Absolute nature of the thermal ambience of accelerated observers

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The blackbody radiation (thermal ambience) surrounding a linearly and uniformly accelerated observer is shown to be totally isotropic. By contrast, an observer who does not merely have this uniform acceleration into some direction, but also (a) drifts uniformly (according to his own clock) into another, or alternatively (b) orbits uniformly around a fixed axis, will be surrounded by vacuum fluctuations with the spectrum of a blackbody endowed with a chemical potential. This potential is proportional to the drift velocity for one observer, and to the angular velocity for the other. These chemical potentials express an anisotropy of empty flat space relative to these observers.

I. INTRODUCTION AND SUMMARY

It is known that in flat space-time the vacuum relative to an observer suffering linear uniform acceleration is distinct from that of an inertial observer.[1] The vacuum fluctuations relative to the accelerated observer have the spectrum of thermal radiation.[2,3] Interestingly enough, the effects of these vacuum fluctuations are directly observable in the presence of accelerated detectors,[3] accelerated conductors,[4] as well as in the presence of accelerated charges.[5]

This paper reports two phenomena. (1) Although a uniformly linearly accelerated observer has a preferred direction (namely, his acceleration), radiation processes on the classical as well as on the quantum level are *totally* isotropic. (2) Relative to more general accelerated observers this *isotropy is broken.* This happens if they have (according to their own clocks) (a) a uniform drift velocity or (b) a uniformly orbiting motion in addition to what otherwise would only be uniform linear acceleration. On the quantum-mechanical level this breaking of the spatial isotropy manifests itself as a nonzero chemical potential of the thermal radiation ("ambience") surrounding these observers. This potential is proportional to the drift velocity or the angular velocity of the respective accelerated observers.

The chemical potential is such that the intensity of wave field modes propagating against (a) the drift or (b) the rotation exceeds that of wave field modes propagating in the same sense. Thus the thermal ambience has a tendency to slow down the drift or angular velocity of an observer until his surrounding is totally isotropic. The novel feature implied by the above-mentioned spatial isotropy together with isotropy breaking by the chemical potentials is this: *Empty flat space has an absolute character relative to uniformly accelerated observers.* Thus, among the set of all accelerated observers, each one of which has a *prima facie* preferred direction associated with his spatial direction of acceleration, there is a subset of accelerated observers for whom this *prima facie* direction is totally absent. This subset of observers consists of uniformly linearly accelerated observers. Relative to them the electromagnetic field of empty Minkowski space-time appears totally isotropic. On a classical level this fact is well known: radiation-reaction forces acting on a detector consisting of a point charge carried along by this observer are identically zero.[6,7] This isotropy, as we shall see, extends itself to the quantum-mechanical level. In fact, for this observer, and others like him, the thermal ambience has a stress energy tensor which is that of a perfect isotropic nonhomogeneous fluid. More graphically, he is immersed in a liquid ocean whose bottom is the event horizon, where the pressure becomes infinite. In the presence of purely orbiting motion, i.e., in the absence of an event horizon there is no thermal radiation and no chemical potential.[8,9]

II. DRIFTING AND ROTATING RINDLER OBSERVERS

Observers possessing (a) uniform drift or (b) uniform orbiting motion in addition to only uniform linear acceleration are generalizations[10] of those observers whose world lines generate the well-known Rindler coordinates on flat space-time. If Schwarzschild space-time is identified with Rindler's representation of Minkowski[11] space-time, then both of the aforementioned generalizations are similar to Kerr space-time.

Drifting and rotating generalizations of Rindler representations are characterized, respectively, by additional vector fields, a translational Killing vector field in one case and a rotational Killing field in the other. The two space-times are so similar that in this paper we shall consider only the drifting space-time in any detail. The results for the rotating space-time we merely quote near the end.

