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Gluon-gluon interactions in the bag model

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An effective spin-dependent interaction Hamiltonian for low-lying gluon modes is calculated to $\mathscr{O}(\alpha_s)$ in the MIT bag model. We give expressions for the energy shifts of low-lying glueballs.

I. INTRODUCTION

From considerations based on QCD one expects hadrons consisting only (or mainly) of glue.[1] The possibility of identifying as glueballs the states $\iota(1440)$ and $\theta(1660)$ recently discovered in $\psi\to\gamma X$ (Refs. 2 and 3) has made it even more interesting to get precise predictions from QCD. Since one still cannot compute the hadron mass spectrum from first principles one must resort to phenomenological models keeping as many as possible of the properties of the full theory. In the case of glueballs, it is of special importance that the model can handle massless particles, and also treat gauge invariance in a satisfactory way.[4] One such model, and the one to be used here, is the MIT bag. The aim of this work, which is in essence technical, is to calculate to $O(\alpha_s)$ the spin-dependent energy shift due to gluon-gluon interactions in the bag. Several authors have already dealt with the properties of glueballs in the bag model,[5-8] and we shall comment on our relationship to their work below.

Following the argument of Ref. 7, we will assume that spherical glueballs exist in the bag model. Hence we can use the static-spherical-cavity approximation which has been successful in the case of low-lying mesons and baryons. The effective Hamiltonian in the n-gluon sector takes the form[9,10]

$$H=\frac{4\pi R^3}{3}B+\sum_i \frac{n_i x_i}{R}+H_{\rm int}-\frac{C_{\rm c.m.}}{R}+\frac{C_{\rm Cas}}{R}\,, \tag{1}$$

where $H_{\rm int}$ has a nontrivial color and spin dependence. The first two terms (volume and kinetic energy) are well known, and the gluon-gluon interaction $H_{\rm int}$, which also includes self-energies, is the subject of this paper. The "center-of-mass" term $(C_{\rm c.m.}/R)$ can be estimated using the method of Donoghue and Johnson,[10] but it is still not clear whether the zero-point or "Casimir" energy $(C_{\rm Cas}/R)$ is of importance. [In earlier works,[9] the two last terms in Eq. (1) were lumped together with the self-energy part of $H_{\rm int}$ in a purely phenomenological term Z_0/R with $Z_0\simeq-1.8$.]

In this paper we shall derive explicit expressions for the two-gluon-interaction part of $H_{\rm int}$ in Eq. (1). This involves calculating the diagrams shown in Figs. 1 and 2. The color magnetic and electric fields generated by the interaction must, of course, satisfy the appropriate bag boundary condition. The interaction magnetic fields generated by each constituent gluon do satisfy the boundary condition, thus causing no problem. The same is not true of the interaction electric fields. Only the sum of the electric fields generated by all the constituent gluons in a color-singlet glueball can satisfy the boundary condition. Hence we must include (at least) the static Coulomb self-energies in order to have the energy shifts due to all the electric fields needed for the boundary condition. This is exactly analogous to what was found when the quark-quark interaction mediated by gluons was considered for mesons and baryons in Ref. 9. Incidentally, the electric energy shift is in general exactly zero for a quarkic hadron with all quarks in the same cavity eigenstate[9]; the analogous statement is not true for glueballs.

Several earlier workers have dealt with spin-dependent splittings among glueballs in the bag model. Thorn, as reported in Ref. 7, calculated the splittings for the $(\mathrm{TE})^2$ 0^{++} and 2^{++} states. He considered only the magnetic energy; to his results should be added the Coulomb interaction energy and self-energy. It happens that for the 0^{++} $(\mathrm{TE})^2$ [or

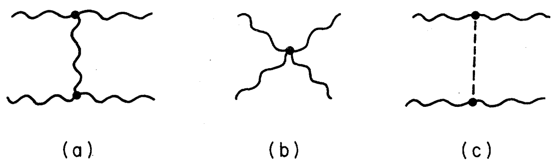

FIG. 1. Gluon-gluon interactions to order α_s. The dashed line represents a Coulomb interaction.

$(TM)^2$, incidentally] glueball, the Coulomb interaction energy and static self-energy cancel exactly. So we agree with Thorn's 0^{++} result, but not his 2^{++} result. (Our magnetic energy alone does agree with Thorn's 2^{++} result.)

Konoplich and Schepkin calculated the spin-dependent mass shift for 0^{++} $(TE)^2$ and 0^{-+} (TE)(TM) glueballs. They also considered only the magnetic energy. We agree with their calculation as it stands (taking note of the approximateness of their numerical evaluations and of their definition of α_s, which differs from the standard one by a factor of 4) but note that the Coulomb interaction energy and the static self-energy should be added. While the Coulomb interaction energy and static self-energy cancel for the 0^{++} $(TE)^2$, they do not cancel for the 0^{-+} (TE)(TM).

Barnes, Close, and Monaghan[5] considered the spin-dependent splittings for 0^{++} and 2^{++} $(TE)^2$ and 0^{-+} and 2^{-+} (TE)(TM) states. They do include the Coulomb interaction energy but do not consider the self-energies. Since the gluon modes do not individually satisfy the $\hat{n}\cdot\vec{E}=0$ boundary condition (cf. their Appendix 3), the Coulomb interaction energy obtained by integrating $\rho_i\phi_j$ (charge density times scalar potential, with i and j labeling different quarks) is gauge variant. Including the static Coulomb self-energy, the case when i and j are the same, makes the result gauge invariant. However, the gauge-variant contribution is the same for any states made from the same gluon modes, so we may compare some of our energy splittings to Barnes *et al.* We do agree on the *differences* of energy shifts between 0^{++} and 2^{++} $(TE)^2$ glueballs and (after some omissions and numerical errors in Ref. 5 are corrected) between 0^{-+} and 2^{-+} (TE)(TM) glueballs.

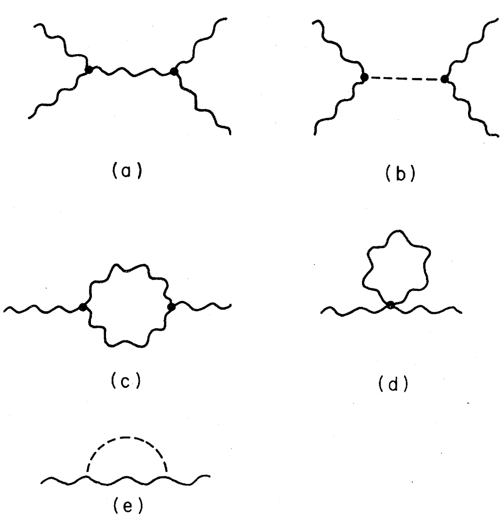

FIG. 2. Annihilation and self-energy diagrams.

To our knowledge, the $(TM)^2$ and $(TE)^3$ glueballs' mass splittings have not been previously considered. Also, Konoplich and Schepkin[8] and Barnes *et al.*[5] calculate via a truncated mode sum. Our technique is rather different and should correspond to having done the whole sum. We do find agreement between the two methods when calculating the same quantities.

The next section outlines the calculation of the effective Hamiltonian leaving most of the technicalities to the Appendices. In the last section we consider some special cases of phenomenological interest.

II. $O(\alpha_s)$ GLUON-GLUON EFFECTIVE HAMILTONIAN

The QCD interaction Hamiltonian density of $O(g^2)$ is in Coulomb gauge given by[11]

$$\begin{aligned} H_I &= H_I^{3g}+H_I^{4g}+H_I^{\rm Coul} \\ &= \tfrac{1}{2} g f^{abc} F^a_{jk} A^b_j A^c_k + \tfrac{1}{4} g^2 f^{abc} f^{ade} A^b_j A^c_k A^d_j A^e_k \\ &\quad + \tfrac{1}{2} g^2 f^{abc} f^{ade} F^b_{0k} A^c_k D_{\rm Coul} F^d_{0l} A^e_l \ , \end{aligned} \tag{2}$$

where the operator $D_{\rm Coul}$ is defined below. The bag-model interaction Hamiltonian

$$H_I = \int_{\rm bag} d^3x\, H_I(x) \tag{3}$$

operates on n-gluon cavity states $(1,2,\ldots,n)$, which are direct products of one-gluon "cavity modes"

$$|\, i\,\rangle = |\, a_i, l_i, m_i, \chi_i \,\rangle \ ,$$

where a denotes color, (l,m) orbital angular momentum, and χ radial quantum number as well as TE or TM (transverse electric or transverse magnetic). We shall consider the $l=1$ modes only, for which

$$|\, a,l,m,\chi\,\rangle \equiv |\, a,\alpha,\chi\,\rangle \ ,$$

where α is the polarization index. A general n-gluon state built from these modes is specified by wave functions

$$\Psi^{(R)}_{(S,M)} = \eta^{(R)}_{a_1\cdots a_N}\phi^{\alpha_1\cdots\alpha_N}_{(S,M)}\vec{A}^{\chi_1}_{\alpha_1}(x_1)\cdots\vec{A}^{\chi_N}_{\alpha_N}(x_N) \ , \tag{4}$$

where (R) and (S,M) denote color and spin, respectively. The relevant cavity modes A are given in Appendix A.

