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Analysis of $\pi^- p \to \pi^0 n$ scattering at high energies

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The differential cross sections for $\pi^- p \to \pi^0 n$ including the recent measurements up to $-t \approx 2.2$ (GeV/c)^2, the polarization, and the difference $\Delta\sigma = \sigma_{\text{tot}}(\pi^+ p) - \sigma_{\text{tot}}(\pi^- p)$ are fitted in the region of high energy by using a ρ+cut model. The $I_t = 1$ amplitudes deduced from the model for $\pi^- p \to \pi^0 n$ are found to be in good agreement with those obtained by amplitude-analysis results of $\pi N \to \pi N$ at 6 GeV/c.

Charge-exchange processes in the region of high energy have gained considerable attention in the recent past. In particular, the pion-nucleon charge-exchange (CEX) reaction $\pi^- p \to \pi^0 n$ has been extensively studied both experimentally and theoretically. This reaction has for a long time been considered one of the simplest examples of the Regge mechanism, being dominated by the exchange of a single Regge trajectory ρ. Its importance stems from the fact that the study of this reaction allows one to choose between various models which describe the pion-nucleon interactions as an exchange of meson states in the crossed channel. However, as the neutral particles produced in a scattering process are difficult to detect, the experimental results on the CEX process are considerably less precise than the results on elastic scattering. At high energies, although the detection of the neutral particles becomes easier, the CEX cross section becomes very small, and the background from multi-π events becomes important. Detailed studies of this reaction up to 18 GeV/c were made at CERN, BNL, and ANL accelerators.[1-6] Stirling *et al.*[1] measured at CERN the differential cross sections at laboratory momenta equal to 5.9, 9.8, 13.3, and 18.2 GeV/c and in the momentum-transfer-squared interval $0.01 < -t < 0.80$ (GeV/c)^2. The bin width varied up to 0.07 (GeV/c)^2. They also obtained by extrapolation the forward differential cross sections at these momenta. The same group[2] has extended the domain of four-momentum transfer squared up to 3 (GeV/c)^2 for the laboratory momenta between 3 and 18.2 GeV/c. The data below 4 GeV/c are found to have roughly the same shape and variation with energy as the high-energy data in spite of the vicinity of the highest πN resonance which should noticeably distort the "asymptotic behavior" of CEX scattering at these energies. Measurements up to 18 GeV/c have also been carried out by other groups.[3-6] At higher energies, this reaction has been extensively studied at Serpukhov[7,8] and Fermilab.[9] With an aim to determine differential cross sections of pion CEX on protons at momenta higher than 20 GeV/c, Bolotov *et al.*[7] measured the $d\sigma/dt$ for $\pi^- p \to \pi^0 n$ for momenta up to 50 GeV/c and in the range $0 < -t < 1.5$ (GeV/c)^2. The measurements at still higher energies were carried out at Fermilab by Barnes *et al.*[9] from 20–200 GeV/c and for $-t$ in the range $0.002 < -t < 1.3$ (GeV/c)^2. The data show a characteristic forward peak with a small dip at $t = 0$ and a break around $-t \simeq 0.6$ (GeV/c)^2. These features are similar to those observed at lower energies although the dip at $t = 0$ is more pronounced in their data than in the highest-energy data (48 GeV/c) of Bolotov *et al.*[7] The presence of a forward dip in the data from 20 to 200 GeV/c suggests that the strong spin-flip contribution which is evident at energies below 20 GeV/c persists at high energies. The position of the break near $-t = 0.6$ (GeV/c)^2 shows no obvious systematic variation with energy over the range from 40 GeV/c to 200 GeV/c and appears to be consistent with the position of the dip observed in data for this reaction down to an incident momentum of 3 GeV/c.[1,2] The value of $d\sigma/dt$ at $t = 0$ for each energy has been extrapolated by Barnes *et al.*[9] Recently, Apel *et al.*[8] have reported results on the same reaction at 70-GeV Serpukhov accelerator. These measurements were carried out with high statistical accuracy for five values of the incident pion momentum in the range 15–40 GeV/c and over a wide range of four-momentum transfer squared up to $-t \simeq 2.2$ (GeV/c)^2. The cross section for this reaction has been measured in a number of experiments[1-7,9] using samples not exceeding a few tens of thousands of events. The results reported in Ref. 8 are however based on a total of about 3×10^6 recorded π^0. These large statistics have allowed them to extend the range of $-t$ and to study in detail the behavior of the cross section in the region of the forward dip, which reflects the importance of the spin-flip amplitude.