The coordinates that mold themselves naturally to the generalized Rindler representation with uniform drift are obtained from Minkowski coordinates by

$$\begin{aligned} t &= \xi \sinh gT \ , \\ x &= \xi \cosh gT \ , \\ y &= Y + vT \ , \\ z &= z \ . \end{aligned} \tag{1}$$

For ξ, Y,z held constant the resultant world line has constant proper acceleration

$$g\frac{\xi g}{\xi^2 g^2 - v^2}$$

whose direction precesses with proper angular velocity

$$g\frac{v}{\xi^2 g^2 - v^2}$$

relative to a set of (Fermi-Walker transported) gyroscopes. Furthermore the world line has a constant drift $dy/dT = v$ into the y direction and traces out a Killing trajectory as is evident from the form of the metric relative to these coordinates:

$$\begin{aligned} ds^2 = &-(\xi^2 g^2 - v^2) dT^2 + 2v\, dY\, dT + dY^2 \\ &+ d\xi^2 + dz^2 \ . \end{aligned} \tag{2}$$

The noncompact event horizon is at $\xi = 0$ and the "ergo" slab, the region where Killing trajectories are spacelike, extends over the interval $0 < \xi < v/g$.

III. THERMAL AMBIENCE OF RINDLER REPRESENTATION WITH DRIFT

We now pose and solve the following initial-value problem: Given a spacelike hypersurface in Minkowski space-time, specify random initial conditions on this surface. Let these initial-value data evolve according to some wave equation, for now the Klein-Gordon equation. Determine the wave field, in particular its Fourier spectrum, as seen by an accelerated observer with uniform drift.

Quantum mechanics enters the picture only through the initial-value data. The evolution and the field amplitude seen by the accelerated observer are considered here strictly within classical wave field theory.

The thermal ambience of the Minkowski vacuum is trivial. It is characterized by zero temperature. Its spectrum is simply that of the vacuum fluctuations. These fluctuations are an incoherent superposition of normal modes of the appropriate wave field in question. Consider a zero-rest-mass scalar wave field in a large cavity of length a and cross-section area L^2. The allowed normal modes are

$$\begin{aligned} \psi_n = A_n \sin(\omega_n t + \beta_n) &\cos\frac{n_1 \pi x}{a} \sin\frac{n_2 \pi y}{L} \\ &\times \sin\frac{n_3 \pi z}{L} \ , \end{aligned} \tag{3}$$

where

$$\omega_n{}^2 = \left[\frac{n_1\pi}{a}\right]^2 + \left[\frac{n_2\pi}{L}\right]^2 + \left[\frac{n_3\pi}{L}\right]^2 \ , \qquad n_1, n_2, n_3 = 1, 2, \ldots$$

is the frequency, and

$$\beta_n = \beta(n_1, n_2, n_3)$$

is the phase angle, a random function of the modes. The modes are oscillators in their ground states whose energies are $\frac{1}{2}\hbar\omega_n$. Consequently, the amplitude A_n of each has the value

$$A_n = \left[\frac{2\hbar}{a\omega_n}\right]^{1/2} \frac{1}{2L} \ .$$

This completes the specification of the Minkowski vacuum as random initial-value data.

An accelerated observer does not use Minkowski normal modes to describe the vacuum fluctuations he sees. Instead he uses the modes associated with the generalized Rindler coordinates of Eq. (1). An important question which all observers following the trajectories of topologically distinct timelike Killing vector fields must ask and answer is this: How are the vacuum fluctuations, in particular their intensity spectra, related to each other?

The answer is obtained by having an accelerated observer Fourier analyze the fluctuating Minkowski normal modes, Eq. (3). He uses, of course, the Fourier basis associated with his own Killing vector field, instead of that based on the inertial world lines of the Minkowski space-time. The procedure is straightforward. Have the observer evaluate Eq. (3) along his own world line, Eq. (1), and then Fourier analyze the resultant signal. The result is strikingly simple and far reaching.

