Bag-inspired gluonic and relativistic effects in a harmonic-oscillator model

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The harmonic-oscillator (HO) quark model, wherein Pauli spinors are replaced by Dirac spinors and quark masses are less than $\frac{1}{3}$ of the nucleon mass, is studied in the spirit of the bag model. In particular, we have calculated G_A/G_V and the magnetic moment and charge radius of the proton; the variation of these quantities with quark mass and spring constant is also investigated. In the case of the magnetic moment we find that the relativistic correction is of the right order of magnitude, while in the case of charge radius the result is better than the nonrelativistic HO value. For G_A/G_V we find that the relativistic corrections bring the nonrelativistic value of 1.667 down to ~ 1.50 only; reasons for this less-than-anticipated fall are investigated in detail. Furthermore, we have calculated certain gluon probability integrals; the trend of our results is the same as those of Golowich and of Close and Horgan.

I. INTRODUCTION

Since the emergence of the quark model[1] great advances have been made in our understanding of the structure of hadrons. The prevalent ideas can be summarized as follows. Baryons and mesons are supposed to be three-quark and quark-antiquark systems, respectively. Quarks come in five flavors (perhaps six), with each possessing three color degrees of freedom. Quantum chromodynamics,[2] which is supposed to be the theory of the q-q interaction, has registered remarkable successes in the deep-inelastic region and has provided vital clues to the confinement problem through lattice calculations[3] and computer experiments.[4] However, at present the theory is not in a position to provide a viable description of hadron spectroscopic data. The spectroscopic data, broadly speaking, is analyzed in terms of two phenomenological models, viz., the bag model and the nonrelativistic oscillator model; both essentially incorporate the quark concept in one form or the other.

The bag model consistently incorporates several of the basic ingredients of QCD, such as asymptotic freedom and quark confinement. In this model the quarks are supposed to be free massless Dirac particles within the bag. Quark as well as gluonic currents are extinguished at the bag surface by imposing boundary condition as in the MIT bag,[5] or by additional interaction at the bag surface as in the Columbia[6] or SLAC[7] bag. The bag provides a reasonable fit to the low-lying spectroscopic data, such as[5,8] magnetic moments of the fundamental octet, G_A/G_V, charge radii, etc. The development of Feynman rules for quarks and gluons in a spherical cavity by Lee[6] leads to a natural and convenient framework for the inclusion of gluonic corrections in this model.[9,10] However, the model faces certain intractable difficulties when applied to resonance spectroscopy. Any attempt to describe resonance states immediately leads to problems connected with the nonspherical shape of the bag. In spite of a few bold attempts,[11] the question of shape of the surface of the bag containing quarks in excited states is still far from settled.[12] The problem of spurious states and that of the center of mass still persists.

On the other hand, the nonrelativistic harmonic-oscillator (HO) model has shown remarkable success in fitting a wide range of spectroscopic data.[13] Two-body correlation, built into the model, successfully demonstrates the treatment of the baryons with unequal quark masses. Perhaps it is the only model in which the center-of-mass motion is treated exactly and spurious excitations are simply separated. Gluonic[14,15] and relativistic[16] corrections have also been incorporated in one form or another, which further sharpens the successes of the model. However, these corrections are not naturally included in the model; e.g., the same quark mass is used for both nonrelativistic and relativistic calculations. Although many do not agree with the basic tenets of the nonrelativistic quark model, still there is a general consensus that the range of its success is surely more than a mere coincidence.[17]

A comparison of the bag model and the HO model suggests that the strength of the bag model lies in its well defined inputs as well as the inclusion of gluonic and relativistic corrections in a more natural fashion. However, the remarkable success and wider applicability of the HO model has to be kept in mind while developing any further refinements in the phenomenological models. In this context, therefore, it seems natural to examine whether certain crucial features from the bag model could be incorporated in the HO model or *vice versa.*

In the present work, we have investigated the question whether certain features of the bag model pertaining to gluonic corrections, obtained through the Feynman rules inside a spherical cavity as developed in Ref. 6, can be incorporated in the HO model or not. In this direction we note a crucial feature of the HO model, viz., the Gaussian factor which quickly dampens the quark wave function. It is easily seen that most of the wave function and hence the quark probabilities, in the case of HO model, are confined within an effective radius. This simulates the central feature of the bag model where the quark current is extinguished by imposing a boundary condition. In fact, DeGrand *et al.*[5] have noted that making the quarks massive makes its wave function damp heavily in the peripheral region; a feature already present in HO-model wave functions. It is worth mentioning that a bag model with massive quarks[8] compares well with the massless-quark bag model.[5] Le Yaouanc *et al.*[16] have noted another aspect of the HO model, viz., when Pauli spinors are replaced by Dirac spinors the non-relativistic HO model faithfully reproduces the relativistic corrections. With the above two points in mind, it becomes interesting to examine a model incorporating the following ingredients:

(i) The quark wave functions are given by the standard nonrelativistic HO model with Pauli spinors replaced by Dirac spinors.