Now, write the effective interaction Hamiltonian $H_{\rm int}$ in Eq. (1) as,[12]

$$H_{\rm int} = \sum_{m<n} H_{mn} + \sum_m H_m^{\rm self} . \tag{5}$$

Although we shall compute the effect of the interaction terms H_{mn} shown in Figs. 1(a), 1(b), and 1(c), the self-energies [Figs. 2(c)—2(e)] might be important as will be briefly commented upon later. Using lowest-order perturbation theory Eqs. (2) and (3) immediately yield

$$H_{mn} = H_{mn}^{3g} + H_{mn}^{4g} + H_{mn}^{\rm Coul} = (-i)S_B \int_{-\infty}^{0} dt \int_{\rm bag} d^3x\, d^3y\, \langle m'n' | \mathscr{H}^{3g}(\vec{x},t)\mathscr{H}^{3g}(\vec{y},0) | mn \rangle$$
$$+ S_B \int_{\rm bag} d^3x\, \langle m'n' | \mathscr{H}^{4g} | mn \rangle + S_B \int_{\rm bag} d^3x\, \langle m'n' | \mathscr{H}^{\rm Coul} | mn \rangle . \tag{6}$$

Here $\chi = \chi'$ but in general $a' \neq a$ and $\alpha' \neq \alpha$. Thus H_{mn} is still an operator in color and spin space although for notational simplicity we suppressed the corresponding indices ($\alpha_N, \alpha_{N'}, a_N$, etc.). The Bose-statistics factor is $S_B = \frac{1}{2}$ for identical modes, and otherwise $S_B = 1$. The diagrams corresponding to the three terms are shown in Fig. 2. Now introduce the current and charge-density operators[13]

$$j_k^a = (-i)\Lambda^a j_k = g f^{abc}(2F_{jk}^b A_j^c - A_j^b \partial_j A_k^c) \tag{7a}$$

$$\rho^a = (-i)\Lambda^a \rho = g f^{abc} F_{0k}^b A_k^c , \tag{7b}$$

where Λ^a is the ath color generator. The corresponding antisymmetrized matrix elements are given by

$$\vec{j}_{mn} = \langle m | \vec{j} | n \rangle - \langle n | \vec{j} | m \rangle , \tag{8a}$$

$$\rho_{mn} = \langle m | \rho | n \rangle - \langle n | \rho | m \rangle . \tag{8b}$$

After some algebra and after carrying out the t integration one gets

$$H_{mn}^{3g} = -\Lambda_m^a \Lambda_n^a S_B \int_{\rm bag} d^3x\, d^3y\, [j_{mm}^k(\vec{x}) D(\vec{x},\vec{y};\omega) j_{nn}^k(\vec{y}) + j_{mn}^k(\vec{x}) D(\vec{x},\vec{y};\omega) j_{nm}^k(\vec{y})] , \tag{9}$$

$$H_{mn}^{\rm Coul} = -\Lambda_m^a \Lambda_n^a S_B \int_{\rm bag} d^3x\, d^3y [\rho_{mm}(\vec{x}) D_{\rm Coul}(\vec{x},\vec{y};\omega) \rho_{nn}(\vec{y}) + \rho_{mn}(\vec{x}) D_{\rm Coul}(\vec{x},\vec{y};\omega) \rho_{nm}(\vec{y})] , \tag{10}$$

where the "exchange" term $j_{mn} j_{nm}$ is absent for identical modes. Here the "confined" propagators D and $D_{\rm Coul}$ differ from the "free" ones by boundary terms.[14,15] Instead of using explicit expressions for the cavity propagators, we follow the original MIT approach and directly calculate the potentials[9,16]

$$\vec{a}_{mn}(\vec{x}) = -\int d^3y\, D(\vec{x},\vec{y};\omega) \vec{j}_{mn}(\vec{x}) , \tag{11a}$$

$$\phi_{mn}(\vec{x}) = -\int d^3y\, D_{\rm Coul}(\vec{x},\vec{y};\omega) \rho_{mn}(\vec{x}) \tag{11b}$$

subject to the boundary conditions

$$\hat{r}\cdot(\vec{\nabla}\times\vec{a}) = 0 , \tag{12a}$$

$$\left.\begin{array}{l} \hat{r}\cdot\dfrac{d\vec{a}}{dt} = 0, \\[2ex] \hat{r}\cdot\vec{\nabla}\phi = 0 , \end{array}\right\} \text{ on the surface .} \tag{12b, 12c}$$

We can then write H_{mn} as

$$H_{mn} = -\Lambda_m^a \Lambda_n^a S_B \int_{\rm bag} d^3x [\vec{j}_{mm}(\vec{x})\cdot\vec{a}_{nn}(\vec{x}) + \vec{j}_{mn}(\vec{x})\cdot\vec{a}_{nm}(\vec{x})] - \Lambda_m^a \Lambda_n^a S_B \int_{\rm bag} d^3x\, \langle mn | H^{4g}(\vec{x},0) | mn \rangle$$
$$+ \Lambda_m^a \Lambda_n^a S_B \int_{\rm bag} d^3x [\rho_{mm}(\vec{x})\phi_{nn}(\vec{x}) + \rho_{mn}(\vec{x})\phi_{nm}(\vec{x})] . \tag{13}$$

Since we consider the lowest TE and TM modes only, there are just three possible combinations (TE)(TE), (TE)(TM), and (TM)(TM). The wave functions ($\vec{A}_\alpha$) are given in Appendix A, the relevant current and charge densities ($\vec{j}$ and ρ) (as calculated in Appendix C) in Table I and the corresponding potentials ($\vec{a}$ and ϕ) in Appendix D. Substituting all this in Eq. (13) we get (see Appendices

TABLE I. Current and charge densities j_{mn} and ρ_{mn} for $l=1$ gluon modes.

	$\vec{j}$	ρ
TE-TE	$3g\frac{N_E^2R}{\rho x_E^2}j_1^2(x_E\rho)\hat{r}\times\vec{S}$	$-2g\frac{N_E^2R}{x_E}j_1^2(x_E\rho)(U-\frac{2}{3}I)$
TM-TM	$g\frac{N_M^2R}{\rho x_M^2}[4j_1^2(x_M\rho)-j_2^2(x_M\rho)]\hat{r}\times\vec{S}$	$2g\frac{N_M^2}{x_M}\frac{R}{3}j_2(x_M\rho)[j_2(x_M\rho)+4j_0(x_M\rho)]U$
		$+\frac{2}{3}[j_2^2(x_M\rho)+2j_0^2(x_M\rho)]I$
TE-TM	$-g\frac{N_EN_M}{x_Ex_M}R\{f_2(\rho)[\hat{r}(\hat{r}\cdot\vec{S})-\frac{1}{3}\vec{S}]$	$-ig\frac{N_EN_MR(x_E+x_M)}{3x_Ex_M}j_1(x_E\rho)$
	$+\tilde{f}_2(\rho)\vec{T}+f_0(\rho)\vec{S}\}$	$+[2j_0(x_M\rho)-j_2(x_M\rho)]\hat{r}\cdot\vec{S}$

E and F) the general result

$$H_{mn}=-\Lambda_1^a\Lambda_2^a\frac{\alpha_s}{R}(a_{mn}\vec{S}_1\cdot\vec{S}_2+b_{mn}T_{12}+\tilde{c}_{mn}I_{12})+\Delta E(\text{self})+\Delta E(\text{ann})\,. \tag{14}$$

For specific color-singlet glueball states, this becomes the following:

(i) $(\text{TE})^2$ glueballs

$$H_{EE}=-\frac{\alpha_s}{R}\Lambda_1^a\Lambda_2^a[a_{EE}\vec{S}_1\cdot\vec{S}_2+b_{EE}(T_{12}-4I_{12})]\,,$$
$$a_{EE}=a_{EE}^{3g}+a_{EE}^{4g}=0.263\,, \tag{15}$$
$$b_{EE}=b_{EE}^{\text{Coul}}=-0.041\,;$$

(ii) (TE)(TM) glueballs

$$H_{EM}=-\frac{\alpha_s}{R}\Lambda_1^a\Lambda_2^a\left[a_{EM}\vec{S}_1\cdot\vec{S}_2+b_{EM}T_{12}+\left[\tilde{c}_{EM}-\frac{\tilde{c}_{EE}+\tilde{c}_{MM}}{2}-2b_{EE}-2b_{MM}\right]I_{12}\right],$$
$$a_{EM}=0.255\,,$$
$$b_{EM}=-0.017\,, \tag{16}$$
$$\tilde{c}_{EM}-\tfrac{1}{2}(\tilde{c}_{EE}+\tilde{c}_{MM})=+0.077\,;$$

(iii) $(\text{TM})^2$ glueballs

$$H_{MM}=-\frac{\alpha_s}{R}\Lambda_1^a\Lambda_2^a[a_{MM}\vec{S}_1\cdot\vec{S}_2+b_{MM}(T_{12}-4I_{12})]\,,$$
$$a_{MM}=a_{MM}^{3g}+a_{MM}^{4g}=0.247\,, \tag{17}$$
$$b_{MM}=b_{MM}^{\text{Coul}}=-0.007\,,$$

where $\alpha_s=g^2/4\pi$, $\vec{S}$ is the spin operator, and T_{12} is a symmetric tensor operator in spin space which can be given by

$$T_{12}=2[(\vec{S}_1\cdot\vec{S}_2)^2-I_{12}]+\vec{S}_1\cdot\vec{S}_2\,. \tag{18}$$

We will also quote some results for $(\text{TE})^3$ glueballs, noting first that the annihilation diagrams illustrated in Figs. 2(a) and 2(b) now can contribute. These are calculated in Appendix F, and then for this fourth case, we obtain the following:

(iv) $(\text{TE})^3$ glueballs

$$H_{EE}=-\frac{\alpha_s}{R}\left[a_{EE}\sum_{i<j}\Lambda_i^a\Lambda_j^a\vec{S}_i\cdot\vec{S}_j+b_{EE}\left[18I_{12}+\sum_{i<j}\Lambda_i^a\Lambda_j^aT_{ij}\right]\right]-d_{EE}\frac{\alpha_s}{R}P_{8A}P_1\,,$$
$$d_{EE}=0.529 \tag{19}$$

with a_{EE} and b_{EE} given above.

