) yields

$$\begin{aligned} (V_4)_1 = & E_0 \sum_A \left| \frac{1}{2} f_{Aab} z_1^a z_1^b + 4\pi G z_0^{A*} \sum_{BC} z_1^B z_1^C + 8\pi G \tilde{f}_0 z_1^{A*} \right|^2 + E_0 \sum_\alpha \left| \frac{1}{2} f_{aab} z^\alpha z^\alpha + 8\pi G z^\alpha \tilde{f}_0 \right|^2 \\ & + 8\pi G E_0 \left[8\pi G \sum_h |\bar{z}^h|^2 - 3 \right] \left| \frac{1}{2} \sum_{AB} f_{AB} z_1^A z_1^B \right|^2 \end{aligned}$$

$$\begin{aligned}
& + 16\pi G E_0 \operatorname{Re} \left\{ \left[\sum_h \left[\bar{z}^h \frac{\partial \tilde{f}}{\partial \bar{z}^h} \right]_0 + \left[8\pi G \sum_h |\bar{z}_0^h|^2 - 3 \right] \tilde{f}_0 \right] \frac{1}{6} \sum_{abc} f_{abc} z_1^a z_1^b z_1^c \right\} \\
& + \sum_K \left| \frac{1}{2} \sum_{ab} f_{Kab} z_1^a z_1^b \right|^2 + \frac{1}{2} \sum_K |z_1^\dagger t_K z_1|^2 + \frac{1}{2} \sum_\kappa |z_1^\dagger t_\kappa z_1|^2.
\end{aligned} \tag{A38}$$

Also, (A30) and (A32) give

$$\begin{aligned}
\sum_{AB} \left[\frac{\partial^2 V_2}{\partial \phi^A \partial \phi^{B*}} \right]_1 z_2^A z_2^{B*} &= E_0 \sum_A \left| \sum_B f_{AB} z_2^B \right|^2 \\
&= E_0 \sum_A \left| \frac{1}{2} \sum_{ab} f_{Aab} z_1^a z_1^b + 4\pi G z_0^A \sum_{CD} f_{CD} z_1^C z_1^D \right. \\
&\quad \left. + 8\pi G z_1^{A*} \left[\sum_h \left[\bar{z}^h \frac{\partial \tilde{f}}{\partial \bar{z}^h} \right]_0 + \left[8\pi G \sum_h |z_0^h|^2 - 2 \right] \tilde{f}_0 \right] \right|^2,
\end{aligned} \tag{A39}$$

$$\begin{aligned}
\frac{1}{2} \sum_{KL} \left[\frac{\partial^2 V_2}{\partial \phi^K \partial \phi^L} \right]_1 z_2^K z_2^L &= 2 \sum_{KL} \mu_{KL}^2 z_2^K z_2^L \\
&= \frac{1}{2} \sum_K (z_1^\dagger t_K z_1)^2.
\end{aligned} \tag{A40}$$

Part of (A39) cancels the first term in (A38), and (A40) cancels the next-to-last term in (A38), leaving us with

$$\begin{aligned}
V_{\text{eff}} &= -16\pi G E_0 \operatorname{Re} \left[\sum_h \left[\bar{z}^h \frac{\partial f}{\partial \bar{z}^h} \right]_0 + \left[8\pi G \sum_h |\bar{z}_0^h|^2 - 3 \right] \tilde{f}_0 \right]^* \\
&\quad \times \left[\frac{1}{2} \sum_{abA} f_{Aab} z_1^A z_1^a z_1^b + 4\pi G \sum_A z_1^A z_0^{A*} \sum_{CD} f_{CD} z_1^C z_1^D + 8\pi G \tilde{f}_0 \sum_A |z_1^A|^2 \right] \\
&\quad - 64\pi^2 G^2 \sum_A |z_1^A|^2 \left| \sum_h \bar{z}^h \frac{\partial \tilde{f}_0}{\partial \bar{z}^h} + \left[8\pi G \sum_h |\bar{z}_0^h|^2 - 3 \right] \tilde{f}_0 \right|^2 + E_0 \sum_\alpha \left| \frac{1}{2} \sum_{ab} f_{\alpha ab} z_1^a z_1^b + 8\pi G z_1^\alpha \tilde{f}_0 \right|^2 \\
&\quad + 8\pi G E_0 \left[8\pi G \sum_h |\bar{z}^h|^2 - 3 \right] \left| \frac{1}{2} \sum_{AB} f_{AB} z_1^A z_1^B \right|^2 \\
&\quad + 16\pi G E_0 \operatorname{Re} \left[\sum_h \left[z^h \frac{\partial \tilde{f}}{\partial \bar{z}^h} \right]_0 + \left[8\pi G \sum_h |\bar{z}^h|^2 - 3 \right] \tilde{f}_0 \right]^* \\
&\quad \times \frac{1}{6} \sum_{abc} f_{abc} z_1^a z_1^b z_1^c + \sum_K \left| \frac{1}{2} \sum_{ab} f_{Kab} z_1^a z_1^b \right|^2 + \frac{1}{2} \sum_\kappa |z_1^\dagger t_\kappa z_1|^2.
\end{aligned} \tag{A41}$$

