PHYSICAL REVIEW D VOLUME 27, NUMBER 9 1 MAY 1983

Radiative corrections to the leptonic decays of pseudoscalar mesons: Contributions of W and Z mesons

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(Received 10 August 1982)

Using the current-algebra formalism we calculate, within the Salam-Weinberg model, the contribution of the W and Z mesons to the radiative corrections to the leptonic decays of pseudoscalar mesons.

I. INTRODUCTION

Ever since the development of gauge theories of weak and electromagnetic interactions the radiative corrections to semileptonic decays of hadrons have received renewed attention. However, the presence of strong-interaction effects makes these studies difficult. As was pointed out by Sirlin, a way[1] to control these effects is to use a current-algebra formulation of radiative corrections. He has applied these techniques to superallowed Fermi transitions and to pion β decay. Motivated by Sirlin's results we calculate the radiative corrections to the leptonic decays of pseudoscalar mesons (M_{l2}) within the context of a gauge model, the Weinberg-Salam (WS) model.[2] In an earlier paper[3] we calculated the radiative corrections to the total decay rate of M_{l2} decays, without resorting to any model of weak and strong interactions. We showed that all the model dependence is contained, to order α, in an effective decay constant. In this paper we now concentrate our attention on such model-dependent contributions to radiative corrections to M_{l2} decays. We will show that the analogous results to Fermi decays are obtained in these Gamow-Teller decays. That is, the photonic corrections induced by the vector current (in what is initially a process mediated by the axial-vector current only) consist of two terms, one asymptotic piece proportional to $\ln m_W/\Lambda$, where m_W is the mass of the charged intermediate vector boson and Λ is a hadronic mass introduced to avoid the infrared divergence, and a finite[4] nonasymptotic term (in the $m_W\rightarrow\infty$ limit). Both contributions are model dependent. The asymptotic photonic corrections arising from the axial-vector current are independent of strong-interactions effects. The nonphotonic corrections, those proceeding from virtual Z exchange, are practically identical to those arising from the vector current.

The problem of the radiative corrections to the axial-vector part of the hadronic weak current was treated by some authors several years ago. They found that, if the pion is massless, the radiative corrections to the G_A/G_V rate is finite, but, of course, model dependent.[5] Recently, the dynamical strong-interactions effects in the leptonic decays of the pion were studied by Goldman and Wilson, within the WS model and the static quark model for the pion. Their results show that the coefficient of the logarithmic lepton-mass singularity is not affected by the strong-interactions and the structure-dependent terms are very small,[6] in accordance with a theorem of Marciano and Sirlin,[7] yielding support to the present status of the μ-e universality. The conclusion of such calculations was that the strong-interaction effects are very small. This conclusion is adopted in this paper.

The plan of the paper is as follows. In Sec. II we present the relevant piece of the interaction Lagrangian density of the WS model. In Sec. III the photonic and nonphotonic corrections, to order G_F, are calculated in the 't Hooft—Feynman gauge. Section IV is devoted to collect our results and discuss their applications.

II. LAGRANGIAN DENSITY AND CURRENTS

The interaction Lagrangian density in the SU(2)$\times$U(1) model with hadrons is given by[8]

$$L_{\text{int}}=L_{lV}+L_{hV}+L_{VV}+\cdots\ , \tag{1}$$

where

$$L_{lV}=-\frac{g}{\sqrt{2}}(\bar{\nu}_l\gamma^\mu a_- l W_\mu^- +\text{H.c.})-\tfrac{1}{2}(g^2+g'^2)^{1/2}Z_\mu[\bar{\nu}_l\gamma^\mu\nu_l+\bar{l}(2\sin^2\theta_W-a_-)l]+eA_\mu\bar{l}\gamma^\mu l\ , \qquad (2)$$

$$L_{hV}=-\frac{g}{\sqrt{2}}(W_\mu^+J_W^\mu+\text{H.c.})-(g^2+g'^2)^{1/2}Z_\mu J_Z^\mu-eA_\mu J_\gamma^\mu\ , \qquad (3)$$

and

$$L_{VV}=-e[\partial^\mu W^{+\nu}(A_\mu W_\nu^- - W_\mu^- A_\nu)-\partial^\mu W^{-\nu}(A_\mu W_\nu^+ - W_\mu^+ A_\nu)]$$
$$+\frac{g}{\sqrt{2}}[\partial^\mu W^{+\nu}(Z_\mu W_\nu^- - W_\mu^- Z_\nu)-\partial^\mu W^{+\nu}(Z_\mu W_\nu^+ - W_\mu^+ Z_\nu)]\ .$$

The dots in Eq. (1) denote the Lagrangian densities of the Higgs mesons and their interactions with leptons and hadrons, and the allowable counterterms which have to be added to cancel out some divergences arising from corrections to the hadronic vertex.

