Branching-ratio predictions for the $\iota(1440)$

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(Received 17 November 1982)

A simple pole model is used to predict $\iota\to\delta\pi$, $\rho\gamma$, $\omega\gamma$, $\phi\gamma$, $\rho\pi\pi$, and $\eta\pi\pi$. The rates $\iota\to\eta\pi\pi$ and $\iota\to K\bar{K}\pi$ are examined in detail. In the pole model the rate $\iota\to\eta\pi\pi$ is compared to $\eta'\to\eta\pi\pi$ and we have the prediction $B(\iota\to\eta\pi\pi)/B(\iota\to K\bar{K}\pi)=10\%$. A direct calculation that takes into account the cancellation between $\iota\to\delta\pi\to(\eta\pi)\pi$ and $\iota\to\eta\epsilon\to\eta(\pi\pi)$, the $K\bar{K}$ threshold, and SU(3) violations seen in the decay of the δ, predicts $20\%\le B(\iota\to\eta\pi\pi)/B(\iota\to K\bar{K}\pi)\le 110\%$. Both calculations are consistent with the experimental limit of 50%.

I. INTRODUCTION

$\iota(1440)$ is a $J^P=0^-$ resonance seen in the $K\bar{K}\pi$ mass distribution in radiative decays of the ψ, $\psi\to\gamma K\bar{K}\pi$, with $K\bar{K}$ primarily in the δ state. The mass, width, production, and decay parameters are given by the Mark II and Crystal Ball groups at SPEAR[1-4]:

$$\text{Mass}=1440\pm15\ \text{MeV},\quad \Gamma=55\pm25\ \text{MeV}\ ,$$
$$B(\psi\to\gamma\iota)B(\iota\to K\bar{K}\pi)=(4.1\pm1.5)\times10^{-3}\ , \qquad (1)$$
$$B(\iota\to\delta\pi)B(\delta\to K\bar{K})/B(\iota\to K\bar{K}\pi)=0.8\pm0.2\ .$$

It has been widely argued that $\iota(1440)$ is a quarkless state of matter, a bound state of gluons, colloquially known as gluonium or a "glueball."[5-7] Certainly $\iota(1440)$ qualitatively satisfies all criteria for glueball status: It is an isosinglet preferentially produced in the hard-gluon (g) channels ($\psi\to\gamma gg\to\gamma\iota$) which mediates in an SU(3)-symmetric way processes which violate the Okubo-Zweig-Iizuka (OZI) rule.

Recently, however, doubt has been cast on this glueball assignment for $\iota(1440)$ by Crystall Ball data which places an upper limit on the decay $\iota\to\eta\pi\pi$,[8]

$$\frac{\Gamma(\iota\to\eta\pi\pi)}{\Gamma(\iota\to K\bar{K}\pi)}<\tfrac{1}{2}\ (\text{Crystal Ball})\ . \qquad (2)$$

The doubt arises from the following argument: By SU(3) symmetry, the couplings of $\eta\pi$ and $K\bar{K}$ to δ should be comparable and thus the $\eta\pi\pi$ rate from ι should be comparable to the $K\bar{K}$ rate. Let us look at this argument more closely. The most naive assumption is that $\Gamma(\iota\to\eta\pi\pi)/\Gamma(\iota\to K\bar{K}\pi)$ proceeds only via the δ and is given by the SU(3) values of the coupling of $\eta\pi$ and $K\bar{K}$ to the δ. This assumption leads to a ratio

$$\frac{\Gamma(\iota\to\eta\pi\pi)}{\Gamma(\iota\to K\bar{K}\pi)}=\frac{4}{3}\ . \qquad (3)$$

This calculation includes two coherent δ's in the two $\eta\pi$ channels. A second symmetry prediction results when these δ's are regarded as incoherent. This reduces Eq. (3) by a factor of 2.

A third symmetry prediction comes from assuming pure SU(3) for the four-point amplitude $\iota(1440)$ to three pseudoscalars, with ι a pure singlet. In the language of the intermediate-state calculation leading to Eq. (3), this coupling includes the other scalar resonances such as the $K\pi$ resonance κ and the $\pi\pi$ resonance ϵ. The prediction is

$$\frac{\Gamma(\iota\to\eta\pi\pi)}{\Gamma(\iota\to K\bar{K}\pi)}=\frac{1}{3}\ . \qquad (4)$$

It is interesting to note that the pure SU(3) prediction of Eq. (4) is not far from the limit set by the data.

