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Inflationary universe with gravity

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The original idea of the inflationary universe is shown to be consistent with observation, if the effects of spacetime curvature are considered. Specifically, a combined grand-unified—gravitational tunneling solution exists, which is homogeneous over the entire spatial section and has lower action than the flat-space, inhomogeneous bubble. Thus, the exit from the metastable de Sitter phase can occur gracefully, without generating large spatial inhomogeneities or magnetic monopoles. The large expansion factor is generated naturally by the tunneling amplitude and does not require fine tuning the initial conditions of the post-tunneling evolution. Fluctuations in this model may differ substantially from the roll-over picture.

I. INTRODUCTION

An essential element in current ideas of unification of the different interactions observed in nature is spontaneous symmetry breaking. However, the symmetry of the ground state of a system generally depends on its temperature. A system with a high degree of symmetry may undergo one or more phase transitions to states of lower or spontaneously broken symmetry as its temperature is lowered. This simple observation leads to dramatic conclusions when grand unified theories[1] are applied to cosmology. For, if the universe was once in an early high-temperature phase, with complete symmetry between the fundamental interactions, its expansion and cooling may have "trapped" it in a supercooled false vacuum state. Since the energy density of the false vacuum is positive, the universe would have passed through an exponentially expanding de Sitter phase, greatly diluting any curvature inhomogeneities, while greatly expanding the size of a causally connected domain.[2] If the exit from this de Sitter phase is via quantum tunneling, which is typically a slow process, then the large expansion factor arises naturally and provides a resolution of the horizon and flatness problems of the standard model.[3]

Just as this idea was proposed it was understood to have significant problems. The mechanism for the first-order phase transition is the spontaneous materialization of bubbles of true vacuum within false.[4] Since this process is completely random, there is no correlation between the different asymmetric states within different bubbles. When the bubble walls collide a violent process of vacuum rearrangement must take place and topological defects are produced. In grand unified theories (GUT's) these defects are magnetic monopoles, which would have been produced in numbers so enormous as to dominate the mass energy of the universe.[5] The present universe would then have large inhomogeneity and anisotropy, contrary to observation. On the other hand, if the bubble walls do not collide then the supercooled universe could never rethermalize, in the simplest picture, and all of the successful predictions of the standard model at later times would be lost.

Recently, several authors have pointed the way out of this dilemma.[6] The collision of the bubble walls is essential to rethermalization only within the thin-wall approximation of vacuum decay. This approximation, in turn, is justified only if the energy-density difference between the false vacuum and true is small compared to the mass scales in the problem. If this condition is not satisfied, rethermalization may occur *within* each bubble by a process of particle creation and interaction, independently of collisions with other bubble walls. There is then no large production of monopoles or associated inhomogeneities and the original inflationary-universe idea has been rescued, provided the entire universe can fit in one such bubble.

Preliminary work has focused on the Coleman-Weinberg mechanism for spontaneous symmetry breaking and has not included a complete treatment of curvature effects. Despite one's initial thought that the Planck mass M_P is 4 orders of magnitude

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larger than the GUT scale, Hawking and Moss[7] observed that gravitational effects are relevant in one important respect. Since de Sitter spacetime has a cosmological event horizon, Hawking radiation does not permit the supercooling to continue below the Hawking-de Sitter temperature T_H.[8]

In addition, curvature has another important effect. The $R\phi^2$ coupling required for renormalizability in curved spacetime[9] can generate an effective barrier to tunneling out of the false vacuum[10]: This barrier is sufficient to produce the large dimensionless number needed for the inflationary picture to work—without any fine adjustment of initial conditions in the post-tunneling expansion. Furthermore we shall show that this tunneling takes place *homogeneously* over the entire universe. In contrast to previous authors, who attempt to gain a sufficiently large expansion of the universe in a post-tunneling evolution down a Coleman-Weinberg plateau, we gain the large expansion factor in the *pre*-tunneling phase. The point is that the universe is expanding in the pre-tunneling false-vacuum stage, and when a homogeneous tunneling takes place it fills this already expanded universe. This is in contradistinction to a scenario where an inhomogeneous bubble tunneling event takes place—because even though the universe may be large at the time of tunneling the bubble nucleates at a typical small size which must then grow in a post-tunneling evolution to a size consistent with the present physical universe.

Also in this homogeneous scenario there are no bubble walls to worry about and monopole production is suppressed. Rethermalization takes place throughout the universe by conversion of the vacuum energy density to matter.

Thus, the consistent inclusion of gravitational effects leads to a small probability for barrier penetration which allows us to achieve a natural solution of the horizon and flatness problems of the standard model without the inhomogeneities of the original inflationary scenario, i.e., curvature effects permit a graceful exit from the symmetric phase.