The electromagnetic and weak currents are, respectively, given by

$$J_\gamma^\mu=\bar{\psi}\gamma^\mu Q\psi\ ,\quad J_W^\mu=\bar{\psi}\gamma^\mu a_- C_-\psi\ ,\quad J_Z^\mu=\tfrac{1}{2}\bar{\psi}\gamma^\mu a_- C_3\psi-\sin^2\theta_W J_\gamma^\mu\ , \qquad (4)$$

where the matrices Q, C_-, and C_3, acting on the SU(4) indices, are

$$Q=\text{diag}(\tfrac{2}{3},\tfrac{2}{3},-\tfrac{1}{3},-\tfrac{1}{3})\ ,\quad C_3=\text{diag}(1,1,-1,-1)\ ,\quad C_-=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ -\sin\theta & \cos\theta & 0 & 0\\ \cos\theta & \sin\theta & 0 & 0\end{bmatrix}$$

and $\psi_i=$ col $(c,u,d,s)_i$ and $a_\pm=\frac{1}{2}(1\pm\gamma_5)$. The index in ψ_i refers to the $SU(3)_c$ degree of freedom. We have chosen the scheme in which the quarks transform according to the fundamental representation of the $SU(4)\times SU(3)_c$ group, but the strong interactions are treated nonperturbatively. θ_W and θ are the Weinberg and Cabibbo angles, respectively. The commutation relations between the hadronic currents are easily shown to be

$$[J_W^\mu(x),J_\gamma^\nu(x')]_{x^0=x^{0'}}=\delta_{0\rho}S^{\mu\rho\nu\sigma}J_{W\sigma}(x)\delta(\vec{x}-\vec{x}\,')-i\delta_{0\rho}\epsilon^{\mu\rho\nu\sigma}[2\bar{\psi}\gamma_\sigma a_- Q_- C_-\psi+J_{W\sigma}(x)]\delta(\vec{x}-\vec{x}\,')+\text{ST}\ , \qquad (5a)$$

$$[J_W^\mu(x),J_Z^\nu(x')]_{x^0=x^{0'}}=\delta_{0\rho}S^{\mu\rho\nu\sigma}\cos^2\theta_W J_{W\sigma}(x)\delta(\vec{x}-\vec{x}\,')$$
$$+i\delta_{0\rho}\epsilon^{\mu\rho\nu\sigma}\sin^2\theta_W[2\bar{\psi}\gamma_\sigma a_- QC_-\psi+J_{W\sigma}(x)]\delta(\vec{x}-\vec{x}\,')+\text{ST}\ , \qquad (5b)$$

where

$$S^{\mu\rho\nu\sigma}=g^{\mu\rho}g^{\nu\sigma}-g^{\mu\nu}g^{\rho\sigma}+g^{\rho\nu}g^{\mu\sigma}\ ,$$

$\epsilon^{\mu\rho\nu\sigma}$ is the totally antisymmetric tensor, and $\epsilon^{0123}=+1$; ST represents a c-number Schwinger term. Since the currents in Eq. (4) are invariant under local $SU(3)_c$, the commutation relations are free of operator Schwinger terms, and no operator anomalies in these commutators appear because the currents are (partially) conserved and the strong interactions are asymptotically free in the $SU(4)\times SU(3)_c$ theory.[9] The c-number Schwinger terms do not contribute to the connected amplitudes.

III. VIRTUAL CORRECTIONS

In this section we calculate the virtual corrections arising from γ, W, or Z exchange. The diagrams we consider are those of Fig. 1.