Very recently, Daum *et al.*[10] have measured the total and differential cross sections for this reaction at a beam momentum of 63 GeV/c. They have compared their results with some of the data points measured at 64 GeV/c by Barnes *et al.*[7] and normalized to the data of Ref. 10 at $-t=0.1$ (GeV/c)^2. The agreement is very good up to $-t\approx 0.5$ (GeV/c)^2. The cross section for $-t>0.5$ (GeV/c)^2, as measured in Ref. 10, is however, about a factor of 2 larger, while the slope for the differential cross section is the same. The cause of discrepancy between the results obtained at almost equal momenta in the two laboratories—Fermilab and CERN—could not be detected.

We note that the main characteristics of the high-energy CEX data are the following.

(1) The angular distribution has a turnover near the forward direction with first maximum near $-t=0.04$ (GeV/c)^2.

(2) The differential cross section then decreases rapidly with an increase in energy up to $-t\simeq 0.6$ (GeV/c)^2 and for $-t>0.6$ (GeV/c)^2 goes through a second maximum which decreases with increasing laboratory momentum for the incident pions.

(3) The angular-distribution curve has two dips: one at $t=0$ (GeV/c)^2 and the other at $-t\approx 0.6$ (GeV/c)^2.

(4) The differential cross section decreases with an increase in energy.

(5) The effective trajectory for the Serpukhov data is linear while that for the Fermilab data is nonlinear. This shows that the effective trajectory for this reaction should be a function of the energy range.

The energy dependence of the forward CEX cross section is of special interest. This value is linked with the difference of the total cross section for $\pi^{\pm}p$ interactions, $\Delta\sigma(\pi^{\pm}p)$, via the optical theorem and dispersion relations. A comparison of $\Delta\sigma(\pi^{\pm}p)$ differences, experimentally measured[11] in the momentum range up to 240 GeV/c with cross sections of reaction $\pi^- p \to \pi^0 n$, allows one to check the consistency of the data on pion-nucleon interactions at high energies.

The polarization parameter at high energy has been measured by Bonamy *et al.*[12] in the interval $0.02 \le -t \le 0.3$ (GeV/c)^2 and is of the order of 10% beyond $-t=0.03$ (GeV/c)^2.

Attempts have been made by several authors to fit the pion-proton CEX data. Hohler *et al.*[13] and Arbab and Chiu[14] were the pioneers who fitted the $d\sigma/dt$ experimental data up to 18 GeV/c assuming that the amplitudes are dominated by ρ-Regge-pole exchange. Desai[15] has tried to explain the angular-distribution data for the $\pi^- p \to \pi^0 n$ reaction at 9.8 and 18.2 GeV/c using the Regge-pole model and single-ρ-exchange trajectory. With the residues consistent with the $\pi^{\pm}p$ total- and differential-cross-section data, good fits were obtained by them but for only $-t \le 0.4$ (GeV/c)^2. Chiu *et al.*[16] have fitted the polarization data at 5.9 and 11.2 GeV/c using the Regge model with ρ and (conspiring) ρ' trajectories without a zero in the residue functions of either ρ or ρ' at the crossover point. Regge-pole fits to the $\pi^- p \to \pi^0 n$ differential-cross-section and polarization data were also obtained by Ahmadzadeh and Jackson[17] at moderately high energies and over a wide range of momentum transfer. They used, besides the ρ trajectory, whose slope and intercept were determined by exchange degeneracy, a conspiring trajectory of the same slope but near zero intercept. They were able to obtain a four-parameter good fit with the experimental data. Siddique and Enz[18] fitted the differential-cross-section data from 5.9 and 18.2 GeV/c in the small momentum transfer region by using a modified absorption model. (For a review see Refs. 19 and 20.) Collins and Swetman[21] have used the absorption model to investigate whether the $\rho p'$ as well as ρp cuts should be included to get a fit over a range of energies. They found that the absorption model with fixed-pole coupling for the ρ with rather strong cuts gave an excellent account of the CEX data up to $p_L=18.2$ GeV/c, including the polarization measured at 5 and 8 GeV/c. Chia[22] has attempted to fit the angular-distribution and polarization data between 5 and 18 GeV/c using a pole + cut model. Barger and Phillips[23] fitted the data of CERN and preliminary results of Fermilab[24] for 5—18 and 20—101 GeV/c using ρ and ρ' trajectories with $\alpha_p=0.506+0.85t$ and $\alpha'_\rho=0.29t$. They have simultaneously fitted the $d\sigma/dt$ data to a $\rho+\rho'$ parametrization together with polarization, cross-section differences $\Delta\sigma$, and continuous-moment sum rules. Although they have omitted the earlier Serpukhov data[7] because of the much higher Fermilab statistics, a small discrepancy with $\Delta\sigma$ remains, possibly due to systematic error or a charge-symmetry breakdown. Navelet and Stevens[25] have proposed a parametrization of the ρ s-channel flip and nonflip helicity amplitudes in terms of Regge and effective cuts to give a good fit to all the data available at that time. An analysis of the problem by Leader and Nicolescu[26] showed that the structure of polarization required that the ρ and ρ' trajectories intersect at $-t\approx 1$ (GeV/c)^2, in contrast to the parallel trajectories of other work, for example, Refs. 27—29. By using this additional, rather flat, trajectory ρ', they were able to fit the experimental data. A similar attempt has also been made by Girardi *et al.*[30] Chu *et al.*[31] have shown that the absorptive correction to the ρ Regge pole, obtained by using a prescription consistent with the bare perturbation expansion of the Regge field theory to second order, gives a cut contribution with the slope $\frac{1}{2}(\alpha'+\alpha'_c)$ instead of the typical cut slope α'_c. With this absorptive cut contribution, whose energy dependence is much closer to that of the pole than the usual cut, they were able to describe high-energy differential-cross-section data for $\pi^- p \to \pi^0 n$ between 20 and 100 GeV/c up to $-t \approx 1.5$ (GeV/c)^2. Joynson *et al.*[32] have shown that the experimental data on the CEX differential cross section and on the difference of the $\pi^+ p$ and $\pi^- p$ total cross sections in the range $p_{\text{lab}} = 5$ GeV/c to 200 GeV/c are incompatible with conventional Regge asymptotic behavior. It is shown that an additional term is required to fit the data simultaneously. This term corresponds to a singularity in the complex angular momentum plane. The precise form of the new term cannot be ascertained but it may correspond to (i) a non-Regge behavior associated with the maximal growth permitted in axiomatic field theory or (ii) a hyper-Regge term, i.e., an odd-signatured analog of the Pomeron. Bialkowski *et al.*[33] have tried to simultaneously fit the experimental data on forward differential cross sections and the difference in total cross sections of $\pi^+ p$ and $\pi^- p$ by assuming the exchange of such a pole in addition to the ρ trajectory.