(ii) The mass of the u or d quark is not $\frac{1}{3}$ of the nucleon mass as in the nonrelativistic HO model. The quarks are not ultrarelativistic as in the case of the bag model but have masses sufficiently small to be treated relativistically.

(iii) The Feynman rules for gluons and quarks are considered as worked out by Lee[6] in the case of the spherical cavity.

(iv) The matrix elements are calculated in the spirit of the bag model.

Thus we observe that the scheme suggested above, although not a dynamical model but a parametrization of the nucleon wave function, essentially simulates the massive quark bag model—however, without sacrificing some of the essential features of the nonrelativistic HO model. We have found that the relativistic as well as gluonic corrections, considered in the spirit of the bag model, can be reproduced in the present scheme with appropriate magnitudes which have a parametric dependence on the quark mass and the spring constant. Therefore, we have also studied in detail some of the low-energy parameters, for example, G_A/G_V, proton magnetic moment (μ_p), and charge radius (r_p^2) as functions of quark mass and the spring constant. Furthermore, the present scheme reduces to the bag model and to the HO model in suitable limits.

It would be appropriate to note that similar ideas have already been considered by other authors[18] with a modified harmonic-oscillator potential. The spinor structure of the wave function in these approaches is similar to one used in bag models; however, the spatial part of the wave function varies in a Gaussian manner in contrast with the MIT bag model where the wave function goes abruptly to zero at the bag surface. The present work differs significantly in emphasis as well as in details from Ref. 18.

The paper is organized as follows. In Sec. II we calculate certain key low-energy parameters, such as G_A/G_V, μ_p, and $\langle r_p^2\rangle$. A critical discussion of these parameters is also carried out in Sec. II. In Sec. III, we calculate certain gluon probability integrals and also the gluon radiative corrections to G_A/G_V. In Sec. IV, we give the summary and conclusion.

II. WAVE FUNCTION AND STATIC LOW-ENERGY PARAMETERS

For a better appreciation of inputs of the present model as well as for the sake of notations and conventions, we present some of the details pertaining to the wave functions. Our starting point is the nonrelativistic HO ground-state wave function given by

$$|(56,L=0^+)_{N=0},\underline{8},S_q=\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}(\chi'\phi'+\chi''\phi'')\psi^S , \quad (1)$$

where

$$\psi^S=\left[\frac{\alpha}{\pi}\right]^{1/2}\exp-\tfrac{1}{2}\alpha(\rho^2+\lambda^2) ,$$

$$\vec{\rho}=\frac{1}{\sqrt{2}}(\vec{r}_1-\vec{r}_2) ,$$

$$\vec{\lambda}=\frac{1}{\sqrt{6}}(\vec{r}_1+\vec{r}_2-2\vec{r}_3) .$$

The notation is from Mitra and Ross.[19] Following Le Yaouanc, Oliver, Pene, and Raynal[16] (LOPR), wherein the relativistic effects are included through the replacement of Pauli spinors by Dirac spinors, we write the spinor part of the wave function, expressed in spherical coordinates, in the following way:

$$U(\vec{r}_i)=\frac{N}{4\pi}\begin{bmatrix}\left[\dfrac{\epsilon_i+m_i}{\epsilon_i}\right]^{1/2} j_0(p_ir_i) \\ -\left[\dfrac{\epsilon_i-m_i}{\epsilon_i}\right]^{1/2} j_1(p_ir_i)\sigma\cdot\hat{r}_i\end{bmatrix}\chi\,. \tag{2}$$