We have chosen the decay of a negatively charged meson. The blob represents schematically strong-interaction effects. The corresponding transition amplitudes are given by[10]

$$M_0=\frac{ig}{2}\frac{ap^\mu}{p^2-m_W{}^2}L_\mu\ , \qquad (6)$$

$$M_{(\gamma)}^{(b)}=-\frac{ig^2}{m_W{}^2}\frac{\alpha}{(2\pi)^3}\int\frac{d^4k}{k^2}\frac{m_W{}^2}{m_W{}^2-(k-p)^2}T_{(\gamma)}^{\lambda\rho}(k)\bar{u}_l\frac{1}{\not{k}-\not{l}-m_l}O_\rho\gamma_\lambda v_\nu\ , \qquad (7a)$$

$$M^{(b)}_{(Z)} = -\frac{ig^2(g^2+g'^2)}{4(2\pi)^4}\int \frac{d^4k}{k^2}\frac{1}{(k^2-m_Z^2)[(k-p)^2-m_W^2]}T^{\lambda\rho}_{(Z)}(k)\bar{u}_l\frac{1}{\not{k}-\not{l}-m_l}O_\rho\gamma_\lambda(2\sin^2\theta_W-a_-)v_\nu \,, \tag{7b}$$

$$M^{(c)}_{(Z)} = \frac{ig^2(g^2+g'^2)}{4(2\pi)^4}\int \frac{d^4k}{(k^2-m_Z^2)[(k-p)^2-m_W^2]}T^{\gamma\rho}_{(Z)}(k)\bar{u}_l\frac{1}{\not{k}}O_\rho\gamma_\lambda v_\nu \,, \tag{8}$$

$$M^{(d)}_{(\gamma)} = -\frac{ig^4\sin^2\theta_W}{2(2\pi)^4}\frac{L^\mu}{p^2-m_W^2}\int\frac{d^4k}{k^2}\frac{1}{(p-k)^2-m_W^2}[(2p-k)_\lambda g_{\mu\rho}+(2k-p)_\mu g_{\lambda\rho}-(p+k)_\rho g_{\mu\lambda}]T^{\lambda\rho}_{(\gamma)}(k) \,, \tag{9a}$$

$$M^{(d)}_{(Z)} = -\frac{ig^4}{2(2\pi)^4}\frac{L^\mu}{p^2-m_W^2}\int\frac{d^4k}{k^2-m_W^2}\frac{1}{(p-k)^2-m_W^2}[(2p-k)_\lambda g_{\mu\rho}+(2p-k)_\mu g_{\lambda\rho}-(p+k)_\rho g_{\mu\lambda}]T^{\lambda\rho}_{(Z)}(k) \,, \tag{9b}$$

$$M^{(e)} = -\frac{ig^2}{m_W^2}\frac{\alpha}{(2\pi)^3}\int\frac{d^4k}{k^2}\frac{m_W^2}{m_W^2-p^2}ap_\mu\bar{u}_l\frac{(2l_\sigma-\gamma_\sigma\not{k})\not{l}}{(k^2-2l\cdot k)^2}\frac{(2l^\sigma-\not{k}\gamma^\sigma)}{2m_l^2}(\not{l}+m_l)O^\mu v_\nu \,, \tag{10}$$

$$M^{(f)} = \frac{ig^2}{2}\frac{1}{2}[F(p^2)p_\mu]\frac{L^\mu}{p^2-m_W^2} \,, \tag{11}$$

where p, l, and k denote the four-momenta of the meson, the electron or muon, and the virtual vector bosons, respectively. g and g' are the coupling constants of the model related by $g'=g\tan\theta_W$; α, the fine-structure constant, is related to g by the expression

$$\alpha=(g^2/4\pi)\sin^2\theta_W \,;$$

the masses of the lepton and W and Z mesons are denoted by m_l, m_W, and m_Z. L_μ is the leptonic covariant,

$$L_\mu=\bar{u}_l O_\mu v_\nu \,,$$

and

$$O_\mu=\gamma_\mu(1-\gamma_5) \,.$$

Finally the meson decay constant is denoted by a.