The typical real standing-wave mode, Eq. (3), is composed of complex exponential modes. They refer to particles of positive ($\hbar\omega_n$) as well as negative ($-\hbar\omega_n$) energies. Each has momenta $\pm\hbar k_y$ parallel and antiparallel to the drift velocity $v = dy/dT$ of the observer. Thus the typical standing-wave mode, Eq. (3), is the superposition of two parts: particle modes having momentum parallel and those having momentum antiparallel to the drift velocity of the observer. This superposition is

$$\psi_n = \psi_{n,k_y} + \psi_{n,-k_y} ,$$

where

$$\psi_{n,\pm k_y} = \pm \frac{A_n}{2i} \sin(\omega_n t + \beta_n) \cos\frac{n_1 \pi x}{L} \times \sin\frac{n_3 \pi z}{L} e^{\pm i n_2 \pi y / L} .$$

The spectral intensity of a single wave is[12]

$$| \psi_{n,\pm k_y}(\omega,\xi,Y,z) |^2 = g_n(\omega,z) \, | K_{\varpi/i}(k\xi) |^2 \times 2(\cosh\pi\varpi + \cos 2\beta_n) , \tag{4}$$

where

$$g_n(\omega,z) = | \mp i A_n \cos\theta_n (\omega \pm k_y v) \sin k_z z |^2 .$$

Here K is the modified Bessel function of pure imaginary order

$$\frac{\varpi}{i} = \frac{1}{i}(\omega \pm k_y v)$$

whose argument is proportional to the magnitude of the transverse propagation vector:

$$k = \left[\left[\frac{n_2 \pi}{L} \right]^2 + \left[\frac{n_3 \pi}{L} \right]^2 \right]^{1/2} \equiv (k_y{}^2 + k_z{}^2)^{1/2} . \tag{5}$$

The angle θ_n characterizes the Minkowski mode by

$$\omega_n = k \cosh\theta_n , \quad k_x = k \sinh\theta_n .$$

What is so important about the spectral intensity Eq. (4)? The answer is threefold: (i) thermal nature; (ii) isotropy for uniformly linearly ($v=0$) accelerated observers; and (iii) anisotropy, via a chemical potential, for accelerated observers with drift ($v \neq 0$).

The first feature, the thermal nature of the thermal ambience, is directly revealed by Eq. (4) when it refers to oscillating wave modes, i.e., when the waves are primarily propagating along the ξ direction: $(k\xi)^2 \ll (\omega \pm v k_y)^2$. In that case the spectrum is

$$| \psi_{n,\pm k_y}(\omega,\xi,Y,z) |^2 = g_n(\omega,z) h(\omega,k_y,\xi) \left\{ \frac{1}{2} + \frac{1}{\exp[\hbar(\omega \pm k_y v)/kT] - 1} + \left[\frac{1}{\exp[\hbar(\omega \pm k_y v)/kT] - 1} + \left[\frac{1}{\exp[\hbar(\omega \pm k_y v)/kT] - 1} \right]^2 \right]^{1/2} \cos 2\beta_n \right\} , \tag{6}$$

where

$$h(\omega,k_y,\xi) = \frac{4\pi}{\omega \pm k_y v} \sin^2 \left[(\omega \pm k_y v) \ln\frac{k\xi}{2} + \arg\Gamma(i\omega \pm i k_y v) \right]$$

and Γ is the gamma function. The importance of the spectrum Eq. (6) lies in the quantity in curly brackets. It consists of three terms. The first two have the form of the Minkowski zero-point energy spectrum augmented by a blackbody spectrum, whose temperature is that of Unruh[3] and Davies,[2]

$$kT = \frac{\hbar}{2\pi} \frac{g}{c} .$$

[Temperature T is not to be confused with the world-line parameter in Eqs. (1) and (2). Neither is Boltzmann's constant k to be confused with the magnitude of the transverse propagation vector Eq. (5).] The grand total spectral intensity seen by the accelerated observer is the sum of contributions, Eq. (6), due to all relevant Minkowski modes. In such a sum the fluctuating third term of Eq. (6) averages to zero. It (i) constitutes the fluctuations away from thermal equilibrium, (ii) is demanded by the thermal nature of blackbody radiation, and (iii) has its roots in the randomness of the Minkowski zero-point fluctuations, i.e., in the phase incoherence of the set of Planckian oscillators, Eq. (3), in their ground states.