A critical survey of literature on theoretical models for the process $\pi^- p \to \pi^0 n$ shows that attempts have been made to describe its characteristics on the following lines:

(1) The earlier experimental data on the characteristics of $\pi^- p \to \pi^0 n$ which were confined to measurements of $d\sigma/dt$ up to $p_{\text{lab}} \approx 18$ GeV/c can be fitted by using a Regge-pole model with the exchange of a single trajectory.

(2) The measurements of $d\sigma/dt$ were then extended through 20—50 GeV/c at Serpukhov to 20—200 GeV/c at Fermilab. From these measurements, the forward differential cross sections could be extrapolated. In addition to that, $\Delta\sigma$ and polarization P were also determined. The polarization was found to be significantly different from zero. The discovery of a sizable polarization in $\pi^- p \to \pi^0 n$ emphasized the need for some additional exchange other than the ρ trajectory. This trajectory, labeled ρ', had the same quantum numbers as ρ but its equation varied from author to author. It was also found that the data for $d\sigma/dt$ at $t=0$ and $\Delta\sigma$ at different momenta could be separately fitted by using different equations for the ρ trajectory, but a simultaneous fit for $d\sigma/dt\,|_{t=0}$ and $\Delta\sigma$ with the same equation for the trajectory was not possible; thus the additional trajectory was again required. In fact the resulting ρ intercept value required for such a description differs substantially in the two cases (the difference in intercept values is approximately 0.1—this is, roughly speaking, a discrepancy of 20%). However, the predictions of the models involving ρ and ρ' were in serious disagreement with the preliminary data at Fermilab energies combined with the CERN, BNL, and Fermilab data at lower energies. The Serpukhov data for $d\sigma/dt$ being considered as incompatible with the Fermilab data had been discarded because to utilize both sets of data would render meaningless any attempt to deduce information about the asymptotic behavior. Joynson *et al.*[32] discarded the Serpukhov data on the grounds that since the Serpukhov results on $\pi^- p$ reactions have been subject to re-evaluation, it is better to assume that there would be an eventual re-evaluation of the $\pi^- p \to \pi^0 n$ results. Barger and Phillips,[23] however, ignored the Serpukhov data because of the much higher Fermilab statistics.

(3) Joynson *et al.*[32] have examined the problem in detail and, assuming the validity of the published data, have shown that it is impossible to achieve compatibility between the data and any conventional form of Regge theory even when the Serpukhov data is ignored. Adopting a philosophy in which as much as possible of the classical Regge model is retained, they have shown that an excellent fit to the