In this Brief Report we would like to point out, moreover, that all of the symmetry predictions [Eqs. (3) and (4)] for sizable $\Gamma(\iota\to K\bar{K}\pi)$ are unreliable. The dynamics of symmetry breaking and varying thresholds for intermediate states strongly influence these rates and and reduce the prediction of Eqs. (3) and (4). Hence there is no reason to doubt the glueball status of $\iota(1440)$ merely on the grounds of the experimental limit, Eq. (2).

We will calculate the rate $\iota\to\eta\pi\pi$ in two ways. First, in Sec. II, we will use a simple pole model[7,9] in which the $\iota(1440)$ is a glueball and mediates OZI-rule-violating processes. In this calculation $\iota\to\eta\pi\pi$ is first related to $\eta'\to\eta\pi\pi$ and then indirectly to $\iota\to K\bar{K}\pi$. The result is

$$\frac{\Gamma(\iota\to\eta\pi\pi)}{\Gamma(\iota\to K\bar{K}\pi)}=10\%\ . \qquad (5)$$

Then, in Sec. III, we directly calculate the $\iota\to K\bar{K}\pi$ and $\iota\to\eta\pi\pi$ rates, taking into account the following.

(i) Cancellation between $\iota\to\delta\pi\to(\eta\pi)\pi$ and $\iota\to\eta\epsilon(700)\to\eta(\pi\pi)$ amplitudes. This cancellation is observed in the related decay $s(1275)$ (Ref. 10) and is predicted by chiral-Lagrangian models (CLM's).[11]

(ii) SU(3) breaking in the coupling of δ which favors $K\bar{K}$ over $\eta\pi$. This is observed to be the case even in the rough data available for on-shell decay of δ to $\eta\pi$ and $K\bar{K}$,[12] and predicted in CLM's.[11]

(iii) The effect of the nearby $K\bar{K}$ threshold on the δ propagator. The works of Achasov *et al.*[13] and Tornquist[14] strongly suggest that the δ is not a Breit-Wigner resonance. They fit phase shifts with a modified propagator which is among the dynamical assumptions we make in our detailed calculation.

The result of this calculation is, incorporating experimental errors,

$$20\% < \frac{\Gamma(\iota \to \eta\pi\pi)}{\Gamma(\iota \to K\bar{K}\pi)} < 110\% \ , \tag{6}$$

well within the experimental limit, Eq. (2).

II. POLE MODEL FOR ι PRODUCTION AND DECAY

If the ι is a glueball resonance in two-gluon channels, it should mediate processes such as $\psi \to \gamma\eta$ or $\gamma\eta'$, as shown in Fig. 1(a). A pole model for the OZI-rule-violating propagation function yields for the relative rates[7,9]

$$\frac{B(\psi\to\gamma\eta)}{B(\psi\to\gamma\eta')} = 1.2\left(\frac{f_\eta}{f_{\eta'}}\right)^2\left(\frac{m_\iota^2 - m_{\eta'}^2}{m_\iota^2 - m_\eta^2}\right)^2 , \tag{7}$$

where the factor of 1.2 represents the effects of phase space. The result is

$$\frac{f_\eta}{f_{\eta'}}\frac{m_\iota^2 - m_{\eta'}^2}{m_\iota^2 - m_\eta^2} = +0.42 \pm 0.3 \tag{8}$$

when the sign of the relative amplitude has been determined by appeal to SU(3) and the known η,η' mixing.[9] f_η and $f_{\eta'}$ represent the amplitudes for ι to mix with η or η'.

The absolute value of the amplitudes is determined by ι production [see Fig. 1(b)] relative to η' production[7,9]:

$$\frac{B(\psi\to\gamma\eta')}{B(\psi\to\gamma\iota)} = 1.5\frac{f_{\eta'}^2}{(m_\iota^2 - m_{\eta'}^2)^2} \ . \tag{9}$$

Using the data of Eq. (1) and the branching ratio

$$B(\psi\to\gamma\eta') = (3.8 \pm 0.8)\times 10^{-3} \ ,$$

we find

$$\frac{f_{\eta'}^2}{(m_\iota^2 - m_{\eta'}^2)^2} = (0.62 \pm 0.30) B(\iota \to K\bar{K}\pi) \ . \tag{10}$$