The above expressions for the energy shifts are gauge invariant. We have included the static Coulomb self-energies. In addition, there exist the

TABLE II. The coefficients a, b, and $\tilde{c}$ for the various cases. (The undetermined constants in $\tilde{c}$ are not indicated since they self-cancel for color-singlet glueball states.) The notation $C=\pm$ refers to the symmetry of the color wave function in the (TE)(TM) case. Energy shifts are given by

$$\Delta E = -\Lambda_1^a \Lambda_2^a \frac{\alpha_s}{R}[a\vec{S}_1\cdot\vec{S}_2 + bT_1] + \tilde{c}I_{12}] + \Delta E(\text{self}) + \Delta E(\text{ann}) .$$

See text, Eq. (14).

		a	b	$\tilde{c}$
$(\text{TE})^2$	$3g$	0.34069		
	$4g$	−0.07762		
	Coul		−0.04076	−1.15490
	Total	0.263	−0.041	−1.155
$(\text{TM})^2$	$3g$	0.32788		
	$4g$	−0.08084		
	Coul		−0.00715	−1.5333
	Total	0.247	−0.007	−1.533
(TE)(TM)	$3g$	0.36197	−0.07725	0.15794
$C=+$	$4g$	−0.03432	−0.01074	−0.02568
	Coul	−0.07287	+0.07130	−1.39727
	Total	0.255	−0.017	−1.265
(TE)(TM)	$3g$	0.21091	0.07725	−0.15794
$C=-$	$4g$	−0.01144	0.01074	0.02568
	Coul	+0.07287	−0.07445	−1.10581
	Total	0.272	0.014	−1.238

magnetic and the nonstatic Coulomb contributions to the self-energy. The magnetic and electric fields leading to these contributions satisfy the bag boundary conditions with no difficulty. Nonetheless, calculating them *ab initio* is technically difficult and must for now be deferred. Some work has been reported on the full self-energy calculation for ground-state quark modes.[17]

As explained in Appendix B, the general form of the effective Hamiltonian involves three linearly independent tensors in two-particle spin space. The above results expressed in the tensors $\vec{S}_m\cdot\vec{S}_n$ and T_{mn} can easily be transformed to any other basis by using the formulas in Appendix B.

Note added. It appears more common[18] to use a traceless tensor $\vec{T}_1\cdot\vec{T}_2$ related to ours by

$$\vec{T}_1\cdot\vec{T}_2 = \tfrac{1}{2}T_{12} - \tfrac{1}{3}I_{12} .$$

If we write

$$\Delta E = -\Lambda_1^a\Lambda_2^a\frac{\alpha_s}{R}[\bar{a}\vec{S}_1\cdot\vec{S}_2 + \bar{b}\vec{T}_1\cdot\vec{T}_2 + \bar{c}I_{12}] + \Delta E(\text{self}) + \Delta E(\text{ann}) ,$$

then

$$\bar{a}=a, \quad \bar{b}=2b$$

and

$$\bar{c}=\tilde{c}+\tfrac{2}{3}b .$$

The (TE)(TM) coefficients listed in Table II are valid for the 1^{-+} as well as the 0^{-+} and 2^{-+}. If we are interested only in the latter two states, then a shuffling of coefficients is allowed,

$$a \to a' = \text{arbitrary} = a + (a'-a) ,$$

$$b \to b' = b + (a'-a) ,$$

$$\tilde{c} \to \tilde{c}' = \tilde{c} - 2(a'-a) .$$

This facilitates showing agreement between ourselves and Ref. 5 (as corrected), where the results are only applied to the 0^{-+} and 2^{-+} states. (See also Appendix 2 of Ref. 18.)

III. LEVEL SPLITTINGS IN LOW-LYING GLUEBALLS

We close by using Eqs. (14)–(16) and (18) to get the energy shifts for the lowest-lying glueballs, made from the lowest-lying $l=1$ TE and/or TM modes. The relevant states are the color singlets

TABLE III. Expectation values for the operators $\Lambda_1^a\Lambda_2^a$, $\vec{S}_1\cdot\vec{S}_2$, and T_{12} for the lowest-lying glueball states.

	$\Lambda_1^a\Lambda_2^a$	$\vec{S}_1\cdot\vec{S}_2$	T_{12}
$0^{(\mp)+}$	-3	-2	4
$2^{(\mp)+}$	-3	1	1

$$\begin{aligned}&\text{(i) (TE)}^2,\ J^{PC}=0^{++},2^{++}\ ,\\ &\text{(ii) (TE)(TM)},\ J^{PC}=0^{-+},2^{-+}\ ,\\ &\text{(iii) (TM)}^2,\ J^{PC}=0^{++},2^{++}\end{aligned} \tag{20}$$

for which the expectation values of the operators $\Lambda_1^a\Lambda_2^a$, $\vec{S}_1\cdot\vec{S}_2$, and T_{12} are listed in Table III. Thus we have

$$\text{(i) (TE)}^2,\ \Delta E_{0++}=-1.58\frac{\alpha_s}{R}\ ,\qquad \Delta E_{2++}=1.16\frac{\alpha_s}{R}\ ; \tag{21a}$$

$$\text{(ii) (TE)(TM)},\ \Delta E_{0-+}=-1.22\frac{\alpha_s}{R}\ ,\qquad \Delta E_{2-+}=1.23\frac{\alpha_s}{R}\ ; \tag{21b}$$

$$\text{(iii) (TM)}^2,\ \Delta E_{0++}=-1.48\frac{\alpha_s}{R}\ ,\qquad \Delta E_{2++}=0.80\frac{\alpha_s}{R}\ . \tag{21c}$$

With an increasing amount of group-theoretical labor similar calculations can be performed for a general n-gluon state. For $(\mathrm{TE})^3$ glueballs, we have $J^{PC}=0^{++}$, 1^{+-}, and 3^{+-} and

$$\text{(iv) (TE)}^3,\ \Delta E_{0++}=1.33\frac{\alpha_s}{R}\ ,\qquad \Delta E_{1+-}=0.28\frac{\alpha_s}{R}\ ,\qquad \Delta E_{3+-}=1.74\frac{\alpha_s}{R}\ . \tag{21d}$$

There are two dangers in obtaining the splittings among the physical glueball states by simply adding the above energy shift to the lowest-order terms. One is that there will be mixing between the listed states and the nonglueball states with the same quantum numbers. This mixing problem is probably most severe for the $(\mathrm{TE})^2$ 0^{++} state, which is expected to mix strongly with the vacuum.[19] The second uncertainty comes from the $O(\alpha_s/R)$ spin-independent energy shifts and the self-energies. The value of R, and hence the spin-dependent splittings [Eq. (21)], depends on these contributions. Also, the self-energies are different for different gluon modes (most successful bag calculations for quark-based hadrons have quarks only in the lowest state so the mode dependence is often not mentioned). Glueballs containing gluons in the same mode have, of course, the same self-energies to the extent that the radii are the same. Thus one can for example predict

$$M_{2-+}-M_{0-+}=\Delta E_{2-+}-\Delta E_{0-+}=2.45\frac{\alpha_s}{R}\ . \tag{22}$$

Further predictions are possible after considering the vacuum-0^{++} mixing and the self-energies; this work is sufficiently extensive to be reported separately.[20]

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APPENDIX A

The wave functions for the lowest $l=1$ TE and TM modes with $P=+$ and $P=-$, respectively, are given by

$$\vec{A}_\alpha^E(\vec{r},t)=-\frac{N_E}{\omega_E}j_1(x_E\rho)(\hat{r}\times\hat{e}_\alpha)e^{-i\omega_E t}\ , \tag{A1a}$$

$$\vec{A}_\alpha^M(\vec{r},t)=\frac{N_M}{\omega_M}[j_2(x_M\rho)(\hat{r}_\alpha\hat{r}-\tfrac{1}{3}\hat{e}_\alpha)+\tfrac{2}{3}j_0(x_M\rho)\hat{e}_\alpha]e^{-i\omega_M t}\ , \tag{A1b}$$

where

$$N_E{}^2=\frac{3}{8\pi}\frac{1}{R^4 j_0{}^2(x_E)}\frac{x_E}{x_E{}^2-2}\ , \tag{A2a}$$

$$N_M{}^2=\frac{3}{8\pi}\frac{x_M}{R^4 j_0{}^2(x_M)} \tag{A2b}$$

also R is the bag radius, $\omega_{E(M)}=x_{E(M)}/R$, $x_E=2.744$, $x_M=4.493$, and $\rho=r/R$. The spherical unit (or polarization) vectors are denoted by $\hat{e}_\alpha$ and $\hat{r}_\alpha=\hat{r}\cdot\hat{e}_\alpha$. The relation to spherical harmonics is

$$Y_1^\alpha(\Omega)=\left[\frac{3}{4\pi}\right]^{1/2}\hat{r}_\alpha \,. \tag{A3}$$

The corresponding magnetic fields

$$\vec{B}^{E(T)}=\nabla\times\vec{A}^{E(T)}$$

are given by ($t=0$)

$$\vec{B}^E_\alpha(\vec{r})=\frac{N_E}{x_E\rho}[2j_1(x_E\rho)\hat{r}_\alpha\hat{r}-\frac{d}{d\rho}(\rho j_1(x_E\rho))(\hat{r}\times(\hat{r}\times\hat{e}_\alpha))]$$

$$=N_E[j_2(x_E\rho)(\hat{r}_\alpha\hat{r}-\tfrac{1}{3}\hat{e}_\alpha)+\tfrac{2}{3}j_0(x_E\rho)\hat{e}_\alpha]\,, \tag{A4a}$$

$$\vec{B}^M_\alpha(r)=-N_M j_1(x_M\rho)(\hat{r}\times\hat{e}_\alpha)\,. \tag{A4b}$$

APPENDIX B

In this appendix we define the various operators acting in spin space and also give some useful relations.