The paper is organized as follows. In Sec. II we consider the effects of the $R\phi^2$ coupling in the SU(5) GUT with Coleman-Weinberg symmetry breaking and calculate the action for various escape paths from the symmetric vacuum. In Sec. III we calculate the decay rate or time of tunneling and present the explicit analytic continuations necessary to demonstrate that this time is large enough to expand the universe by as much as 60 orders of magnitude. In Sec. IV we consider the post-tunneling evolution and estimate the temperature at rethermalization, which is important for baryon asymmetry calculations. We conclude with a discussion of the possibility of relaxing the pure Coleman-Weinberg potential and fluctuations about the homogeneous background.

II. $R\phi^2$ BARRIER PENETRATION IN SU(5)

In the SU(5) theory,[11] the Higgs field Φ relevant for spontaneous symmetry breaking is in the $\underline{24}$ representation of SU(5). Assuming this breaking is directly to $SU(3)\times SU(2)\times U(1)$ we can write the expectation value of Φ in the form

$$\langle\Phi\rangle=(\tfrac{2}{15})^{1/2}\phi\,\mathrm{diag}(1,1,1,-\tfrac{3}{2},-\tfrac{3}{2})\ . \tag{2.1}$$

The Coleman-Weinberg[12] one-loop effective potential for ϕ is then (cf. Fig. 1)

$$V_{\mathrm{CW}}(\phi)=B\phi^4[\ln(\phi^2/\sigma^2)-\tfrac{1}{2}]+\tfrac{1}{2}B\sigma^4\ , \tag{2.2}$$

where

$$B=\tfrac{25}{16}\alpha_{\mathrm{GUT}}{}^2=7.7\times10^{-4} \tag{2.3}$$

and σ is the expectation value at zero temperature,

$$\sigma=1.2\times10^{15}\ \mathrm{GeV}\ , \tag{2.4}$$

with $\alpha_{\mathrm{GUT}}=\frac{1}{45}$, at this energy scale.

When the theory is considered in curved spacetime, a counterterm of the form $-\eta R\phi^2/2$ is required to obtain finite results. Here R is the Ricci scalar and η is a free dimensionless parameter. Since η evolves according to renormalization-group changes in scale, it cannot be set equal to zero for all scales.[9] Now, the Coleman-Weinberg potential is derived by assuming scale invariance at the tree level; thus, the most natural choice for η is $\frac{1}{6}$ (at the GUT scale) for a classically conformally invariant theory. We shall leave η unspecified but implicitly

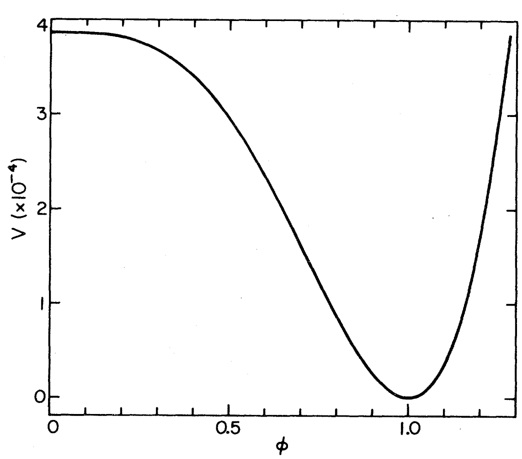

FIG. 1. The form of the one-loop Coleman-Weinberg effective potential of Eq. (2.2), in units in which $\sigma=1$. The $R\phi^2$ barrier near $\phi=0$ is totally negligible on this scale.

assume it to be positive and of order $\frac{1}{10}$ to 1 in what follows. When the theory is considered at temperatures greater than $T_{\rm GUT}$, the unique ground state is at $\phi=0$. As the temperature falls, the asymmetric state becomes energetically favored. Escape from the $\phi=0$ vacuum is highly improbable at high temperatures even in flat space.[13] For low temperatures the form of the effective potential is

$$V(\phi,T)\cong V_{\rm CW}(\phi)+\frac{\pi^2}{30}N(T)T^4+\tfrac{5}{2}\pi\alpha T^2\phi^2\ , \tag{2.5}$$

where $N(T)$ is the number of massless particle species at temperature T. When T falls below σ the thermal energy density quickly becomes negligible compared to $V(0)$ and the universe begins the de Sitter expansion. This defines both the time direction and origin. It is not necessary to assume anything about earlier times, particularly the existence of any singularity, provided that the temperature was once high enough to place the universe in the symmetric vacuum. Since the approach to the de Sitter phase is exponentially rapid, any previous inhomogeneity or detailed history becomes irrelevant.

The fact that the vacuum energy density $V(0)$ dominates, implies that the scalar curvature is fixed at

$$R_0=-\frac{32\pi}{M_P{}^2}V(0)\equiv-4\kappa V(0) \tag{2.6}$$

by Einstein's equations, with cosmological constant $\kappa V(0)$. Thus the $R\phi^2$ term acts as an effective mass term, with mass of order of the de Sitter scale,

$$H=\left[\frac{\kappa V(0)}{3}\right]^{1/2}=6.7\times10^9\ \text{GeV}\ . \tag{2.7}$$

This mass term presents a larger effective barrier than the temperature term in Eq. (5) as soon as T falls below 10^{10} GeV. This is approximately $10T_H$ where

$$T_H=H/2\pi=1.1\times10^9\ \text{GeV} \tag{2.8}$$

is the Hawking-de Sitter temperature.