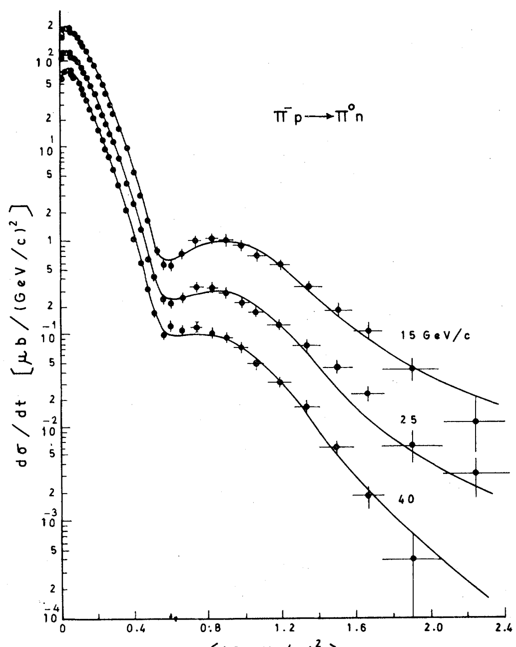

FIG. 1. Fits of the present model (curves) to the experimental differential-cross-section data of Apel *et al.* (Ref. 8) for the $\pi^- p \to \pi^0 n$ reaction at incident pion momenta $15 \le p_{\text{lab}} \le 40$ GeV/c and for the interval $0 \le -t \le 2.2$ (GeV/c)^2.

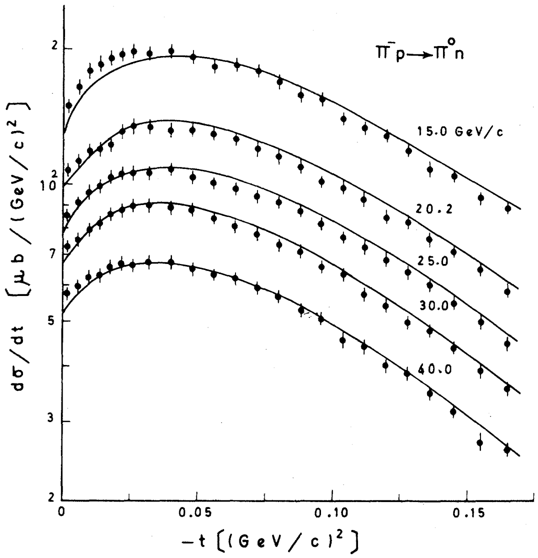

FIG. 2. The same as in Fig. 1 but for small $-t$.

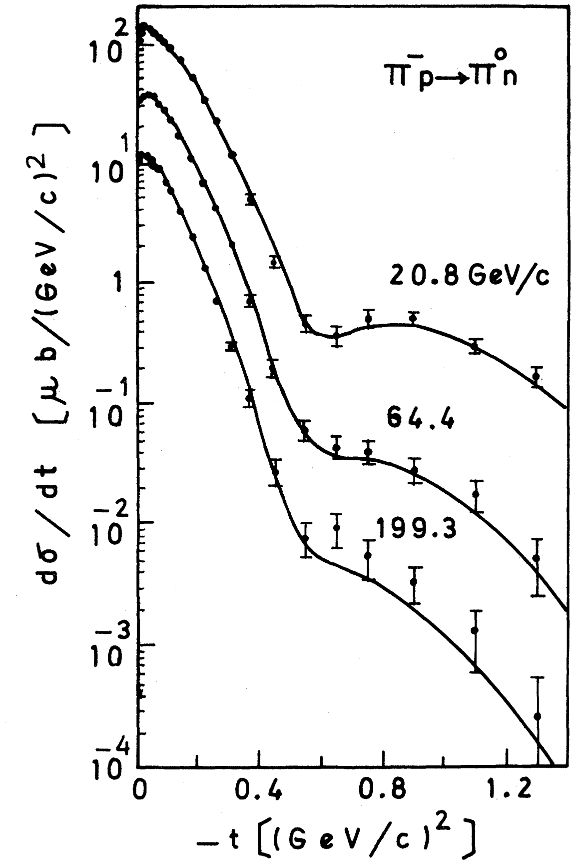

FIG. 3. Fits of the present model (curves) to the experimental differential-cross-section data of Barnes (Ref. 9) for the $\pi^- p \to \pi^0 n$ reaction at incident pion momenta $20 \leq p_{\rm lab} \leq 200$ GeV/c and for the interval $0 \leq -t \leq 1.3$ (GeV/c)^2.

data, both statistically and quantitatively, can be obtained if the spin-nonflip amplitude contains an additional term that grows in importance with energy and that will dominate the amplitude at ultrahigh energies. The data are not good enough to pin down the precise form of the additional term but by examining a fairly wide class of possibilities, they have shown that the new term will very likely correspond to a singularity at $j=1$ in the complex angular momentum plane. They have associated this singularity with a non-Regge or hyper-Regge term.

(4) Recently, Apel *et al.*[8] have measured the $d\sigma/dt$ for $\pi^- p \to \pi^0 n$ *with very high statistics.* These data, wherever they overlap the Fermilab results, are consistent with them. We have found that at high energy the world data for $\pi^- p \to \pi^0 n$ can be fitted within the framework of Regge theory provided a (ρ+cut) model with phenomenological residue functions is introduced. The necessity of introducing a non-Regge or hyper-Regge term therefore does not arise.