First consider operators $\mathscr{O}_{\alpha\beta}$ acting in one-particle spin space with (polarization) vectors $\hat{e}_\alpha$. In addition to the usual antisymmetric spin vector operator

$$\vec{S}_{\alpha\beta}=-i\hat{e}_\alpha\times\hat{e}_\beta \tag{B1}$$

we also use the symmetric pseudovector

$$\vec{T}_{\alpha\beta}=i(\hat{r}_\beta\hat{r}\times\hat{e}_\alpha+\hat{r}_\alpha\hat{r}\times\hat{e}_\beta) \tag{B2}$$

and the symmetric tensor operator

$$U_{\alpha\beta}=\hat{r}_\alpha\hat{r}_\beta-\tfrac{1}{3}\delta_{\alpha\beta} \tag{B3}$$

($\hat{r}_\alpha$ is the α component of the unit vector $\hat{r}$). Since $\int d\Omega\,\vec{T}_{\alpha\beta}=0$ and $\hat{r}\cdot\vec{T}_{\alpha\beta}=0$, we can conclude that $\vec{T}_{\alpha\beta}$ has purely $l=2$ orbital angular momentum, and the same holds up for $U_{\alpha\beta}$. On the other hand, $\vec{S}_{\alpha\beta}$ obviously has $l=0$ only. A useful expression for $\vec{T}_{\alpha\beta}$ is

$$\vec{T}_{\alpha\beta}=(-i)[(\hat{r}\cdot\vec{S})\hat{r}\times\vec{S}+\hat{r}\times\vec{S}(\hat{r}\cdot\vec{S})]_{\alpha\beta}\,, \tag{B4}$$

where the order of the spin operators is important.

Next consider scalar ($\hat{r}$-independent) operators $\mathscr{O}_{(\alpha\gamma),(\beta\delta)}$ acting on the direct-product spin space with vectors $\hat{e}_\alpha\hat{e}_\beta$. There are three linearly independent operators of this type, namely, $\delta_{\alpha\beta}\delta_{\gamma\delta}$, $\delta_{\alpha\gamma}\delta_{\beta\delta}$, and $\delta_{\alpha\delta}\delta_{\gamma\beta}$. A more convenient basis is

$$I_{12}=\delta_{\alpha\beta}\delta_{\gamma\delta}\,, \tag{B5a}$$

$$\vec{S}_1\cdot\vec{S}_2=\vec{S}_{\alpha\beta}\cdot\vec{S}_{\gamma\delta}=\delta_{\alpha\delta}\delta_{\beta\gamma}-\delta_{\alpha\gamma}\delta_{\beta\delta}\,, \tag{B5b}$$

$$T_{12}=\delta_{\alpha\delta}\delta_{\beta\gamma}+\delta_{\alpha\gamma}\delta_{\beta\delta}\,, \tag{B5c}$$

where I_{12} is the unit operator, while T_{12} can be expressed as

$$T_{12}=2[(\vec{S}_1\cdot\vec{S}_2)^2-I_{12}]+\vec{S}_1\cdot\vec{S}_2\,. \tag{B6}$$

One can, of course, use other basis than Eq. (B4). If we, e.g., use the "quadrupole-quadrupole" tensor

$$(S^iS^j-\tfrac{2}{3}\delta^{ij}I)_1(S^iS^j-\tfrac{2}{3}\delta^{ij}I)_2=(\vec{S}_1\cdot\vec{S}_2)^2-\tfrac{4}{3}I_{12}\,, \tag{B7}$$

we have

$$a\vec{S}_1\cdot\vec{S}_2+bT_{12}+c=(a+b)\vec{S}_1\cdot\vec{S}_2+2b[(\vec{S}_1\cdot\vec{S}_2)^2-\tfrac{4}{3}]+c+\tfrac{2}{3}b\,, \tag{B8}$$

where I_{12} is understood in the constant terms. In Appendix E we also need the angular integral

$$\frac{1}{4\pi}\int d\Omega\,\vec{T}_1\cdot\vec{T}_2=\tfrac{4}{15}-\tfrac{2}{5}T_{12}\,. \tag{B9}$$

APPENDIX C

Here we calculate the current and charge densities $\vec{j}^{\,EE}$, ρ^{EE}, etc. The current operator in Eq. (7a) can be written as

$$\vec{j}=ig[\vec{A}\times\vec{B}-(\vec{A}\cdot\vec{\nabla})\vec{A}]\,. \tag{C1}$$

The antisymmetrical expectation value of j [cf., Eq. (8)] which is an operator in spin space, takes the form

$$\vec{j}_{\alpha\beta}=ig[\vec{A}_\alpha\times\vec{B}_\beta+\vec{B}_\alpha\times\vec{A}_\beta+\vec{\nabla}\times(\vec{A}_\alpha\times\vec{A}_\beta)]\,. \tag{C2}$$

(i) The TE-TE current:

$$\vec{A}^E_\alpha\times\vec{B}^E_\beta+\vec{B}^E_\alpha\times\vec{A}^E_\beta=-\frac{2N_E{}^2R}{x_E{}^2\rho}j_1^{E^2}[(\hat{r}\times\hat{e}_\alpha)\times\hat{r}\hat{r}_\beta-(\hat{r}\times\hat{e}_\beta)\times\hat{r}\hat{r}_\alpha]=-2i\frac{N_E{}^2R}{x_E{}^2\rho}j_1^{E^2}\hat{r}\times\vec{S}_{\alpha\beta}\,, \tag{C3}$$

where the last step follows from Eq. (B2) and we introduced the notation $j_1^E = j_1(x_E\rho)$, $j_1^M = j_1(x_M\rho)$, etc.,

$$\vec{\nabla}\times(\vec{A}_\alpha^E\cdot\vec{A}_\beta^E)=\frac{N_E^2R^2}{\rho x_E^2}j_1^{E^2}\nabla\times[(\hat{r}\times\hat{e}_\alpha)\times(\hat{r}\times\hat{e}_\beta)]=-i\frac{N_E^2R}{\rho x_E^2}j_1^{E^2}\hat{r}\times\vec{S}_{\alpha\beta}\ . \tag{C4}$$

So for the current $\vec{j}^{\,EE}$ we get (suppressing polarization indices)

$$\vec{j}^{\,EE}=3g\frac{N_E^2R}{\rho x_E^2}j_1^{E^2}\hat{r}\times\vec{S}\ . \tag{C5}$$

(ii) The TM-TM current:

$$\vec{A}_\alpha^M\times\vec{B}_\beta^M+\vec{B}_\alpha^M\times\vec{A}_\beta^M=-2i\frac{N_M^2R}{x_M^2}j_1^{M^2}\hat{r}\times\vec{S}_{\alpha\beta}\ , \tag{C6}$$

$$\vec{\nabla}\times\vec{A}_\alpha^M\times\vec{A}_\beta^M=\frac{N_M^2R^2}{x_M^2}\vec{\nabla}\times\{f(\rho)[\hat{r}(\hat{r}\cdot\vec{S}_{\alpha\beta})-\vec{S}_{\alpha\beta}]-ig(\rho)\vec{S}_{\alpha\beta}\}\ , \tag{C7}$$

where $f(\rho)$ and $g(\rho)$ are defined as

$$f(\rho)=\tfrac{1}{3}j_2^M(2j_0^M-j_2^M)\ , \tag{C8a}$$

$$g(\rho)=\tfrac{1}{9}(2j_0^M-j_2^M)^2\ . \tag{C8b}$$

The current $\vec{j}^{\,MM}$ is then given by

$$\vec{j}^{\,MM}=g\frac{N_M^2R}{x_M^2\rho}(4j_1^{M^2}-j_2^{M^2})\hat{r}\times\vec{S}\ . \tag{C9}$$

(iii) The TE-TM current:

$$\vec{A}_\alpha^E\times\vec{B}_\beta^M=i\frac{N_EN_M}{x_E}Rj_1^Mj_1^E(\hat{r}\times\vec{S}_{\alpha\beta})\hat{r} \tag{C10}$$

$$\vec{A}_\beta^M\times\vec{B}_\alpha^E=i\frac{N_EN_M}{3x_M}R\{a_1(\rho)[\hat{r}\times(\hat{r}\times\vec{S}_{\alpha\beta})]+a_2(\rho)\vec{T}_{\alpha\beta}+a_3(\rho)\vec{S}_{\alpha\beta}\}\ , \tag{C11}$$

where we used the definitions Eq. (B1), (B3), and (B6) and

$$a_1(\rho)=j_2^Ej_0^M+j_2^Mj_0^E-j_2^Ej_2^M\ , \tag{C12a}$$

$$a_2(\rho)=j_2^Ej_0^M-j_2^Mj_0^E\ , \tag{C12b}$$

$$a_3(\rho)=-\tfrac{1}{3}(4j_0^Ej_0^M-2j_0^Ej_2^M-2j_2^Ej_0^M+j_2^Ej_2^M)\ , \tag{C12c}$$

$$\vec{A}_\alpha^E\times\vec{A}_\beta^M=\frac{N_EN_M}{x_Ex_M}R^2j_1^E[j_2^M\hat{r}_\alpha\hat{r}_\beta\hat{r}+\tfrac{1}{3}(2j_0^M-j_2^M)\hat{r}\delta_{\alpha\beta}-\tfrac{2}{3}(j_0^M+j_2^M)\hat{r}_\beta\hat{e}_\alpha]\ , \tag{C13}$$

$$\vec{\nabla}\times(\vec{A}_\alpha^E\times\vec{B}_\beta^M)=i\frac{N_EN_M}{x_Ex_M}R\left[2\frac{j_1^Ej_1^M}{x_M\rho^2}\vec{S}+\frac{1}{\rho}\left[j_1^Ej_2^M+\frac{x_E}{x_M}j_2^Ej_1^M\right]\hat{r}\times(\hat{r}\times\vec{S})-\frac{1}{\rho}\frac{x_E}{x_M}j_2^Ej_1^M\vec{T}\right]_{\alpha\beta}\ . \tag{C14}$$