If (as we shall argue below) $B(\iota\to K\bar{K}\pi)$ is 30%, we have

$$\left[f_{\eta'}^2/(m_\iota^2 - m_{\eta'}^2)^2\right] \sim \tfrac{1}{5} \ .$$

This is the basic coupling strength of the quarks to quarkless states. Having determined its parameters, a test of the pole model is now afforded by a calculation of $\iota\to\delta\pi$ relative to $\delta\to\pi\eta$ as given by the diagrams of Fig. 2. Since the basic coupling strength of gluonium to quarks is given in terms of $B(\iota\to K\bar{K}\pi)$, the result of this calculation is the ratio

$$\frac{B(\iota\to\delta\pi)}{B(\iota\to K\bar{K}\pi)} = 88\% \ , \tag{11}$$

a value consistent with the data. Thus encouraged, we can use our model to calculate the rates in Table I. Collecting all rates and assuming we have accounted for 90% of all decays, we find $B(\iota\to K\bar{K}\pi)$

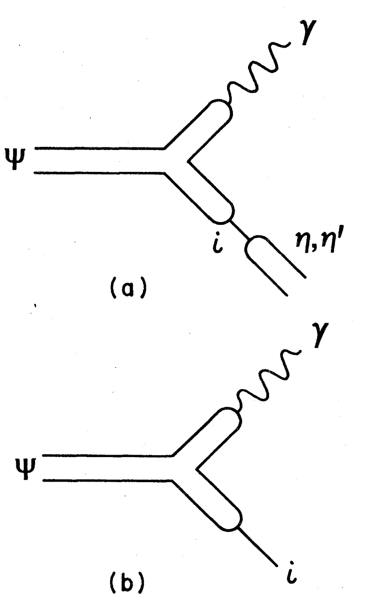

FIG. 1. (a) Pole-model diagram for $\psi\to\gamma\eta$ and $\psi\to\gamma\eta'$. (b) Pole-model diagram for $\psi\to\gamma\iota(1440)$.

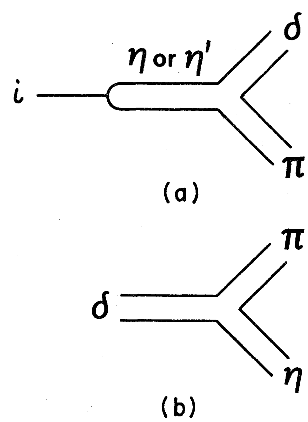

FIG. 2. (a) Pole-model diagram for $\iota(1440)\to\delta\pi$. (b) Diagram for $\delta\to\eta\pi$.

=33%. [The large $\iota \to \rho\gamma$ rate deserves further comment. The $\rho\gamma$ channel is very interesting, not only for the information it will yield on the $\iota(1440)$, but also for other gluonium states, such as the $J^P=1^{-+}$ state predicted in the 1-GeV region by the QCD sum rules. Additionally the as yet unobserved $f \to \rho\gamma$ may be found[15] ($\Gamma \approx 1$ MeV).]

The last rate in Table I,

$$\frac{B(\iota \to \eta\pi\pi)}{B(\iota \to K\bar{K}\pi)} = 9\% \quad , \tag{12}$$

is based on the simple picture in Fig. 3, which yields

$$\frac{\Gamma(\iota \to \eta\pi\pi)}{\Gamma(\eta' \to \eta\pi\pi)} = \left(\frac{m_{\eta'}}{m_\iota}\right)^2 \frac{S_G}{S_{\eta'}} \left(\frac{f_{\eta'}}{m_\iota^2 - m_\eta^2}\right)^2 \left(1 + \frac{f_\eta}{f_{\eta'}} \frac{g_{\eta\delta\pi}}{m_\iota^2 - m_{\eta'}^2}\right)^2 , \tag{13}$$

where $S_G/S_{\eta'}$ is the ratio of squared amplitudes with couplings and mass factors scaled out. The coupling-constant ratio $g_{\eta\delta\pi}/g_{\eta'\delta\pi}$ is given by 0.83 for an octet-singlet mixing angle of 15°. Since the basic coupling strength [Eq. (10)] is given in terms of $B(\iota \to K\bar{K}\pi)$, we can express Eq. (13) as

$$\frac{B(\iota \to \eta\pi\pi)}{B(\iota \to K\bar{K}\pi)} = 10\% \quad . \tag{14}$$

The basic reason why the rate $\iota \to \eta\pi\pi$ is small in this calculation is that the rate $\eta' \to \eta\pi\pi$, to which it is scaled, is small. $B(\iota \to K\bar{K}\pi)$ enters into the calculation indirectly via Eq. (13). Thus we have little insight here into why $\iota \to \eta\pi\pi$ (and $\eta' \to \eta\pi\pi$) is so small relative to $\iota \to K\bar{K}\pi$. (In Sec. III we will calculate $\iota \to K\bar{K}\pi$ directly, answering that question.) However, it is clear that $\iota \to \eta\pi\pi$ is *expected* to be small within the context of this simple model for the ι as a glueball.