We may take this opportunity to point out that for $-t > 0.5$ (GeV/c)^2 the recent measurements of Daum *et al.*[10] cannot be fitted simultaneously with the data of Barnes *et al.*,[9] obtained at almost the same energy, being higher by a factor of 2.

We will show in this paper that at high energy the $d\sigma/dt$ and P for $\pi^- p \to \pi^0 n$ and the $\Delta\sigma(\pi^{\pm} p)$ can be described within the Regge framework with the ρ as exchanged trajectory and $p \otimes p$ as a phenomenological cut.

Using the general expression for the cut[34] the two independent helicity amplitudes for this reaction may be written as

$$T_{++}(s,t) = \left[\gamma^{\rho}_{++}(t)\xi_{\rho}(t) s^{\alpha_\rho(t)} - i\gamma^{c}_{++}(t) e^{-i\pi\alpha_c(t)/2} \frac{s^{\alpha_c(t)}}{\ln s} \right] (\mu\text{b})^{1/2}\ \text{GeV}\ ,$$

$$T_{+-}(s,t) = \sqrt{-t} \left[\gamma^{\rho}_{+-}(t)\xi_{\rho}(t) s^{\alpha_\rho(t)} - i\gamma^{c}_{+-}(t) e^{-i\pi\alpha_c(t)/2} \frac{s^{\alpha_c(t)}}{\ln s} \right] (\mu\text{b})^{1/2}\ \text{GeV}\ ,$$

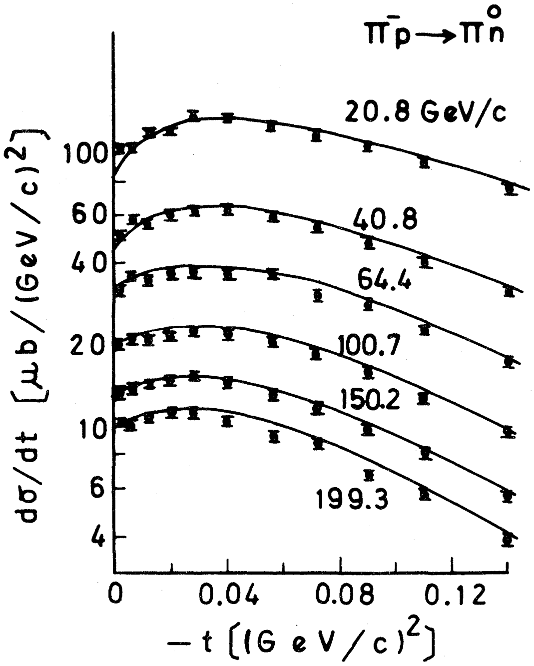

FIG. 4. The same as in Fig. 3 but for small $-t$.

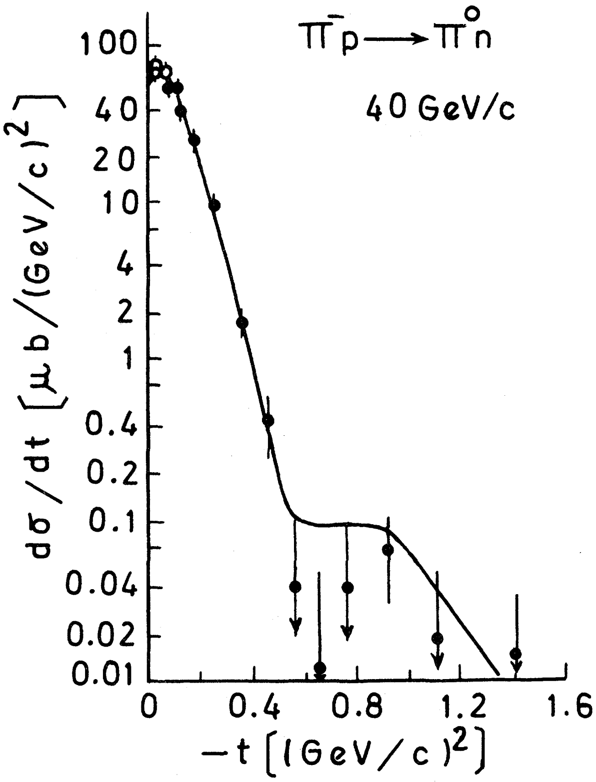

FIG. 6. Fits of present model to the experimental differential-cross-section data of Bolotov *et al.* (Ref. 7) for $\pi^- p \rightarrow \pi^0 n$ at incident pion momentum $p_{\text{lab}}=40$ GeV/c.