We note also that

$$f_{K\alpha\beta} = \sum_A f_{K\alpha A} z_1^A = 0 \tag{A42}$$

so the sum over a and b in the next-to-last term in (A41) runs only over heavy scalar indices A and B , and this term is therefore independent of light scalars.

(*Proof:* From the gauge-invariance condition on the superpotential

$$\sum_C \frac{\partial f}{\partial z^c} (t_K z)^c = 0$$

we have by differentiating twice and setting $z = z_0$

$$\sum_c [f_{abc} (t_K z)^c + f_{ac} (t_K)_b^c + f_{bc} (t_K)_a^c] = 0.$$

Using (A7) and (A8) allows us to solve for f_{Kab} :

$$f_{Kab} = - \sum_{Lc} \mu^{-1}{}_{KL} [f_{ac} (t_L)_b^c + f_{bc} (t_L)_a^c].$$

Equation (A5) then yields

$$f_{K\alpha\beta} = 0$$

and

$$f_{K\alpha A} = - \sum_{LB} \mu^{-1}{}_{KL} f_{AB} (t_L)_\alpha^B$$

so

$$\sum_A f_{K\alpha A} z_1^A = + 8\pi G \tilde{f} \sum_{LB} z_0^{B*} (t_L)_\alpha^B$$

which vanishes by (A7.)

It is very convenient to rewrite (A41) by introducing an effective superpotential

$$f_{\text{eff}} = \frac{E_0^{1/2}}{6} \sum_{abc} f_{abc} z_1^a z_1^b z_1^c. \quad (\text{A43})$$

The other z^α -dependent quantities in (A41) can be written

$$\frac{1}{2} \sum_{abA} f_{Aab} z_1^A z_1^a z_1^b = 3f^{(0)} + 2f^{(1)} + f^{(2)},$$

$$\frac{1}{2} \sum_{ab\alpha} f_{\alpha ab} z_1^\alpha z_1^a z_1^b = f^{(1)} + 2f^{(2)} + 3f^{(3)},$$

$$\frac{1}{2} \sum_{aab} f_{aab} z_1^a z_1^b = \frac{\partial f_{\text{eff}}}{\partial z^\alpha},$$

where $f^{(n)}$ is the term in f_{eff} of n th order in light scalars. Equation (A41) thus takes the form

$$\begin{aligned} V_{\text{eff}} = & \sum_\alpha \left| \frac{\partial f_{\text{eff}}}{\partial z^\alpha} \right|^2 + 2 \operatorname{Re} \{ m_g^* f^{(3)} \} + 4 \operatorname{Re} \{ m_g^* f^{(2)} \} + 2 \operatorname{Re} \{ (4m_g - m_g')^* f^{(1)} \} + |m_g|^2 \sum_\alpha |z^\alpha|^2 \\ & + \frac{1}{2} \sum_\kappa \left| \sum_{\alpha\beta} z^\alpha z^\beta (t_\kappa)_\beta^\alpha \right|^2 + V_0, \end{aligned} \quad (\text{A44})$$

where m_g , m_g' , and V_0 are the constants

$$m_g = 8\pi G E_0^{1/2} \tilde{f}_0, \quad (\text{A45})$$

$$m_g' = 8\pi G E_0^{1/2} \left| \sum_h \tilde{z}_0^h \left[\frac{\partial \tilde{f}}{\partial \tilde{z}^h} \right]_0 + 8\pi G \tilde{f}_0 \sum_h |\tilde{z}_0^h|^2 \right|, \quad (\text{A46})$$