III. DIRECT CALCULATION OF $\iota \to \eta\pi\pi$ AND $\iota \to K\bar{K}\pi$ RATES

We assume an isobar model in which the intermediate states are δ ($\to \eta\pi$ or $K\bar{K}$) and the low-mass "ϵ" ($\to \pi\pi$). These are the resonance or phase shifts which should be most significant given the masses and thresholds involved.

We will parametrize the low-mass phase-shift behavior in the 0^+ partial wave as a wide bump with a phase of 90°. The form we use is

$$\frac{i}{(t - m_\epsilon^2) + m_\epsilon^2\Gamma_\epsilon^2} , \tag{15}$$

with $m_\epsilon^2 = 0.5\ \text{GeV}^2$ and $m_\epsilon\Gamma_\epsilon = 0.42\ \text{GeV}^2$ (Ref. 16).

The δ resonance will be parametrized in two ways. First we will use the δ propagator of Achasov *et al.*,[13] in which finite-width and threshold effects are incorporated in a theoretical form which is fit to phase-shift data. Their claim is that δ is really a broad non-Breit-Wigner resonance which appears narrow because of $K\bar{K}$ threshold effects. We will also use a conventional Breit-Wigner form with various values for the δ width.

The diagrams we calculate are $\iota \to \delta\pi \to (\eta\pi)\pi$, $\iota \to \eta\epsilon \to \eta(\pi\pi)$, and $\iota \to \delta\pi \to (K\bar{K})\pi$. The SU(3) partner to $\iota \to \epsilon\eta \to \pi\pi\eta$, $\iota \to \kappa\bar{K} \to (K\pi)\bar{K}$, would ordinarily appear but since the κ is such a high-mass

TABLE I. Pole-model predictions for all two- and three-body decays of the $\iota(1440)$ (Ref. 7).

KK^*	Suppressed by generalized G parity
$\rho\gamma$	$\frac{\Gamma(\iota \to \rho^0\gamma)}{\Gamma(\iota \to K\bar{K}\pi)} = 14\%$
$\omega\gamma$	$\frac{\Gamma(\iota \to \omega\gamma)}{\Gamma(\iota \to K\bar{K}\pi)} = 1.6\%$
$\phi\gamma$	$\frac{\Gamma(\iota \to \phi\gamma)}{\Gamma(\iota \to K\bar{K}\pi)} = 0.6\%$
$\gamma\gamma$	(6 to 17 keV) $\times B(G \to K\bar{K}\pi)$
$K\bar{K}\pi$	Input
$\rho\pi\pi$	$\frac{\Gamma(\iota \to \rho\pi\pi)}{\Gamma(\iota \to K\bar{K}\pi)} = 150\%$
$\eta\pi\pi$	$\frac{\Gamma(\iota \to \eta\pi\pi)}{\Gamma(\iota \to K\bar{K}\pi)} = 9\%$

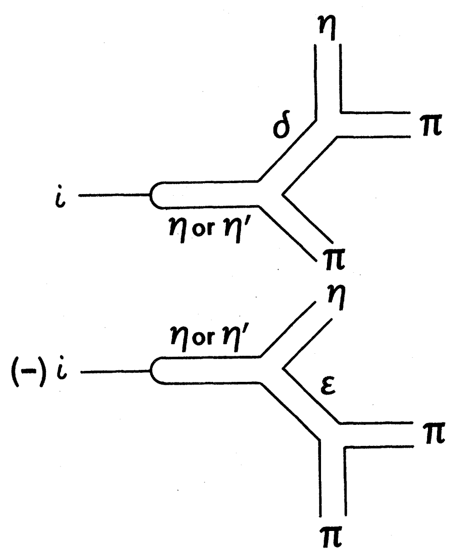

FIG. 3. (a) Pole-model diagram for $\iota(1440) \to \delta\pi \to (\eta\pi)\pi$. (b) Pole-model diagram for $\iota(1440) \to \eta\epsilon \to \eta(\pi\pi)$.

enhancement, its contribution to $\iota\to K\bar{K}\pi$ is negligible.