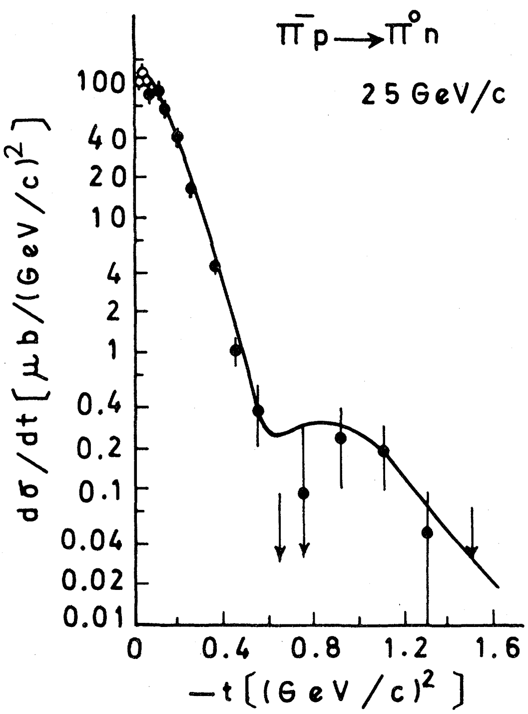

FIG. 5. Fits of the present model to the experimental differential-cross-section data of Bolotov *et al.* (Ref. 7) for $\pi^- p \rightarrow \pi^0 n$ at incident pion momentum $p_{\text{lab}}=25$ GeV/c.

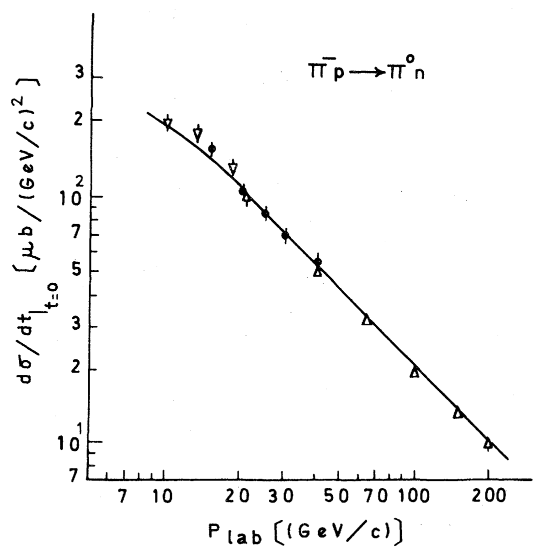

FIG. 7. The forward differential cross section for $\pi^- p \rightarrow \pi^0 n$ plotted versus t. The solid curve represents the results obtained by using the model described in the text. The experimental points have been taken from Refs. 1, 8, and 9.

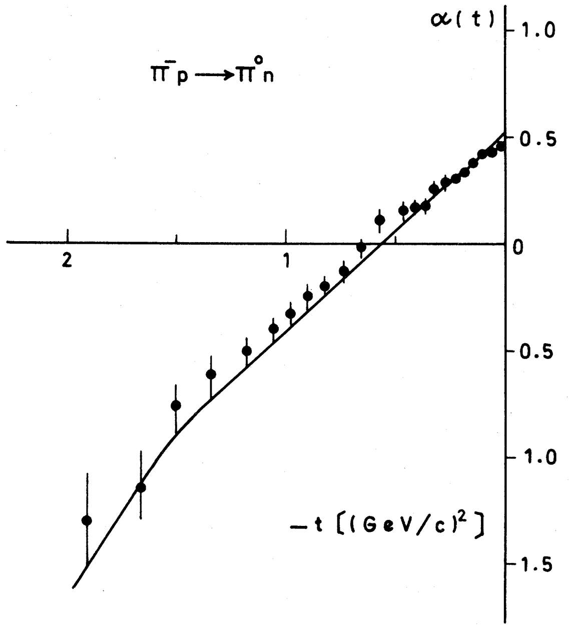

FIG. 8. The effective trajectory $\alpha(t)$ plotted versus $-t$. The experimental values of $\alpha(t)$ have been taken from the data of Apel *et al.* (Ref. 8).

where $\gamma_{++}(t)$ and $\gamma_{+-}(t)$ are the residue functions which are unknown theoretically, $\gamma^c_{++}(t)$ and $\gamma^c_{+-}(t)$ are the coefficients in the cut terms, $\xi_\rho(t)$ is the signature factor, and $\alpha_\rho(t)$ stands for the Regge trajectory ρ. The $\alpha_c(t)$ denotes the branch point. The scaling factor s_0 has been chosen as 1 GeV^2. Then the differential cross section $d\sigma/dt$, the polarization P, and the difference $\Delta\sigma$ are given by

$$\frac{d\sigma}{dt}=\frac{|T_{++}(s,t)|^2+|T_{+-}(s,t)|^2}{sp^2}$$

$$P=2\,\text{Im}(T_{++}T^*_{+-})\Big/\left[sp^2\frac{d\sigma}{dt}\right],$$

$$\Delta\sigma=\frac{0.1979}{\sqrt{sp}}\,\text{Im}T(s,t=0)\text{ mb}.$$

Here p is the c.m. momentum of the incident pion. The turnover near the forward direction suggests that the nonflip helicity amplitude plays a significant role near $t=0$.