In order to discuss the tunneling through the $R\phi^2$ barrier we must consider the Euclidean section. Since the geometry is essentially fixed to be de Sitter space, the global properties of that space on the Euclidean section are relevant. The line element may be written in the form

$$ds^2=d\xi^2+\rho^2(\xi)(d\chi^2+\sin^2\chi\, d\Omega^2)\ , \tag{2.9}$$

with

$$\rho(\xi)=H^{-1}\cos(H\xi-H\xi_1)\ , \tag{2.10}$$

ξ_1 an arbitrary constant. This is the line element on a four-sphere of radius H^{-1}.

Now field theory at finite temperature is defined by its periodicity in the Euclidean time coordinate. Since it is impossible for any function defined on S_4 to have a periodicity greater than $2\pi/H$, for any choice of time coordinate, it is impossible to define a temperature lower than T_H on this manifold. Physically, this is so because the cosmological event horizon of de Sitter space implies a loss of information accessible to an observer following the trajectory of the standard timelike Killing vector. By the homogeneity of de Sitter space, a cosmological observer whose constant time slices are surfaces of homogeneity is in a similar situation. This manifests itself in the form of a bath of thermal radiation at the Hawking temperature T_H—independently of the growth of the scale factor $a(t)$. That is, the usual assumption that the temperature is inversely proportional to $a(t)$, which grows exponentially in the de Sitter phase, does not apply. The extreme supercooling of the original inflationary model ignores the event horizon of de Sitter space: radiation cannot be red-shifted by expansion further than the infinite wavelengths approached near the horizon. Thus the temperature cannot fall below T_H.

The concept of temperature makes sense only in equilibrium or when the system is slowly evolving. Since the Coleman-Weinberg potential is very flat, the scalar field and $V(\phi)$ are slowly varying.

Also, if there were no barrier ($\eta=0$) the homogeneity and isotropy of the de Sitter phase would persist as long as ϕ remained on the plateau of the potential. When $\eta\sim\frac{1}{6}$ the height of the barrier is of order $10^{-16}V(0)$ so that we should expect that the system would prefer to tunnel through this very small obstacle and then continue its quasiclassical evolution to the new vacuum $\phi=\sigma$, without destroying its homogeneity and isotropy, rather than tunneling directly to $\phi=\sigma$ by the nucleation of small bubbles of new vacuum. We shall see that this expectation is borne out.

To analyze the tunneling process in detail we must find the classical Euclidean solution of least action. In flat space it is well known that the minimal action solution is rotationally invariant.[14] No proof of this statement in curved space is known, but it is reasonable to assume that it remains true in the presence of gravity. The most general rotationally invariant Euclidean metric may then be considered. It has already been given by Eq. (2.9). Furthermore, since the barrier is so much smaller than the scale of $V(0)$, Eq. (2.10) remains valid to good approximation. If, as we have argued is plausible, the tunneling does not destroy the rotational invariance, then there must exist a choice of ξ such that the Higgs field ϕ is also a function only of ξ. With $\phi=\phi(\xi)$ the Euclidean equations become

$$\phi''+\frac{3\rho'}{\rho}\phi'=\frac{dV}{d\phi}-\eta R\phi\ , \tag{2.11}$$

$$\rho^3(V-\tfrac{1}{2}\phi'^2)+\frac{3}{\kappa}\rho(\rho'^2-1)=\text{const}\ , \tag{2.12}$$

where a prime denotes $d/d\xi$ and the closed universe has been chosen for definiteness. Equation (2.12) is the conservation equation corresponding to the translation symmetry of the metric (2.9),

$$\xi\rightarrow\xi+\xi_1\ . \tag{2.13}$$

The other components of Einstein's equations are simply consequences of (2.11) and (2.12). Since the Lorentzian form of the general homogeneous, isotropic metric is the Robertson-Walker line element,

$$ds^2=-d\tau^2+a^2(\tau)(d\chi^2+\sin^2\chi\, d\Omega^2)\ , \tag{2.14}$$

it is natural to identify ξ with Euclidean time and Eq. (2.12) as the conservation of energy condition. The precise relation of this time to that fixed at the beginning of the de Sitter expansion will be given in the next section. The identification of Eq. (2.12) with conservation of Euclidean energy allows us to fix the constant by comparison with the de Sitter solution on the Lorentzian section:

$$\begin{aligned}&\phi=0\ ,\\ &a(\tau)=H^{-1}\cosh(H\tau)\ .\end{aligned} \tag{2.15}$$