The hadronic covariant tensor $T^{\lambda\rho}_{(a)}$ is given by the Fourier transform of the time-ordered product of two currents:

$$T^{\lambda\rho}_{(a)}=\int d^4k\, e^{ik\cdot x}\langle 0\,|\,T[J^\lambda_{(a)}(x)J^\rho_W(0)]\,|\,p\rangle \quad (a=\gamma \text{ or } Z) \tag{12}$$

and

$$J^\rho_W=V^\rho+A^\rho \,.$$

The meson form factor $F(p^2)$ in Eq. (11) can be expressed, to order α, as

$$F(p^2)=F^0(p^2)+\sum_a \delta F^{(a)}(p^2) \quad (a=\gamma,\ Z,\text{ or } W) \,, \tag{13}$$

where $F^0(p^2)$ is the form factor to zero order in α, and $\delta F^{(a)}$ is the correction due to the perturbation $a=\gamma, Z$, or W. This correction can be expressed in terms of the Fourier transform of the product of three currents.[11] Therefore Eq. (11) includes the uncorrected amplitude of Eq. (6). In the graph of Fig. 1(f) both ends of the γ, Z, or W line are attached to

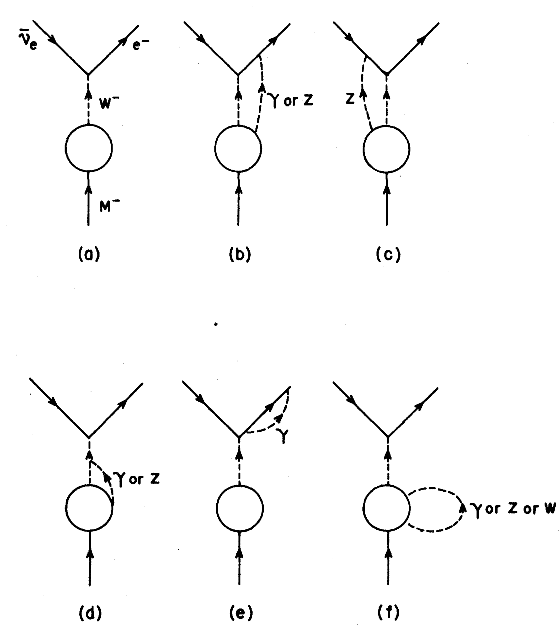

FIG. 1. Graphs contributing to virtual radiative corrections to M_{l2} decays mediated by the W meson. The blob represents strong-interaction effects.

the incoming meson. The graphs of Fig. 1(d) correspond to meson—W-boson vertex corrections.

In order to get the photonic corrections of the local $V-A$ theory we have to consider the graphs in which the virtual exchange particle is a photon; the ultraviolet cutoff is replaced[12] by m_W and the infrared divergence is canceled by that of the bremsstrahlung diagrams. These last diagrams are the same as in the $V-A$ calculation because m_W is much greater than M, the mass of the pseudoscalar meson. The result is the Φ function of Eq. (10) in Ref. 3. We now evaluate the asymptotic limit of Eq. (7a), i.e., the limit in which $M \ll m_W$. We will show that in this limit corrections arising from the axial-vector current in Eq. (7a) are independent of the dynamics of strong interactions, whereas those induced by the vector current are model dependent. For the nonphotonic corrections, those of Eqs. (8) and (9b), we will get a result identical to that in the Fermi transitions.

To begin with, let us consider the contribution of the axial-vector current to the amplitude of Eq. (7a),

$$M_{(\gamma)}^{(\mathrm{b})}=\frac{ig^2}{m_W{}^2}\frac{\alpha}{(2\pi)^3}\int\frac{d^4k}{k^2}\frac{m_W{}^2}{[m_W{}^2-(k-p)^2][(k^2-2l\cdot k+i\epsilon)]}$$
$$\times\bar{u}_l[(k_\lambda\gamma_\rho+k_\rho\gamma_\lambda)A_{(\gamma)}^{\lambda\rho}(k)-\not{k}A_{(\gamma)}^{\lambda}(k)-2l_\lambda\gamma_\rho A_{(\gamma)}^{\lambda\rho}(k)-ik_\alpha\gamma_\beta\gamma_5\widetilde{V}_{(\gamma)}^{\alpha\beta(k)}]a_-v_\nu\ , \qquad (14)$$

where $\widetilde{V}^{\alpha\beta}$ is the dual of $V_{\lambda\rho}$. The last term in Eq. (14) is known as the induced one by the vector part of the weak current. The Bjorken limit of $V_{(\gamma)}^{\lambda\rho}$ and $A_{(\gamma)}^{\lambda\rho}$ is evaluated with the help of the commutation relations of Eqs. (5a) and (5b). These are

$$V_{(\gamma)}^{\lambda\rho}(k)=-2\overline{Q}\frac{k_\beta}{k^2-\Lambda^2}\epsilon^{\lambda\beta\rho\alpha}\langle 0|A_\alpha(0)|p\rangle\ , \qquad (15a)$$

$$A_{(\gamma)}^{\lambda\rho}(k)=-\frac{ik_\beta}{k^2-2p\cdot k}S^{\rho\beta\lambda\alpha}\langle 0|A_\alpha(0)|p\rangle\ , \qquad (15b)$$

where Λ^2 is a hadronic mass introduced to avoid the infrared divergences, $\Lambda^2 \ll m_W{}^2$; $\overline{Q}$ is the average charge of the u and d quarks.