The cancellation between the two $\eta\pi\pi$ diagrams deserves comment. The related amplitudes $s(1275)\to\delta\pi$ and $s(1275)\to\epsilon\eta$ show this cancellation in the phase-shift analysis.[10] This cancellation also occurs in a chiral-Lagrangian symmetry-breaking model.[11] We expect it also to occur in the corresponding amplitudes for $\eta'\to\eta\pi\pi$.

Assuming isospin invariance of the amplitudes, we find

$$\frac{\Gamma(\iota\to\eta\pi\pi)}{\Gamma(\iota\to K\bar{K}\pi)}=\frac{\frac{3}{2}\Gamma(\iota\to\eta\pi^+\pi^-)}{6\Gamma(\iota\to K^+K^-\pi^0)}=\frac{1}{4}\left[\frac{g_{\delta^+\eta\pi^+}}{g_{\delta^0K\bar{K}}}\right]^2\frac{S(\iota\to\eta\pi^+\pi^-)}{S(\iota\to K^+K^-\pi^0)}\,, \tag{16}$$

where S represent the amplitudes integrated over phase space with coupling strengths removed. The SU(3) value for $(g_{\delta^+\eta\pi^+}/g_{\delta^0K^+K^-})^2$ is $\frac{4}{3}$. This ratio may be estimated from what is known about $\Gamma(\delta\to\eta\pi)$ and $\Gamma(\delta\to K\bar{K})$,[12]

$$\left[\frac{g_{\delta^+\eta\pi^-}}{g_{\delta^0K^+K^-}}\right]^2=2\frac{P_K}{P_\eta}\frac{\Gamma(\delta\to\eta\pi)}{\Gamma(\delta\to K\bar{K})}=0.086-0.44\ . \tag{17}$$

This is considerably less than the SU(3) estimate.

Our numerical results for $R=S(\iota\to\eta\pi^+\pi^-)/S(\iota\to K\bar{K}\pi)$ are tabulated in Table II for various assumptions concerning the δ propagator. To display the effect of the cancellation between the δ and ϵ, we also show R without the ϵ intermediate state. The cancellation is as large as 50% in the squared matrix element. Taking $S(\iota\to\eta\pi^+\pi^-)/S(\iota\to K^+K^-\pi^0)$ at a midrange value of 10 yields

$$\frac{\Gamma(\iota\to\eta\pi\pi)}{\Gamma(\iota\to K\bar{K}\pi)}=\frac{1}{4}(0.086-0.44)10=0.2-1.1\ . \tag{18}$$

The range is due to the uncertainty in $\Gamma(\delta\to K\bar{K})$ and $\Gamma(\delta\to\eta\pi)$.

TABLE II. Numerical results (Ref. 16) for the direct calculation of $\Gamma(\iota\to\eta\pi\pi)/\Gamma(\iota\to K\bar{K}\pi)$. g_ϵ is the ϵ coupling strength. $R=S(\iota\to\eta\pi^+\pi^-)/S(\iota\to K^+K^-\pi^0)$. R (no ϵ) is the result for $g_\epsilon=0$.

	g_ϵ	R	R (no ϵ)
Ref. 13 Solution K	0.24	22.2	34.6
Ref. 13 Solution J	0.6	12.9	23.5
Ref. 13 Solution I	0.9	9.4	17.0
Simple pole $\Gamma_\delta=50$ MeV $m_\epsilon^2=0.5$ GeV^2, $m_\epsilon\Gamma_\epsilon=0.42$ GeV^2	1.2	14.2	18.7
Simple pole $\Gamma_\delta=100$ MeV $m_\epsilon^2=0.5$ GeV^2, $m_\epsilon\Gamma_\epsilon=0.42$ GeV^2	1.1	8.6	14.8

ACKNOWLEDGMENTS

The authors would like to thank K. Milton for many helpful comments and for his collaboration on the early stages of this work. This work was supported in part by the Department of Energy.

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[15]The authors would like to thank J. Rosner for calling our attention to the importance of the $\rho\gamma$ rate.

[16]Our results are not sensitive to 20% variations in these parameters.