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ψ and Υ systems in a consistent quark model

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The η_b, η_b', P, and D states of the Υ system as well as the missing ${}^1P_1$, ${}^3D_2$, and ${}^3D_3$ states of the ψ system are calculated based on a parameter system obtained from the known masses of states of the charmonium system. The results are consistent with flavor independence of the potential; however, the compositions of the states are found to differ in the ψ and Υ systems.

I. INTRODUCTION

The general framework for models based on quantum chromodynamics (QCD) has been described by De Rújula, Georgi, and Glashow.[1] This consists of short-range q-q (or q-$\bar{q}$) forces dominated by one-gluon exchange and at large distances a scalar confining potential. Within this framework Isgur and Karl[2–4] introduced a model based on a harmonic-oscillator confining potential to describe the spectra and decay couplings of the ground-state and low-lying excited baryon states; here each set of levels of alternating parity has been analyzed with a different parameter set. The effects of a Coulomb-type force derived from QCD and of deviations from the harmonic-oscillator form at large distances are incorporated in an interaction term $U(r_{ij})$. Kalman and Hall[5] and Kalman[6] have discussed modifications of this latter term so that a uniform parameter set can be developed for all baryon calculations. Such a consistent quark model was used by Kalman, Hall, and Misra[7] and by Kalman and Misra[8] to examine baryoniums. A more interesting application is to mesons. Recently Kalman and Mukerji[9] successfully applied the basic Isgur-Karl model to a study of the low-lying S states of charmonium. It is the purpose of this paper to present a full calculation of the masses of all of the low-lying states of the ψ and Υ systems.

II. CONSISTENT MODEL

The model employs a Hamiltonian of the form

$$H = 2m_Q - (H_0 + H_{\text{hyp}} + H_{\text{SO}}) \sum_\alpha \Lambda_1^\alpha \Lambda_2^{*\alpha} , \quad (2.1)$$

where m_Q is the mass of the c quark for the ψ system and of the b quark for the Υ system; also

$$H_0 = \sum_i P_i^2/2m_Q + V - \left[\sum_i P_i \right]^2 \Big/ 4m_Q , \quad (2.2a)$$

$$H_{\text{hyp}} = \frac{4_{\alpha_s}}{3m_Q^2} \left\{ \frac{8\pi}{3} (\vec{S}_1 \cdot \vec{S}_2) \delta^{(3)}(\vec{r}) + \frac{1}{r^3} \left[\frac{3(\vec{S}_1 \cdot \vec{r})(\vec{S}_2 \cdot \vec{r})}{r^2} - \vec{S}_1 \cdot \vec{S}_2 \right] \right\} , \quad (2.2b)$$

$$H_{\text{SO}} = H_{\text{SO}(1G)} + H_{\text{SO(HO)}} , \quad (2.2c)$$

$$H_{\text{SO}(1G)} = \frac{2_{\alpha_s}}{3m_Q^2 r^3} (\vec{S}_1 \cdot \vec{r} \times \vec{P}_1 - \vec{S}_2 \cdot \vec{r} \times \vec{P}_2 + 2\vec{S}_1 \cdot \vec{r} \times \vec{P}_2 - 2\vec{S}_2 \cdot \vec{r} \times \vec{P}_1) , \quad (2.2d)$$

$$H_{\text{SO(HO)}} = -\frac{k}{m_Q^2} (\vec{S}_1 \cdot \vec{r} \times \vec{P}_1 - \vec{S}_1 \cdot \vec{r} \times \vec{P}_2) , \quad (2.2e)$$

where $\vec{r}$ is the interquark distance and $\vec{P}$, $\vec{S}$, and Λ^α $(-\Lambda^{*\alpha})$ are the momenta, spins, and color vectors of the quark (antiquark). Finally

$$V = [\tfrac{1}{2} k r^2 + U(r)] , \quad (2.2f)$$

where $U(r)$ is some unknown potential which incorporates an attractive potential at short range (a Coulomb-type piece derived from QCD) and deviations from the harmonic-oscillator interaction at large distances.