The complete spatial part of the wave function is written as

$$\Psi(\{\vec{r}_i\})=\prod_{i=1}^{3}\frac{N_p}{4\pi}\begin{bmatrix}\left[\dfrac{\epsilon_i+m_i}{\epsilon_i}\right]^{1/2} j_0(p_ir_i)\chi \\ -\left[\dfrac{\epsilon_i-m_i}{\epsilon_i}\right]^{1/2} j_1(p_ir_i)\sigma\cdot\hat{r}_i\chi\end{bmatrix}\exp(-\tfrac{1}{2}\alpha\vec{r}_i^{\,2})\,. \tag{3}$$

The spatial wave function (3) is normalized with respect to the measure

$$\prod_{i=1}^{3}d\vec{r}_i\delta\left[\sum_{i=1}^{3}\vec{r}_i\right]$$

to give

$$\left[\frac{N_p}{4\pi}\right]^{-2}=\frac{1}{4p_i^2}\left[\frac{\pi}{\alpha}\right]^{1/2}\left\{\frac{\epsilon_i+m_i}{\epsilon_i}(1-e^{-2p_i^2/3\alpha})+\frac{\epsilon_i-m_i}{\epsilon_i}\left[2e^{-2p_i^2/3\alpha}+\left[1-\frac{3\alpha}{2p_i^2}\right](1-e^{-2p_i^2/3\alpha})\right]\right\}. \tag{4}$$

Here ϵ_i is the total energy of the ith quark, and we take it as $\frac{1}{3}$ of the nucleon mass. α is the usual flavor-independent spring constant. In the bag model, the quark's momentum values (p_i) are discrete because of the bag boundary condition. However, in the absence of any such explicit boundary condition in the present framework, the quarks in the ground state are assigned a certain characteristic momentum given by $\langle p_i\rangle=1/R$, determined implicitly by the spring constant α.

To investigate the role of the quark mass and the spring constant in hadronic matrix elements, we evaluate certain low-energy parameters, viz., μ_p, G_A/G_V, and $\langle r_p^2\rangle$. The matrix element of a one-body operator acting on the third quark is

$$\int d\vec{r}_1d\vec{r}_2d\vec{r}_3d\vec{r}\,'_3\psi^\dagger(\vec{r}_1,\vec{r}_2\vec{r}\,'_3)\mathscr{O}(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}\,'_3)\psi(\vec{r}_1,\vec{r}_2,\vec{r}_3)\delta(\vec{r}_1+\vec{r}_2+\vec{r}\,'_3)\delta(\vec{r}_1+\vec{r}_2+\vec{r}_3)\,. \tag{5}$$

The operator $\mathscr{O}$ acting on the third quark corresponding to the G_A/G_V magnetic moment and the proton charge radius is

$$(\tau_3\vec{\sigma})_z\,,\quad(\vec{r}_3\times\vec{\nabla}Q_3)_z\,,\quad\text{and}\quad|\vec{r}_3|^2\,, \tag{6}$$

respectively. To facilitate the simplification of the expressions for the various matrix elements, to be considered in the present work as well as subsequent works, we define below certain integrals:

$$\mathscr{I}_1(p',p)\equiv\int_0^\infty r^2j_0(p'r)j_0(pr)e^{-\alpha r^2}dr=\frac{1}{4p'p}\left[\frac{\pi}{\alpha}\right]^{1/2}\left[\exp\frac{-(p'-p)^2}{4\alpha}-\exp\frac{-(p'+p)^2}{4\alpha}\right], \tag{7a}$$

$$\begin{aligned}\mathscr{I}_2(p',p)&=\int_0^\infty r^2j_1(p'r)j_1(pr)e^{-\alpha r^2}dr\\&=\frac{1}{4p'p}\left\{\left[\frac{\pi}{\alpha}\right]^{1/2}\left[\left[1+\frac{2\alpha}{p'p}\right]\exp\frac{-(p'+p)^2}{4\alpha}+\left[1-\frac{2\alpha}{p'p}\right]\exp\frac{-(p'-p)^2}{4\alpha}\right]-\frac{2\pi}{p}\operatorname{erf}\frac{p'-p}{2\sqrt{\alpha}}\right\},\end{aligned} \tag{7b}$$

$$\begin{aligned}\mathscr{I}_3(p',p;k)&\equiv\int_0^\infty r^2j_0(p'r)j_1(kr)j_1(pr)e^{-\alpha r^2}dr\\&=\frac{1}{p'pk}\left[\frac{\pi}{8}X_++\frac{p'\pi}{4k}X_-+\frac{\pi\alpha}{8pk}\left[X_--2\operatorname{erf}\frac{k}{2\sqrt{\alpha}}\right]-\frac{\sqrt{\pi\alpha}}{2k}\sinh\frac{p'k}{\alpha}\exp\frac{-(4p'+k^2)}{4\alpha}\right],\end{aligned} \tag{7c}$$