Following Saleem and Fazal-e-Aleem,[35] we assign phenomenological values to the γ's. We then find that a good fit with experiment is obtained by the following choice of γ's:

$$\gamma^\rho_{++}(t)=40.1e^{11.5t}$$
$$-0.148e^{-9.18t-3.68t^2}\,(\mu\text{b})^{1/2}\text{ GeV},$$

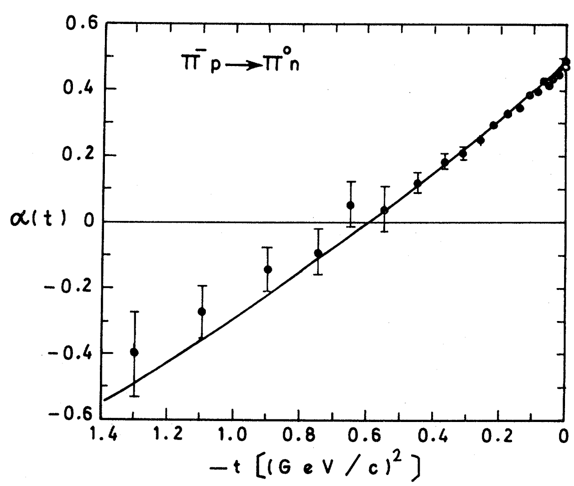

FIG. 9. The effective trajectory $\alpha(t)$ plotted versus $-t$. The experimental values of $\alpha(t)$ have been taken from the data of Barnes *et al.* (Ref. 9).

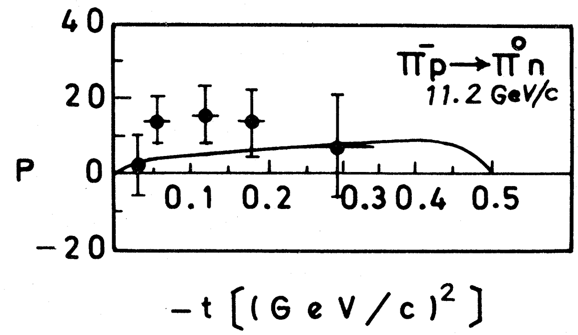

FIG. 10. The polarization for the reaction $\pi^-p\to\pi^0n$ at $p_{\text{lab}}=11.2$ GeV/c. The experimental data have been taken from Ref. 12.

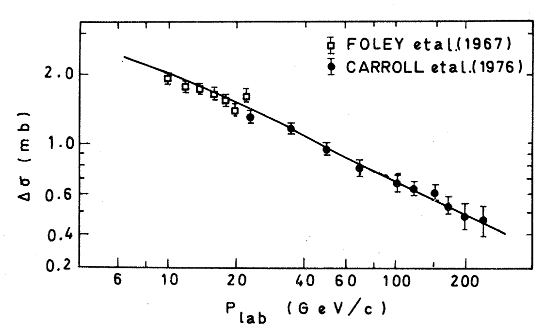

FIG. 11. Comparison of the difference $\Delta\sigma$ between the π^-p and π^+p total cross sections as obtained from the model described in the text with the experimental data from Ref. 11.

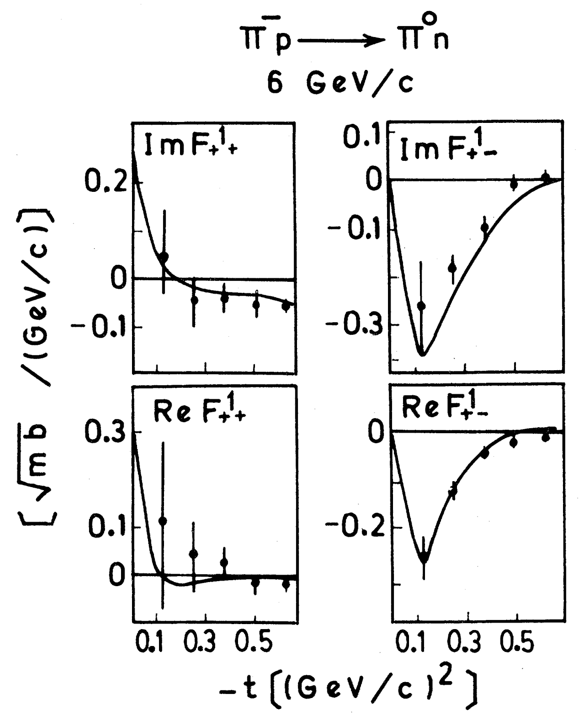

FIG. 12. Fits of the present model (curves) to the compilation of results from three different amplitude analyses of $\pi N \to \pi N$ at 6 GeV/c, as given in Ref. 36.