$$\begin{aligned}
 V_0 = & -2 \operatorname{Re}(m'_g - 3m_g^*) m_g \sum_A |z_1^A|^2 - \sum_A |z_1^A|^2 |m'_g - 3m_g|^2 + E_0 \sum_\alpha \left| \frac{1}{2} \sum_{AB} f_{\alpha AB} z_1^A z_1^B \right|^2 \\
 & - 4E_0^{1/2} \operatorname{Re}(m'_g - 3m_g) f^{(0)} + E_0 \sum_K \left| \frac{1}{2} \sum_{AB} f_{KAB} z_1^A z_1^B \right|^2 + v_0, \tag{A47}
 \end{aligned}$$

where v_0 is the part of (A35) that arises from any additive constant of order m_g^3 in \tilde{f} . Equation (A44) is our desired result, quoted in Sec. III as Eq. (3.11).

APPENDIX B: CONDITIONS FOR LIGHT SCALAR INDEPENDENCE IN LOW ORDERS

Consider a general potential $V(z)$, expressed as a power series in a small parameter ϵ :

$$V(z) = V_0(z) + \epsilon V_1(z) + \epsilon^2 V_2(z) + \dots \tag{B1}$$

Suppose that the zeroth order potential has a minimum at $z^a = z_0^a$:

$$\frac{\partial V_0(z)}{\partial z^a} = 0 \text{ at } z^a = z_0^a. \tag{B2}$$

Choose a basis in which a runs over values A and α , with

$$V_{0\alpha\beta} = V_{0A\alpha} = 0, \tag{B3}$$

$$V_{0AB} \text{ nonsingular}, \tag{B4}$$

where subscripts denote differentiation with respect to z^a :

$$V_{nab\dots} = \left[\frac{\partial^n V_n(z)}{\partial z^a \partial z^b \dots} \right]_{z=z_0}. \tag{B5}$$

We write the scalars as

$$z^a = z_0^a + \epsilon \phi^a \tag{B6}$$

and expand

$$\begin{aligned}
 V(z) = & C + \epsilon^2 \left[\frac{1}{2} \sum_{AB} V_{0AB} \phi^A \phi^B + \sum_a V_{1a} \phi^a + \epsilon^3 \left[\frac{1}{6} \sum_{abc} v_{0abc} \phi^a \phi^b \phi^c + \frac{1}{2} \sum_{ab} V_{1ab} \phi^a \phi^b \right] + \sum_a V_{2a} \phi^a \right] + O(\epsilon^4), \tag{B7}
 \end{aligned}$$

where C is a z^a -independent constant. We “integrate out” the heavy scalars ϕ^A , by imposing the condition that

$$\frac{\partial V}{\partial \phi^A} = 0 \text{ at } \phi^A = \phi^A(\phi^\alpha). \tag{B8}$$

This has a power-series solution

$$\phi^A = z_1^A + \epsilon z_2^A + \dots, \tag{B9}$$

with

$$z_1^A = - \sum_B V^{-1}_{0AB} V_{1B}. \tag{B10}$$

Inserting this back into (B7) yields

$$\begin{aligned}
 V_{\text{eff}}(\phi^\alpha) = & C' + \epsilon^2 \sum_\alpha V_{1\alpha} \phi^\alpha + \epsilon^3 \left[\frac{1}{6} \sum_{\alpha\beta\gamma} V_{0\alpha\beta\gamma} \phi^\alpha \phi^\beta \phi^\gamma + \frac{1}{2} \sum_{\alpha\beta A} V_{0\alpha\beta A} \phi^\alpha \phi^\beta z_1^A \right. \\
 & \left. + \frac{1}{2} \sum_{\alpha AB} V_{0\alpha AB} \phi^\alpha z_1^A z_1^B + \frac{1}{2} \sum_{\alpha\beta} V_{1\alpha\beta} \phi^\alpha \phi^\beta + \sum_{\alpha A} V_{1\alpha A} \phi^\alpha z_1^A + \sum_\alpha V_{2\alpha} \phi^\alpha \right] + O(\epsilon^4) \tag{B11}
 \end{aligned}$$

with C' a ϕ^α -independent constant. The conditions for the z^α -dependent terms in V_{eff} to vanish in orders ϵ^2 and ϵ^3 are therefore

$$V_{1\alpha} = 0, \quad (\text{B12})$$

$$V_{0\alpha\beta\gamma} = 0, \quad (\text{B13})$$

$$\sum_A V_{0\alpha\beta A} z_1^A + V_{1\alpha\beta} = 0, \quad (\text{B14})$$

$$\frac{1}{2} \sum_{AB} V_{0\alpha AB} z_1^A z_1^B + \sum V_{1\alpha A} z_1^A + V_{1\alpha} = 0. \quad (\text{B15})$$