$$\mathscr{I}_4(p',p;k)\equiv \int_0^\infty r^2 j_0(p'r)j_1(kr)j_0(pr)e^{-\alpha r^2}dr\ , \tag{7d}$$

$$\mathscr{I}_5(p',p',k)\equiv \int_0^\infty r^2 j_1(p'r)j_0(kr)j_1(pr)e^{-\alpha r^2}dr\ , \tag{7e}$$

$$\mathscr{I}_6(p',p;k)\equiv \int_0^\infty r j_1(p'r)j_1(kr)j_1(pr)e^{-\alpha r^2}dr\ , \tag{7f}$$

where

$$X_\pm(p,p';k)\equiv\left[\operatorname{erf}\frac{p+p'}{2\sqrt{\alpha}}\pm\operatorname{erf}\frac{p-p'}{2\sqrt{\alpha}}\right],\quad \operatorname{erf}\eta=\frac{2}{\sqrt{\pi}}\int_0^\eta e^{-t^2}dt$$

and

$$p=\sqrt{\tfrac{2}{3}}p_3\ .$$

In terms of the above integrals the expression for the normalization (4) simplifies to

$$\left[\frac{N_p}{4\pi}\right]^{-2}=\left[\frac{\pi}{\alpha}\right]^{1/2}[\delta\mathscr{I}_1(p,p)+\eta\mathscr{I}_2(p,p)]\ , \tag{8}$$

where

$$\delta=\frac{\epsilon_i+m_i}{\epsilon_i}\ \text{and}\ \eta=\frac{\epsilon_i-m_i}{\epsilon_i}\ .$$

For numerical evaluation of the different integrals, we have used $p'=p\simeq\sqrt{\alpha}$. We obtain the following expressions for the various low-energy parameters considered here:

$$G_A/G_V=N_pN_{p'}[(\delta'\delta)^{1/2}\mathscr{I}_1(p',p)-\tfrac{1}{3}(\eta'\eta)^{1/2}\mathscr{I}_2(p',p)]\ , \tag{9}$$

$$\mu_p=\frac{N_p^{\,2}}{6p^3}\left[\frac{2\pi}{3\alpha}\right]^{1/2}(\delta\eta)^{1/2}[1-(1+p^2/\alpha)\exp(-p^2/\alpha)]\ , \tag{10}$$

$$\langle r_p^{\,2}\rangle=4\pi N_p^{\,2}\left[\delta\int_0^\infty r^4 j_0^{\,2}(pr)e^{-\alpha r^2}dr+\eta\int_0^\infty r^4 j_1(pr)e^{-\alpha r^2}dr\right]\ . \tag{11}$$

A. G_A/G_V

In Table I, we have displayed a few numerical values of G_A/G_V for some typical values of quark mass and spring constant. Before we discuss our results, it is essential to recall some of the facts known about G_A/G_V in the context of the nonrelativistic HO model and the standard bag model. The nonrelativistic quark model with unbroken SU(6) predicts G_A/G_V to be 1.66, a result which is too large compared with the experimental value. It is known that the relativistic effects[20] as well as the configuration mixing[16,21] brings down its value. That the relativistic effects bring down the result is also borne out by the fact that the bag model with massless quarks predicts $G_A/G_V=1.09$, which is rather small. However, this value presumably can be pushed up to the experimental level by considering massive quarks in the bag.[8]

TABLE I. Present results of G_A/G_V for some typical values of quark mass (m_q) and spring constant.

m_q (MeV) \ α (GeV^2)	0.02	0.05	0.10	0.20
50	1.472	1.515	1.507	1.509
100	1.526	1.559	1.553	1.554
200	1.609	1.622	1.620	1.620

In our case although the relativistic effects has lowered the nonrelativistic quark model value of G_A/G_V, but the calculated values are still appreciably higher as compared with the data. Table I also indicates that the effect of quark mass variation is not appreciable on G_A/G_V. At this point a question which naturally arises is why the bag model with light massive quarks fits the data.[8] To understand this behavior it becomes essential to investigate certain not-so-apparent features of the bag model.