For the current j^{EM} we thus get

$$\vec{j}^{\,EM}=-g\frac{N_EN_M}{x_Ex_M}R\{f_2(\rho)[\hat{r}(\hat{r}\cdot\vec{S})-\tfrac{1}{3}\vec{S}]+\tilde{f}_2(\rho)\vec{T}+f_0(\rho)\vec{S}\} \tag{C15}$$

where

$$f_2(\rho)=x_Mj_1^Ej_1^M+x_Ej_2^Ej_2^M\ , \tag{C16a}$$

$$\tilde{f}_2(\rho)=\tfrac{1}{3}x_E(j_2^Mj_0^E-j_2^Ej_2^M-2j_2^Ej_0^M)\ , \tag{C16b}$$

$$f_0(\rho)=\tfrac{1}{3}(x_Mj_1^Ej_1^M+2x_Ej_0^Ej_0^M-x_Ej_2^Ej_2^M)\ . \tag{C16c}$$

(iv) The TE-TE charge:

$$\rho_{\alpha\beta}^{EE}=2g\frac{x_E}{R}\vec{A}_\alpha^E\cdot\vec{A}_\beta^E=-2g\frac{N_E{}^2R}{x_E}j_1^{E^2}(U_{\alpha\beta}-\tfrac{2}{3}\delta_{\alpha\beta}) , \quad \text{(C17)}$$

where $U_{\alpha\beta}$ is defined in Eq. (B3).

(v) The TM-TM charge:

$$\rho_{\alpha\beta}^{MM}=2g\frac{x_M}{R}\vec{A}_\alpha^M\cdot\vec{A}_\beta^M=2g\frac{N_M{}^2}{x_M}\frac{R}{3}[j_2^M(j_2^M+4j_0^M)U_{\alpha\beta}+\tfrac{2}{3}(j_2^{M^2}+2j_0^{M^2})\delta_{\alpha\beta}] . \quad \text{(C18)}$$

(vi) The TE-TM charge:

$$\rho_{\alpha\beta}^{EM}=g\frac{x_E+x_M}{R}\vec{A}_\alpha^E\cdot\vec{A}_\beta^M=-igN_EN_MR\frac{x_E+x_M}{3x_Ex_M}j_1^E(2j_0^M-j_2^M)\hat{r}\cdot\vec{S}_{\alpha\beta} . \quad \text{(C19)}$$

APPENDIX D

In this section we calculate the potentials $\vec{a}$ and Φ.

1. The $\vec{a}$ potentials

Generally one has

$$\vec{a}(\vec{x})=\int_{\text{bag}}d^3y\,D(\vec{x},\vec{y};\omega)\,\vec{j}(\vec{y}) . \quad \text{(D1)}$$

We use the free Green's functions and impose the boundary conditions later. The currents j^{EE} and j^{MM} have no time dependence and hence the appropriate expression for $D(\vec{x},\vec{y};\omega)$ in Eq. (D1) is

$$D(\vec{x},\vec{y};\omega)=\sum_{l,m}\frac{1}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y_{l,m}(\Omega)Y_{l,m}^*(\Omega') , \quad \text{(D2)}$$

whereas in the TE-TM case one has

$$D(\vec{x},\vec{y};\omega)=-\omega\sum_{l,m}j_l(x\rho_<)n_l(x\rho_>)\times Y_{l,m}(\Omega)Y_{l,m}^*(\Omega') , \quad \text{(D3)}$$

where $\omega R=x=x_M-x_E$.

(i) The TE-TE case. From Eqs. (C5), (D1), and (D2) we get

$$\vec{a}^{EE}=g\frac{N_E{}^2R^3}{x_E{}^2}\left[\frac{1}{\rho^2}J_1^E(\rho)+\rho N_1^E(\rho)\right]\hat{r}\times\vec{S} , \quad \text{(D4)}$$

where

$$J_1^E(\rho)=\int_0^\rho d\xi\,\xi^2j_1{}^2(x_E\xi) , \quad \text{(D5a)}$$

$$N_1^E(\rho)=a_1^E+\int_\rho^1 d\xi\frac{1}{\xi}j_1{}^2(x_E\xi) . \quad \text{(D5b)}$$

The constant a_1^E is determined by the boundary condition Eq. (12) and found to be

$$a_1^E=\tfrac{1}{2}J_1^E(1) . \quad \text{(D6)}$$

(ii) The TM-TM case. From Eqs. (C9), (D1), and (D2) we get

$$\vec{a}^{MM}=g\frac{N_M{}^2R^3}{3x_M{}^2}\left[\frac{1}{\rho^2}J_1^M(\rho)+\rho N_1^M(\rho)\right]\hat{r}\times\vec{S} , \quad \text{(D7)}$$

where

$$J_1^M(\rho)=\int_0^\rho d\xi\,\xi^2[4j_1{}^2(x_M\xi)-j_2{}^2(x_M\xi)] , \quad \text{(D8a)}$$

$$N_1^M(\rho)=a_1^M+\int_\rho^1 d\xi\frac{1}{\xi}[4j_1{}^2(x_M\xi)-j_2{}^2(x_M\xi)] . \quad \text{(D8b)}$$

The boundary condition [Eq. (12)] gives

$$a_1^M=\tfrac{1}{2}J_1^M(1) . \quad \text{(D9)}$$

(iii) The TE-TM case. Here we face the complication that $\vec{j}^{EM}$ is not transverse ($\vec{\nabla}\cdot\vec{j}^{EM}\neq 0$). Care must be taken because in the Coulomb gauge the vector potential $\vec{a}_T^{EM}$ satisfies the equation

$$-(\nabla^2+\omega^2)\vec{a}_T^{EM}=\vec{j}_T^{EM}=\vec{j}^{EM}-\vec{\nabla}\frac{1}{\nabla^2}\vec{\nabla}\cdot\vec{j}^{EM}=\vec{j}^{EM}-\vec{j}_L^{EM} . \quad \text{(D10)}$$

Note that

$$\vec{j}_L^{EM}=i\omega\vec{\nabla}\phi^{EM} , \quad \text{(D11a)}$$

where $\omega R=x_M-x_E$ and ϕ^{EM} is given in Eq. (D22) below. Also

$$\hat{r}\cdot\vec{j}_L^{EM}=0 \quad \text{(D11b)}$$

on the bag surface.

Rather than calculating a_T^{EM} directly we first compute $\vec{a}^{EM}$ defined by

$$-(\nabla^2+\omega^2)\vec{a}^{EM}=\vec{j}^{EM} \quad \text{(D12a)}$$

and then obtain $\vec{a}_T^{EM}$ from

$$\vec{a}_T^{EM}=\vec{a}^{EM}-\vec{\nabla}\frac{1}{\nabla^2}\vec{\nabla}\cdot\vec{a}^{EM}$$

$$=\vec{a}^{EM}+\vec{\nabla}\frac{1}{\omega^2}\vec{\nabla}\cdot\vec{a}^{EM}+\frac{i}{\omega}\vec{\nabla}\phi^{EM} . \quad \text{(D12b)}$$

(This procedure is equivalent to first calculating the vector potential in Lorentz gauge and then returning to Coulomb gauge using a gauge transformation.)