To perform the integrations in Eq. (14) we assume some approximations valid in the asymptotic limit; we set $l \approx p \approx 0$, and perform a Wick rotation. Neglecting terms of order $G_F{}^2$, we get

$$M_{(\gamma)}^{(\mathrm{b})}=\frac{\alpha}{2\pi}M_0\left[\tfrac{3}{2}\ln\frac{M}{m_W}+\tfrac{1}{2}(6\overline{Q})\ln\frac{m_W}{\Lambda}\right]. \qquad (16)$$

In Eq. (16) we have neglected terms proportional to $M_l{}^2$, which are in fact very small.

The second term between the square brackets is induced by the vector part of the weak current; it depends on the quark model used to describe the hadron. In the model used $6\overline{Q}=1$. The first one is independent of any model of quarks and it proceeds from the axial-vector current. For the nonphotonic corrections we follow the same procedure after Eq. (14); the Bjorken limit of $T_{(Z)}^{\lambda\rho}$ is evaluated using the commutation relations and a Wick rotation is performed. Adding Eqs. (8) and (7b) the result, denoted as $M_{(Z)}$ is, neglecting terms $O(G_F{}^2)$,

$$M_{(Z)}=\frac{\alpha}{4\pi}M_0\left[\frac{R}{R-1}\right]\ln R\left(\tfrac{5}{2}\cot^2\theta_W+3\overline{Q}\tan^2\theta_W\right) \qquad (17)$$

with $R=(m_W/m_Z)^2$.

Equation (17) is identical with Eq. (4.35) of Ref. 1. Thus, Eq. (17) is valid for leptonic and semileptonic decays of a hadron, and of course, to purely leptonic decay (μ decay).

For the rest of the contributions, Eqs. (9a) and (11), we proceed as in Ref. 1. The contributions of Eq. (11) are handled using Ward identities to reduce the three-current correlation functions into residual three-current correlation functions, and two-current correlation functions, in Sirlin's terminology. The last functions are combined with those contributions arising from the corrections of Fig. (1d). The residual three-current correlation functions are combined with the (*ad hoc*) order-α counterterms and tadpole diagrams and give contributions to $O(G_F{}^2)$ (Ref. 13). The correction of Fig. (1d), Eq. (9a), is evaluated following the procedure after Eq. (14). Adding the contributions of the two-current correlation functions and that of Eq. (9a), we get the corrections for the meson—W-boson vertex,

$$M_{M-W}=\frac{\alpha}{8\pi}M_0\cot^2\theta_W\left[2+\frac{1+R}{1-R}\ln R\right]. \qquad (18)$$

In Eq. (18) we have absorbed a divergent strong-interaction-independent term in a redefinition of the

TABLE I. Numerical values for the different terms of Eq. (19). The values of Φ have been taken from Ref. 3.

	$\frac{\alpha}{\pi}\Phi(\Delta k=1\ \mathrm{MeV})$	$\frac{\alpha}{2\pi}\left[3\ln\frac{m_W}{M}\right]$	$\frac{\alpha}{2\pi}\left[6\bar{Q}\ln\frac{m_W}{\Lambda}\right]$	$\frac{\alpha}{2\pi}\left[3\ln\frac{m_W}{m_Z}\right](1+2\bar{Q})$
π_{e2}	−0.0998	0.0212	0.0048	0.0009
K_{e2}	−0.1607	0.0161	0.0048	0.0009
D_{e2}	−0.2408	0.0122	0.0048	0.0009
$\pi_{\mu 2}$	−0.0018	0.0212	0.0048	0.0009
$K_{\mu 2}$	−0.0214	0.0161	0.0048	0.0009
$D_{\mu 2}$	−0.0656	0.0122	0.0048	0.0009

weak coupling constant, and we have neglected terms of $O(G_F{}^2)$. The hadronic corrections of the W propagator and those involving Higgs scalars are of $O(G_F{}^2)$ also.[14]