In the case of the supergravity potential (A1), we can regard \tilde{f} and $\sum_n \tilde{z}^h \partial \tilde{f} / \partial \tilde{z}^h$ as parameters of order ϵ (with \tilde{z}^h regarded as a fixed parameter). With $z_0^a = 0$, we can easily verify that $V_{1\alpha}$, $V_{0\alpha\beta\gamma}$, and $V_{2\alpha}$ all vanish, while the first two terms in (B14) and (B15) cancel. This verifies again that V_{eff} is independent of z^α in orders ϵ^2 and ϵ^3 .

APPENDIX C: THEORIES WITHOUT SUPERHEAVY SCALARS

We suppose in this appendix that there are no superheavy scalars, but that the superpotential $f(z)$ involves a light mass scale, of order m_g . That is, the scalar field labels a, b, \dots run only over values α, β, \dots denoting light scalars, and the superpotential now has nonvanishing but small second derivatives with respect to these scalars:

$$f_{\alpha\beta} \equiv \left[\frac{\partial^2 f}{\partial z^\alpha \partial z^\beta} \right]_0 \sim m_g. \quad (\text{C1})$$

To order m_g^4 , Eq. (2.2) then gives

$$V = \sum_\alpha \left| \frac{\partial f_{\text{eff}}}{\partial z^\alpha} + 8\pi G E^{1/2} z^\alpha \tilde{f} \right|^2 + \sum_h \left| E^{1/2} \left[\frac{\partial \tilde{f}}{\partial \tilde{z}^h} + 8\pi G \tilde{z}^h \tilde{f} \right] + 8\pi G \tilde{z}^h f_{\text{eff}} \right|^2 - 24\pi G |E^{1/2} \tilde{f} + \tilde{f}_{\text{eff}}|^2 + \sum_\kappa D_\kappa^2 \quad (\text{C2})$$

with z^α and \tilde{f} taken as of order m_g and

$$f_{\text{eff}} = E^{1/2} \left[\frac{1}{2} \sum_{\alpha\beta} f_{\alpha\beta} z^\alpha z^\beta + \frac{1}{6} \sum_{\alpha\beta\gamma} f_{\alpha\beta\gamma} z^\alpha z^\beta z^\gamma \right] \sim m_g^3. \quad (\text{C3})$$

The quantities within the absolute value signs in (C2) are purely of order m_g^2 in the first term, but of order m_g and m_g^3 in the second and third terms. Discarding the terms in V of order m_g^6 , we have then to order m_g^4 :

$$V = E\tilde{V} + \sum_\alpha \left| \frac{\partial f_{\text{eff}}}{\partial z^\alpha} \right|^2 + 16\pi G E^{1/2} \text{Re} \tilde{f}^* \sum_\alpha z^\alpha \frac{\partial f}{\partial z^\alpha} + 64\pi^2 G^2 E |\tilde{f}|^2 \sum_\alpha |z^\alpha|^2 + 16\pi G E^{1/2} \text{Re} \sum_h \left[\tilde{z}^h \frac{\partial \tilde{f}}{\partial \tilde{z}^h} + 8\pi G \tilde{f} |\tilde{z}^h|^2 \right]^* f_{\text{eff}} - 48\pi G E^{1/2} \text{Re} \tilde{f}^* f_{\text{eff}} + \sum_\kappa D_\kappa^2. \quad (\text{C4})$$

To this order the minimization of V with respect to \tilde{z}^h gives $\tilde{z}^h = \tilde{z}_0^h$, where \tilde{V} vanishes. Equation (C4) is then the same as the result quoted in Eq. (3.14).