The value of G_A/G_V [Eq. (9)] in the present work as well as in the bag model[22] is basically controlled by the integrals $\mathscr{I}_1$ and $\mathscr{I}_2$ which represent the overlaps of upper with upper and lower with lower components of the Dirac spinors, respectively. In the case of the nonrelativistic model $\mathscr{I}_2$ is identically zero, whereas in the usual massless-quark bag model $\mathscr{J}_2$ is 35% of $\mathscr{J}_1$ for $r=5$ GeV^{-1} and the factors δ and η are both unity. Crucially, in the massive-quark bag model[8] the integrals corresponding to $\mathscr{J}_1$ and $\mathscr{J}_2$ become comparable for $r\sim 8$ GeV^{-1} (the bag radius) thereby lowering the numerical value of G_A/G_V. In Ref. 8, while fitting G_A/G_V, this increased contribution from lower-lower components is appropriately compensated by having nonzero quark mass. On the other hand, in our case, $\mathscr{J}_2$ continues to be $\sim 10\%$ of $\mathscr{J}_1$ over a wide range of α. This is reflected in the negligible effect of variation of α on G_A/G_V. Furthermore, an increase in the quark masses suppresses the lower components via the factor η and thus tends to increase G_A/G_V.

B. Magnetic moment (μ_p)

Figures 1(a) and 1(b) depict the behavior of μ_p as a function of quark mass and the spring constant, respectively. To understand the shown trend and the low numerical values of μ_p, it seems necessary to go into certain details pertaining to the calculations of magnetic moment in the nonrelativistic and bag models. In the nonrelativistic model the magnetic moment of a hadron is taken to be the sum total of the individual moments of the constituent quarks.

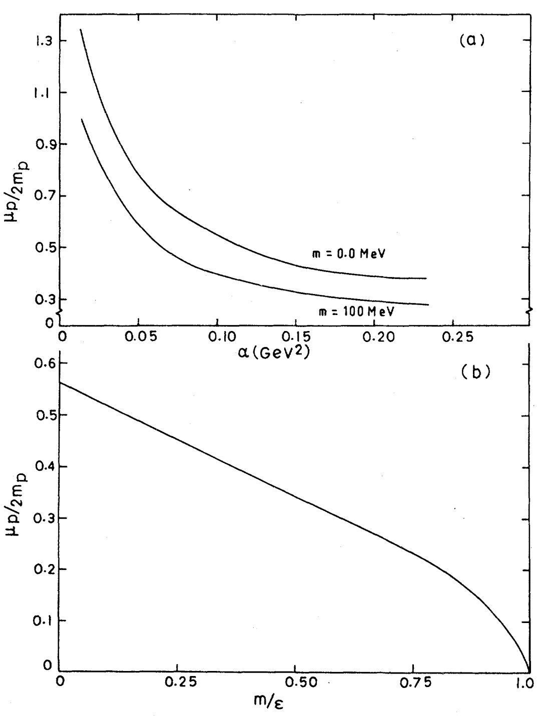

FIG. 1. (a) $\mu_p/2m_p$ plotted against spring constant. (b) $\mu_p/2m_p$ plotted against quark mass.

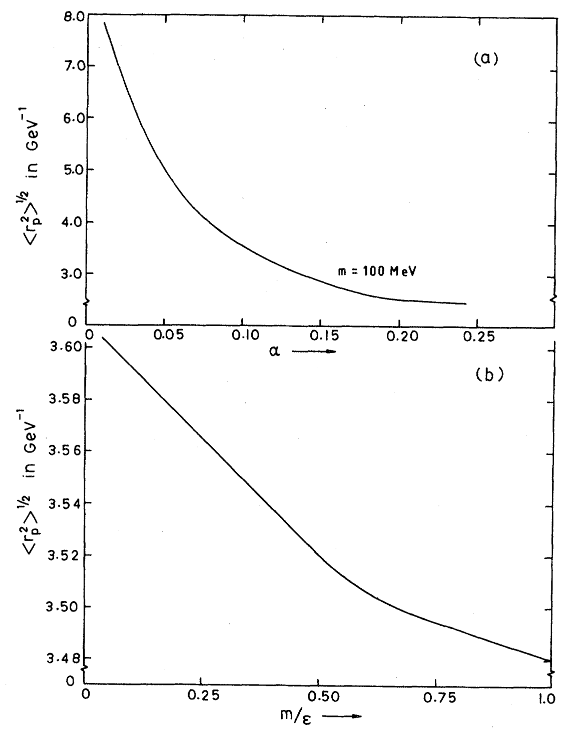

FIG. 2. (a) Proton charge radius $(\langle r_p^2\rangle)^{1/2}$ versus spring constant. (b) Proton charge radius $(\langle r_p^2\rangle^{1/2})$ versus quark mass.