We get

$$\vec{a}_T^{EM}=g\frac{xN_EN_M}{x_Ex_M}R^3\Bigg[[j_2(x\rho)\tilde{N}_2^{EM}(\rho)+n_2(x\rho)\tilde{J}_2^{EM}(\rho)]\vec{T} + \left[j_2(x\rho)\hat{N}^{EM}(\rho)+n_2(x\rho)\hat{J}^{EM}(\rho)+\frac{1}{x^3}[\tfrac{2}{3}f_2(\rho)+f_0(\rho)]\right][(\hat{r}\cdot\vec{S})\hat{r}-\tfrac{1}{3}\vec{S}] + \frac{2}{3}\left[j_0(x\rho)\hat{N}^{EM}(\rho)+n_0(x\rho)\hat{J}^{EM}(\rho)+\frac{1}{2x^3}[\tfrac{2}{3}f_2(\rho)+f_0(\rho)]\right]\vec{S}+\frac{i}{\omega}\vec{\nabla}\phi^{EM}\Bigg] , \quad \text{(D13a)}$$

where

$$\tilde{J}_2^{EM}(\rho)=\int_0^\rho d\xi\,\xi^2 j_2(x\xi)\tilde{f}_2(\xi) , \quad \text{(D13b)}$$

$$\tilde{N}_2^{EM}(\rho)=\tilde{a}_2^{EM}+\int_\rho^1 d\xi\,\xi^2 n_2(x\xi)\tilde{f}_2(\xi) , \quad \text{(D13c)}$$

$$\hat{J}^{EM}(\rho)=\int_0^\rho d\xi\,\xi^2[\tfrac{1}{3}j_2(x\xi)f_2(\xi)+j_0(x\xi)f_0(\xi)] , \quad \text{(D13d)}$$

$$\hat{N}^{EM}(\rho)=\hat{a}^{EM}+\int_\rho^1 d\xi\,\xi^2[\tfrac{1}{3}n_2(x\xi)f_2(\xi)+n_0(x\xi)f_0(\xi)] . \quad \text{(D13e)}$$

The constants $\tilde{a}_2^{EM}$ and $\hat{a}^{EM}$ are determined by Eq. (12) and given by

$$\tilde{a}_2^{EM}=-\frac{n_2(x)+xn_2'(x)}{j_2(x)+xj_2'(x)}\tilde{J}_2^{EM}(1) , \quad \text{(D13f)}$$

$$\hat{a}^{EM}=-\frac{n_1(x)}{j_1(x)}\hat{J}^{EM}(1) . \quad \text{(D13g)}$$

2. The ϕ potentials

Generally one has

$$\phi(\vec{x})=\int_{\text{bag}} d^3y\, D_{\text{Coul}}(\vec{x},\vec{y})\rho(\vec{y}) . \quad \text{(D14)}$$

(i) The TE-TE case. Here both the $l=2$ and $l=0$ waves in Eq. (D14) contribute and we get

$$\phi^{EE}=2g\frac{N_E{}^2R^3}{x_E}\left\{\frac{1}{5}\left[\frac{1}{\rho^3}G_2^E(\rho)+\rho^2H_2^E(\rho)\right]U - \frac{2}{3}\left[\frac{1}{\rho}G_0^E(\rho)+H_0^E(\rho)\right]I\right\} , \quad \text{(D15)}$$

where

$$G_2^E(\rho)=\int_0^\rho d\xi\,\xi^4 j_1{}^2(x_E\xi) , \quad \text{(D16a)}$$

$$H_2^E(\rho)=s_2^E+\int_\rho^1 d\xi\,\frac{1}{\xi}j_1{}^2(x_E\xi) , \quad \text{(D16b)}$$

$$G_0^E(\rho)=\int_0^\rho d\xi\,\xi^2 j_1{}^2(x_E\xi) , \quad \text{(D16c)}$$

$$H_0^E(\rho)=s_0^E+\int_\rho^1 d\xi\,\xi j_1{}^2(x_E\xi) . \quad \text{(D16d)}$$

Following the same procedure as above the constants s_0^E and s_2^E should be determined by the boundary condition [Eq. (12)]:

$$\hat{r}\cdot\vec{\nabla}\phi=0 \ \text{ for } \rho=1 . \quad \text{(D17)}$$

This equation gives for the "tensor" contribution to ϕ

$$s_2^E=\tfrac{3}{2}G_2^E(1) . \quad \text{(D18)}$$

For the unit tensor term in ϕ, the condition (D17) is, however, identically fulfilled, which means that the constant s_0^E remains undetermined. This is related to a residual gauge freedom as discussed in the main text.

(ii) The TM-TM case. Using Eqs. (C9), (D1), and (D2) we get

$$\phi^{MM}=2g\frac{N_M{}^2}{x_M}\frac{R^3}{3}\left\{\frac{1}{5}\left[\frac{1}{\rho^3}G_2^M(\rho)+\rho^2H_2^M(\rho)\right]U + \left[\frac{1}{\rho}G_0^M(\rho)+H_0^M(\rho)\right]I\right\} , \quad \text{(D19)}$$

where

$$G_2^M(\rho)=\int_0^1 d\xi\,\xi^4 g(x_M\xi) , \quad \text{(D20a)}$$

$$H_2^M(\rho)=s_2^M+\int_\rho^1 d\xi\,\frac{1}{\xi}g(x_M\xi) , \quad \text{(D20b)}$$

$$G_0^M(\rho)=\int_0^1 d\xi\,\xi^2 h(x_M\xi) , \quad \text{(D20c)}$$

$$H_0^M(\rho)=s_0^M+\int_\rho^1 d\xi\,\xi h(x_M\xi)\,, \tag{D20d}$$

and

$$g(x_M\xi)=j_2(x_M\xi)[j_2(x_M\xi)+4j_0(x_M\xi)]\,, \tag{D20e}$$

$$h(x_M\xi)=\tfrac{2}{3}[j_2{}^2(x_M\xi)+2j_0{}^2(x_M\xi)]\,. \tag{D20f}$$

Again only s_2^M can be determined by the boundary conditions [see Eq. (D17)]. One finds

$$s_2^M=\tfrac{3}{2}G_2^M(1)\,. \tag{D21}$$

(iii) The TE-TM case. From Eqs. (C19), (D2), and (D14) we get

$$\phi^{EM}=-i\frac{g}{3}N_EN_M\tfrac{1}{3}R^3\frac{x_E+x_M}{x_Ex_M}\left[\frac{1}{\rho^2}G_1^{EM}(\rho)+\rho H_1^{EM}(\rho)\right]\hat{r}\cdot\vec{S}\,, \tag{D22}$$

where

$$G_1^{EM}(\rho)=\int_0^\rho d\xi\,\xi^3 j_1(x_E\xi)[2j_0(x_M\xi)-j_2(x_M\xi)]\,, \tag{D23a}$$

$$H_1^{EM}(\rho)=s_1^{EM}+\int_\rho^1 d\xi\,j_1(x_E\xi)[2j_0(x_M\xi)-j_2(x_M\xi)]\,. \tag{D23b}$$

s_1^{EM} is determined by Eq. (12) to be

$$s_1^{EM}=2G_1^{EM}(1)\,. \tag{D24}$$

APPENDIX E

Here we calculate the quantities a, b, and $\tilde{c}$ occurring in Eqs. (14)–(16). In general, they get contributions from three sources: the three-gluon, four-gluon, and Coulomb terms, e.g.,

$$a=a^{3g}+a^{4g}+a^{\text{Coul}}\,. \tag{E1}$$

1. TE-TE

From Eqs. (13), (C5), and (D4) one obtains

$$H_{EE}^{3g}=-\Lambda_1^a\Lambda_2^a\int_{\text{bag}}d^3x\,\vec{j}^{\,EE}\cdot\vec{a}^{\,EE}=-\Lambda_1\Lambda_2\frac{\alpha_s}{R}a_{EE}^{3g}\vec{S}_1\cdot\vec{S}_2\,, \tag{E2}$$

where

$$a_{EE}^{3g}=\tfrac{9}{2}\gamma_E\int_0^1 d\rho\,j_1{}^2(x_E\rho)\left[\frac{1}{\rho}J_1^E(\rho)+\rho^2N_1^E(\rho)\right]\approx 0.341 \tag{E3a}$$

with

$$\gamma_E=\left[\frac{8\pi N_E{}^2R^4}{3x_E{}^2}\right]^2. \tag{E3b}$$

The Bose factor $S_B=\frac{1}{2}$ was canceled in Eq. (E2) because of the two identical terms in Eq. (9). The four-gluon contribution H_{EE}^{4g} is given by

$$H_{EE}^{4g}=-\Lambda_1^a\Lambda_2^a2\int_{\text{bag}}d^3x\,\vec{I}_1^{\,E}(\vec{x})\cdot\vec{I}_2^{\,E}(\vec{x})=-\Lambda_1\Lambda_2\frac{\alpha_s}{R}a_{EE}^{4g}\vec{S}_1\cdot\vec{S}_2\,, \tag{E4}$$

where

$$\vec{I}_1(x)=2\vec{A}_\alpha^E\times\vec{A}_\beta^E \tag{E5}$$

and

$$a_{EE}^{4g}=-\tfrac{3}{4}\gamma_E\int_0^1 d\rho\,\rho^2 j_1{}^4(x_E\rho)\approx-0.078\,. \tag{E6}$$

For the Coulomb part one gets

$$H_{EE}^{\text{Coul}}(\text{int})=\Lambda_1^a\Lambda_2^a\int d^3x\,\rho^{EE}\phi^{EE}=-\Lambda_1\Lambda_2\frac{\alpha_s}{R}(b_{EE}^{\text{Coul}}T_{12}+\tilde{c}_{EE}^{\text{Coul}}T_{12})\,. \tag{E7}$$

We may write (D15) as

$$\phi^{EE}=\tfrac{4}{3}g\frac{N_E{}^2R^3}{x_E}s_0^E+\bar{\phi}^{EE}\equiv-\frac{g}{4\pi R}S_0^E+\bar{\phi}^{EE} \tag{E8}$$

and then get

$$b_{EE}^{\text{Coul}}=-\tfrac{3}{25}x_E{}^2\gamma_E\int_0^1 d\rho\,j_1{}^2(x_E\rho)\left[\frac{1}{\rho}G_2^E(\rho)+\rho^4H_2^E(\rho)\right]=-0.041 \tag{E9}$$

and

$$\tilde{c}_{EE}^{\text{Coul}}=S_0^E-\tfrac{2}{3}b_{EE}^{\text{Coul}}-4x_E{}^2\gamma_E\int_0^1 d\rho\,j_1{}^2(x_E\rho)[\rho G_0^E(\rho)+\rho^2\bar{H}_0^E(\rho)]=S_0^E-1.155\,, \tag{E10}$$

where $\bar{H}_0^E$ is H_0^E without the s_0^E term. In getting the last equation we have used

$$\int d^3x\,\rho^{EE}(x)=\int d^3x\,\rho^{MM}(x)=g\ . \tag{E11}$$

The constant S_0^E is not determined, but it gives no contributions to the energy shifts of color-singlet glueball states. In fact, if we restrict our attention to color-singlet (TE)^n states, then $c_{EE}^{\rm Coul}$ itself gives no contributions.