In applications to the baryons, the spin-orbit force is neglected from the beginning. This is based on calculations by Isgur and Karl[2] which indicate "that spin-orbit forces, if present at all, are at a level much reduced over naive expectations." Isgur and

Karl[2] suggest that this result is due in part to a cancellation between that part of the spin-orbit interaction arising from one-gluon exchange [Eq. (2.2d)] and that arising from the harmonic potential [Eq. (2.2e)]. This suggestion is considered in detail by Schnitzer.[10] He notes that the sum of the two spin-orbit terms [Eq. (2.2c)] depends on $\langle r\rangle_{\rm hadron}$: "Since $\langle r\rangle_{\rm baryons}$ is somewhat larger than $\langle r\rangle_{q\bar q}$, one also understands why the coefficient of $\vec{L}\cdot\vec{S}$ is absent in the Isgur-Karl model of baryons, is weakly attractive for ordinary mesons, and more strongly attractive for charmonium." In view of Schnitzers' findings a spin-orbit term has been included in this consistent model for mesons although it was disregarded in the corresponding model for baryons.

The wave functions for the low-lying meson states are as follows:

$$\psi_{000}=\frac{\beta^{3/2}}{\pi^{3/4}}\exp(-\tfrac{1}{2}\beta^2r^2)\,, \tag{2.3}$$

$$\psi_{11m}=(\tfrac{2}{3})^{1/2}\frac{2\beta^{5/2}}{\pi^{1/4}}r\exp(-\tfrac{1}{2}\beta^2r^2)Y_{1m}(\theta,\phi)\,, \tag{2.4}$$

$$\psi_{200}=(\tfrac{2}{3})^{1/2}\frac{\beta^{7/2}}{\pi^{3/4}}(\tfrac{3}{2}\beta^{-2}-r^2)\exp(-\tfrac{1}{2}\beta^2r^2)\,, \tag{2.5}$$

$$\psi_{22m}=\frac{4\beta^{7/2}}{\sqrt{15}\pi^{1/4}}\beta^2r^2\exp(-\tfrac{1}{2}\beta^2r^2)Y_{2m}(\theta,\phi)\,, \tag{2.6}$$

where

$$\beta^4=km_Q\,. \tag{2.7}$$

The contribution of the harmonic-oscillator potential to the energy of the state is given by

$$E_0=(n+\tfrac{3}{2})\omega_Q\,, \tag{2.8}$$

where

$$\omega_Q{}^2=(4k)/m_Q\,. \tag{2.9}$$

Calculations of the nonharmonic part of the potential for different values of β [Eq. (2.7)] have been discussed by Kalman, Hall, and Misra.[7] Based on this work,[11] we set

$$a(t)=(\beta^3t^{3/2}/\pi^{3/2})\times\int d^3r\,U(r)\exp(-t\beta^2r^2)\,, \tag{2.10a}$$

$$b(t)=(\beta^5t^{5/2}/\pi^{3/2})\times\int d^3r\,U(r)r^2\exp(-t\beta^2r^2)\,, \tag{2.10b}$$

$$c(t)=(\beta^7t^{7/2}/\pi^{3/2})\times\int d^3r\,U(r)r^4\exp(-t\beta^2r^2)\,. \tag{2.10c}$$

For charmonium we set $t=1$ and use $a=a(1)$, $b=b(1)$, and $c=c(1)$ as three of the basic parameters needed to be set from experimental data. For the Υ system we use the quadratic approximation

$$a(t)\simeq A+Bt+Ct^2\,, \tag{2.11a}$$

$$b(t)\simeq(3A+Bt-Ct^2)/2\,, \tag{2.11b}$$

and

$$c(t)\simeq(15A+3Bt-Ct^2)/4\,. \tag{2.11c}$$

The values of A, B, and C are obtained from a, b, and c by setting $t=1$ in Eqs. (2.11). Thus the parameters for the Υ system are derived from those of the charmonium system. It then follows from Eqs. (2.1), (2.3)–(2.6), (2.8), and (2.10) that the total contribution to the energy excluding mixing, hyperfine, and spin-orbit effects is then

$$E_0(S)=2m_Q+\tfrac{3}{2}\omega_Q+a(t)\,, \tag{2.12}$$

$$E_0(P)=2m_Q+\tfrac{5}{2}\omega_Q+\tfrac{2}{3}b(t)\,, \tag{2.13}$$

$$E_0(S')=2m_Q+\tfrac{7}{2}\omega_Q+\tfrac{3}{2}a(t)-2b(t)+\tfrac{2}{3}c(t)\,, \tag{2.14}$$

$$E_0(D)=2m_Q+\tfrac{7}{2}\omega_Q+\tfrac{4}{15}c(t)\,, \tag{2.15}$$

where $t=1$ for the charmonium system and

$$t=(m_b/m_c)^{1/2} \tag{2.16}$$

for the Υ system.