It may be noted that since the quarks are heavy, the spatial overlap is nonzero only between upper-upper components of the Dirac spinor.

On the other hand, the expression of the μ_p employed in the bag model, and also in our case, involves an overlap of the upper and lower components of the Dirac spinor, which vanishes in the nonrelativistic limits ($\epsilon\rightarrow m$). Therefore, this contribution is the relativistic correction to the nonrelativistic value of the magnetic moment. A typical value of μ_p is 0.78 (in nuclear magnetons) for $m=0.1$ GeV and $\alpha=0.1$ GeV.^{2} In this context, it is worth mentioning that quark masses of the order of 250 MeV have been used in the nonrelativistic approach for obtaining the magnetic moments of fundamental octet.[23] The relativistic correction obtained in the above manner will be 20% of the nonrelativistic value which is in accordance with the naive findings of Isgur and Karl.[24]

C. Proton charge radius ($\langle r_p^2\rangle$)

In Fig. 2(a) and 2(b) we have plotted $\langle r_p^2\rangle^{1/2}$ versus α and m_q, respectively. The trends can be understood in the following way. The matrix element of the operator $|\vec{r}_3|^2$ (6) involves overlap of upper-upper and lower-lower components of the

Dirac spinor. Our numerical evaluation of (11) shows that the presence of the exponential makes the radial integrals converge faster for larger values of α, while for smaller α the convergence takes place for a larger value of r. The almost exponential fall of $\langle r_p^2\rangle^{1/2}$ with α [Fig. 2(a)], therefore, emerges as a consequence of the Gaussian present in the wave function. On the other hand, $\langle r_p^2\rangle^{1/2}$ falls almost linearly with increasing quark mass in the present model as well as the bag model. However, this variation is quite small, in terms of the total numerical value, because it is the radial overlaps which dominate the numerical results rather than the factors δ or η. It may be noted that the experimental fit (0.88±0.03 fm) (Ref. 25) corresponds to $m=50$ MeV and $\alpha=0.067$ GeV^2.

III. GLUONIC CORRECTIONS

Our numerical calculations of previous sections show that the matrix elements receive their dominant contributions from $r=0$ to 5 GeV^{-1}; in most of the cases the integrals have more than 85% of their value within $r\sim5$ GeV^{-1}. This provides *a posteriori* justification of our model. It therefore becomes interesting to carry out an exercise similar to the one carried out in the bag model for the gluonic corrections to the matrix elements. To this end, we have calculated certain key overlaps involving the gluon wave function, which have earlier been calculated by Golowich[9] [Fig. 3(a)] and Close and Horgan[10] [Fig. 3(c)] within the spherical-cavity approximation of the bag model. As we have already noted that the present model simulates the bag model, it is therefore natural to assume that the same gluon frequencies dominate the overlaps where the gluonic contributions are important. To facilitate the discussion of results as well as comparison with the findings of Refs. 9 and 10, we reproduce certain essential details of the calculations.

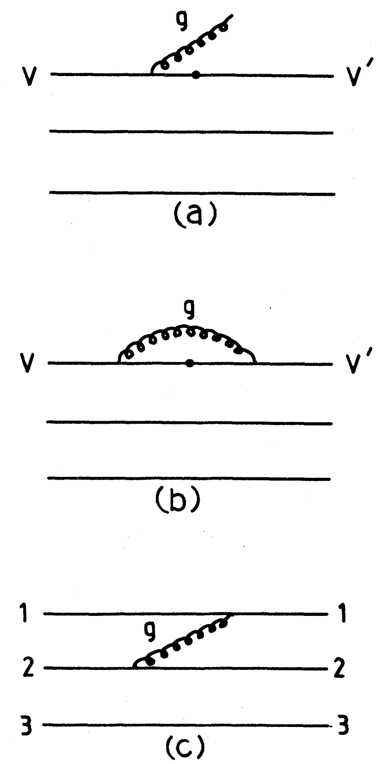

FIG. 3. (a) Three-quark—one-gluon configuration. (b) A typical $O(g^2)$ correction induced by Fig. 3(a). (c) Feynman diagram of a typical gluon-exchange correction.