Replacing $d/d\xi$ by $i\,d/d\tau$ and ρ by a fixes the constant to be zero so that (2.12) can be rewritten as

$$\rho'^2=1-\frac{\kappa\rho^2}{3}(V-\tfrac{1}{2}\phi'^2)\ . \tag{2.16}$$

Equations (2.11) and (2.16) are the equations of the Coleman-de Luccia bounce.[15] The action for a solution of these equations is

$$A=4\pi^2\int d\xi(\rho^3V-3\rho/\kappa)\ . \tag{2.17}$$

To find the solution of least action let us first consider tunneling directly from the symmetric vacuum to $\phi=\sigma$. This corresponds to the nucleation of bubbles of size $\bar\rho\sim(B\sigma^2)^{-1/2}$. For such bubbles, gravitational effects are negligible ($\bar\rho\ll H^{-1}$) and the action is of order

$$A_{\text{bubble}}\sim\bar\rho^4V(0)\sim\frac{1}{B}\cong1300\ . \tag{2.18}$$

Direct numerical integration of the equations was performed and confirms this estimate.[16]

On the other hand Hawking and Moss[7] pointed out that a simple homogeneous solution also exists:

$$\begin{aligned}&\phi=\phi_1\ ,\\ &\rho=H_1{}^{-1}\sin(H_1\xi)\ ,\\ &H_1=\left[\frac{\kappa V(\phi_1)}{3}\right]^{1/2}\ ,\end{aligned} \tag{2.19}$$

where ϕ_1 is the maximum of $V(\phi)+\eta R\phi^2/2$ indicated in Fig. 1. The action for this solution is

$$\begin{aligned}A_H&=\frac{1}{2\kappa}2\pi^2\int d\xi\, R\rho^3\\ &\quad+2\pi^2\int d\xi\,\rho^3V-(\phi_1\rightarrow\phi=0)\\ &=\tfrac{3}{8}M_P{}^4\left[\frac{1}{V(0)}-\frac{1}{V(\phi_1)}\right]\ .\end{aligned} \tag{2.20}$$

Now, the maximum of $V+\eta R\phi^2/2$ is given by

$$\phi_1{}^2\ln(\phi_1{}^2/\sigma^2)=\eta\frac{R(\phi_1)}{4B}=-\frac{\kappa\eta V(\phi_1)}{B}\ . \tag{2.21}$$

Thus

$$V(\phi_1)=V(0)\left[1-\frac{R_0\phi_1{}^2}{2B\sigma^4}\right] \tag{2.22}$$

and

$$A_H=\frac{6\eta\pi^2}{3B\ln(\sigma/\phi_1)}=410(6\eta)\ . \tag{2.23}$$

Note that there is an additional $\ln(\sigma/\phi_1)=10$ in the denominator of the homogeneous action. This accounts for the lower action for the homogeneous solution and implies that the preferred escape from the de Sitter symmetric phase is *not* the nucleation of small bubbles of new vacuum but tunneling of the universe as a whole—without destroying the homogeneity of the symmetric phase.

The solution, Eq. (2.19), represents a sudden "jump" to the top of the $R\phi^2$ barrier due to a homogeneous thermal fluctuation at the Hawking temperature. The number e^{-A_H} measures the probability for this thermal fluctuation. In general, a second solution exists in which $\rho(\xi)$ has qualitatively the same behavior as in (2.10) or (2.19) but ϕ varies from ϕ near zero to ϕ near ϕ_2 as ξ ranges from 0 to $\xi_{\max}$ with

$$\frac{\pi}{H_1}\le\xi_{\max}\le\frac{\pi}{H}\ .$$

This is the Coleman-de Luccia bounce, the action of which has been calculated by Parke in the zero temperature thin-wall approximation.[17] Let

$$\epsilon=V(0)-V(\phi_2)\ , \tag{2.24}$$

where ϕ_2 is the point to the right of the $R\phi^2$ barrier at which the bounce has $\phi'=0$ and the continuation to Lorentzian time is performed. The quantity

$$S_1=\int_0^{\phi_2} d\phi[2V(0)-2V(\phi)]^{1/2} \cong \frac{1}{12B\ln(\sigma/\phi_1)}(-\eta R_0)^{3/2} \quad (2.25)$$

measure the surface tension of the wall. The action of the bounce can be expressed in the form

$$A_B=\frac{27\pi^2 S_1{}^4}{2\epsilon^3}r\ , \quad (2.26)$$

where r is the correction factor due to gravity, computed by Parke.[17] For the case $\epsilon\rightarrow 0$,

$$r\rightarrow\frac{16\epsilon^3}{27S_1{}^3}\left[\frac{3}{2\kappa V(0)}\right]^{3/2} . \quad (2.27)$$

Thus the minimized bounce action in this approximation is

$$(A_B)_{\text{thin wall}}\cong\frac{\sqrt{2}\pi^2(6\eta)^{3/2}}{3B\ln(\sigma/\phi_1)}\cong 570(6\eta)^{3/2} . \quad (2.28)$$

The cancellation of ϵ in this formula is indicative of the fact that this bounce does not have a flat-space analog. Its size is limited by gravitational effects ($\bar{\rho}\sim H^{-1}$) and not by ϵ. The similarity between A_B and A_H indicates that homogeneous tunneling via the zero-temperature bounce is not sharply distinguishable from classically jumping over the barrier by a thermal fluctuation. In fact, numerical integration of the bounce equations yield

$$A_B=421 \quad (2.29)$$

for $\eta=\frac{1}{6}$, instead of (2.28).[16] If we recall that the Hawking temperature is itself a one-loop quantum effect and view the $R\phi^2$ barrier as similar to the $T^2\phi^2$ barrier with $T\sim T_H$, the similarity of (2.29) and (2.23) is not surprising.