In addition to mixing caused by the hyperfine interaction, the nonharmonic potential U itself has an off-diagonal contribution

$$U_0=\langle S\,|\,U\,|\,S'\rangle=\langle S'\,|\,U\,|\,S\rangle=(\tfrac{3}{2})^{1/2}a(t)-(\tfrac{2}{3})^{1/2}b(t)\,. \tag{2.17}$$

The quark and antiquark also interact via gluon exchange giving rise to a color-magnetic force [Eq. (2.2b)]. For the S states, there is no spin-orbit or tensor term [second term in Eq. (2.2b)]. The only contribution is the Fermi contact term [first term in Eq. (2.2b)]. This interaction not only splits (ψ,η_c), (Υ,η_b), (ψ',η_c'), and (Υ',η_b') but also[4–6] (ψ,ψ'), (Υ,Υ'), and (η_b,η_b'). Evaluating this interaction using Eqs. (2.2b), (2.3), and (2.5) for ψ,ψ' or Υ,Υ' yields the mixing matrix

$$\begin{vmatrix} E_0(S)+\dfrac{2^{3/2}}{3}\delta_Q & U_0+\dfrac{2}{\sqrt{3}}\delta_Q \\ U_0+\dfrac{2}{\sqrt{3}}\delta_Q & E_0(S')+\sqrt{2}\delta_Q \end{vmatrix}\,, \tag{2.18}$$

where $E_0(S)$, $E_0(S')$, and U_0 are given by Eqs. (2.12), (2.14), and (2.17), respectively,

$$\delta_c = 4\alpha_s \beta^3 / 3\sqrt{2\pi} m_c^2 \tag{2.19}$$

and

$$\delta_b = (m_c/m_b)^{5/4} \delta_c \, . \tag{2.20}$$

Similarly for (η_c, η_c') and (η_b, η_b') the corresponding mixing matrix is

$$\begin{bmatrix} E_0(S) - 2^{3/2}\delta_Q & U_0 - 2\sqrt{3}\delta_Q \\ U_0 - 2\sqrt{3}\delta_Q & E_0(S') - 3\sqrt{2}\delta_Q \end{bmatrix} . \tag{2.21}$$

The P and D states, on the other hand, have no contributions from the Fermi contact term. Following Isgur and Karl,[2] the tensor term is easily evaluated from the identity

$$\langle LSJ \,|\, r^{-3}(3\vec{S}_1\cdot\hat{r}\vec{S}_2\cdot\hat{r} - \vec{S}_1\cdot\vec{S}_2) \,|\, L'S'J \rangle = (-)^{J-L-S'}[(2L+1)(2S+1)]^{1/2} W(LL'SS';2J) \times \langle L \| \tfrac{1}{2}\sqrt{3} r^{-3}\hat{r}_+\hat{r}_+ \| L' \rangle \langle S \| \tfrac{1}{2}\sqrt{3} S_{1-}S_{2-} \| S' \rangle \, , \tag{2.22}$$

where W is a Racah coefficient and the last two factors are the reduced matrix elements of the tensors whose components are displayed. Thus applying Eqs. (2.2) and (2.6) to Eq. (2.22) yields

$$\langle {}^1P_1 \,|\, H_{\text{tens}} \,|\, {}^1P_1 \rangle = 0 \, , \tag{2.23}$$

$$\langle {}^3P_0 \,|\, H_{\text{tens}} \,|\, {}^3P_0 \rangle = \frac{-4\sqrt{2}}{3}\delta_Q \, , \tag{2.24}$$

$$\langle {}^3P_1 \,|\, H_{\text{tens}} \,|\, {}^3P_1 \rangle = \frac{2\sqrt{2}}{3}\delta_Q \, , \tag{2.25}$$

$$\langle {}^3P_2 \,|\, H_{\text{tens}} \,|\, {}^3P_2 \rangle = \frac{-2\sqrt{2}}{15}\delta_Q \, , \tag{2.26}$$

$$\langle {}^1D_2 \,|\, H_{\text{tens}} \,|\, {}^1D_2 \rangle = 0 \, , \tag{2.27}$$

$$\langle {}^3D_1 \,|\, H_{\text{tens}} \,|\, {}^3D_1 \rangle = -\frac{4\sqrt{2}}{15}\delta_q \, , \tag{2.28}$$

$$\langle {}^3D_2 \,|\, H_{\text{tens}} \,|\, {}^3D_2 \rangle = +\frac{4\sqrt{2}}{15}\delta_q \, , \tag{2.29}$$

$$\langle {}^3D_3 \,|\, H_{\text{tens}} \,|\, {}^3D_3 \rangle = -\frac{8\sqrt{2}}{105}\delta_q \, . \tag{2.30}$$