We can conveniently here calculate the static Coulomb self-energy diagrammed in Fig. 2(e). This is most easily done by appropriately contracting the indices on T_{12} and I_{12} in Eq. (E7), letting $\Lambda_1^a\Lambda_2^a\to\Lambda_i{}^2=3$, where i stands for one of the gluons, and remembering a factor of $\frac{1}{2}$. Then for *each* gluon, we get

$$H_{EE}^{\rm Coul}({\rm self})=-\tfrac{1}{2}\Lambda_i{}^2\frac{\alpha_s}{R}(4b_{EE}^{\rm Coul}+\tilde{c}_{EE}^{\rm Coul})I_{12}\ . \tag{12}$$

The sum of the Coulomb interaction energy and static Coulomb self-energy is then

$$H_{EE}^{\rm Coul}=-3\frac{\alpha_s}{R}b_{EE}^{\rm Coul}(4I_{12}-T_{12}) \tag{E13}$$

for (TE)^2 states (note that this is zero for the 0^{++}) and

$$H_{EE}^{\rm Coul}=-\frac{\alpha_s}{R}b_{EE}^{\rm Coul}\left[6nI_{12}+\sum_{i<j}\Lambda_i^a\Lambda_j^aT_{ij}\right] \tag{E14}$$

for (TE)^n states.

2. TM-TM

As above, one gets

$$H_{MM}^{3g}=-\Lambda_1^a\Lambda_2^a\int_{\rm bag}d^3x\ \vec{j}^{\,MM}\cdot\vec{a}^{\,MM}=-\Lambda_1^a\Lambda_2^a\frac{\alpha_s}{R}a_{MM}^{3g}\vec{S}_1\cdot\vec{S}_2\ , \tag{E15}$$

where

$$a_{MM}^{3g}=\tfrac{1}{2}\gamma_M\int_0^1 d\rho[4j_1{}^2(x_M\rho)-j_2{}^2(x_M\rho)]\left[\frac{1}{\rho}J_1^M(\rho)+\rho^2N_1^M(\rho)\right]\approx 0.328\ , \tag{E16}$$

with

$$\gamma_M=\left[\frac{8\pi N_M{}^2R^4}{3x_M{}^2}\right]^2; \tag{E17}$$

also

$$H_{MM}^{4g}=-\Lambda_1^a\Lambda_2^a2\int_{\rm bag}d^3x\ \vec{1}_1^{\,MM}\cdot\vec{1}_2^{\,MM}=-\Lambda_1^a\Lambda_2^a\frac{\alpha_s}{R}a_{MM}^{4g}\vec{S}_1\cdot\vec{S}_2\ , \tag{E18}$$

with

$$\vec{1}_1^{\,MM}(x)=2A_\alpha^M\times A_\beta^M\ . \tag{E19}$$

One obtains

$$a_{MM}^{4g}=-\frac{1}{36}\gamma_M\int_0^1 d\rho\,\rho^2[2j_0(x_M\rho)-j_2(x_M\rho)]^2[4j_0{}^2(x_M\rho)+4j_0(x_M\rho)j_2(x_M\rho)+3j_2{}^2(x_M\rho)]\approx -0.081\ , \tag{E20}$$

For the Coulomb part one obtains

$$H_{MM}^{\rm Coul}({\rm int})=\Lambda_1^a\Lambda_2^a\int d^3x\,\rho^{MM}\phi^{MM}=-\Lambda_1^a\Lambda_2^a\frac{\alpha_s}{R}(b_{MM}^{\rm Coul}T_{12}+\tilde{c}_{MM}^{\rm Coul}I_{12})\ . \tag{E21}$$

As for the (TE)^2 case we may write

$$\phi^{MM}=-\frac{g}{4\pi R}S_0^M+\bar{\phi}^{MM}\ , \tag{E22}$$

where $\bar{\phi}^{MM}$ is ϕ^{MM} without S_0^M (and $\bar{H}_0^M$ will be similarly related to H_0^M) and proceed to get

$$b_{MM}^{\rm Coul}=-\tfrac{1}{75}\gamma_Mx_M{}^2\int_0^1 d\rho\,\rho^2j_2(x_M\rho)[j_2(x_M\rho)+4j_0(x_M\rho)]\left[\frac{1}{\rho^3}G_2^M(\rho)+\rho^2H_2^M(\rho)\right]\simeq -0.007\ , \tag{E23}$$

$$\tilde{c}_{MM}^{\rm Coul}=S_0^M-\tfrac{2}{3}b_{MM}^{\rm Coul}-\tfrac{2}{3}x_M{}^2\gamma_M\int_0^1 d\rho[j_2{}^2(x_M\rho)+2j_0{}^2(x_M\rho)][\rho G_0^M(\rho)+\rho^2\bar{H}_0^M(\rho)]=S_0^M-1.533\ . \tag{E24}$$

Again, the constant S_0^M is not determined, but does not contribute to energy shifts of color-singlet states.

The expressions for the static Coulomb self-energy and the sum of static Coulomb self-energy plus Coulomb interaction energy are

$$H_{MM}^{\rm Coul}({\rm self}) = -\tfrac{1}{2}\Lambda_i{}^2 \frac{\alpha_s}{R}(4b_{MM}^{\rm Coul}+\tilde{c}\,_{MM}^{\rm Coul})I_{12} \quad \text{(E25)}$$

for each gluon,

$$H_{MM}^{\rm Coul} = -3\frac{\alpha_s}{R}b_{MM}^{\rm Coul}(4I_{12}-T_{12}) \quad \text{(E26)}$$

for $(\rm TM)^2$, and

$$H_{MM}^{\rm Coul} = -\frac{\alpha_s}{R}b_{MM}^{\rm Coul}\left[6nI_{12}+\sum_{i<j}\Lambda_i^a\Lambda_j^a T_{ij}\right] \quad \text{(E27)}$$

for $(\rm TM)^n$.

3. TE-TM

From Eqs. (13), (C15), and (D10) one obtains

$$H_{EM}^{3g} = -\Lambda_1^a\Lambda_2^a \int d^3x(\vec{j}^{\,MM}\cdot\vec{a}^{\,EE}+\vec{j}_T^{\,EM}\cdot\vec{a}_T^{\,ME}) \quad \text{(E28a)}$$

$$\equiv \Lambda_1^a\Lambda_2^a\frac{\alpha_s}{R}(a_{EM}^{3g}\vec{S}_1\cdot\vec{S}_2+b_{EM}^{3g}T_{12}+\tilde{c}\,_{EM}^{3g}I_{12}) \;. \quad \text{(E28b)}$$

The labeling implied for the color and spin indices for the two terms is shown in Fig. 3. Notice that

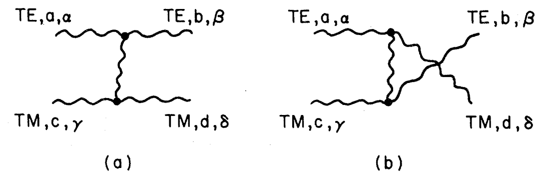

FIG. 3. Labeling for (TE)(TM) pairs. Latin characters are for color and Greek characters are for spin.

$$\int d^3x\, \vec{j}_T^{\,EM}\cdot\vec{a}_T^{\,ME} = \int d^3x\, \vec{j}^{\,EM}\cdot\vec{a}_T^{\,ME} = \int d^3x\, \vec{j}^{\,EM}\cdot\left[\vec{a}^{\,ME}+\vec{\nabla}\frac{1}{\omega^2}\vec{\nabla}\cdot\vec{a}^{\,ME}\right] - \int d^3x\, \rho^{EM}\phi^{ME} \quad \text{(E29)}$$

and that

$$\vec{a}_{\gamma\beta}^{\,ME} = -\vec{a}_{\beta\gamma}^{\,EM} \;. \quad \text{(E30)}$$

We recognize the last term in Eq. (E9) as the Coulomb (or longitudinal electric) energy. In this case it would be simpler to directly calculate the sum of magnetic and electric energies. For consistency we quote the results separately as in the other cases. Also, in this case the $3g$ energy shift has a transverse electric contribution in contrast to the $(\rm TE)^2$ and $(\rm TM)^2$ cases.