IV. RESULTS AND CONCLUSIONS

Collecting the results of Sec. III, the total transition rate for M_{l2} decays is

$$\Gamma=\frac{\hat{g}^2}{2}\frac{|a|^2}{4\pi}\frac{m^2}{M^3}(M^2-m^2)^2\left\{1+\frac{\alpha}{2\pi}\left[2\Phi+3\ln\frac{m_W}{M}+6\bar{Q}\ln\frac{m_W}{\Lambda}-\tfrac{3}{2}(1+2\bar{Q})\ln R+\cdots\right]\right\}, \tag{19}$$

where

$$\hat{g}=g\left[1+\frac{3\alpha}{8\pi}\ln R\tan^2\theta_W+\cdots\right]. \tag{20}$$

In Eq. (19) the first term in the square brackets depends on quantum electrodynamics only. The model-dependent contributions are common to the vector and axial-vector parts of the weak hadronic current. The first and second terms in the brackets are the photonic contributions coming from the axial-vector current. The third term is the asymptotic photonic correction induced by the vector current. The fourth term corresponds to the nonphotonic corrections. A comparison with the calculation of the Fermi transition[15] shows that the third and fourth terms in the square brackets in Eq. (19) appear in both Fermi and Gamow-Teller transitions, within the approximations made here. In Table I we give the numerical values of each contribution in Eq (19) for different M_{l2} decays, with $m_W=63$ GeV, $\Lambda=1$ GeV, and $\sin\theta_W=0.59$, taken from Ref. 1, in order to make the same analysis as in that reference.

It can be seen that the term $\ln(m_Z/M)$ has a coefficient independent of strong interactions, in accordance with a theorem, for semileptonic processes recently proved by Sirlin.[16] From Table I we can also observe that the dominant non-Φ term is precisely this one. Then, for practical applications, the model dependence of radiative corrections to the axial-vector current part of the matrix element is controlled by Sirlin's theorem. We also observe that the non-Φ terms are independent of the lepton mass and, therefore, they are not universality-violating contributions. (Recall that we are neglecting terms proportional to m^2 in the evaluation of the integrals.) In other words, such terms, which are of order $\alpha G_F m^2/m_W{}^2$, are absorbed in a redefinition of the decay constant.

Our main conclusion is then that the model-dependent part of radiative corrections to M_{l2} decays is given mainly for the detailed gauge model of weak interactions and agrees with good precision with the model-dependent radiative corrections to superallowed Fermi transitions. The effects of strong interactions are there but they are expected to be small as indicated by the work of Goldman and Wilson.[6]

ACKNOWLEDGMENTS

This work was submitted by one of us (A.Q.) in partial fulfillment of the requirements for the Dr. Sci. degree at the Escuela Superior de Fisica y Matemáticas, Instituto Politécnico Nacional, 1982. The work of A.Q. was supported in part by COFAA.

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[12]To see how this proceeds we need to write the photon propagator as

$$\frac{1}{k^2}=\frac{1}{k^2-m_W{}^2}+\frac{m_W{}^2}{k^2-m_W{}^2}\frac{1}{k^2}$$

which can be interpreted as formed by a massive photon (m_W) and a massless photon with a convergence factor $m_W{}^2/(m_W{}^2-k^2)$. The second term gives the cutoff factor m_W. The first term contributes with a term independent of model and is absorbed in a redefinition of the weak coupling constant.

[13]The procedure is similar to that described in Sec. V A of Ref. 1. In our case we need more counterterms to cancel the divergences coming from the residual three-current correlation function in which the four-divergence of the axial-vector current, $\partial_\mu A^\mu$, appears (which is absent in the Fermi case, $\partial_\mu V^\mu=0$). A simpler procedure is to assume that the axial-vector current is divergenceless, in which case the situation is reduced to the Fermi case.

[14]The finite contributions arising from exchanges of virtual Higgs scalars are $O(\alpha G_F m_q{}^2/m_W{}^2)$, where m_q is a typical quark mass, and are considered of $O(G_F{}^2)$. See S. Weinberg, Phys. Rev. D 8, 605 (1973).

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