A. Gluonic radiative corrections

The three-quark—one-gluon diagram [Fig. 3(a)] has the following amplitude[9]:

$$|P\rangle_G=\left[\frac{g}{k}\int d^3\vec{r}_i\bar{\psi}_{V'}(\{\vec{r}_i\})\vec{\gamma}\lambda_A\psi_V(\{\vec{r}_i\})\cdot\vec{A}^{(E)}_{A,nlm}(\vec{r}_i)+(E\leftrightarrow M)\right]|P\rangle_V\ , \tag{12}$$

where $|P\rangle_V$ is the normalized three-valence-quark system, λ_A are the usual generators of SU(3)$_{\text{color}}$ gauge group and k's are the energy eigenmodes of the gluons in GeV units. $A^{(E)}$ and $A^{(M)}$ are the gluon (electric and magnetic, respectively) propagators in Coulomb gauge and have been discussed in detail in Ref. 9. A straightforward calculation of the probability density for $l=1$ (angular momentum conservation at the quark-gluon vertex restricts gluons to have $l=1$) associated with the three-quark—one-gluon configuration yields

$$P_r=\frac{128\alpha_s}{3k}N_p{}^2N_{p'}{}^2\{N_G^{(E)2}[(\delta'\eta)^{1/2}\mathcal{F}_3(p',p;k)+(\delta\eta')^{1/2}\mathcal{F}_3(p,p';k)]^2+N_G^{(M)2}[k(\delta\mathcal{F}_5-\eta\mathcal{F}_4)-2\eta\mathcal{F}_6]^2\}\ . \tag{13}$$

To gauge the role of the quark mass and spring constant we have plotted in Fig. 4 the expression (13) as a function of m and α.

A closer scrutiny of (13) indicates that the probability falls linearly with the increasing quark mass. The trend is similar in the bag model, essentially because of the product of upper and lower components in the overlap. A similar quark-mass dependence

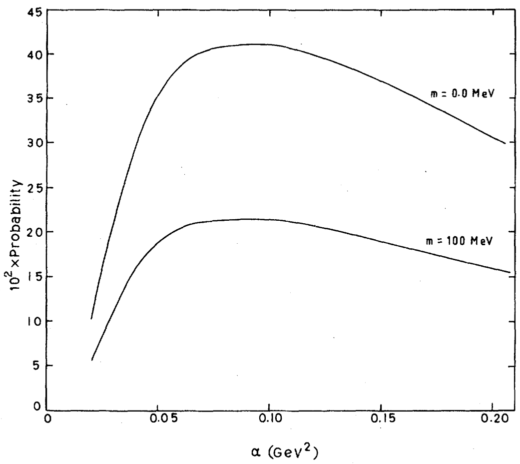

FIG. 4. Gluonic probability as a function of the spring constant α.

can also be seen in Ref. 14, wherein the one-gluon-exchange potential decreases as quarks become more and more massive. Interestingly, the probability plotted against spring constant α shows a broad maximum around $\alpha = 0.1\ \mathrm{GeV}^2$ which is the standard nonrelativistic HO model value of the spring constant. Furthermore we note that our numerical calculations follow the same pattern as that of Ref. 9 as far as the relative strengths of "electric" and "magnetic" gluons modes are concerned. The dominant contribution comes from the "electric" mode of the lowest gluon frequency. Corresponding magnetic mode contributions are smaller by two orders of magnitude. As is seen from a typical set in Table II, our results are of the same order of magnitude as that of the bag model.

In continuation of the above, we consider the effect of a typical $O(g^2)$ diagram [Fig. 3(b)] on G_A/G_V. The shift $\Delta(G_A/G_V)$ is

$$\Delta(G_A/G_V) = -\frac{160\alpha_s}{27k} N_p{}^3 N_{p'}{}^3 [(\delta\delta')^{1/2} \mathscr{I}_1(p',p) - \tfrac{1}{3}(\eta'\eta)^{1/2} \mathscr{I}_2(p',p)] \times \{ N_G^{(E)2} [(\delta'\eta)^{1/2} \mathscr{I}_3(p',p;k) + (\delta\eta')^{1/2} \mathscr{I}_3(p,p';k)]^2 + N_G^{(M)2} [k(\delta\mathscr{I}_5 - \eta\mathscr{I}_4) - 2\eta\mathscr{I}_6]^2 \} . \quad (14)$$

Some of the typical values of (14) are given in Table II. As in Ref. 9, this particular gluonic correction tends to bring down the value of G_A/G_V. However, the small numerical value of (14), as compared to the one in the bag model, can be traced back to the corresponding lower numerical value of the probability density in our case. We, therefore, find that G_A/G_V is not much affected by the gluon radiative corrections.