¹The possibilities of constructing realistic models with low-energy spontaneously broken supersymmetry were extensively explored by P. Fayet, Phys. Lett. **69B**, 489 (1977); **70B**, 461 (1977); in *New Frontiers in High Energy Physics*, edited by A. Perlmutter and L. F. Scott (Plenum, New York, 1978), p. 413; in *Unification of the Fundamental Particle Interactions*, proceedings of the Europhysics Study Conference, Erice, 1980, edited by S. Ferrara, J. Ellis, and P. van Nieuwenhuizen (Plenum, New York, 1980), p. 587; in *Proceedings of the XVIth Rencontre de Moriond: Vol. I, Perturbative QCD and Electroweak Interactions*, edited by J. Trân Thanh Vân

(Editions Frontières, Dreux, France, 1981), p. 347. Also see M. Sohnius, Nucl. Phys. **B122**, 291 (1977). For more recent discussions of the difficulties encountered in these models, see E. Witten, Nucl. Phys. **B188**, 513 (1981); S. Dimopoulos and H. Georgi, *ibid.* **B193**, 150 (1981); N. Sakai, Z. Phys. C **11**, 753 (1982); S. Weinberg, Phys. Rev. D **26**, 287 (1982); L. Alvarez-Gaumé, M. Claudson, and M. Wise, Nucl. Phys. **B207**, 96 (1982); L. Hall and I. Hinchcliffe, Phys. Lett. **112B**, 351 (1982); G. Farrar and S. Weinberg, Phys. Rev. D (to be published).

²Globally supersymmetric models with a partially isolated

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- ¹⁰L. Alvarez-Gaumé, J. Polchinski, and M. Wise, Harvard/CalTech Report No. HUTP-82/A063-CALT-68-990 (unpublished).
- ¹¹J. León, M. Quirós, and M. Ramón Medrano, Madrid report (unpublished).
- ¹²Forerunners of the work of Refs. 3–11 include B. A. Ovrut and J. Wess, Phys. Lett. **112B**, 347 (1982) and R. Barbieri, S. Ferrara, D. V. Nanopoulos, and K. S. Stelle, *ibid.* **113B**, 219 (1982). Ovrut and Wess considered a supergravity model in which supersymmetry was broken in the hidden sector not spontaneously, but through the discard of a cosmological constant. Barbieri *et al.* discussed a supergravity model in which supersymmetry is spontaneously broken by a hidden sector, but there is no observable sector superpotential.
- ¹³The appearance of relatively light gauge fermions in theories like those of Refs. 3–12 is discussed by S. Weinberg, Phys. Rev. Lett. **50**, 387 (1983). R. Arnowitt, A. H. Chamseddine, and P. Nath, *ibid.* **50**, 232 (1983).
- ¹⁴The assumption of a superpotential consisting of a sum of two terms involving different sets of superfields would be more attractive if it could be shown to follow naturally from some symmetry of the theory. This is easy to arrange if the superpotential is limited to be a cubic polynomial; for instance, if the theory has a group $G \times \tilde{G}$ of gauge and/or global symmetries, then if all superfields S^a are non-neutral under G and neutral under \tilde{G} and all superfields \tilde{S}^h are neutral under G and non-neutral under \tilde{G} , the superpotential can contain no terms involving both S^a 's and \tilde{S}^h 's. However, it is doubtful whether the assumption of a cubic superpotential can be justified on the grounds of renormalizability for fields like \tilde{S}^h , whose scalar VEV's are of order of the Planck mass. Alternatively, it would be possible to understand the decomposition of the superpotential into $f(S)$ and $\tilde{f}(\tilde{S})$ if the theory had an R symmetry (a symmetry for which the superfield coordinate θ_L carries some nonvanishing quantum number, say, $+1$); for instance, if all S^a superfields have $R = \frac{2}{3}$ and all \tilde{S}^h superfields have $R = 2$, then the superpotential consists of terms trilinear in the S^a plus terms linear in the \tilde{S}^h . In such a theory there would be no additive constant in the superpotential that could be used to cancel the cosmological constant, but it would still be possible to arrange for a flat space by adjustment of the Kähler potential.
- ¹⁵An alternative with similar consequences is suggested by E. Cremmer, P. Fayet, and L. Girardello, Phys. Lett. **122B**, 41 (1983).
- ¹⁶The independence of the physics of the light Higgs scalars on the grand-unified mass scale was subsequently discussed for a wide class of superpotentials by P. Nath, A. H. Chamseddine, and R. Arnowitt, Harvard-Northeastern Report No. HUPT82/AO57-NUB2579 (unpublished).
- ¹⁷This result was given for nongauge theories with a single chiral superfield by E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello, and P. van Nieuwenhuizen, Phys. Lett. **79B**, 231 (1978); Nucl. Phys. **B147**, 105 (1979). The generalization to theories with arbitrary numbers of chiral scalars was given by E. Witten and J. Bagger, Phys. Lett. **115B**, 202 (1982); R. Barbieri, S. Ferrara, D. V. Nanopoulos, and K. S. Stelle, *ibid.* **113B**, 219 (1982). The results for general gauge theories were given by E. Cremmer, S. Ferrara, L. Girardello, and A. Van Proeyen, *ibid.* **116B**, 231 (1982); J. Bagger, Princeton report (unpublished).
- ¹⁸The breakdown of the grand-gauge group can allow scalars to remain with zero mass and VEV for any one of three possible reasons: either because of unbroken symmetries, or of a fine tuning of the parameters of the theory, or as a more or less accidental property of the potential minimum that follows automatically from the symmetries and cubic polynomiality of the observable sector superpotential. In the case of fine tuning a tiny change in the input parameters could introduce additional mass parameters in the low-energy theory in addition to those produced by supergravity, so in this case it may be argued that it is not gravitation alone that sets the mass scale of known particles. In the other two cases supersymmetry can truly be said to resolve the hierarchy problem, except that the small scale of the hidden-sector superpotential still has to be put in by hand. In this paper we will consider all these cases to-