From the viewpoint of basic issues of principle the importance of the fluctuating (third) term can not be stressed too much. If it were absent, one would be totally unjustified in claiming that

$$2\pi c \frac{\omega}{g} = \frac{\hbar\omega}{kT} ,$$

i.e., that the power spectrum (the second term) seen by the accelerated observer is thermal in nature. After all, a Planckian spectrum can be obtained from a suitable multicolor array of lasers. It is the magnitude of the rms fluctuations in each spectral component away from the Planckian mean that provides distinctive agreement with a thermal spectrum.

The second important feature about the spectral intensity Eq. (4) is its stunning simplicity both in regard to its mean power spectrum and its root-mean-squared fluctuation spectrum. Instead of Klein-Gordon theory, consider now Maxwell theory in the absence of drift ($v=0$). Equation (4) now serves as a scalar potential for transverse electric modes provided the normalization constant gets replaced by $A_n{}^2=32\pi\hbar/a\omega_n k^2L^2$. Transverse magnetic modes follow virtually the same treatment. We have now the following principle: *The thermal ambience is spatially isotropic in every respect, even though the observer is accelerating into a preferred direction.* This isotropy holds with respect to (a) the polarization of the electromagnetic field, (b) the thermal fluctuations away from the mean spectral power, and (c) the electromagnetic stress energy tensor. The proof of these claims, sketched below, follows from the manner in which polarization-sensitive detectors respond to the electromagnetic field.

A polarization-sensitive detector[13] consists of a particle of charge e, mass m, bound harmonically with frequency ω, and constrained to move along one of the three perpendicular spatial directions of the observer. Consequently, each detector responds only to its respective electric field component E_j, $j=\xi,y,z$. The energy deposited into the jth detector of frequency ω by all normal modes, both transverse electric and magnetic, typically given by Eq. (3), having the various propagation directions $(k_y,k_z)=(n_2\pi/L,n_3\pi/L)$ is

$$(\text{energy})_j=(\pi e^2/m)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[E_j(\omega,k_y,k_z)]^2dk_y dk_z\ ,\quad j=\xi,y,z\ . \tag{7}$$

A straightforward evaluation of these integrals shows that the energies in all three polarizations have the same value,[14] namely,

$$(\text{energy})_j=\frac{\pi e^2}{m}\left[\frac{32\pi\hbar}{a\omega_n L^2}\right]\frac{\pi}{L^4}\left[\frac{1}{2}+\frac{1}{\exp(\hbar\omega/kT)-1}\right]\frac{\omega+\omega^3}{3}\ . \tag{8}$$

In order to show that this frequency spectrum has in fact a thermal nature one must show that the rms fluctuations in these three energies have also identical values.

This demonstration is easily achieved by inference from Eq. (8) applied to Eq. (4):

$$[\text{rms fluctuation in }(\text{energy})_j]=\frac{\pi e^2}{m}\left[\frac{32\pi\hbar}{a\omega_n L^2}\right]\frac{\pi}{L^4}\left[\frac{1}{\exp(\hbar\omega/kT)-1}+\left[\frac{1}{\exp(\hbar\omega/kT)-1}\right]^2\right]^{1/2}\frac{\omega+\omega^3}{3}\ ,\quad j=\xi,y,z\ . \tag{9}$$

Thus the fluctuations in all three energies are indeed identical and have in fact the characteristic thermal signature.

An analogous set of statements holds for the magnetic field components. Moreover the remaining electromagnetic (shear) components constructed [like Eq. (7)] from E_iE_j and B_iB_j, $i\neq j$, are zero. Furthermore, the Poynting vector is also zero. An absorber, i.e., a detector sensitive to radiative electromagnetic momentum, would therefore indicate no preferred direction. We thus conclude that the thermal ambience is *totally* isotropic, in its power spectrum as well as in its thermal fluctuation spectrum.