B. Exchange contribution

We have also evaluated the energy shift induced by one-gluon-exchange diagram [Fig. 3(c)], following the work of Close and Horgan.[10] It is readily seen that the evaluation of the diagram in our framework would essentially keep all the details of Ref. 10 intact. The key term affected is the radial overlap I_k^{IF}, which in the context of present ap-

TABLE II. Certain typical numerical results of the present analysis compared with the bag model and nonrelativistic HO model. μ_p in nuclear magnetons, $\langle r^2 \rangle^{1/2}$ in fm.

	Present analysis: $\alpha = 0.09\ \mathrm{GeV}^2$ ($m_q = 100$ MeV)	($m_q = 200$ MeV)	Bag model	Nonrelativistic HO model	Expt.
G_A/G_V	1.55	1.61	1.09[a]	1.667	1.248±0.01[e]
μ_p	1.00	0.978	1.986[a]	0	
$\langle r^2 \rangle^{1/2}$	0.74	0.73	0.73[a]	0.63[d]	0.88 ±0.03[f]
Probability	0.2295	0.0976	1.13[b]		
$\sqrt{4\pi} I^{IF}$	0.4956	0.3232	1.10[c]		
$\Delta(G_A/G_V)$	−0.03	−0.01	−0.1145[b]		

[a]Reference 5.
[b]Reference 9.
[c]Reference 10.
[d]Reference 26.
[e]Reference 25.
[f]Reference 27.

proach modifies to

$$I_k^{IF}=2\left[\frac{\pi}{3}\right]^{1/2}N_pN_{p'}N_G^{(E)}[(\delta'\eta)^{1/2}\mathscr{J}_3(p',p;k)+(\delta\eta')^{1/2}\mathscr{J}_5(p,p';k)]\ . \quad (15)$$

The energy shift (ΔE) of Ref. 10 involves squares of the expression (13). Thus, in principle, ΔE shows the same behavior with the variation in quark mass and the spring constant as the probability in Sec. III A. Again we find that our numerical value is of the same order of magnitude as that of Ref. 10 (Table II).

IV. SUMMARY AND CONCLUSION

Gluon-exchange[10] and radiative gluon[9] corrections have been studied in the bag model, using Feynman rules developed by Lee for a spherical cavity.[6] We have carried out similar calculations in the framework of the nonrelativistic HO model with Pauli spinors replaced by Dirac spinors. This is possible because of the fast damping provided by the Gaussian in the case of the HO wave function. A similar feature has been noted for the massive-quark bag model by DeGrand *et al.*[5] However, before calculating gluonic corrections, we have studied in detail G_A/G_V, μ_p, and $\langle r_p{}^2\rangle$ as functions of quark mass and spring constant. For G_A/G_V we have found that in the present work the value does not go lower than 1.5 (Table I), indicating that configuration mixing[21] is important for bringing down this value. Interestingly, we have observed that the bag-model value with or without massive quarks requires a contribution of small components in Dirac spinors which is as important as the upper-component contribution. In the case of the magnetic moment, our approach can successfully simulate the relativistic corrections to the nonrelativistic quark-model values. The proton charge radius is not much affected in the present approach since it involves the sum of the overlaps of upper-upper and lower-lower components only.

With regard to the gluon corrections, we find that our results are of the same order of magnitude as that of the bag model. In the absence of any clear-cut experimental signals in this regard, it is difficult to choose definite values of the spring constant and the quark mass. In Table II we have summarized our typical values of various parameters along with the corresponding results from the bag and HO models.

Interestingly, we find that the present approach can reproduce the results of the bag model as well as the nonrelativistic HO model in appropriate limits.[22] We believe that the true model may lie between the bag approach and the oscillator model; therefore, the present approach may shed some light in this direction. In this context, it is essential to extend the scope of the present approach to other areas of baryon spectroscopy, which will be the subject of subsequent publications.

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