The tensor interaction also mixes the ${}^3D_1$ state with the ${}^3S_1$ and ${}^3S_1'$ states. Here making use of Eqs. (2.3), (2.5), and (2.6) in conjunction with Eq. (2.22) implies that

$$\langle {}^3D_1 \,|\, H_{\text{tens}} \,|\, {}^3S_1 \rangle = +\frac{4}{\sqrt{15}}\delta_q \, , \tag{2.31}$$

$$\langle {}^3D_1 \,|\, H_{\text{tens}} \,|\, {}^3S_1' \rangle = +\tfrac{2}{3}(\tfrac{2}{5})^{1/2}\delta_q \, . \tag{2.32}$$

Since

$$\langle LSJM \,|\, \vec{L}\cdot\vec{S} \,|\, L'SJM' \rangle = \tfrac{1}{2}[J(J+1) - L(L+1) - S(S+1)]\delta_{JJ'}\delta_{LL'}\delta_{MM'} \, , \tag{2.33}$$

the spin-orbit contribution to the P and D states calculated from Eqs. (2.2b), (2.4), and (2.6) is

$$\langle {}^1P_1 \,|\, H_{\text{SO}} \,|\, {}^1P_1 \rangle = 0 \, , \tag{2.34}$$

$$\langle {}^3P_0 \,|\, H_{\text{SO}} \,|\, {}^3P_0 \rangle = -4\sqrt{2}\delta_Q + \omega_Q^2/m_Q \, , \tag{2.35}$$

$$\langle {}^3P_1 \,|\, H_{\text{SO}} \,|\, {}^3P_1 \rangle = -2\sqrt{2}\delta_Q + \omega_Q^2/2m_Q \, , \tag{2.36}$$

$$\langle {}^3P_2 \,|\, H_{\text{SO}} \,|\, {}^3P_2 \rangle = +2\sqrt{2}\delta_Q - \omega_Q^2/2m_Q \, , \tag{2.37}$$

$$\langle {}^1D_2 \,|\, H_{\text{SO}} \,|\, {}^1D_2 \rangle = 0 \, , \tag{2.38}$$

$$\langle {}^3D_1 \,|\, H_{\text{SO}} \,|\, {}^3D_1 \rangle = -\frac{12\sqrt{2}}{5}\delta_Q + 3\omega_Q^2/2m_Q \, , \tag{2.39}$$

$$\langle {}^3D_2 \,|\, H_{\text{SO}} \,|\, {}^3D_2 \rangle = -\frac{4\sqrt{2}}{5}\delta_Q + \omega_Q^2/2m_Q \, , \tag{2.40}$$

$$\langle {}^3D_3 \,|\, H_{\text{SO}} \,|\, {}^3D_3 \rangle = \frac{8\sqrt{2}}{5}\delta_Q - \omega_Q^2/m_Q \, . \tag{2.41}$$

III. CALCULATIONS

Examination of the previous section shows that there are seven parameters to be calculated, namely, ω, a, b, c, δ_c, m_c, and m_b. All but the last one can be calculated from the charmonium sector. The ${}^1P_1$, ${}^3D_2$, and ${}^3D_3$ states of charmonium have not yet been observed and are predicted in this paper. The ${}^3D_1$ state is just above threshold for disintegration by strong-interaction pair production into charmed

TABLE I. Masses of the low-lying states of ψ and Υ systems in MeV. Input parameters are underlined.

		ψ				Υ	
State	Experiment	Calculation	Deviation (%)	State	Experiment	Calculation	Deviation (%)
ψ	$\underline{3097}$	3096.2	-0.027	Υ	$\underline{9440}$	9441.6	0.018
ψ'	$\underline{3685}$	3684.2	-0.021	Υ'	10 000	9998.2	-0.018
η_c	$\underline{2984}$	2984.8	0.025	η_b		9369.3	
η_c'	$\underline{3592}$	3592.8	0.023	η_b'		9996.8	
${}^1P_1$		3520.8		${}^1P_1$		10 178.9	
${}^3P_0$	$\underline{3414}$	3413.1	-0.026	${}^3P_0$		10 130.6	
${}^3P_1$	$\underline{3507}$	3507.7	0.021	${}^3P_1$		10 169.5	
${}^3P_2$	3551	3550.1	-0.024	${}^3P_2$		10 194.1	
${}^1D_2$		3838.2		${}^1D_2$		10 298.2	
${}^3D_1$	3768	3840.9	1.94	${}^3D_1$		10 285.1	
${}^3D_2$		3866.0		${}^3D_2$		10 303.6	
${}^3D_3$		3829.3		${}^3D_3$		10 304.1	