Since e^{-A_B} is small we expect that the universe must remain trapped in the systematic phase for a long time before tunneling. Then the expansion factor $a(\tau)$ could become very large naturally. However, the scale factor ρ would seem to be less than or on the order of H^{-1}, so that any immediate connection with the Lorentzian metric (2.14) presents a problem: How can a large $a(\tau)$ match onto the Euclidean solution? In the next section we answer this question by exhibiting an explicit continuation between the Lorentzian and Euclidean sections which relates the small probability of tunneling e^{-A_B} to a large expansion factor.

III. THE DURATION OF THE DE SITTER EXPANSION PHASE

In the usual application of the Callan-Coleman bounce[4] at zero temperature one considers the persistence amplitude for the false vacuum. The contribution of the bounce to this amplitude is proportional to

$$i\,|\det{}'(\delta^2A)|^{-1/2}e^{-A_B}\ , \quad (3.1)$$

where δ^2A is the second-order variation of the action and the prime indicates nonzero modes only of δ^2A are to be included in the determinant. The factor of i comes from the negative mode. In the case of the Coleman-de Luccia bounce,[15] there are four zero modes corresponding to arbitrary translations of the solution on S_4. In curved spacetime the collective-coordinate integration multiplying (3.1) must respect general coordinate invariance. Therefore the volume element multiplying (3.1) is proportional to

$$\int d^4x\sqrt{g}=2\pi^2\int d\xi\,\rho^3(\xi)\ . \quad (3.2)$$

In order to calculate the decay rate and follow the real time evolution, we consider the de Sitter metric on the Lorentzian section, given by (2.14). The Lorentzian analogs of (2.11) and (2.16) are

$$\phi''+\frac{3\dot{a}}{a}\phi'=-\frac{dV}{d\phi}+\eta R\phi\ , \quad (3.3)$$

$$a'^2=-1+\tfrac{1}{3}\kappa a^2(V+\tfrac{1}{2}\dot{\phi})^2\ , \quad (3.4)$$

where the overdot denotes $d/d\tau$. If τ is real, $\phi=0$ and

$$a(\tau)=H^{-1}\cosh H\tau \quad (3.5)$$

describes the exponentially expanding de Sitter phase. What determines τ_1, the time of tunneling?

In one-dimensional tunneling through a potential barrier there is a conserved energy and a corresponding time translational symmetry of the action. Hence the time of tunneling cannot be determined by the minimization of an action which is invariant under $t\rightarrow t+t_1$. Instead, the time of tunneling is determined by requiring that the probability of escaping the potential will be of order unity, i.e., $t_1\Gamma=1$ where Γ is the semiclassical decay rate.

For field theory in flat space time the bounce solution of minimal action has O(4) spherical symmetry. The role of the time variable is taken by the O(4)-invariant radius $\rho=(r^2+t^2)^{1/2}$. This variable ranges from 0 to ∞ and so has a definite origin. There is no translational symmetry in ρ and the bubble radius $\bar{\rho}$ is determined by the equations of motion, i.e., the minimization of the action. Homogeneity is necessarily destroyed in the tunneling process by the nucleation of the bubble.

When gravity is added, an additional function $\rho(\xi)$ makes its appearance. The inhomogeneous bounce solution(s) which existed before remain, though slightly reduced in size. However, since $\xi \to \xi + \xi_1$ is a symmetry of the metric, ρ need not have a definite origin. There may exist new solutions to the equations which are more similar to one-dimensional quantum mechanics than to the flat space bounce, in that the "time" of tunneling is *not* determined by the equations of motion but by the barrier-penetration factor. The homogeneous solution (2.19) represents precisely this possibility.

The explicit continuation between the Lorentzian and Euclidean sections is achieved for such homogeneous tunneling just as in the one-dimensional case. Let

$$\tau = t + i\xi \,, \tag{3.6}$$

with $\xi = 0$ until the instant of tunneling, $\tau_1 = t_1$. Then fix $\mathrm{Re}\tau = t_1$ and allow ξ to vary, so that $d\tau = i d\xi$. If we let

$$\begin{aligned} \rho(\xi) &= a(\tau)\,|_{\tau = t_1 + i\xi} \,, \\ \phi(\xi) &= \phi(\tau)\,|_{\tau = t_1 + i\xi} \,, \end{aligned} \tag{3.7}$$

with ξ varying from 0 to $\xi_{\max} < \pi/H$, then Eqs. (3.3) and (3.4) become identical to the Euclidean bounce equations (2.11) and (2.16). If a solution to these equations exists, then we have the desired tunneling solution which connects smoothly[18] onto the pre-tunneling expansion through Eqs. (3.7), by letting $\xi_1 = it_1$ be the zero point of the Euclidean time evolution of Eqs. (2.11) and (2.16).