For the two-gluon (TE)(TM) glueball, the color wave function is of course symmetric. However, to allow consideration of (TE)(TM) pairs within multigluon states, we will also quote results for color antisymmetric wave functions. We then need

$$A = \gamma_{EM}\int_0^1 d\rho\,\rho^2\Big\{\tfrac{1}{2}f_2(\rho)[j_2(x\rho)\hat{N}^{EM}(\rho)+n_2(x\rho)\hat{J}^{EM}(\rho)]+\frac{1}{x^3}[\tfrac{2}{3}f_2(\rho)+f_0(\rho)] + \tfrac{3}{2}f_0(\rho)[j_0(x)\hat{N}^{EM}(\rho)+n_0(x\rho)\hat{J}^{EM}(\rho)]+\frac{1}{2x^3}[\tfrac{2}{3}f_2(\rho)+f_0(\rho)]\Big\} \;, \quad \text{(E31a)}$$

$$B = \tfrac{9}{10}x\gamma_{EM}\int_0^1 d\rho\,\rho^2\tilde{f}_2(\rho)[j_2(x\rho)\tilde{N}_2^{EM}(\rho)+n_2(x\rho)\tilde{J}_2^{EM}(\rho)] \;. \quad \text{(E31b)}$$

Then,

$$a_{EM}^{3g}(\pm) = \tfrac{3}{2}\gamma_{EM}\int_0^1 d\rho[4j_1{}^2(x_M\rho)-j_2{}^2(x_M\rho)]\left[\frac{1}{\rho}J_1^E(\rho)+\rho^2N_1^E(\rho)\right]\pm\tfrac{1}{2}(A-\tfrac{5}{3}B) = 0.2864\pm 0.0755 \;, \quad \text{(E32a)}$$

$$b_{EM}^{3g}(\pm) = \mp\tfrac{1}{2}(A-\tfrac{1}{3}B) = \mp 0.077 \;, \quad \text{(E32b)}$$

$$\tilde{c}\,_{EM}^{3g}(\pm) = \pm(A+B) = \pm 0.158 \;. \quad \text{(E32c)}$$

The $(\pm)$ refers to the color symmetry of the (TE)(TM) pair, and

$$\gamma_{EM} = \left[\frac{8\pi N_E N_M R^4}{3x_E x_M}\right]^2 . \quad \text{(E33)}$$

For the $4g$ interaction one has

$$H_{EM}^{4g}=-\Lambda_1^a\Lambda_2^a\frac{\alpha_s}{R}\int_{\rm bag}d^3x[\vec{l}_1^{EM}(x)\vec{l}_2^{EM}(x)+\vec{l}_1^{E}(x)\vec{l}_2^{M}(x)]=-\Lambda_1^a\Lambda_2^a\frac{\alpha_s}{R}(a_{EM}^{4g}\vec{S}_1\cdot\vec{S}_2+b_{EM}^{4g}T_{12}+c_{EM}^{4g}I_{12})\,, \tag{E34}$$

where

$$l_1^{EM}(x)=A_\alpha^E\times A_\beta^M-A_\alpha^M\times A_\beta^E\,. \tag{E35}$$

Then we find

$$a_{EM}^{4g}(\pm)=\frac{A}{24}(2\pm1)=-0.0114(2\pm1)\,, \tag{E36a}$$

$$b_{EM}^{4g}(\pm)=\pm(\tfrac{1}{24}A-\tfrac{1}{20}B)=\mp0.011\,, \tag{E36b}$$

$$\tilde{c}_{EM}^{4g}(\pm)=\pm(\tfrac{1}{12}A+\tfrac{1}{5}B)=\mp0.026\,, \tag{E36c}$$

where now

$$A=-\gamma_{EM}\int_0^1 d\rho\,\rho^2[j_1(x_E\rho)]^2[2j_0(x_M\rho)-j_2(x_M\rho)]^2\,, \tag{E37a}$$

$$B=-\gamma_{EM}\int_0^1 d\rho\,\rho^2[j_1(x_E\rho)]^2 j_2(x_M\rho)[4j_0(x_M\rho)+j_2(x_M\rho)]\,. \tag{E37b}$$

For the Coulomb part we get

$$H_{EM}^{\rm Coul}=\Lambda_1^a\Lambda_2^a\int d^3x(\tfrac{1}{2}\rho^{MM}\phi^{EE}+\tfrac{1}{2}\rho^{EE}\phi^{MM}+\rho^{EM}\phi^{ME})=-\Lambda_1^a\Lambda_2^a\frac{\alpha_s}{R}(a_{EM}^{\rm Coul}\vec{S}_1\cdot\vec{S}_2+b_{EM}^{\rm Coul}T_{12}+c_{EM}^{\rm Coul}I_{12})\,. \tag{E38}$$

Note that the first two terms of the integral above would be identical if the gluons individually satisfied the $\hat{n}\cdot\vec{E}=0$ boundary condition. The coefficients are

$$a_{EM}^{\rm Coul}(\pm)=\mp\tfrac{1}{72}\gamma_{EM}(x_E+x_M)^2\int_0^1 d\rho\, j_1(x_E\rho)[2j_0(x_M\rho)-j_2(x_M\rho)][G_1^{EM}(\rho)+\rho^3H_1^{EM}(\rho)]=\mp0.073\,, \tag{E39a}$$

$$\begin{aligned}b_{EM}^{\rm Coul}(\pm)&=\tfrac{1}{25}\gamma_{EM}x_Ex_M\int_0^1 d\rho\, j_2(x_M\rho)[4j_0(x_M\rho)+j_2(x_M\rho)]\left[\frac{1}{\rho}G_2^E(\rho)+\rho^4H_2^E(\rho)\right]-a_{EM}^{\rm Coul}(\pm)\\&=-0.002\pm0.073\,,\end{aligned} \tag{E39b}$$

$$\begin{aligned}\tilde{c}_{EM}^{\rm Coul}(\pm)=&\tfrac{1}{2}(S_0^E+S_0^M)-\tfrac{2}{3}b_{EM}^{\rm Coul}(\pm)+\tfrac{5}{3}a_{EM}^{\rm Coul}(\pm)\\&-\gamma_{EM}x_Ex_M\int_0^1 d\rho\{j_1{}^2(x_E\rho)[\rho G_0^M(\rho)+\rho^2\overline{H}_0^M(\rho)]\\&\qquad+\tfrac{2}{3}[j_2{}^2(x_M\rho)+2j_0{}^2(x_M\rho)][\rho G_0^E(\rho)+\rho^2H_0^E(\rho)]\}\\=&\tfrac{1}{2}(S_0^E+S_0^M)-1.2515\pm0.1457\,.\end{aligned} \tag{E39c}$$

Again the undetermined constants S_0^E and S_0^M are canceled when the energy of a color-singlet state is calculated.

These coefficients are summarized in Table II.

APPENDIX F

In this appendix we calculate the contribution to H_{mn} due to s-channel annihilation graphs, Figs. (2a) and (2b). These contributions can be obtained by crossing from the t- and u-channel graphs previously calculated. We note several items before quoting our results.

(a) The most naturally occurring spin-spin operator is now (see the labeling in Fig. 3)

$$\vec{S}_{\alpha\gamma}\cdot\vec{S}_{\beta\delta}=-2P_1^{\rm sp}$$

where $P_1^{\rm sp}$ is the projection operator on spin-1 states. Projection operators for spin-0 and spin-2 states may also appear.

(b) Similarly for color, use

$$\Lambda_{ac}^i\Lambda_{bd}^i=-3P_{8A}^{\rm col}\,,$$

where $P_{8A}^{\rm col}$ is the projection operator on antisymmetric color states.

(c) The gluon-exchange graphs [Fig. (1a) and (c)] are the sum of t- and u-channel contributions. To get the s-channel results crossing we must use only the t-channel (or only the u-channel) piece. For the $(\rm TE)^2$ or $(\rm TM)^2$ cases, the t- and u-channel contributions are equal so we can simply divide by 2.

(d) For the Coulomb diagram, the charge densities vanish identically for the $(\rm TE)^2$ and $(\rm TM)^2$ cases; the results are, for $(\rm TE)^2$ and $(\rm TM)^2$,

$$H_{mn}(\text{ann}) = -\frac{\alpha_s}{R} P_{8A}^{\rm col} P_1^{\rm sp} d_{mn} ,$$

$$d_{EE} = d_{EE}^{3g} + 3a_{EE}^{3g} = -0.296 - 0.234 = -0.529$$

$$d_{MM} = d_{MM}^{3g} + 3a_{MM}^{3g} = -0.312 - 0.243 = -0.555 ,$$

where we have included a term coming from the four-gluon interaction in Fig. (1b) although it is not formally an annihilation contribution. Note that diagram (2a) gives a positive energy shift as expected from mixing with a lower-lying, in this case dominantly lowest, one-gluon, state.

For (TE)(TM) we get

$$H_{EM}(\text{ann}) = -\frac{\alpha_s}{R} P_{8A}^{\rm col}(d_{EM} P_1^{\rm sp} + d_{EM}^{(0)} P_0^{\rm sp} + d_{EM}^{(2)} P_2^{\rm sp}) ,$$

$$d_{EM} = d_{EM}^{3g} + d_{EM}^{4g} + d_{EM}^{\rm Coul} = 0.223 - 0.079 - 0.051 = -0.354 ,$$

$$d_{EM}^{(0)} = d_{EM}^{(0)4g} = 0.275 ,$$

$$d_{EM}^{(2)} = d_{EM}^{(2)4g} = 0.075 .$$

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[1]For a review with references to the literature see, e.g., P. Fishbane, in *Gauge Theories, Massive Neutrinos, and Proton Decays,* proceedings of Orbis Scientiae, Coral Gables, 1981, edited by A. Perlmutter (Plenum, New York, 1981).

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[4]There have been several attempts (see Ref. 1) to construct nonrelativistic potential models for glueballs. Such models, however, require a somewhat arbitrary introduction of an effective "constituent gluon mass." This breaks gauge invariance and special assumptions must be made about the unphysical longitudinal gluon modes.

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[12]This form is appropriate for calculating the diagonal elements of the glueball mass matrix. For the problem of mixing, methods similar to those described by C. Carlsson and T. H. Hansson [Nucl. Phys. B199, 441 (1982)] must be used.

[13]Using this form of the current we obtain

$$\vec{\mu}^a = \frac{1}{2}\int d^3r\, \vec{r} \times \vec{j}^a = \frac{g}{2\omega}\Lambda^a(\vec{L} + 2\vec{S})$$

with obvious notation. This is the result expected for a particle with gyromagnetic ratio $g = 2$.

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