mesons. The mass of this state is presumably affected by this decay and is thus not suitable for fitting to the parameters. This leaves ψ (${}^3S_1$), ψ' (${}^3S_1'$), η_c (${}^1S_0$), η_c' (${}^1S_0'$), $\chi(3415)$ (${}^3P_0$), $\chi(3510)$ (${}^3P_1$), $\chi(3550)$ (${}^3P_2$). From Eqs. (2.22) and (2.33), it is clear that once two of the P states are fit to the parameters, the value of the third state is obtained in a model-independent manner. The values $\omega=390.5$ MeV, $a=-3004.9$ MeV, $b=-4430.3$ MeV, $c=-11349.9$ MeV, $\delta=21.63$ MeV, and $m_c=2749.0$ MeV were obtained by brute-force substitution until the masses of ψ, ψ', η_c, η_c', $\chi(3415)$, and $\chi(3510)$ corresponded to the experimental values within approximately 0.02% error (<1 MeV). The corresponding predicted values of the masses of the ${}^1P_1$, ${}^3D_1$, ${}^3D_2$, and ${}^3D_3$ states are found in Table I. Note that the mass of $\psi(3770)$ (${}^3D_3$) differs from the experimental value by 1.9%. As noted earlier, this is not a test of the model since there are undoubtedly contributions to this mass from the strong decay of this state. The value of m_b is obtained by fitting to Υ (${}^3S_1$). For display purposes it was decided to use the value $m_b=6188.6$ MeV which equalizes the error between Υ (${}^3S_1$) and Υ' (${}^3S_1'$). With this single parameter and those already obtained for the charmonium system, the masses of the entire Υ spectrum up to $n=2$ are obtained. The results are given in Table I. Finally, for all states mixed by the nonharmonic U term and/or the hyperfine interaction, the composition of the states after mixing is as follows:

$$|\psi\rangle=0.9974\,|{}^3S_1\rangle-0.0639\,|{}^3S_1'\rangle+0.0275\,|{}^3D_1\rangle\,, \tag{3.1}$$

$$|\psi'\rangle=0.0649\,|{}^3S_1\rangle+0.9966\,|{}^3S_1'\rangle+0.0508\,|{}^3D_1\rangle\,, \tag{3.2}$$

$$|\eta_c\rangle=0.9725\,|{}^1S_0\rangle-0.2330\,|{}^1S_0'\rangle\,, \tag{3.3}$$

$$|\eta_c'\rangle=0.2330\,|{}^1S_0\rangle+0.9725\,|{}^1S_0'\rangle\,, \tag{3.4}$$

$$|\Upsilon\rangle=0.7413\,|{}^3S_1\rangle-0.6711\,|{}^3S_1'\rangle+0.0142\,|{}^3D_1\rangle\,, \tag{3.5}$$

$$|\Upsilon'\rangle=0.6712\,|{}^3S_1\rangle+0.7413\,|{}^3S_1'\rangle-0.0016\,|{}^3D_1\rangle\,, \tag{3.6}$$

$$|\eta_b\rangle=0.7294\,|{}^1S_0\rangle-0.6840\,|{}^1S_0'\rangle\,, \tag{3.7}$$

$$|\eta_b'\rangle=0.6840\,|{}^1S_0\rangle+0.7294\,|{}^1S_0'\rangle\,. \tag{3.8}$$

IV. DISCUSSION OF THE RESULTS

Two interesting features emerge from the results. First of all, the values of the masses of the Υ and Υ' states are obtained using the potential derived from the charmonium spectrum. The only parameter used is the mass of the b quark. Thus the results are consistent with flavor independence of the potential. Second, the composition of the states as given in Eqs. (3.1)–(3.8) is very different in the charmonium and Υ systems. In the charmonium system the states are essentially unmixed, but in the Υ system both states contain almost equal mixtures of S and S'. In both systems there is very little admixture of D states. Since the ${}^3D_1$ state in the Υ system is below threshold for strong pair production of charmed mesons, a determination of the experimental values of the mass of this state would test the hypothesis that the roughly 2% deviation from the known mass of the corresponding charmonium state is indeed due to its decay properties.

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[11]The notation is somewhat different here than in Ref. 7.