Thus, it is possible to have $\rho \gg H^{-1}$ for the solution of the Euclidean bounce equations by the simple expedient of allowing the integration constant ξ_1 in Eq. (2.10) to be imaginary. This implies that $\rho(\xi)$ for $0 < \xi < \xi_{\max}$ is complex. Complex paths have been utilized in one-dimensional tunneling at finite temperature where they have been shown to be essential to the correct semiclassical limit.[19] In this case, the homogeneous tunneling path which matches smoothly onto the pre-tunneling exponential expansion of $a(\tau)$ in the de Sitter phase *must* be complex, since ρ is bounded by H^{-1} for real paths.

The geometric interpretation of the Euclidean bounce as residing on a real four-sphere is lost if $\rho(\xi)$ is complex. However, nothing actually depends on this geometry. The periodicity in ξ is determined solely by the differential equations (2.11) and (2.16). If $\rho(\xi)$ is a solution to these equations with periodicity $2\xi_{\max}$, then $\rho(\xi - \xi_1)$ is as well and it has the same periodicity, whether ξ_1 is real or not. Because the action is invariant under (2.13) the action for the translated complex solution is identical to that of the real solution with $\xi_1 = 0$. Since $A_H < A_{\text{bubble}}$ [Eqs. (2.18), (2.23), and (2.29)] the complex tunneling path is the preferred path for the (graceful) exit from the symmetric phase.

As ξ varies from zero to $\xi_{\max}$, ϕ varies from zero to ϕ_2 in the bounce solution—over the entire spatial section.[18] At $\xi = \xi_{\max}$ we fix ξ and resume the real time evolution of Eqs. (3.4) and (3.5) with $\tau = t + i\xi_{\max}$, $d\tau = dt$. Since ρ is periodic with period $2\xi_{\max}$, the scale factor is again real and given by $-a(t_1)$ while $\dot a$ has become $-\dot a(t_1)$.

The sign change is not physically significant since a^2 appears in the line element and physical observables such as the curvature scalar

$$R = 6\left[\frac{\ddot a}{a} + \frac{1}{a^2}(1 + \dot a^2)\right] \tag{3.8}$$

are invariant under $a \to -a$.

Thus, the complex Euclidean bounce solution smoothly connects the symmetric vacuum with the post-tunneling evolution to the new vacuum by continuation in the Robertson-Walker time coordinate.

The time of tunneling t_1 is computed by multiplying the Lorentzian form of (3.2),

$$2\pi^2 \int_0^{t_1} dt\, a^3(t) \tag{3.9}$$

by (3.1) and setting the result equal to unity. Since (3.1) must have dimensions $(\text{distance})^{-4}$ and H^{-1} is the only distance relevant in the tunneling process, we have the estimate

$$H^4 \times H^{-4} e^{3Ht_1} e^{-A_B} \cong 1 \,, \tag{3.10}$$

or

$$\begin{aligned} t_1 &\cong A_B/3H \,, \\ a(t_1) &\cong \frac{1}{2H} e^{A_B/3} \,, \end{aligned} \tag{3.11}$$

for $t_1 \gg H^{-1}$, i.e., $A_B \gg 1$, the condition of validity of the semiclassical approximation. An action of 570 [cf. Eq. (2.18)] implies that the universe is trapped in the symmetric phase for 190 de Sitter times, which allows the expansion factor to grow by $e^{190} \cong 10^{82}$. This is more than enough to accommodate the 28 orders of magnitude required by observation in our present universe. An action of the order $A_H = 410$ implies an expansion factor of $e^{138} = 10^{60}$ which is still amply large for accommodating the present universe.

One aspect of this calculation of t_1 may require some further explanation. The assumption of rotation invariance fixes the metric to be (2.9). Thus, the tunneling event is certainly homogeneous and simultaneous in *some* frame. But this frame need not be the same one in which the pre-tunneling evolution is given by (3.5). The situation is analogous to Lorentz invariance in flat space. Bubbles of new vacuum need not form at rest; they may appear with any velocity $v < c$. However, the bubble walls are accelerated to c very rapidly and any original velocity becomes irrelevant.