gether.

¹⁹This is in apparent conflict with the cosmological lower bound of about 10 TeV on the mass of heavy gravitinos; see S. Weinberg, Phys. Lett. **48**, 1303 (1982). However, entropy-producing mechanisms like cosmic inflation that dilute the gravitino (and monopole) densities could reduce the entropy produced in gravitino decay sufficiently so as not to disturb calculations of cosmic nucleosynthesis; see J. Ellis, A. D. Linde, and D. V. Nanopoulos, Phys. Lett. **118B**, 59 (1982); S. Dimopoulos and S. Raby, Los Alamos Report No. LA-UR-82-1282, 1982 (unpublished).

²⁰Theories with a purely trilinear effective low-energy superpotential have a severe phenomenological problem: the electroweak $SU(2) \times U(1)$ symmetry is spontaneously broken only for $|m'_g/m_g| > 3$, but in this case the introduction of quark and lepton superfields leads to a vacuum solution for which quark and/or lepton scalars have nonzero VEV's, as pointed out by J.-M. Frère, D. R. T. Jones, and S. Raby, Michigan Report No. UMHE82-58 (unpublished). However, $SU(2) \times U(1)$ can be spontaneously broken for $|m'_g/m_g| < 3$ if bilinear terms are allowed in the effective superpotential, and in this case it is not clear that VEV's appear for quark and lepton scalars. Also it is not clear that the tunneling from the baryon- and lepton-conserving local

minima of the potential to the one found by Frère *et al.* is fast enough to pose a serious problem; M. Claudson, L. Hall, and I. Hinchcliffe (unpublished).

²¹The problem of radiative corrections has been discussed by R. Arnowitt, A. H. Chamseddine, and P. Nath, Phys. Lett. **120B**, 145 (1983); S. Ferrara, D. V. Nanopoulos, and C. A. Savoy, Ref. 8; H. P. Nilles, M. Srednicki, and D. Wyler, CERN Report No. TH-3461 (unpublished); A. B. Lahanas, CERN Report No. TH-3467 (unpublished); R. Barbieri and S. Cecotti, Pisa Report No. SNS9/1982 (unpublished). Also see J. Polchinski and L. Susskind, Ref. 2.

²²This symmetry was used in a similar way by S. Weinberg, Ref. 13. Also see M. K. Gaillard, talk at the Vanderbilt University Conference on Novel Results in Particle Physics, Berkeley Report No. LBL-14647 (unpublished).

²³The assumption that the scalar fields \mathcal{H} , \mathcal{H}' , and \mathcal{J} do not get large masses or VEV's in the breakdown of the grand-gauge group is, for example, automatically satisfied if we suppose that a discrete symmetry remains unbroken, for which the superfields H , H' , and J are transformed by a factor $\exp(2i\pi/3)$. In this case the only terms in (5.9) are the last two, with coefficient λ and λ' .