The above remarks make it clear that the electromagnetic stress energy tensor constructed from the above quadratic expressions in the electromagnetic field is spatially isotropic. In fact, it is given by

$$T_\mu{}^\nu=\frac{1}{4\pi}\left[\frac{32\pi\hbar}{a\omega_n L^2}\right]\frac{\pi^2}{\xi^4}\int_0^\infty\left[\frac{1}{2}+\frac{1}{\exp(\hbar\omega/kT)-1}\right](\omega+\omega^3)d\omega\,\mathrm{diag}\{-1,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3}\}\ . \tag{10}$$

Note, however, that this tensor is not an event-wise-defined object. In fact, it is composed of the mean-field intensities, which are measured by the rms energies, Eq. (7), of an oscillator. These energies in turn are averages over at least one oscillation, typically,

$$\frac{c}{g}=\frac{1}{2\pi}\frac{\hbar}{kT}\ ,$$

the time during which the accelerated detector changes its velocity by a substantial fraction as seen by a Minkowski observer. This stress energy tensor is therefore defined along a non-negligible segment of the accelerated detector's world line. In short, the "tensor" field is not pointwise defined. It does not have the appropriate coordinate-transformation properties. It is therefore a rather dubious proposition to use such "tensor" fields (after a suitable regularization) as some sort of effective source for the Einstein field equation.

The third important and novel feature of the blackbody spectrum is that it has a chemical potential. It is such that the accelerated observer sees a larger intensity of wave fields running antiparallel to his drift velocity than parallel. If he interacts with this thermal ambience, then it has a tendency to slow him down. This is analogous to the spin down of a Kerr black hole.

IV. THERMAL AMBIENCE OF RINDLER REPRESENTATION WITH ROTATION

The absolute nature of the thermal ambience is evidently present with all accelerated observers that have an event horizon. In fact, let the accelerated observer not be drifting but rather *rotating* uniformly with some angular velocity constant as measured by his own clock. Such an observer traces out a world line ($\xi,\Phi,r=$const, $-\infty<T<\infty$) given by[10]

$$t=\xi\ \sinh gT\ ,$$

$$x=\xi\ \cosh gT\ ,$$

$$y=r\sin(\Phi+\Omega T)\ ,$$

$$z=r\cos(\Phi+\Omega T)\ .$$

In x,y,z coordinate space this observer executes a helical motion. The natural representation of the metric for this observer is

$$ds^2=-(\xi^2g^2-r^2\Omega^2)dT^2+2r^2\Omega\, dT\, d\Phi$$
$$+r^2d\Phi^2+d\xi^2+dr^2\ .$$

The spectrum of this ambience is analogous to that given by Eq. (4). Now, however, the Planck spectrum has the form

$$\frac{1}{\exp[\hbar(\omega\pm m\Omega)/kT]-1}\ .$$

The chemical potential $\pm\hbar\Omega m$ (here m is the azimuthal wave number) is such that the wave field intensity of the thermal ambience is greater for those modes that circulate against the angular velocity Ω of the observer. Suppose the observer interacts with this thermal ambience. Then it will have a tendency to slow his rotation until his thermal ambience coincides with a thermal ambience which has no rotation (and no drift). This thermal ambience is isotropic; i.e., (i) its power spectrum is isotropic, (ii) its fluctuation spectrum is isotropic and gives the power spectrum a thermal signature, (iii) its stress energy tensor is isotropic, and (iv) classical radiation reaction forces are absent from point detectors carried along by the observer. The thermal ambience has therefore an "absolute" status in relation to those ambiences in which the event horizon is rotating (or drifting) relative to the observer.

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[12]For computational convenience we have absorbed the acceleration g into the time T in Eq. (1). This results in the simplification $gT \rightarrow T$, $vg^{-1} \rightarrow v$, $\omega g^{-1} \rightarrow \omega$, $A_n g^{-1} \rightarrow A_n$ in Eqs. (4) and (6). After Eq. (6) we revert back to dimensioned units by simply reversing the arrows.

[13]See, for example, J. D. Jackson, *Classical Electrodynamics,* 2nd ed. (Wiley, New York, 1980), Chap. 13.2.

[14]This fact has also been found independently by T. Boyer, Phys. Rev. D 21, 2137 (1980).