In the present case, we have a de Sitter invariance group. An arbitrary group rotation relates τ in Eq. (3.5) to that in (3.6). For example, consider the group rotation $\xi \rightarrow \xi + \pi/H$ which is the furthest from the identity on the Euclidean section in the sense that antipodal points on S_4 are mapped into each other. On the Lorentzian section this means that a typical large group rotation can change t_1 by π/H and hence $a(t_1)$ by a factor of e^{π}. Thus, in the frame in which the pre-tunneling evolution is given by Eq. (3.5) the post-tunneling universe is not quite homogeneous and isotropic. Rather, considering the spatial sections as three-spheres, one side of the three-sphere may tunnel earlier, by $\Delta t_1 \lesssim \pi/H$, while the other side later by the same amount. Consequently, one part of the universe may experience an expansion factor of $e^{A_H/3-\pi} = 10^{58.6}$ while another part an expansion factor of $e^{A_H/3+\pi} = 10^{61.3}$. This is a trivial difference. It means that the universe is flat to one part in $10^{61.3}$ in one region but only to one part in $10^{58.6}$ in another. Any such inhomogeneity would be undetectable by any current observations. Thus the de Sitter group rotations are as irrelevant here as the Lorentz group boosts in flat-space false-vacuum decay.

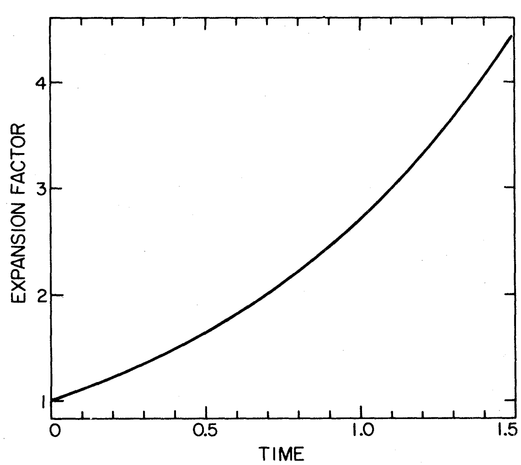

FIG. 2. The post-tunneling evolution of the Robertson-Walker expansion factor $a(t)$ in time units of H^{-1}. a grows exponentially until $t \cong 1.5H^{-1}$ after the phase transition. It then switches to the power law, appropriate for nonrelativistic matter.

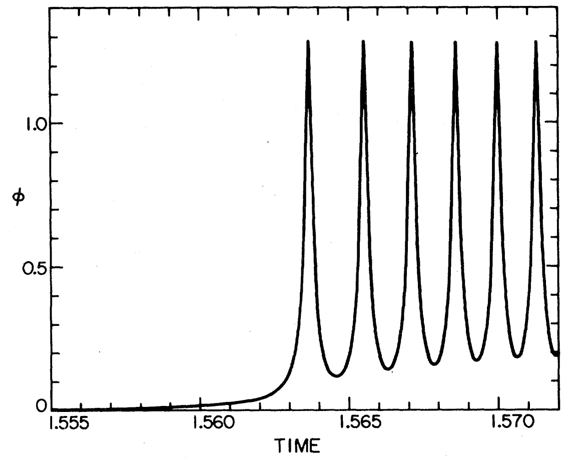

FIG. 3. The oscillations of the ϕ field around $\phi = \sigma$. The slow decrease in the amplitude of the oscillations is due to the cosmological red-shift alone; particle creation effects are not included.

IV. THE POST-TUNNELING EVOLUTION

With the homogeneous exit from the symmetric false vacuum, the real time evolution resumes according to Eqs. (3.3) and (3.4). For the bounce solution, a^2, $\dot{a}^2$, and $\dot{\phi}^2$ are unchanged[18] from their pre-tunneling values. Thus, Eq. (3.4) implies $V(\phi_2) = V(0)$ or

$$\phi_2 = (4.52 \times 10^{-5})\sigma . \tag{4.1}$$

Equations (3.3) and (3.4) were integrated numerically with these initial conditions and the results are presented in Figs. 2 and 3. Note that the additional expansion factor due to the rollover from the plateau of the potential is less than 5, compared to the expansion due to the tunneling suppression, $e^{A_H/3} \cong 10^{60}$. Thus when $\eta \neq 0$ a very natural explanation for the large expansion factor arises, which requires no fine adjustment of the initial conditions of the rollover. In fact, the initial conditions are fixed by the Euclidean bounce equations which do not allow changing ϕ_2 to adjust the rollover time.

Since the large expansion is due to the number e^{A_B} generated by the tunneling process and not the slow rollover from the plateau, it is possible to consider modifying the pure Coleman-Weinberg potential of Eq. (2.2). For example, we might add an explicit $m^2\phi^2$ term to V or allow η to differ from $\frac{1}{6}$. In order for the tunneling picture to be correct the barrier must be large enough ($A_B > 194$) to yield a large enough expansion factor, while still small enough ($A_B < 1300$) to suppress nucleation of small bubbles and breakdown of homogeneity. This gives the limits

$$H^2 < |m^2 - \eta R| < 6H^2 . \quad (4.2)$$

Explicit mass terms of the order of H present no problem but $m \sim \mu$ are ruled out in this picture.

As the classical solution begins to oscillate about the new vacuum, the coherent state energy density $E = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ will be converted to matter by particle production. The effective potential language is no longer adequate to describe this process since ϕ now varies on time scales comparable to the mass of its excited particle states. A correct description of the process would require solving the full nonlocal semiclassical equations derived from the one-loop effective action. It will be sufficient for our purposes, however, to estimate the particle production rate very roughly as follows.