$$\gamma^{\rho}_{+-}(t) = (-250e^{4.74t} + 1.55e^{-2.46t-0.22t^2})\, c\,(\mu b)^{1/2}\ ,$$

$$\gamma^{c}_{++}(t) = 30.5e^{5.77t}\,(\mu b)^{1/2}\ \text{GeV}\ ,$$

$$\gamma^{c}_{+-}(t) = 4.4e^{1.51t}\, c\,(\mu b)^{1/2}\ .$$

The $\cos(\pi\alpha_\rho/2)$ factor in the denominator of ξ_ρ has been absorbed in the residue functions. The equations of the Pomeron and ρ trajectories have, respectively, been taken as

$$\alpha_P(t) = 1 + 0.1t\ ,$$

$$\alpha_\rho(t) = 0.48 + 0.8t$$

while $\alpha_c(t)$ is given by

$$\alpha_c(t) = 0.48 + 0.089t\ .$$

The curves in Figs. 1 and 2 show the results of the fit of the present model to the experimental data of Ref. 8. Figure 1 gives $d\sigma/dt$ plotted versus t. Figure 2 shows the differential cross section in the small momentum transfer region plotted against $-t$. In both the figures, we find good agreement. The curves in Figs. 3 and 4 show the results of the fit of the present model to the data of Barnes *et al.*[9] The agreement is again very good.

The measurements of Bolotov *et al.*[7] for $d\sigma/dt$ can be fitted by using our model. The agreement between the model and the experimental data is very good. This is exhibited graphically for $p_{\text{lab}} = 25$ and 40 GeV/c in Figs. 5 and 6 where the $d\sigma/dt$ has been plotted versus t.

Figure 7 exhibits the forward differential cross section plotted against $-t$. The experimental data have been taken from Refs. 1, 8, and 9. The agreement with experiment is good.

It would be interesting at this stage to determine the effective trajectory from $d\sigma/dt$ measurements at Serpukhov and Fermilab. Within the framework of the Regge-pole model, the energy dependence of the differential cross sections for any reaction can be described in a single-pole approximation as

$$\frac{d\sigma}{dt} = B(t)s^{2\alpha(t)-2}\ .$$

The effective trajectory $\alpha(t)$ along with the values found for it for the reaction $\pi^- p \to \pi^0 n$ in the Serpukhov range is given in Fig. 8. While fitting the $d\sigma/dt$ data obtained by Apel *et al.*,[8] the trajectory and the cut $p \otimes \rho$ have been assumed to contribute. The corresponding effective trajectory for the Fermilab data is shown in Fig. 9. In both cases the agreement with experiment is good.

Since according to our model the ρ trajectory and the $p \otimes \rho$ cut are exchanged in this process, the polarization in general is nonzero. We have calculated the polarization at $p_{\text{lab}} = 11.2$ GeV/c. The results are exhibited in Fig. 10. Experimental data for polarization at $p_{\text{lab}} = 11.2$ GeV/c have been taken from Ref. 12. Theoretical results of our model for polarization for $\pi^- p \to \pi^0 n$ at 11.2 GeV/c are satisfactory.

The difference $\Delta\sigma$ between the $\pi^- p$ and $\pi^+ p$ total cross sections is plotted as a function of the incident pion momentum p_{lab} in Fig. 11. The experimental data have been taken from Ref. 11. The agreement between theory and experiment is excellent. It may be pointed out that in our model the ρ trajectory as well as the $p \otimes \rho$ cut contribute to both the forward differential cross sections and $\Delta\sigma$.

It would be interesting to consider the amplitude structure for this process at 6 GeV/c. We know from the $\pi^\pm p$ elastic scattering data that the imaginary part of the nonflip CEX amplitude has a crossover near $-t = 0.15$ (GeV/c)^2. By using our model, we have calculated the real and imaginary parts of the $I_t = 1$ amplitudes F^1_{++} and $F^1_{\pm}$ for the reaction $\pi^- p \to \pi^0 n$ and compared with those obtained from three amplitude-analysis results of $\pi N \to \pi N$ at 6 GeV/c,[36] including that by Halzen and Michael.[37] The agreement is very good. (See Fig. 12.)

We conclude that the ρ+cut model can be used to fit the differential cross sections for $\pi^- p \to \pi^0 n$ including the recent measurements up to $-t \approx 2.2$ $(\mathrm{GeV}/c)^2$, the polarization, and the difference $\Delta\sigma = \sigma_{\mathrm{tot}}(\pi^+ p) - \sigma_{\mathrm{tot}}(\pi^- p)$.

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