Let $\phi = \phi_{cl} + \phi_{qu}$ where ϕ_{cl} is the solution of Eqs. (3.3) and (3.4) and ϕ_{qu} is responsible for the particle production. Expanding to second order in ϕ_{qu} yields

$$\phi_{qu}{}^2 V''(\phi_{cl}) \sim B\phi_{qu}{}^2 \phi_{cl}{}^2 .$$

Thus B characterizes the coupling between the classical field, viewed as a time dependent source and the quantum field. Now, the energy density E decays into particles at a rate proportional to E, to the coupling B, and to the frequency of oscillation about $\phi = \sigma$ which is $\mu = \sqrt{8B}\,\sigma$:

$$\frac{dE}{dt} \sim -B\mu E . \quad (4.3)$$

Therefore, the time scale for full conversion of the vacuum energy density into matter is not μ^{-1} but

$$t_{conv} \sim B^{-1}\mu^{-1} \sim 1300\mu^{-1} . \quad (4.4)$$

This is the same order of magnitude as the rollover time from the potential plateau.

As the Higgs particles are created they spontaneously decay into lighter species. Since B is $O(\alpha_{GUT}{}^2)$ the decay time is also of order t_{conv}. Finally, the various particle species must interact in order to rethermalize. The interaction time can be estimated from the interaction rate, nSv, where n is the number density of created particles $[\sim V(0)/\mu]$, S is the interaction cross section $(\sim \alpha^2/\mu^2)$, and v is the average particle velocity $(\lesssim c)$. The interaction time is then

$$t_{int} = \frac{1}{nSv} \sim 10\mu^{-1} . \quad (4.5)$$

This must be compared with the characteristic expansion time of the universe at this epoch,

$$t_{exp} \sim H^{-1} \sim \mu^{-1}\left[\frac{M_P}{\sigma}\right] \sim 10\,000\mu^{-1} . \quad (4.6)$$

If t_{exp} were less than t_{int}, the universe would be expanding faster than the particles could interact and equilibrium could not be reestablished. Since t_{exp} involves M_P/σ this is not the case and the matter does rethermalize. The rethermalization temperature T_* may be estimated by equating the energy density E of the classical solution to $(\pi^2/30)N_* T_*{}^4$ where N_* is the number of massless particle species at $T = T_*$ $(N_* \sim 100)$. Since $T_* \sim E^{1/4} \sim 1/t^{1/2}$, T_* is not strongly dependent on the time t that we choose to evaluate E. Taking t to be $5t_{conv}$ and $10t_{conv}$ gives

$$T_* = 5.9 \times 10^{13}\ \text{GeV}$$

or (4.7)

$$T_* = 4.4 \times 10^{13}\ \text{GeV} ,$$

respectively.

Once equilibrium is reestablished at $T = T_*$ the universe resumes the homogeneous and isotropic radiation-dominated expansion of the standard model. In this model baryon asymmetry is generated when the density becomes low enough that the interaction rate falls below the expansion rate.[20] Because of (4.5) and (4.6) the temperature at which this occurs is lower than T_* by roughly an order of magnitude. Thus the phase transition and particle creation epoch has a negligible effect on the baryon asymmetry calculations in the standard model. This conclusion is somewhat model dependent, however.

The magnetic monopole density predicted in this picture is the very small value associated with the Boltzmann equilibrium distribution e^{-M/T_*} where M is the monopole mass.

The entropy generated in the vacuum decay and particle production process is of order of the number of particles created,

$$S \sim \left[\frac{M_P}{\mu}\right]^3 e^{A_B} \sim 10^{75} . \quad (4.8)$$

Thus the same large factor from the tunneling process accounts for the large entropy of the universe.

We conclude that the solution of the horizon, flatness (and possibly the singularity) problems of the standard model through homogeneous vacuum decay does not destroy any of its successful features.

Finally, there is the issue of inhomogeneous perturbations on the homogeneous classical background evolution. It is known that the spectrum of quantum fluctuations in a de Sitter background is white.[21] This scale invariance appears to be precisely what is required for the subsequent development of irregularities into galaxies.[22] However, in the rollover inflationary scenario the *amplitude* of these fluctuations has been shown[23] to be far too large to be in accordance with the isotropy of the microwave background.

This large result depends on the slow rollover in a crucial way, through the large value of $(\dot{\phi})^{-1}$. In the tunneling transition or "old inflationary universe" considered by us, $\dot{\phi}$ is zero classically during the relevant large expansion era. It is not clear that the same result for the fluctuation amplitude remains valid in this very different circumstance. Linde[24] has argued that in the case of interest (his case E) the one-loop approximation is not valid so that the magnitude of the fluctuations may be quite different than in the rollover picture. The issue of the fluctuations in the homogeneous inflationary universe is examined in the following paper.

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